

- **Matrix** : Any rectangular arrangement or any array of numbers enclosed in brackets such as $[]$ or $()$ is called a matrix.

$$A = [a_{ij}]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{m1} & a_{m2} & a_{m3} \dots & a_{mn} \end{bmatrix} \text{ is an } m \times n \text{ matrix.}$$

Here, a_{ij} is the element of i th row and j th column.

- **Equal Matrices** : $A = B \Leftrightarrow a_{ij} = b_{ij}$ for matrices $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{m \times n}$, $\forall i = 1, 2, 3 \dots m$ and $j = 1, 2, 3 \dots n$

- **Types of Matrices** :

(1) **Row Matrix** : A $1 \times n$ matrix $[a_{11} \ a_{12} \ a_{13} \dots \ a_{1n}]$ is called a row matrix.

(2) **Column Matrix** : An $m \times 1$ matrix $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \cdot \\ \cdot \\ \cdot \\ a_{m1} \end{bmatrix}$ is called a column matrix.

(3) **Square Matrix** : An $n \times n$ matrix is called a square matrix.

(4) **Diagonal Matrix** : If we have $a_{ij} = 0$ whenever $i \neq j$ in a square matrix $A = [a_{ij}]$, then A is called a diagonal matrix. In this matrix, all entries are zero except possibly those on the principal diagonal. It is denoted as $\text{diag} [a_{11} \ a_{22} \dots \ a_{nn}]$.

(5) $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n} \Rightarrow A + B = [a_{ij} + b_{ij}]_{m \times n}$

(6) **Zero Matrix** : If all elements of a matrix are zero, then the matrix is known as zero matrix. It is denoted as $O_{m \times n}$ or O .

[O is the identity matrix for addition. $A + O = O + A = A$]

- If A and B both are $m \times n$ matrices, then the sum of them is possible. They are called compatible for addition.
- **Additive Inverse** : If $A = [a_{ij}]$ is any matrix, then $-A = [-a_{ij}]$ is the additive inverse of A .
- **Product of a Matrix with a Scalar** : If $A = [a_{ij}]_{m \times n}$ is a matrix and k is any real number, then the matrix $[ka_{ij}]$ is called the product of the matrix A by the scalar k .

Thus, if $A = [a_{ij}]_{m \times n}$, then $kA = [ka_{ij}]_{m \times n}$

- (1) $A + B = B + A$. (2) $(A + B) + C = A + (B + C)$
 (3) $A + O = O + A = A$. (4) $A + (-A) = (-A) + A = O$
 (5) $k(A+B) = kA + kB$ (6) $(k + l) A = kA + lA$
 (7) $lA = A$ (8) $(-1) A = -A$

- **Transpose of a Matrix** : If all the rows of matrix $A = [a_{ij}]_{m \times n}$ are converted into corresponding columns, the matrix so obtained is called the transpose of A. It is denoted by A^T or A' .

If $A = [a_{ij}]_{m \times n}$, then $A^T = [a_{ji}]_{n \times m}$

- **Symmetric Matrix** : If $A^T = A$ for a square matrix A, then A is called a symmetric matrix.
- **Skew-Symmetric Matrix** : If $A^T = -A$ for a square matrix A, then A is called a skew-symmetric matrix.

For a skew-symmetric matrix $A^T = [a_{ji}]_{n \times n}$, $a_{ij} = -a_{ji} \forall i$ and j .

Now, if $i = j$ then $2a_{ii} = 0$, so $a_{ii} = 0$, $i = 1, 2, \dots, n$.

Thus, all the elements on the principal diagonal of a skew-symmetric matrix are zero.

- (1) $(A + B)^T = A^T + B^T$ (2) $(A^T)^T = A$ (3) $(kA)^T = kA^T$

- **Multiplication of Matrices** : $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$ are two matrices. Their product $AB = [c_{ij}]_{m \times p}$ is defined by,

$$c_{ij} = \sum_{k=1}^n a_{ik} b_{kj} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj}$$

Thus, if $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$, then the product of $AB = \left[\sum_{k=1}^n a_{ik} b_{kj} \right]_{m \times p}$.

- The product of A and B is possible if $A = [a_{ij}]_{m \times n}$ and $B = [b_{ij}]_{n \times p}$. They are compatible for product.

Properties of Matrix Multiplication :

(1) Distributive Laws :

- (i) If $A = [a_{ij}]_{m \times n}$; $B = [b_{ij}]_{n \times p}$ and $C = [c_{ij}]_{n \times p}$, then $A(B + C) = AB + AC$
 (ii) If $A = [a_{ij}]_{m \times n}$; $B = [b_{ij}]_{m \times n}$ and $C = [c_{ij}]_{n \times p}$, then $(A + B)C = AC + BC$

(2) Associative Law : If matrices $A = [a_{ij}]_{m \times n}$; $B = [b_{ij}]_{n \times p}$ and $C = [c_{ij}]_{p \times q}$, then $A(BC) = (AB)C$

- **Identity Matrix (Unit Matrix)** : A square matrix in which all elements on principal diagonal are 1 and the rest of them are zero is called an identity or a unit matrix. Identity matrix is denoted by I.

$$I_n = I_{n \times n} = I = [a_{ij}]_{n \times n} \text{ where } a_{ij} = \begin{cases} 1, & \text{If } i = j \\ 0, & \text{If } i \neq j \end{cases}$$

- If $A = [a_{ij}]_{n \times n}$, then for the identity matrix I_n we have $A I_n = I_n A = A$
- **Scalar Matrix** : If $k \in \mathbb{R}$, then $k I_n$ is called a scalar matrix.
- **The Determinant of a Square Matrix** : If all the entries of a square matrix are kept in their respective places and the determinant of this array is taken, then the determinant so obtained is called the determinant of the given square matrix. If A is a square matrix, then determinant of A is denoted by $|A|$ or $\det A$.
- For square matrices A and B, $|AB| = |A| |B|$

- **Adjoint of a Matrix :**

For a given square matrix A, if we replace every entry in A by its cofactor as in $|A|$ and then the transpose of this matrix is taken, then the matrix so obtained is called the adjoint of A and is denoted by $\text{adj } A$.

A_{ij} is the cofactor of the element a_{ij} in $|A|$.

$$\text{If } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \text{ then } \text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

- **Inverse of a Matrix :** If there exists another square matrix B for a square matrix A such that $AB = I = BA$, then B is called an inverse of A. Inverse of A is denoted by A^{-1} .
- If an inverse of A exists, then it is unique.
- For a square matrix A, $A \cdot \text{adj } A = \text{adj } A \cdot A = |A| I$
- **Non-Singular Matrix :** A square matrix is said to be non-singular, if it has an inverse matrix.
- **Singular Matrix :** A matrix which is not a non-singular matrix is called a singular matrix.
- A square matrix A is non-singular if and only if $|A| \neq 0$

Inverse of matrix A is $A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$ (if it exists.)

Some Important Results :

- For a non-singular matrix A, the value of the reciprocal of the determinant of A is same as the value of the determinant of the inverse of A. This means $|A^{-1}| = |A|^{-1}$.
- If A and B are non-singular matrices, then AB is also non-singular and $(AB)^{-1} = B^{-1}A^{-1}$.
- For $m \times n$ matrices A and B, $(AB)^T = B^T A^T$
- A^T is non-singular if and only if A is non-singular and $(A^T)^{-1} = (A^{-1})^T$
- $\text{adj } A^T = (\text{adj } A)^T$
- **Unique solution of a system of linear equations using inverse of a matrix :**

Suppose, $a_1x + b_1y + c_1z = d_1$

$$a_2x + b_2y + c_2z = d_2$$

$a_3x + b_3y + c_3z = d_3$ is a system of three linear equations in x, y, z.

$$\text{If we take } A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix},$$

then the system can be written as $AX = D$.

If A is a non-singular matrix, then A^{-1} exists.

$$\therefore X = A^{-1}D = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}. \text{ So, } x = p_1, y = p_2, z = p_3$$

is the unique solution of the given system of linear equations.

Note : This result is also true for a system of two linear equations in two variables.

Multiple Choice Questions

(1) If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(A) $\alpha = a^2 + b^2, \beta = a^2 - b^2$ (B) $\alpha = a^2 + b^2, \beta = 2ab$ (C) $\alpha = 2ab, \beta = a^2 + b^2$ (D) $\alpha = a^2 + b^2, \beta = ab$

Sol. : $A^2 = A \cdot A = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$

$$\therefore \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$$

$$\therefore \alpha = a^2 + b^2, \beta = 2ab$$

Ans. : (B)

(2) Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. The only correct statement about the matrix A is (AIEEE : 2004)

(A) A is zero matrix

(B) $A^2 = I$

(C) A^{-1} does not exist

(D) $A = (-1)I$, where I is the unit matrix

Sol. : $A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Ans. : (B)

(3) Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $(10) B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of matrix A, then $\alpha = \dots$

(AIEEE : 2004)

(A) -2

(B) 5

(C) 2

(D) -1

Sol. : Here, $B = A^{-1}$ is given

$$\therefore AB = I$$

$$10(AB) = 10I$$

$$\therefore A(10B) = 10I$$

$$\therefore \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = 10 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 4+5+1 & 2+0-2 & 2-\alpha+3 \\ 8-5-3 & 4+0+6 & 4+\alpha-9 \\ 4-5+1 & 2+0-2 & 2+\alpha+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}. \text{ So, } \begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha-5 \\ 0 & 0 & \alpha+5 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\therefore \alpha + 5 = 10$$

$$\therefore \alpha = 5 \quad (\text{If } \alpha = 5, \text{ then } 5 - \alpha = \alpha - 5 = 0)$$

Ans. : (B)

