193

I

Total No. of Questions—24

Total No. of Printed Pages—4 Regd. No. | | | | | | | | | | |

Part III

MATHEMATICS

Paper I-B

(English Version)

Time : 3 Hours ***Max. Marks : 75***

Note :— This question paper consists of THREE Sections A, B and C.

SECTION A 10×2=20

(I) Very short answer type questions :

(*i*) Attempt ALL questions.

(*ii*) Each question carries TWO marks.

1. Find the ratio in which the straight line $3x + 4y = 6$ divides the line joining the points (2, –1) and (1, 1). State whether the points lie on the same side or on either side of the straight line.

2. Find the value of p, if the straight lines $3x + 7y - 1 = 0$ and $7x - py + 3 = 0$ are mutually perpendicular.

3. Show that the points A(3, 2, –4), B(5, 4, –6) and C(9, 8, –10) are collinear and find the ratio in which B divides $\overline{AC}$.

4. Find the equation to the plane parallel to the ZX-plane and passing through (0, 4, 4).

5. Show that :

$$\lim_{x \to 2} -\frac{|x-2|}{x-2} = -1.$$

6. Compute :

$$\lim_{x \to -\infty} \frac{5x^3 + 4}{\sqrt{2x^4 + 1}}.$$

7. If $f(x) = \cos[\log(\cot x)]$, then find $f'(x)$.

8. If $f(x) = 2x^2 + 3x - 5$, then prove that $f'(0) + 3f'(-1) = 0$.

9. The time t, of a complete oscillation of a simple pendulum of length l is given by $t = 2\pi\sqrt{\frac{l}{g}}$ where g is gravitational constant. Find the approximate percentage of error in t when the percentage of error in l is 1%.

10. Define stationary point with example.

SECTION B

5×4=20

(II) Short answer type questions :

(*i*) Attempt ANY FIVE questions.

(*ii*) Each question carries FOUR marks.

11. Find the equation of locus of P, if the line segment joining (2, 3) and (–1, 5) subtends a right angle at P.

12. When the origin is shifted to the point (2, 3), the transformed equation of a curve is :

$$x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0.$$

Find the original equation of the curve.

13. A straight line passing through A(1, –2) makes an angle $\tan^{-1}\left(\frac{4}{3}\right)$ with the positive direction of the X-axis in the anticlockwise sense. Find the points on the straight line whose distance from A is 5.

14. Check the continuity of the following function at 2 :

$$f(x) = \begin{cases} \frac{1}{2}(x^2 - 4) & \text{if } 0 < x < 2 \\ 0 & \text{if } x = 2 \\ 2 - 8x^{-3} & \text{if } x > 2 \end{cases}$$

15. Find the derivative of $f(x) = \cot x$ using the first principle.

16. A stone is dropped into a quiet lake and ripples move in circles at the speed of 5 cm/sec. At the instant when the radius of circular ripple is 8 cm, how fast is the enclosed area increases ?

17. Find the angle between the curves $y^2 = 8x$; $4x^2 + y^2 = 32$.

SECTION C

5×7=35

(III) Long answer type questions :

(*i*) Attempt ANY FIVE questions.

(*ii*) Each question carries SEVEN marks.

18. Find the equations of the straight lines passing through the point (1, 2) and making an angle of 60° with the line $\sqrt{3}x + y + 2 = 0$.

19. Find the centroid and the area of the triangle formed by the lines $3x^2 - 4xy + y^2 = 0$, $2x - y = 6$.

20. Show that the product of the perpendicular distances from the origin to the pair of straight lines represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is :

$$\frac{|c|}{\sqrt{(a-b)^2 + 4h^2}}.$$

21. Find the angle between the lines whose direction cosines are given by the equations $3l + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$.

22. Find the derivative of the function :

$$x^x + (\cot x)^x.$$

23. Find the equations of the tangents to the curve $y = 3x^2 - x^3$, where it meets the X-axis.

24. From a rectangular sheet of dimensions 30 cm × 80 cm, four equal squares of side x cm are removed at the corners, and the sides are then turned up so as to form an open rectangular box. Find the value of x, so that the volume of the box is the greatest.