	COMPLEX NUMBERS Class XI	
Q.11)	Where does z lies, if $\left\|\frac{z-5i}{z+5i}\right\| = 1$?	
Sol.11)	Let $z = x + iy$ We have, $\left\|\frac{z-5i}{z+5i}\right\| = 1$ $\Rightarrow \frac{\|z-5i\|}{\|z+5i\|} = 1$ $\Rightarrow \|z-5i\| = \|z+5i\|$ $\Rightarrow \|x+iy-5i\| = \|x+iy+5i\|$ $\Rightarrow \|x-i(y-5)\| = \|x+i(y+5)\|$ $\Rightarrow \sqrt{x^2+(y-5)^2} = \sqrt{x^2+(y+5)^2}$ Squaring $\Rightarrow x^2+y^2+25-10y = x^2+y^2+25+10y$ $\Rightarrow 20y = 0$ $\Rightarrow y = 0$ $\Rightarrow z = x$ lies on $x - axis$	
Q.12)	Evaluate: $1 + i^2 + i^4 + i^6 + \cdots \ldots \ldots . i^{2n}$	
Sol.12)	$= 1 - 1 + 1 - 1 + \cdots \ldots \ldots (-1)^n$ This cannot be evaluated unless value of n is known	
Q.13)	If $\|z_1\| = \|z_2\|$, is it necessary that $z_1 = z_2$?	
Sol.13)	No. Example $z_1 = 3 + 4i$ & $z_2 = 4 + 3i$ $\Rightarrow \|z_1\| = \sqrt{9+16} = 5$ $\Rightarrow \|z_2\| = \sqrt{16+9} = 5$ Clearly $\|z_1\| = \|z_2\|$ but $\|z_1\| \neq \|z_2\|$	
Q.14)	What is the polar form of complex number $= (i^{25})^3$	
Sol.14)	Let $z = (i^{25})^3$ $\Rightarrow z = (i^{-1})^3$ $\Rightarrow z = -i$ $\Rightarrow z = 0 - 1i$ Here, $a = 0, b = -1$ $\Rightarrow r = \sqrt{0+1} = 1$ $\Rightarrow \tan\alpha = \left\|\frac{-1}{0}\right\| = \left\|\frac{1}{0}\right\| = 0$ $\alpha = \frac{\pi}{2}$ since z is in 4th quadrant $\Rightarrow \theta = -\alpha \Rightarrow \theta = \frac{-\pi}{2}$ Polar form $\Rightarrow z = r(\cos\theta + i\sin\theta)$ $\Rightarrow z = 1\left(\cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)\right)$ $\Rightarrow z = \cos\left(\frac{\pi}{2}\right) - i\sin\left(\frac{\pi}{2}\right)$ ans.	
Q.15)	Find the complex number satisfying the equation $z + \sqrt{z}\|z+1\| + i = 0$	
Sol.15)	We have, $z + \sqrt{z}\|z+1\| + i = 0$ Let $z = x + iy$	

	$\Rightarrow x + iy + \sqrt{2}\lvert x + iy + 1\rvert + i = 0$ $\Rightarrow x + iy + \sqrt{2}\lvert (x+1) + iy\rvert + i = 0$ $\Rightarrow x + iy + \sqrt{2}\sqrt{(x+1)^2 + y^2} + i = 0$ $\Rightarrow x + \sqrt{2}\sqrt{(x+1)^2 + y^2} + i(y+1) = 0 + 0i$ Equating real & imaginary parts We get $y + 1 = 0$ $\Rightarrow x + \sqrt{2}\sqrt{(x+1)^2 + y^2} = 0$ Put $y = -1$ $\Rightarrow x + \sqrt{2}\sqrt{x^2 + 1 + 2x + 1} = 0$ $\Rightarrow x + \sqrt{2}\sqrt{x^2 + 2x + 2} = 0$ $\Rightarrow \sqrt{2}\sqrt{x^2 + 2x + 2} = -x$ Squaring, $2(x^2 + 2x + 2) = x^2$ $\Rightarrow x^2 + 4x + 4 = 0$ $\Rightarrow (x+2)^2 = 0$ $\Rightarrow x = -2$ $\Rightarrow z = x + iy = -2 - i$ ans.	
Q.16)	If $\arg(z-1) = \arg(z+3i)$ then find $x - 1 : y$ if $z = x + iy$.	
Sol.16)	We have, $z = x + iy$ $\Rightarrow z - 1 = z = x + iy - 1 = (x-1) + iy$ And $z + 3i = x + iy + 3i = x + i(y+3)$ $\Rightarrow \arg(z-1) = \tan\alpha = \frac{b}{a} = \frac{y}{x-1}$ $\Rightarrow \arg(z+3i) = \tan\alpha = \frac{b}{a} = \frac{y+3}{x}$ Given $\arg(z-1) = \arg(z+3i)$ $\Rightarrow \frac{y}{x-1} = \frac{y+3}{x}$ $\Rightarrow xy = xy + 3x - y - 3$ $\Rightarrow y = 3(x-1)$ $\Rightarrow \frac{x-1}{y} = \frac{1}{3}$ $\Rightarrow 1:3$ ans.	
Q.17)	If $a = \cos\theta + i\sin\theta$, find the value of $\frac{1+a}{1-a}$.	
Sol.17)	We have, $\frac{1+a}{1-a} = \frac{(1+\cos\theta) + i\sin\theta}{(1-\cos\theta) - i\sin\theta}$ $= \frac{2\cos^2\frac{\theta}{2} + i2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2} - i2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}$ $= \frac{2\cos^2\left(\frac{\theta}{2}\right)\left[\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right]}{2\sin^2\left(\frac{\theta}{2}\right)\left[\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}\right]}$ Rationalize $= \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} . \left[\frac{1}{-1}\frac{\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}\right]$ $= i\cot\left(\frac{\theta}{2}\right)$ ans.	

Q.18)	Evaluate: $i^n + i^{n+1} + i^{n+2} + i^{n+3}$	
Sol.18)	$i^n + i^n . i + i^n . i^2 + i^n i^3$ $= i^n\left(1 + i + (i^2 + i^3)\right)$ $= i^n(1 + i - 1 - i)$ $= i^n = 0$ $= 0$ ans.	