(4) If $A^2 - A + I = O$, then inverse of A is

(A) $A + I$

(B) A

(C) $A - I$

(D) $I - A$

(AIEEE : 2005)

Sol. : $I = A - A^2$

$$IA^{-1} = (A - A \cdot A) A^{-1}$$

$$\therefore A^{-1} = AA^{-1} - (A \cdot A)A^{-1}$$

$$= I - A(AA^{-1})$$

$$= I - A$$

Ans. : (D)

(5) If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which of the following holds for all $n \in \mathbb{N}$ by the principle of mathematical induction ? (AIEEE : 2005)

(A) $A^n = nA - (n-1)I$

(B) $A^n = 2^{n-1}A - (n-1)I$

(C) $A^n = nA + (n-1)I$

(D) $A^n = 2^{n-1}A + (n-1)I$

Sol. : $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

Suppose $A^{n-1} = \begin{bmatrix} 1 & 0 \\ n-1 & 1 \end{bmatrix}$. This is true for $n = 2$.

$$A^n = A^{n-1} \cdot A = \begin{bmatrix} 1 & 0 \\ n-1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}. \text{ So, } A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} \quad \forall n \in \mathbb{N}$$

$$\text{Now, } nA - (n-1)I = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix} - \begin{bmatrix} n-1 & 0 \\ 0 & n-1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} = A^n$$

Ans. : (A)

- (6) If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true? (AIEEE : 2006)

- (A) $A = B$ (B) $AB = BA$
(C) either of A or B is a zero matrix (D) either of A or B is an identity matrix

Sol. : $A^2 - B^2 = (A - B)(A + B)$
 $\therefore A^2 - B^2 = AA + AB - BA - BB$
 $\therefore A^2 - B^2 = A^2 + AB - BA - B^2$
 $\therefore O = AB - BA$

$$BA = AB$$

Ans. : (B)

- (7) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $a, b \in \mathbb{N}$, then (AIEEE : 2006)

- (A) there cannot exist any B such that $AB = BA$
 (B) there exists more than one but finite number of B 's such that $AB = BA$
 (C) there exists exactly one B such that $AB = BA$
 (D) there exists infinitely many B 's such that $AB = BA$

Sol. : $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$$\therefore AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix} \text{ and } BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$$

$\therefore$ If $a = b$, then $AB = BA$

As $a, b \in \mathbb{N}$, there exist infinitely many B 's such that $AB = BA$.

Ans. : (D)

- (8) If $A = \begin{bmatrix} \alpha^2 & 5 \\ 5 & -\alpha \end{bmatrix}$ and $|A^{10}| = 1024$, then $\alpha = \dots\dots$ (AIEEE : 2007)

- (A) -3 (B) 3 (C) 2 (D) -2

Sol. : $|A^{10}| = 1024$

$$\therefore |A|^{10} = 2^{10}$$

$$\therefore |A| = 2$$

$$A = \begin{bmatrix} \alpha^2 & 5 \\ 5 & -\alpha \end{bmatrix} \quad \text{So, } |A| = \begin{vmatrix} \alpha^2 & 5 \\ 5 & -\alpha \end{vmatrix}$$

$$\therefore -\alpha^3 - 25 = 2$$

$$\therefore -\alpha^3 = 27. \quad \text{So, } \alpha = -3$$

Ans. : (A)

: Assertion and Reason Type Questions :

Explanation : Each of these questions contains statement 1 (Assertion) and statement 2 (Reason). Each question has four choices, option (A), (B), (C) and (D), which are as follows. You have to select the correct choice.

- (A) Statement 1 is true, statement 2 is true. Statement 2 is the correct explanation for statement 1.
 (B) Statement 1 is true, statement 2 is true. Statement 2 is not a correct explanation for statement 1.
 (C) Statement 1 is true, statement 2 is false.
 (D) Statement 1 is false, statement 2 is true.

- (9) Let A be a 2×2 matrix with real entries and I be the 2×2 identity matrix. Denote by $tr(A)$, the sum of the principal diagonal entries of A , assume that $A^2 = I$.

Statement : 1 If $A \neq I$ and $A \neq -I$, then $\det A = -1$

Statement : 2 If $A \neq I$ and $A \neq -I$, then $\text{tr}(A) \neq 0$

(AIEEE : 2008)

Sol. : If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

Now, $A^2 = I$.

$$\therefore a^2 + bc = 1; \quad b(a + d) = 0$$

$$\therefore bc + d^2 = 1 \quad c(a + d) = 0$$

$$a^2 + bc = bc + d^2 \Rightarrow a = -d \text{ or } a = d$$

Let $a + d \neq 0$. Then $b = c = 0$

$$\therefore a^2 = 1 = d^2. \text{ So, } a = d = 1 \text{ as } a \neq -d \text{ or } a = d = -1. \text{ So, } A = \pm I.$$

But $A \neq \pm I$.

$$\therefore a + d = 0. \text{ So, } |A| = \begin{vmatrix} a & b \\ c & -a \end{vmatrix} = -a^2 - bc = -1$$

$\therefore$ Statement 1 is true.

Statement 2 is not true, because $\text{Tr}(A) = a + d = 0$

Ans. : (C)

(10) Suppose A is a 2×2 matrix.

Statement : 1 $\text{adj}(\text{adj } A) = A$

Statement : 2 $|\text{adj } A| = |A|$

Sol. : We know that $|\text{adj } A| = |A|^{n-1}$

Taking $n = 2$, $|\text{adj } A| = |A|$

Statement 2 is true.

$$\text{Also, let } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \text{ So, } \text{adj } A = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\therefore \text{adj}(\text{adj } A) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = A$$

$\therefore$ Statement 1 is true.

But statement 2 is not a correct explanation of statement 1.

Ans. : (B)

(11) Let A be 2×2 matrix with non-zero entries and let $A^2 = I$; where I is a 2×2 identity matrix.

Define $\text{Tr}(A)$ = The sum of the principal diagonal elements of A and $|A|$ = determinant of matrix A

Statement : 1 $|A| = -1$

Statement : 2 $\text{Tr}(A) = 0$

Sol. : $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $a, b, c, d \neq 0$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

But $A^2 = I$.

$$a^2 + bc = 1 \quad b(a + d) = 0$$

$$bc + d^2 = 1 \quad c(a + d) = 0$$

as $b \neq 0, c \neq 0, a + d = 0$

$\text{tr}(A) = a + d = 0$. So statement 2 is true.

$$a = -d \quad (1)$$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = -a^2 - bc = -1$$

Statement 1 is true.

Statement 1 is true, statement 2 is true. Statement 2 is the correct explanation for statement 1

Ans. : (A)

(12) Suppose A and B are 3×3 symmetric matrices.

Statement : 1 A (BA) and (AB)A are symmetric matrices.

Statement : 2 If $AB = BA$, then AB is symmetric.

(AIEEE : 2011)

Sol. : A and B are symmetric, so $A^T = A, B^T = B$

$$\begin{aligned} [A(BA)]^T &= (BA)^T \cdot A^T \\ &= (A^T B^T) A^T \\ &= (AB) A = A(BA) \end{aligned}$$

Similarly, $[(AB)A]^T = (AB)A$

$\therefore$ A (BA) and (AB) A are symmetric matrices.

$\therefore$ Statement 1 is true.

If $AB = BA$, then $(AB)^T = B^T A^T = BA = AB$. So, AB is symmetric matrix.

Statement 2 is true.

But statement 2 is not the correct explanation of statement 1.

Ans. : (B)

(13) **Statement : 1** Determinant of a 3×3 skew-symmetric matrix is zero.

Statement : 2 $|A| = |A^T|$ and $|-A| = -|A|$ for any matrix A.

Sol. :

Statement 1 is true.

$\det A = \det (A^T)$ is true.

$\det (-A) = -\det (A)$ is true for 3×3 matrix.

But it is not true for any matrix.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad -A = \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix}$$

$$\therefore \det A = ad - bc \quad \therefore \det (-A) = \begin{vmatrix} -a & -b \\ -c & -d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\therefore \det (-A) = ad - bc \neq -\det A$$

$\therefore$ Statement 2 is false.

Ans. : (C)

- (14) Let a relation R be defined on the set of 3×3 matrices;
 $R : \{(A, B) | A = P^{-1}BP, P \text{ is some non-singular matrix.}\}$ (We call A and B are similar matrices.)

Statement : 1 R is an equivalence relation.

Statement : 2 If inverses of 3×3 matrices M and N exist then $(MN)^{-1} = N^{-1} M^{-1}$

Sol. : (1) Similarity is reflexive, because $A = I^{-1}AI$ for $P = I$.

(2) If $A = P^{-1}BP$, then $B = PAP^{-1}$

$$B = (P^{-1})^{-1}A(P^{-1}) \quad (\text{relation for } P^{-1}) \quad (\text{Symmetry})$$

(3) Let $A = P^{-1}BP$, $B = Q^{-1}CQ$, for some non-singular matrices P and Q.

$$A = (P^{-1}BP) = P^{-1}(Q^{-1}CQ)P = (QP)^{-1}C(QP) \quad (\text{Transitivity})$$

Statement 1 is true. Statement 2 is true.

But statement 2 is not a correct explanation of statement 1.

