

PE-254-CV-19
M.A./M.Sc. MATHEMATICS (2nd Semester)
Examination, June 2022
Paper-II
REAL ANALYSIS (II)

Time : Three Hours]

[Maximum Marks : 80
[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Answer the following questions:

1x10=10

- (a) Write property of countable subadditivity for outer measure of a set.
- (b) When an algebra is called b-algebra?
- (c) "An open interval is not necessarily a measurable set." True or False?
- (d) "What does $E(f \leq \alpha)$ denote?"
- (e) "Step function and simple function are measurable function." True or False.
- (f) Write an example of a function which is Lebesgue measurable function.
- (g) Write any the property of non-negative measurable function.
- (h) When is a non-negative measurable function f defined in a measurable set E called integrable?
- (i) "A function of bounded variation is necessarily continuous." True or False?
- (j) What is Lebesgue point?

2. Answer the following questions:

2x5=10

- (a) What is the first definition of measurable set given by H. Lebesgue?
- (b) What is Cantor set?
- (c) Give an example to show that a measurable function need not continuous.
- (d) Define Convergence in measure.
- (e) Define upper Lebesgue integral of a function.

Section-B

15x4=60

Answer the four questions of the following:

- 3. State and prove any three properties of outer measure.**
- 4. Prove that a countable union of measurable sets is a measurable set.**
- 5. a). State and prove the property "translation invariance" with the Lebesgue measure.**
b). Let $\{E_i\}$ be an infinite increasing sequence of measurable sets, that is, a sequence with $E_{i+1} \supset E_i$ for each $i \in \mathbb{N}$. Then prove that. $m(\bigcup_{i=1}^{\infty} E_i) = \lim_{n \rightarrow \infty} m(E_n)$
- 6. a). State and prove a theorem showing relation between continuity and measurability of a function.**
b). Let f and g be two functions defined on a common domain E such that $f=g$ a.e. and g is measurable, prove that f is measurable.
- 7. a). Prove that a continuous function defined on a Borel set in Borel measurable and the converse may not be true.**
b). State and prove a theorem showing a relation between point wise convergence and convergence in measure of a sequence of measurable functions.
- 8. a). State and prove any two properties of the Lebesgue integral for non-negative measurable functions.**
b). Let f be a nonnegative function which is integrable over a set E prove that for given $\epsilon > 0$ such that for Every set $A \subset E$ with $m(A) < \delta$, we have $\int_A f < \epsilon$
- 9. a). State and prove any two properties of a function of bounded variation.**
b). Let f be an integral function on $[a, b]$ of $\int_a^x f(t) dt = 0$ for all $x \in [a, b]$, then prove that $f = 0$ a. e. in $[a, b]$.