Ans. : (B)

- (15) If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then $\alpha = \dots$ (JEE Main : 2013)

- (A) 0 (B) 5 (C) 4 (D) 11

Sol. : $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ So, $|P| = -\alpha(4-6) + 3(4-6) = 2\alpha - 6$

$$\text{adj } A = P$$

$$\therefore |\text{adj } A| = |P|$$

$$\therefore |A|^2 = 2\alpha - 6$$

$$\therefore 16 = 2\alpha - 6$$

$$\text{So, } \alpha = 11$$

Ans. : (D)

- (16) If $A = \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$, then $A^{-1} = \dots$

- (A) $\begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$ (B) $\begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}$ (C) $\begin{bmatrix} -\sin \theta & \cos \theta \\ \cos \theta & -\sin \theta \end{bmatrix}$ (D) $\begin{bmatrix} -\sin \theta & \cos \theta \\ -\cos \theta & -\sin \theta \end{bmatrix}$

Sol. : $|A| = \begin{vmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{vmatrix} = \sin^2 \theta + \cos^2 \theta = 1 \neq 0$

$$\text{adj } A = \begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}$$

Ans. : (B)

- (17) If $a_{ij} = \frac{(i-2j)^2}{3}$, then $A = [a_{ij}]_{2 \times 2} = \dots$

- (A) $\begin{bmatrix} \frac{1}{3} & 3 \\ 0 & \frac{4}{3} \end{bmatrix}$ (B) $\begin{bmatrix} -\frac{1}{3} & 3 \\ 0 & -\frac{4}{3} \end{bmatrix}$ (C) $\begin{bmatrix} \frac{1}{3} & -3 \\ 0 & \frac{4}{3} \end{bmatrix}$ (D) $\begin{bmatrix} -\frac{1}{3} & -3 \\ 0 & -\frac{4}{3} \end{bmatrix}$

$$\text{Sol. : } a_{11} = \frac{(1-2)^2}{3} = \frac{1}{3}, \quad a_{21} = \frac{(2-2)^2}{3} = 0$$

$$a_{12} = \frac{(1-4)^2}{3} = 3, \quad a_{22} = \frac{(2-4)^2}{3} = \frac{4}{3}$$

$$\text{So, } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 3 \\ 0 & \frac{4}{3} \end{bmatrix}$$

Ans. : (A)

(18) Solving $x - 2y - 4 = 0$ and $3x - 5y + 7 = 0$, we get $x = \dots\dots$ and $y = \dots\dots$

(A) 34, 19

(B) -34, 19

(C) 34, -19

(D) -34, -19

Sol. : $x - 2y = 4$ and $3x - 5y = -7$

$$\therefore \begin{bmatrix} 1 & -2 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix} \text{ i.e. } AX = D$$

$|A| = -5 + 6 = 1 \neq 0$ So, A^{-1} exists. So system has a unique solution.

$$A^{-1} = \frac{1}{|A|} \text{adj } A, \quad \text{adj } A = \begin{bmatrix} -5 & 2 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -5 & 2 \\ -3 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1}D = \begin{bmatrix} -5 & 2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -7 \end{bmatrix} = \begin{bmatrix} -34 \\ -19 \end{bmatrix}$$

$$\therefore x = -34, y = -19$$

Ans. : (D)

(19) If $\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}$ is an identity matrix, then $x = \dots\dots$

(A) $\frac{1}{5}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

$$\text{Sol. : } \begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -2x+0+7x & 28x+0-28x & 14x+0-14x \\ 0 & 1 & 0 \\ -x+0+x & 14x-2-4x & 7x+0-2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore 5x = 1 \text{ and } 10x - 2 = 0$$

$$\therefore x = \frac{1}{5}$$

Ans. : (A)

(20) $A = \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix}$ and $A^{-1} = mA + nI$, then $m = \dots$ and $n = \dots$

(A) $\frac{-1}{11}, \frac{2}{11}$

(B) $\frac{2}{11}, \frac{-1}{11}$

(C) $\frac{-2}{11}, \frac{-1}{11}$

(D) $\frac{2}{11}, \frac{1}{11}$

Sol. : $|A| = \begin{vmatrix} 1 & 2 \\ -5 & 1 \end{vmatrix} = 1 + 10 = 11 \neq 0$

$$\therefore A^{-1} = \frac{1}{11} \begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix} = m \begin{bmatrix} 1 & 2 \\ -5 & 1 \end{bmatrix} + n \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{1}{11} & \frac{-2}{11} \\ \frac{5}{11} & \frac{1}{11} \end{bmatrix} = \begin{bmatrix} m+n & 2m \\ -5m & m+n \end{bmatrix}$$

$$\therefore m + n = \frac{1}{11}, \quad 2m = \frac{-2}{11}, \quad -5m = \frac{5}{11}$$

$$m = -\frac{1}{11} \quad n = \frac{2}{11}$$

Ans. : (A)

(21) If ω is a cube root of unity and $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then $A^{37} = \dots$ ($\omega \neq 1$)

(A) 0

(B) $-A$

(C) A

(D) A^2

Sol. : $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \Rightarrow A^{37} = \begin{bmatrix} \omega^{37} & 0 \\ 0 & \omega^{37} \end{bmatrix}$

Now $\omega^{37} = (\omega^3)^{12} \cdot \omega = (1)^{12} \omega = \omega$

$$\therefore A^{37} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = A$$

Ans. : (C)

(22) Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 3\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha| = \dots$

(AIEEE 2007)

(A) 5^2

(B) 1

(C) $\frac{1}{5}$

(D) 5

Sol. : $|A^2| = 25$

$$\therefore |A|^2 = 25. \quad \text{So, } |A| = \pm 5$$

$$\therefore \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 3\alpha \\ 0 & 0 & 5 \end{vmatrix} = \pm 5$$

$$\therefore 25\alpha = \pm 5$$

$$\therefore \alpha = \pm \frac{1}{5}. \quad \text{So, } |\alpha| = \frac{1}{5}$$

Ans. : (C)

- (23) If $\begin{bmatrix} 1 & -\cot\theta \\ \cot\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \cot\theta \\ -\cot\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $a = \dots$ and $b = \dots$; $\theta \in \mathbb{R} - \{k\pi \mid k \in \mathbb{Z}\}$
- (A) 1, 1 (B) $-\cos 2\theta, \sin 2\theta$ (C) $-\cos 2\theta, -\sin 2\theta$ (D) $\cos 2\theta, -\sin 2\theta$

Sol. : $A = \begin{bmatrix} 1 & \cot\theta \\ -\cot\theta & 1 \end{bmatrix} \Rightarrow |A| = 1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$A^{-1} = \frac{1}{\operatorname{cosec}^2\theta} \begin{bmatrix} 1 & -\cot\theta \\ \cot\theta & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\cot\theta \\ \cot\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \cot\theta \\ -\cot\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\therefore \frac{1}{\operatorname{cosec}^2\theta} \begin{bmatrix} 1 & -\cot\theta \\ \cot\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & -\cot\theta \\ \cot\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\therefore \frac{1}{\operatorname{cosec}^2\theta} \begin{bmatrix} 1 - \cot^2\theta & -2\cot\theta \\ 2\cot\theta & 1 - \cot^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\therefore \begin{bmatrix} \sin^2\theta - \cos^2\theta & -2\sin\theta \cos\theta \\ 2\cos\theta \sin\theta & \sin^2\theta - \cos^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\therefore \begin{bmatrix} -\cos 2\theta & -\sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\therefore a = -\cos 2\theta, b = \sin 2\theta$$

Ans. : (B)

- (24) If A is 3×3 matrix and if $|A| = 3$, then $|\operatorname{adj} A| = \dots$
- (A) 27 (B) 9 (C) -3 (D) 3

Sol. : $|A \operatorname{adj} A| = |A|^n$

Here, A is 3×3 matrix. Taking $n = 3$, $|A \operatorname{adj} A| = |A|^3$

$$|A| |\operatorname{adj} A| = 3^3$$

$$|\operatorname{adj} A| = 3^2 = 9$$

or $|\operatorname{adj} A| = |A|^2 = 9$ (for 3×3 matrix)

Ans. : (B)

- (25) If $P = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$, then $P^{-1} = \dots$

- (A) P^T (B) P (C) $-P^T$ (D) $-P$

Sol. : $|P| = \begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = \sin^2\theta + \cos^2\theta = 1$

$$\therefore P^{-1} = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} \text{ and } P^T = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$\therefore P^{-1} = P^T$$

Ans. : (A)

(26) If matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ satisfies $AA^T = 9I$, then $(a, b) = \dots$ (JEE Main : 2015)

- (A) $(2, -1)$ (B) $(-2, 1)$ (C) $(2, 1)$ (D) $(-2, -1)$

Sol. : $AA^T = 9I$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 9 & 0 & a+4+2b \\ 0 & 9 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\therefore a + 4 + 2b = 0 \Rightarrow a + 2b = -4 \quad (1)$$

$$2a + 2 - 2b = 0 \Rightarrow 2a - 2b = -2 \quad (2)$$

So, $3a = -6$. So, $a = -2$ and $b = -1$

$(a, b) = (-2, -1)$ (See that $a^2 + 4 + b^2 = 9$) **Ans. : (D)**

(27) For 3×3 matrices M and N , which of the following statement(s) is/are false ? (JEE adva : 2013)

- (A) $N^T M N$ is symmetric or skew-symmetric according as M is symmetric or skew-symmetric.
 (B) $MN - NM$ skew-symmetric, for all symmetric matrices M and N .
 (C) MN is symmetric, for all symmetric matrices M and N .
 (D) $(adj M)(adj N) = adj(MN)$, for all invertible matrices M and N .

Sol. : (A) If M is symmetric, then $M^T = M$

$$(N^T M N)^T = N^T M^T (N^T)^T = N^T M N$$

$\therefore N^T M N$ is symmetric.

If M is skew-symmetric, then $M = -M^T$

$$\begin{aligned} (N^T M N)^T &= N^T M^T (N^T)^T \\ &= N^T (-M) N \\ &= -N^T M N \end{aligned} \quad (M^T = -M)$$

$\therefore N^T M N$ is skew-symmetric, so statement (A) is true.

(B) Here, M and N are symmetric matrices, so $M = M^T$ and $N = N^T$

$$(MN - NM)^T = N^T M^T - M^T N^T = NM - MN = -(MN - NM)$$

$\therefore MN - NM$ is skew-symmetric.

$\therefore$ Statement B is true.

(C) M and N are symmetric matrices.

$$(MN)^T = N^T M^T = NM \neq MN. \text{ So, (C) is false.}$$

(D) $(adj M) (adj N) = adj (NM) \neq adj (MN)$.

So, (D) is false.

Ans. : (C), (D)

(28) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ and for column matrices u_1 and u_2 , $Au_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, then $u_1 + u_2 = \dots\dots$

(A) $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

Sol. : $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ Let $u_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ So, $Au_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$$Au_1 = \begin{bmatrix} a \\ 2a+b \\ 3a+2b+c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore a = 1, 2a + b = 0$ giving $b = -2$ and $3a + 2b + c = 0$ giving $c = 1$. So, $u_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

Similarly, let $u_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$. So, $Au_2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} p \\ 2p+q \\ 3p+2q+r \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore p = 0, 2p + q = 1, 3p + 2q + r = 0$. So, $p = 0, q = 1, r = -2$

$$u_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \text{ So, } u_1 + u_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

Ans. : (A)

(29) If $\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} X = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$, then 2×2 matrix $X = \dots\dots$

(A) $\begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$

(B) $\begin{bmatrix} -1 & 4 \\ -3 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 4 \\ 3 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & -4 \\ -3 & 2 \end{bmatrix}$

Sol. : Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5a-7c & 5b-7d \\ -2a+3c & -2b+3d \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$5a - 7c = -16$$

$$5b - 7d = -6$$

$$-2a + 3c = 7$$

$$-2b + 3d = 2$$

Solving the equations; we get $a = 1$, $c = 3$ and $b = -4$, $d = -2$

$$\therefore X = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$$

Ans. : (A)

Second method : $X = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}^{-1} \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$

(30) If $[1 \ x \ 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = [0]$, then $x = \dots\dots$

(A) 2, 14

(B) -2, 14

(C) -2, -14

(D) 2, -14

Sol. : $[1 \ x \ 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = [0]$

$$\therefore [1 + 2x + 15 \quad 3 + 5x + 3 \quad 2 + x + 2] \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = [0]$$

$$\therefore [16 + 2x \quad 6 + 5x \quad 4 + x] \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = [0]$$

$$\therefore [16 + 2x + 12 + 10x + 4x + x^2] = [0]$$

$$\therefore x^2 + 16x + 28 = 0$$

$$\therefore x = -2 \text{ or } x = -14$$

Ans. : (C)

(31) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ and U_1, U_2, U_3 are column matrices such that $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$;

$AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$. If U is 3×3 matrix having as columns U_1, U_2, U_3 in the sequence, then $|U| = \dots\dots$

(A) 2

(B) $\frac{3}{2}$

(C) -3

(D) 3

Sol. : Let, $U_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

So, $AU_1 = \begin{bmatrix} a \\ 2a+b \\ 3a+2b+c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\therefore a=1, 2a+b=0, 3a+2b+c=0$

$\therefore a=1, b=-2, c=1$

$\therefore U_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

Let, $U_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

So, $AU_2 = \begin{bmatrix} x \\ 2x+y \\ 3x+2y+z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$

$\therefore x=2, 2x+y=3, 3x+2y+z=0$

$\therefore x=2, y=-1, z=-4$

$\therefore U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}$

Let, $U_3 = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$

$AU_3 = \begin{bmatrix} p \\ 2p+q \\ 3p+2q+r \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

$\therefore p=2, 2p+q=3, 3p+2q+r=1$

$\therefore p=2, q=-1, r=-3$

$\therefore U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

$\therefore U = [U_1 \ U_2 \ U_3] = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}$

$\therefore |U| = \begin{vmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{vmatrix} = 1[3-4] - 2[6+1] + 2[8+1] = -1 - 14 + 18 = 3$

Ans. : (D)

Second method : $AU = B$, where $B = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore U = A^{-1}B$. So, $|U| = |A|^{-1}|B| = 1.3 = 3$

(32) If matrices $A_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$, $r = 1, 2, 3, \dots$, then $|A_1| + |A_2| + \dots + |A_{101}| = \dots$

(A) 101

(B) $(101)^2$

(C) 100

(D) $(100)^2$

Sol. : $A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow |A_1| = 1, A_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow |A_2| = 3, A_3 = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \Rightarrow |A_3| = 5 \dots$

$$|A_1| + |A_2| + \dots + |A_{101}| = 1 + 3 + 5 + \dots + 201 = (101)^2$$

Ans. : (B)

(33) If $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$, then $A^3 = \dots$

(A) 27 A

(B) 81 A

(C) 243 A

(D) 729 A

Sol. : $A^2 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$

$$= 9 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 9 \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 27 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = 27 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

$$= 81 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 81 \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 81 A$$

Ans. : (B)

(34) If $A^n = \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 2^n \end{bmatrix}$, then $A^{10} = \dots$

(A) 512 I

(B) 256 I

(C) 1024 I

(D) 128 A

Sol. : $A = 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 2I$

$$A^2 = (2I)(2I) = 4I^2 = 4I$$

$$A^3 = (2I)(2I)(2I) = 8I^3 = 8I$$

$$A^{10} = 2^{10} I = 1024 I$$

Ans. : (C)

(35) If $A = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$, then $B^{-1}AB = \dots$

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}$

(B) $\begin{bmatrix} 5 & 0 \\ 0 & 30 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 \\ 0 & 6 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & -6 \end{bmatrix}$

Sol. : $AB = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ -1 & 12 \end{bmatrix}$

Also, $|B| = 4 + 1 = 5$

So, $B^{-1} = \frac{1}{|B|} \text{adj } B = \frac{1}{5} \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

Now $B^{-1}AB = \frac{1}{5} \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ -1 & 12 \end{bmatrix}$

$= \frac{1}{5} \begin{bmatrix} 5 & 0 \\ 0 & 30 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}$

Ans. : (A)

(36) If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ and $A^2 - mA + nI = 0$, then

(A) $m = -9, n = -14$

(B) $m = -9, n = 14$

(C) $m = 9, n = 14$

(D) $m = 9, n = -14$

Sol. : $A^2 = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$

$A^2 - mA + nI = 0$. So, $A^2 = mA - nI$

$\therefore \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} = m \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$

$= \begin{bmatrix} 4m - n & 3m \\ 2m & 5m - n \end{bmatrix}$

$\therefore$ From $4m - n = 22$ and $3m = 27$, $m = 9$, $n = 14$, which also satisfy $2m = 18$, $5m - n = 31$.

Ans. : (C)

Note : If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^2 - (a + d)A + |A|I = O$

$\therefore m = a + d = 9$ $n = |A| = 14$

(37) If $A = \begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix}$, then $A^n = \dots$

(A) $\begin{bmatrix} 1 & m^n \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & mn \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} n & n \\ 0 & n \end{bmatrix}$

(D) $\begin{bmatrix} n & mn \\ 0 & n \end{bmatrix}$

$$\text{Sol. : } A^2 = \begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2m \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 2m \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3m \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^4 = A^3 \cdot A = \begin{bmatrix} 1 & 3m \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4m \\ 0 & 1 \end{bmatrix}$$

$$\text{Similarly, } A^n = \begin{bmatrix} 1 & mn \\ 0 & 1 \end{bmatrix}$$

Ans. : (B)

$$(38) \quad \text{If } A = \begin{bmatrix} 4 & 4 \\ -3 & 4 \end{bmatrix}; B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \text{ then } (B^{-1}A^{-1})^{-1} = \dots\dots$$

$$(A) \frac{1}{10} \begin{bmatrix} 4 & -4 \\ 4 & 3 \end{bmatrix} \quad (B) \begin{bmatrix} 4 & -4 \\ 4 & 4 \end{bmatrix} \quad (C) \begin{bmatrix} 4 & -4 \\ 4 & 3 \end{bmatrix} \quad (D) \frac{1}{10} \begin{bmatrix} 3 & 4 \\ -4 & 4 \end{bmatrix}$$

$$\text{Sol. : } (B^{-1}A^{-1})^{-1} = (A^{-1})^{-1} (B^{-1})^{-1} = AB = \begin{bmatrix} 4 & 4 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 4 & 3 \end{bmatrix}$$

Ans. : (C)

$$(39) \quad \text{If } A = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}, \text{ then } AA^T = \dots\dots$$

$$(A) I \quad (B) A \quad (C) A^T \quad (D) \text{None of these}$$

$$\text{Sol. : } AA^T = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\theta + \cos^2\theta & -\sin\theta\cos\theta + \sin\theta\cos\theta \\ -\cos\theta\sin\theta + \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(If $AA^T = I$ or $A^T = A^{-1}$, then A is known as an orthogonal matrix.)

Ans. : (A)

$$(40) \quad \text{The unique solution of } x + 3y + 4z = 8, \quad 2x + y + 2z = 5, \quad 5x + y + z = 7, \text{ is } \dots\dots$$

$$(A) x = 1, y = 1, z = 1 \quad (B) x = 1, y = 2, z = 1 \quad (C) x = -1, y = 1, z = 1 \quad (D) x = 1, y = 1, z = -1$$

$$\text{Sol. : } x + 3y + 4z = 8$$

$$2x + y + 2z = 5$$

$$5x + y + z = 7$$

$$\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} \Rightarrow AX = D$$

$$\text{where } A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad D = \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{vmatrix} = 1(1-2) - 3(2-10) + 4(2-5) = -1 + 24 - 12 = 11 \neq 0$$

$\therefore A^{-1}$ exists.

$$A_{11} = -1, A_{12} = 8, A_{13} = -3, A_{21} = 1, A_{22} = -19, A_{23} = 6, A_{31} = 2, A_{32} = 6, A_{33} = -5$$

$$\therefore \text{adj } A = \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix}, \quad A^{-1} = \frac{1}{11} \text{adj } A$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -1 & 1 & 2 \\ 8 & -19 & 6 \\ -3 & 14 & -5 \end{bmatrix} \begin{bmatrix} 8 \\ 5 \\ 7 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -8 + 5 + 14 \\ 64 - 95 + 42 \\ -24 + 70 - 35 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 11 \\ 11 \\ 11 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore x = 1, y = 1, z = 1$$

Ans. : (A)

Note : Clearly (1, 1, 1) is the solution by verification.

$$(41) \quad \text{If } A = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \text{ and } A + A' = I, \text{ then } \alpha = \dots\dots\dots . \quad 0 < \alpha < \frac{\pi}{2}$$

$$(A) \frac{\pi}{3} \quad (B) \frac{\pi}{6} \quad (C) \frac{\pi}{2} \quad (D) 0$$

$$\text{Sol. : } A = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}, \quad A' = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$$

$$\text{Now } A + A' = I$$

$$\begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} + \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad \text{So, } \begin{bmatrix} 2\sin \alpha & 0 \\ 0 & 2\sin \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore 2 \sin \alpha = 1$$

$$\therefore \sin \alpha = \frac{1}{2}. \quad \text{So, } \alpha = \frac{\pi}{6} \quad (0 < \alpha < \frac{\pi}{2}) \quad \text{Ans. : (B)}$$

$$(42) \quad \text{If square matrices } A \text{ and } B \text{ are such that } AB = A \text{ and } BA = B, \text{ then } B^2 = \dots\dots\dots$$

$$(A) B \quad (B) 0 \quad (C) I \quad (D) A$$

$$\text{Sol. : } B^2 = B \cdot B$$

$$= (BA) B \quad (B = BA)$$

$$= B (AB) \quad (\text{Associative law})$$

$$= BA \quad (AB = A)$$

$$\therefore B^2 = B \quad (BA = B)$$

Ans. : (A)

(43) If $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$, then $X = \dots\dots$

(A) $2 \begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$ (B) $2 \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

Sol. : $2X = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$

$\therefore X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

Ans. : (D)

(44) If $A = \begin{bmatrix} 1 & \cot\theta \\ -\cot\theta & 1 \end{bmatrix}$ satisfies $(A^{-1})^2 = f(\theta) \begin{bmatrix} -\cos 2\theta & -\sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$, then $f(\theta) = \dots\dots$

(A) $\sin^2 \theta$ (B) $\cos^2 \theta$ (C) $\cos \theta$ (D) $\sin \theta$

Sol. : $|A| = \begin{vmatrix} 1 & \cot\theta \\ -\cot\theta & 1 \end{vmatrix} = 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$

$A^{-1} = \frac{1}{\operatorname{cosec}^2 \theta} \begin{bmatrix} 1 & -\cot\theta \\ \cot\theta & 1 \end{bmatrix}$

$(A^{-1})^2 = \sin^4 \theta \begin{bmatrix} 1 & -\cot\theta \\ \cot\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & -\cot\theta \\ \cot\theta & 1 \end{bmatrix}$

$= \sin^4 \theta \begin{bmatrix} 1 - \cot^2 \theta & -2\cot\theta \\ 2\cot\theta & -\cot^2 \theta + 1 \end{bmatrix}$

$= \sin^2 \theta \begin{bmatrix} \sin^2 \theta - \cos^2 \theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \sin^2 \theta - \cos^2 \theta \end{bmatrix}$

$= \sin^2 \theta \begin{bmatrix} -\cos 2\theta & -\sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

$\therefore f(\theta) = \sin^2 \theta$

Ans. : (A)

(45) If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - 5A = kI$, then $k = \dots\dots$

(A) 8 (B) 7 (C) 5 (D) 6

Sol. : $A^2 = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$

$$A^2 - 5A = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} + \begin{bmatrix} -5 & -15 \\ -15 & -20 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 5I$$

$$\therefore k = 5$$

Ans. : (C)

Note : If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^2 - (a+d)A + (ad - bc)I = O$, so $A^2 - 5A - 5I = O$

(46) If $A = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, $X = \begin{bmatrix} 0 & 2 & 3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$, then $AB + XY = \dots\dots$

(A) [20]

(B) [-20]

(C) 20

(D) -20

Sol. : $AB + XY = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$

$$= [6 - 6 + 4] + [0 + 4 + 12] = [4] + [16] = [20]$$

Ans. : (A)

(47) If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \dots\dots$ satisfy $AB = BA$. where $B \neq I$

(A) $\begin{bmatrix} x & x \\ y & 0 \end{bmatrix}$

(B) $\begin{bmatrix} x & y \\ 0 & x \end{bmatrix}$

(C) $\begin{bmatrix} x & y \\ 0 & y \end{bmatrix}$

(D) $\begin{bmatrix} x & x \\ 1 & x \end{bmatrix}$

Sol. : Let $B = \begin{bmatrix} x & y \\ p & q \end{bmatrix}$

Here, $AB = BA$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x & y \\ p & q \end{bmatrix} = \begin{bmatrix} x & y \\ p & q \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x+p & y+q \\ p & q \end{bmatrix} = \begin{bmatrix} x & x+y \\ p & p+q \end{bmatrix}$$

$$x+p = x \quad y+q = x+y \quad q = p+q$$

$$\therefore p = 0 \quad q = x$$

$$\therefore B = \begin{bmatrix} x & y \\ 0 & x \end{bmatrix}$$

Ans. : (B)

(48) If $A = \text{diag} [3 \ 2 \ 1]$, then the sum of the elements of $A^{-1} = \dots\dots$.

- (A) $\frac{5}{6}$ (B) $\frac{11}{6}$ (C) $\frac{1}{6}$ (D) 6

Sol. : $A = \text{diag} [3 \ 2 \ 1]$

$$A^{-1} = \text{diag} \left[\frac{1}{3} \ \frac{1}{2} \ 1 \right]$$

$$\therefore \text{The sum of the elements of } A^{-1} = \frac{1}{3} + \frac{1}{2} + 1 = \frac{3+2+6}{6} = \frac{11}{6}$$

Ans. : (B)

(49) If $\lambda x + 2y - 2z = 1$, $4x + 2\lambda y - z = 2$ and $6x + 6y + \lambda z = 3$, have unique solution, then

- (A) $\lambda \neq -2$ (B) $\lambda \neq 3$ (C) $\lambda \neq 2$ (D) $\lambda \neq 1$

Sol. : $\lambda x + 2y - 2z = 1$

$$4x + 2\lambda y - z = 2$$

$$6x + 6y + \lambda z = 3$$

$$\begin{bmatrix} \lambda & 2 & -2 \\ 4 & 2\lambda & -1 \\ 6 & 6 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

If $|A| = \begin{vmatrix} \lambda & 2 & -2 \\ 4 & 2\lambda & -1 \\ 6 & 6 & \lambda \end{vmatrix} \neq 0$, then A^{-1} exists and we get a unique solution.

$$\lambda (2\lambda^2 + 6) - 2 (4\lambda + 6) - 2 (24 - 12\lambda) \neq 0$$

$$\therefore 2\lambda^3 + 22\lambda - 60 \neq 0$$

$$\therefore \lambda^3 + 11\lambda - 30 \neq 0$$

$$\therefore (\lambda - 2) (\lambda^2 + 2\lambda + 15) \neq 0$$

For real roots $\lambda \neq 2$ is required

Ans. : (C)

(50) If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$ is an idempotent matrix, then $x = \dots\dots$

- (A) -1 (B) -5 (C) -4 (D) -3

$$\text{Sol. : } A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -2 & -16-4x \\ -1 & 3 & 16+4x \\ 4+x & -8-2x & -12+x^2 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix} = A$$

$$4 + x = 1 \quad 16 + 4x = 4, \quad -12 + x^2 = x$$

$\therefore$ All the equations are true for $x = -3$.

Ans. : (D)

(51) If $A = \begin{bmatrix} 3a & b & c \\ b & 3c & a \\ c & a & 3b \end{bmatrix}$; $a, b, c \in \mathbb{R}$, $abc = 1$, $AA^T = 64I$ and $|A| > 0$, then $(a^3 + b^3 + c^3)^3 = \dots$

(A) 343

(B) 729

(C) 256

(D) 512

Sol. : Here, $A^T = A$ and $AA^T = 64I$

$$\therefore A^2 = 64I$$

$$\therefore |A|^2 = 64 \Rightarrow |A| = 8 \quad (|A| > 0)$$

$$\text{Now, } |A| = \begin{vmatrix} 3a & b & c \\ b & 3c & a \\ c & a & 3b \end{vmatrix}$$

$$= 3a[9bc - a^2] - b[3b^2 - ca] + c[ab - 3c^2]$$

$$\therefore |A| = 8 \Rightarrow 29abc - 3(a^3 + b^3 + c^3) = 8$$

$$\therefore a^3 + b^3 + c^3 = 7 \quad (\text{taking } abc = 1)$$

$$\therefore (a^3 + b^3 + c^3)^3 = 343 \quad \text{Ans. : (A)}$$

(52) If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then $\text{adj } A = \dots$

(A) A

(B) A^T

(C) $3A$

(D) $3A^T$

Sol. : $\text{adj } A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} = 3 \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = 3A^T \quad \text{Ans. : (D)}$

(53) If a, b, c are positive real numbers, then the system of equations,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \quad -\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1, \quad -\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \text{ have } \dots$$

(A) unique solution (B) no solution (C) more than one finite solution (D) infinite solution.

Sol. : $\begin{bmatrix} \frac{1}{a^2} & -\frac{1}{b^2} & -\frac{1}{c^2} \\ -\frac{1}{a^2} & \frac{1}{b^2} & -\frac{1}{c^2} \\ -\frac{1}{a^2} & -\frac{1}{b^2} & \frac{1}{c^2} \end{bmatrix} \begin{bmatrix} x^2 \\ y^2 \\ z^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} \frac{1}{a^2} & -\frac{1}{b^2} & -\frac{1}{c^2} \\ -\frac{1}{a^2} & \frac{1}{b^2} & -\frac{1}{c^2} \\ -\frac{1}{a^2} & -\frac{1}{b^2} & \frac{1}{c^2} \end{vmatrix}$$

$$= \frac{1}{a^2 b^2 c^2} \begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix} \quad C_1 (a^2), C_2 (b^2), C_3 (c^2)$$

$$= -\frac{4}{a^2 b^2 c^2} \neq 0$$

∴ The equations have unique solution.

Ans. : (A)

(54) 3×3 matrix A satisfies the equation $A^2 - 5A + 7I = 0$. If $A^5 = aA + bI$, then $2a - 3b = \dots$

(A) 3135

(B) 1453

(C) 1435

(D) 4135

Sol. : $A^3 = A \cdot A^2 = A[5A - 7I] = 5A^2 - 7A = 5[5A - 7I] - 7A = 18A - 35I$ ($A^2 = 5A - 7I$)

$$A^4 = A \cdot A^3 = A[18A - 35I] = 18A^2 - 35A$$

$$\therefore A^4 = 18[5A - 7I] - 35A = 55A - 126I$$

$$A^5 = A \cdot A^4 = A[55A - 126I] = 55A^2 - 126A = 55[5A - 7I] - 126A$$

$$\therefore A^5 = 149A - 385I = aA + bI$$

$$\therefore a = 149, b = -385$$

$$2a - 3b = 2(149) - 3(-385) = 298 + 1155 = 1453$$

Ans. : (B)

(55) If $[x]$ is the greatest integer function, then the value of $\begin{vmatrix} [e] & [\pi] & [\pi^2 - 6] \\ [\pi] & [\pi^2 - 6] & [e] \\ [\pi^2 - 6] & [e] & [\pi] \end{vmatrix}$ is

(A) 8

(B) 0

(C) 1

(D) -8

Sol. : We know that,

$$2 < e < 3 \Rightarrow [e] = 2$$

$$3 < \pi < 4 \Rightarrow [\pi] = 3$$

$$3 < \pi^2 - 6 < 4 \Rightarrow [\pi^2 - 6] = 3$$

$$\therefore \text{The given determinant} = \begin{vmatrix} 2 & 3 & 3 \\ 3 & 3 & 2 \\ 3 & 2 & 3 \end{vmatrix} = -8$$

Ans. : (D)

(56) A unit matrix for the set of matrices $= \left\{ \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}, x \in R, x \neq 0 \right\}$ under multiplication is

(A) $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$

(B) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Sol. : Let $\begin{bmatrix} k & k & k \\ k & k & k \\ k & k & k \end{bmatrix}$ be the required unit matrix.

$$\therefore \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \begin{bmatrix} k & k & k \\ k & k & k \\ k & k & k \end{bmatrix} = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \quad \forall x \quad \text{So, } \begin{bmatrix} 3kx & 3kx & 3kx \\ 3kx & 3kx & 3kx \\ 3kx & 3kx & 3kx \end{bmatrix} = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$$

$$\therefore 3kx = x \quad \forall x$$

$$\therefore k = \frac{1}{3} \quad (x \neq 0)$$

$$\therefore \text{ Required unit matrix is } \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \quad \text{Ans. : (A)}$$

(57) If $A = \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$, then $(I + A)(I - A)^{-1} = \dots\dots$

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Sol. : $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix}$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 2 \end{bmatrix}$$

$$(I - A)^{-1} = \frac{1}{|I - A|} \text{adj}(I - A) = \frac{1}{4} \begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\begin{aligned} \therefore (I + A)(I - A)^{-1} &= \frac{1}{4} \begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -2 & 0 \end{bmatrix} \\ &= \frac{1}{4} \begin{bmatrix} 0 & 4 \\ -4 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \end{aligned}$$

Ans. : (B)

(58) If a non-zero solution of $x + 3y + z = 0$, $2x - y - z = 0$, $kx + 2y + 3z = 0$ exist, then $k = \dots\dots$ ($x \neq 0, y \neq 0, z \neq 0$)

(A) $\frac{13}{2}$

(B) $\frac{9}{2}$

(C) $-\frac{15}{2}$

(D) $-\frac{13}{2}$

Sol. : $\begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ k & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow AX = D \quad \text{where } A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ k & 2 & 3 \end{bmatrix}$

As non-zero solution does not exist.

$$\therefore |A| = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ k & 2 & 3 \end{vmatrix} = 0$$

$$1[-3+2]-3[6+k]+1[4+k]=0$$

$$-1-18-3k+4+k=0$$

$$-15-2k=0$$

$$\therefore k = -\frac{15}{2}$$

Ans. : (C)

(59) If non-zero solution of the equations $y+z=-ax$, $z+x=-by$, $x+y=-cz$ exists, then

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = \dots (x+y+z \neq 0)$$

(A) 1

(B) 2

(C) -1

(D) -2

Sol. : $y+z=-ax$, $z+x=-by$, $x+y=-cz$

$$\therefore -a = \frac{y+z}{x} \quad -b = \frac{z+x}{y} \quad -c = \frac{x+y}{z}$$

$$1-a = 1 + \frac{y+z}{x}; \quad 1-b = \frac{z+x}{y} + 1; \quad 1-c = \frac{x+y}{z} + 1$$

$$1-a = \frac{x+y+z}{x}; \quad 1-b = \frac{x+y+z}{y}; \quad 1-c = \frac{x+y+z}{z}$$

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = \frac{x}{x+y+z} + \frac{y}{x+y+z} + \frac{z}{x+y+z} = 1$$

Ans. : (A)

(60) If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then $A^{100} = \dots$

(A) $2^{99}A$

(B) $2^{101}A$

(C) $2^{100}A$

(D) I

Sol. : $A^2 = AA = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$A^3 = A^2A = 2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2^2 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\text{Similarly, } A^{100} = 2^{99} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2^{99}A$$

Ans. : (A)

(61) If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, then $A^T A^{-1} = \dots$

(A) $\begin{bmatrix} -\cos 2x & \sin 2x \\ -\sin 2x & \cos 2x \end{bmatrix}$

(B) $\begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$

(C) $\begin{bmatrix} \sin 2x & \cos 2x \\ \cos 2x & \sin 2x \end{bmatrix}$

(D) $\begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$

Sol. : $|A| = 1 + \tan^2 x = \sec^2 x \neq 0$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \cos^2 x \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$\begin{aligned} A^T A^{-1} &= \cos^2 x \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix} = \cos^2 x \begin{bmatrix} 1 - \tan^2 x & -2\tan x \\ 2\tan x & 1 - \tan^2 x \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 x - \sin^2 x & -2\sin x \cos x \\ 2\sin x \cos x & \cos^2 x - \sin^2 x \end{bmatrix} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix} \quad \text{Ans. : (B)} \end{aligned}$$

(62) If $A = \begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix}$, then $I + 2A + 3A^2 + 4A^3 + \dots \infty = \dots$

(A) $\begin{bmatrix} 9 & 1 \\ -9 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 1 \\ -9 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 7 & 2 \\ -18 & -5 \end{bmatrix}$ (D) $\begin{bmatrix} 7 & 2 \\ -5 & -18 \end{bmatrix}$

Sol. : $A^2 = \begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -9 & -3 \end{bmatrix} = \begin{bmatrix} 9-9 & 3-3 \\ -27+27 & -9+9 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \dots$

$$\therefore I + 2A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ -18 & -6 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ -18 & -5 \end{bmatrix}$$

$$I + 2A + 3A^2 + 4A^3 + \dots \infty = I + 2A = \begin{bmatrix} 7 & 2 \\ -18 & -5 \end{bmatrix} \quad \text{Ans. : (C)}$$

(63) If a symmetric matrix X and a skew-symmetric matrix Y satisfy $X + Y = A = \begin{bmatrix} 1 & 6 & 7 \\ 2 & 5 & 9 \\ 3 & 4 & 8 \end{bmatrix}$, then $X = \dots$

(A) $\begin{bmatrix} 0 & 4 & 4 \\ -4 & 0 & 6 \\ -4 & -5 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 2 & 2 \\ -2 & 0 & \frac{5}{2} \\ -2 & -\frac{5}{2} & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 8 & 10 \\ 8 & 5 & 13 \\ 10 & 13 & 8 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 4 & 5 \\ 4 & 5 & \frac{13}{2} \\ 5 & \frac{13}{2} & 8 \end{bmatrix}$

Sol. : $A = X + Y \Rightarrow A^T = X^T + Y^T = X - Y$

$$\therefore X = \frac{1}{2} [A + A^T]$$

$$= \frac{1}{2} \left\{ \begin{bmatrix} 1 & 6 & 7 \\ 2 & 5 & 9 \\ 3 & 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 7 & 9 & 8 \end{bmatrix} \right\} = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 5 & \frac{13}{2} \\ 5 & \frac{13}{2} & 8 \end{bmatrix} \quad \text{Ans. : (D)}$$

(64) If $A = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$, then $(AB)^{-1} = \dots$

(A) $\begin{bmatrix} -\frac{61}{2} & \frac{47}{2} \\ \frac{87}{2} & -\frac{67}{2} \end{bmatrix}$

(B) $\begin{bmatrix} \frac{61}{2} & -\frac{47}{2} \\ -\frac{87}{2} & \frac{67}{2} \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{61}{2} & \frac{47}{2} \\ \frac{87}{2} & \frac{67}{2} \end{bmatrix}$

(D) $\begin{bmatrix} \frac{61}{2} & -\frac{47}{2} \\ \frac{87}{2} & -\frac{67}{2} \end{bmatrix}$

Sol. : $|A| = 54 - 56 = -2 \neq 0$. Now $A^{-1} = \frac{1}{|A|} \text{adj } A$. So, $A^{-1} = -\frac{1}{2} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$

$$(AB)^{-1} = B^{-1} A^{-1} = -\frac{1}{2} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} 61 & -47 \\ -87 & 67 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{61}{2} & \frac{47}{2} \\ \frac{87}{2} & -\frac{67}{2} \end{bmatrix}$$

Ans. : (A)

(65) If $[x - 5 - 1] \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = [0]$, then $x = \dots$

(A) $\pm 2\sqrt{3}$

(B) $\pm \sqrt{3}$

(C) $\pm 4\sqrt{3}$

(D) $\sqrt{3}$

Sol. : $[x - 5 - 1] \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = [0]$

$$[(x - 2) - 10 - (2x - 5 - 3)] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = [0]$$

$$\therefore [x - 2 - 10 - 2x + 8] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = [0]$$

$$x^2 - 2x - 40 + 2x - 8 = 0$$

$$x^2 = 48.$$

$$\text{So, } x = \pm \sqrt{48} = \pm 4\sqrt{3}$$

Ans. : (C)

(66) If $A^2 - 3A = I$, then $A^{-1} = \dots$

(A) $I - 3A$

(B) $3A - I$

(C) $A - 3I$

(D) $3I - A$

Sol. : $A^2 - 3A = I$

$$A^{-1} (A^2 - 3A) = A^{-1} I$$

$$(A^{-1}A) A - 3(A^{-1}A) = A^{-1}I$$

$$IA - 3I = A^{-1}$$

$$\therefore A^{-1} = A - 3I$$

Ans. : (C)

(67) If $A = \begin{bmatrix} \frac{1}{2} & 2 \\ 0 & 2x-3 \end{bmatrix}$; $B = \begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix}$ and $AB = I$, then $x = \dots$

(A) 2

(B) -1

(C) 1

(D) 0

Sol. : $|AB| = |I|$

$$|A| |B| = 1$$

$$\therefore \frac{1}{2} [2x - 3] (-2) = 1. \text{ So, } 2x - 3 = -1$$

$$\therefore x = 1$$

Ans. : (C)

(68) If $A = \begin{bmatrix} a & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ and $A^2 = B$, then $a = \dots$

(A) -1

(B) 1

(C) 4

(D) a does not exist

Sol. : $A^2 = \begin{bmatrix} a & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a^2 & 0 \\ a+1 & 1 \end{bmatrix}. \quad A^2 = B \Rightarrow \begin{bmatrix} a^2 & 0 \\ a+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$$\therefore a^2 = 1, \quad a + 1 = 5$$

$$a = 4. \text{ Given equations are inconsistent.}$$

Ans. : (D)

(69) If a skew-symmetric matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is given, then $(I + A) (I - A)^{-1} = \dots$

(A) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(B) I

(C) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Sol. : $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$(I - A)^{-1} = \frac{1}{|I - A|} \text{adj} (I - A) = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$\text{Now, } (I + A) (I - A)^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Ans. : (A)

$$(70) \quad A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}, \text{ then } (I + A) (I - A)^{-1} = \dots\dots .$$

$$(A) \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (B) \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (C) \begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (D) \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$$

Sol. : I is a 2×2 unit matrix.

$$I + A = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \quad \text{and} \quad I - A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$(I - A)^{-1} = \cos^2 \frac{\theta}{2} \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$\therefore (I + A) (I - A)^{-1} = \cos^2 \frac{\theta}{2} \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$= \cos^2 \frac{\theta}{2} \begin{bmatrix} 1 - \tan^2 \frac{\theta}{2} & -2\tan\frac{\theta}{2} \\ 2\tan\frac{\theta}{2} & -\tan^2 \frac{\theta}{2} + 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} & -2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \\ 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} & \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \text{Ans. : (B)}$$

(71) If $AA^T = A^T A$ and $B = A^{-1}A^T$, then $BB^T = \dots\dots$ where A is a 3×3 non-singular matrix.

$$(A) I + B \quad (B) I \quad (C) B^{-1} \quad (D) (B^{-1})^T$$

$$\text{Sol. : } BB^T = (A^{-1}A^T) (A^{-1}A^T)^T = (A^{-1}A^T) (A (A^{-1})^T) = [A^{-1}(A^T A)] (A^{-1})^T$$

$$\therefore BB^T = [(A^{-1}(AA^T))] (A^{-1})^T = [(A^{-1}A) (A^T) (A^{-1})^T] = A^T (A^T)^{-1} = I \quad \text{Ans. : (B)}$$

$$(72) \quad \text{If } [1 \ x \ 1] \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = [0], \text{ then } x = \dots\dots$$

$$(A) \frac{-9 \pm \sqrt{35}}{2} \quad (B) \frac{-7 \pm \sqrt{53}}{2} \quad (C) \frac{-9 \pm \sqrt{53}}{2} \quad (D) \frac{-7 \pm \sqrt{35}}{2}$$

$$\text{Sol. : } [1 \quad 3 + 5x + 3 \quad 2 + x + 2] \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = [0]$$

$$\therefore [1 \quad 6+5x \quad 4+x] \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = [0]$$

$$\therefore 1 + 6 + 5x + 4x + x^2 = 0$$

$$\therefore x^2 + 9x + 7 = 0$$

$$x = \frac{-9 \pm \sqrt{81-4(7)}}{2(1)} = \frac{-9 \pm \sqrt{53}}{2}$$

Ans. : (C)

$$(73) \quad \text{If } \begin{bmatrix} 3 & -1+x & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{bmatrix} \text{ is singular and } x \in [-4, -1], \text{ then } x = \dots\dots .$$

(A) -4

(B) 3

(C) 0

(D) 2

Sol. : Given matrix is singular.

$$\therefore \begin{vmatrix} 3 & x-1 & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & x & -x \\ -x & 0 & x \\ x+3 & -1 & 2 \end{vmatrix} = 0$$

$R_{21} (-1), R_{32} (-1)$

$$\therefore x^2 \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ x+3 & -1 & 2 \end{vmatrix} = 0$$

$$\therefore x^2 \{-1(-2-x-3) - 1(1)\} = 0$$

$$\therefore x^2 (x+4) = 0$$

$$\therefore x = 0 \quad \text{or} \quad x = -4$$

$$\therefore 0 \notin [-4, -1] \quad \text{and} \quad -4 \in [-4, -1].$$

$$\text{So, } x = -4$$

Ans. : (A)

$$(74) \quad P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } Q = PAP^T \text{ then } P^T Q^{205} P = \dots\dots$$

$$(A) \begin{bmatrix} 1 & 205 \\ 0 & 1 \end{bmatrix}$$

$$(B) \begin{bmatrix} 205 & 205 \\ 0 & 1 \end{bmatrix}$$

$$(C) \begin{bmatrix} 1 & 205 \\ 0 & 205 \end{bmatrix}$$

(D) none of these

$$\text{Sol. : } PP^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} + \frac{1}{4} & -\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} & \frac{1}{4} + \frac{3}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore P^T = P^{-1}. \text{ Also, } P^T P = PP^T$$

$$(PP^{-1} = P^{-1}P = I)$$

$$\text{Now } P^T Q^{205} P = P^T [Q \times Q \times \dots \times Q \text{ 205 times}] P$$

$$= P^T [(PAP^T)(PAP^T) \times \dots \times 205 \text{ times}] P$$

$$= (P^T P)A (P^T P)A (P^T P)A \dots 205 \text{ times} = A \times A \times \dots 205 \text{ times} = A^{205}$$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\text{Similarly, } A^4 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \quad \text{So, } A^{205} = \begin{bmatrix} 1 & 205 \\ 0 & 1 \end{bmatrix}$$

Ans. : (A)

$$(75) \quad A = \begin{bmatrix} \cos \frac{7\pi}{6} & \sin \frac{11\pi}{6} \\ \sin \frac{11\pi}{6} & -\cos \frac{7\pi}{6} \end{bmatrix}, \text{ then } A^2 = \dots$$

$$(A) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(B) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$(C) \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$(D) \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\text{Sol. : } A = \begin{bmatrix} \cos\left(\pi + \frac{\pi}{6}\right) & \sin\left(2\pi - \frac{\pi}{6}\right) \\ \sin\left(2\pi - \frac{\pi}{6}\right) & -\cos\left(\pi + \frac{\pi}{6}\right) \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} + \frac{1}{4} & -\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} & \frac{1}{4} + \frac{3}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Ans. : (A)

$$(76) \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix} = \dots$$

$$(A) \begin{bmatrix} 20 & 25 & 16 \\ 32 & 40 & 28 \\ 44 & 55 & 40 \end{bmatrix}$$

$$(B) \begin{bmatrix} -12 & -15 & 4 \\ 32 & 40 & 28 \\ 44 & 55 & 40 \end{bmatrix}$$

$$(C) \begin{bmatrix} 12 & 15 & 18 \\ -24 & -40 & 48 \end{bmatrix}$$

(D) none of these

$$\text{Sol. : Required product is } \begin{bmatrix} -1-4 & -2-12 \\ -2-6 & -4-16 \\ -3-8 & -6-20 \end{bmatrix} \begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -14 \\ -8 & -20 \\ -11 & -26 \end{bmatrix} \begin{bmatrix} -4 & -5 & -6 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 25 & 16 \\ 32 & 40 & 28 \\ 44 & 55 & 40 \end{bmatrix}$$

Ans. : (A)

$$(77) \quad \text{If } 2A + 3B = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix} \text{ and } A + 2B = \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix}, \text{ then } B = \dots$$

$$(A) \begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$$

$$(B) \begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & 1 \end{bmatrix}$$

$$(C) \begin{bmatrix} 8 & -1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$$

$$(D) \begin{bmatrix} -8 & 1 & -2 \\ -1 & 10 & -1 \end{bmatrix}$$

$$\text{Sol. : } 2A + 4B = \begin{bmatrix} 10 & 0 & 6 \\ 2 & 12 & 4 \end{bmatrix} \quad (1)$$

$$2A + 3B = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix} \quad (2)$$

Subtracting (2) from (1),

$$B = \begin{bmatrix} 10 & 0 & 6 \\ 2 & 12 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix} \quad \text{Ans. : (A)}$$

$$(78) \text{ If } A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, \text{ then } A^{12} = \dots$$

$$(A) \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad (B) \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \quad (C) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (D) \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\text{Sol. : } A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{12} = (A^4)^3 = I^3 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{Ans. : (C)}$$

$$(79) \text{ If } A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \text{ and } A + A^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ then } \alpha = \dots \quad 0 < \alpha < \frac{\pi}{2}$$

$$(A) \frac{\pi}{6} \quad (B) \frac{\pi}{4} \quad (C) \frac{\pi}{3} \quad (D) \frac{\pi}{2}$$

$$\text{Sol. : } A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \text{ and } A^T = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore 2 \cos \alpha = 1. \quad \text{So, } \alpha = \frac{\pi}{3} \quad \text{Ans. : (C)}$$

(80) If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then $A^8 = \dots$

(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Sol. : $A^2 = A \cdot A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^8 = A^4 \cdot A^4 = I$$

Ans. : (C)

(81) If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ and $A + A^T = \sqrt{3} I$, then $\theta = \dots$

$$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $-\frac{\pi}{6}$

(D) $-\frac{\pi}{3}$

Sol. : $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, $A^T = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

Now $A + A^T = \sqrt{3} I$

$$\therefore \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \sqrt{3} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2\cos \theta & 0 \\ 0 & 2\cos \theta \end{bmatrix} = \begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{bmatrix}$$

$$\therefore \cos \theta = \frac{\sqrt{3}}{2}$$

$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, So, $P(\theta)$ is in the first or in fourth quadrant.

$$\therefore \theta = \frac{\pi}{6} \text{ or } \frac{-\pi}{6}$$

Ans. : (A), (C)

(82) Find the inverse of $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ by using reduced row echelon method.

Sol. : Let the matrix $A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ and $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now $A = I_2 A$

$$\therefore \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \quad \text{So, } \begin{bmatrix} 1 & \frac{5}{4} \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & 1 \end{bmatrix} A$$

$$R_1\left(\frac{1}{4}\right)$$

$$\therefore \begin{bmatrix} 1 & \frac{5}{4} \\ 0 & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 \\ -\frac{3}{4} & 1 \end{bmatrix} A \quad (R_2 \rightarrow R_2 - 3R_1) \quad \text{So, } \begin{bmatrix} 1 & \frac{5}{4} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 \\ -3 & 4 \end{bmatrix} A \quad (R_2 \rightarrow 4R_2)$$

$$\therefore \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A \quad R_2(4), R_{21} \left(\frac{-5}{4} \right)$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

(83) Find the inverse of $\begin{bmatrix} 2 & 1 & 4 \\ -3 & 4 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ by using reduced row echelon method.

Sol. : Let matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ -3 & 4 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now $A = I_3 A$

$$\therefore \begin{bmatrix} 2 & 1 & 4 \\ -3 & 4 & -1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ -3 & 4 & -1 \\ 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A \quad R_1 \leftrightarrow R_3$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 10 & 8 \\ 0 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 3 \\ 1 & 0 & -2 \end{bmatrix} A \quad R_2 \rightarrow R_2 + 3R_1, \quad R_3 \rightarrow R_3 - 2R_1$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{4}{5} \\ 0 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{10} & \frac{3}{10} \\ 1 & 0 & -2 \end{bmatrix} A \quad R_2 \rightarrow \frac{1}{10} R_2$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{4}{5} \\ 0 & 0 & \frac{2}{5} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{10} & \frac{3}{10} \\ 1 & \frac{3}{10} & \frac{-11}{10} \end{bmatrix} A \quad R_3 \rightarrow R_3 + 3R_2$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{4}{5} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{10} & \frac{3}{10} \\ \frac{5}{2} & \frac{3}{4} & \frac{-11}{4} \end{bmatrix} A \quad R_3 \rightarrow \frac{5}{2} R_3$$

$$\therefore \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-15}{2} & \frac{-9}{4} & \frac{37}{4} \\ -2 & -\frac{1}{2} & \frac{5}{2} \\ \frac{5}{2} & \frac{3}{4} & \frac{-11}{4} \end{bmatrix} A \quad R_2 \rightarrow R_2 - \frac{4}{5} R_3, R_1 \rightarrow R_1 - 3 R_3$$

$$\therefore \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-7}{2} & \frac{-5}{4} & \frac{17}{4} \\ -2 & -\frac{1}{2} & \frac{5}{2} \\ \frac{5}{2} & \frac{3}{4} & \frac{-11}{4} \end{bmatrix} A \quad (R_1 \rightarrow R_1 - 2 R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{-7}{2} & \frac{-5}{4} & \frac{17}{4} \\ -2 & -\frac{1}{2} & \frac{5}{2} \\ \frac{5}{2} & \frac{3}{4} & \frac{-11}{4} \end{bmatrix}$$

(84) $A = \begin{bmatrix} 1 & 5 & 40 \\ 7 & 9 & 72 \\ 5 & 3 & 29 \end{bmatrix}$. To find A^{-1} using elementary row operations, after some steps, we have

$$\begin{bmatrix} 1 & 5 & 40 \\ 0 & 1 & 8 \\ 0 & -22 & -171 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \beta & \alpha & 0 \\ -5 & 0 & 1 \end{bmatrix} A. \text{ Find } \alpha \text{ and } \beta.$$

(A) $\alpha = \frac{7}{26}, \beta = \frac{-1}{26}$ (B) $\alpha = \frac{-1}{26}, \beta = \frac{7}{26}$ (C) $\alpha = \frac{1}{26}, \beta = \frac{-7}{26}$ (D) $\alpha = \frac{1}{26}, \beta = \frac{7}{26}$

Sol. : $\begin{bmatrix} 1 & 5 & 40 \\ 7 & 9 & 72 \\ 5 & 3 & 29 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$$\therefore \begin{bmatrix} 1 & 5 & 40 \\ 0 & -26 & -208 \\ 0 & -22 & -171 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix} A \quad R_2 \rightarrow R_2 - 7 R_1, R_3 \rightarrow R_3 - 5 R_1$$

$$\therefore \begin{bmatrix} 1 & 5 & 40 \\ 0 & 1 & 8 \\ 0 & -22 & -171 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{7}{26} & -\frac{1}{26} & 0 \\ -5 & 0 & 1 \end{bmatrix} A \quad \left(R_2 \rightarrow -\frac{1}{26} R_2 \right)$$

$\therefore \alpha = \frac{-1}{26}, \beta = \frac{7}{26}$ **Ans. : (B)**

(85) If for $A = \begin{bmatrix} 1 & 5 & 40 \\ 7 & 9 & 72 \\ 5 & 3 & 29 \end{bmatrix}$, using elementary row transformations to obtain inverse of A, we get

$$\begin{bmatrix} 1 & 0 & x \\ 0 & 1 & 8 \\ 0 & -22 & -171 \end{bmatrix} = \begin{bmatrix} -\frac{9}{26} & y & 0 \\ \frac{7}{26} & -\frac{1}{26} & 0 \\ -5 & 0 & 1 \end{bmatrix} A, \text{ then } x \text{ and } y \text{ are and respectively.}$$

(A) 1 and $\frac{5}{26}$

(B) $\frac{5}{26}$ and 1

(C) 1 and $-\frac{5}{26}$

(D) -1 and $-\frac{5}{26}$

Sol. : $\begin{bmatrix} 1 & 5 & 40 \\ 7 & 9 & 72 \\ 5 & 3 & 29 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$$\therefore \begin{bmatrix} 1 & 5 & 40 \\ 0 & -26 & -208 \\ 0 & -22 & -171 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & 0 \\ -5 & 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 - 7R_1, R_3 \rightarrow R_3 - 5R_1$$

$$\therefore \begin{bmatrix} 1 & 5 & 40 \\ 0 & 1 & 8 \\ 0 & -22 & -171 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{7}{26} & -\frac{1}{26} & 0 \\ -5 & 0 & 1 \end{bmatrix} A$$

$$(R_2 \rightarrow \frac{-1}{26}R_2)$$

$$\therefore \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 8 \\ 0 & -22 & -171 \end{bmatrix} = \begin{bmatrix} \frac{-9}{26} & \frac{5}{26} & 0 \\ \frac{7}{26} & -\frac{1}{26} & 0 \\ -5 & 0 & 1 \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - 5R_2$$

$$\therefore x = 1 \text{ and } y = \frac{5}{26}$$

Ans. : (A)

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