

PE-480
(514) M.A./M.Sc. MATHEMATICS (FOURTH SEMESTER)
Examination JUNE 2022
Compulsory/Optional
Group-
Paper-I

Name/Title of Paper- INTEGRATION THEORY AND FUNCTIONAL ANALYSIS-II

Time: Three Hours

Maximum Marks-080

Minimum Passing Marks-029

नोट: दोनों खण्डों से निर्देशानुसार उत्तर दीजिए। प्रश्नों के अंक उनके दाहिनी ओर अंकित हैं।

Note: Answer From Both the Section as Directed. The Figures in the right-hand margin indicate marks.

Section-A

1- Fill in the blanks of the following:-

1x10=10

- a. If d is the natural metric on a normed linear space $(X, \|\cdot\|)$, then $d(0, x) = \underline{\hspace{2cm}}$
- b. If Q is normed subspace of R , then limit of the sequence $\{0.1, 0.101, 0.101001, 0.1010010001, \dots\}$ is $\underline{\hspace{2cm}}$ number.
- c. Two equivalent norms on a linear space X induces $\underline{\hspace{2cm}}$ topology on X .
- d. Any linear transformation between finite dimensional spaces is $\underline{\hspace{2cm}}$
- e. The norm $\|\cdot\|: X \rightarrow \mathbb{R}$ on a linear space X is $\underline{\hspace{2cm}}$ linear functional.
- f. If X is a real normed linear space and $x \in X$ is such that $f(x) = 0$ for all $f \in X^*$, then $x = \underline{\hspace{2cm}}$
- g. A normed linear space is reflexive if $J(X) = \underline{\hspace{2cm}}$
- h. The sequence l_2 is an inner-product space with the inner product $\langle x, y \rangle = \underline{\hspace{2cm}}$
- i. Every inner-product space X $\underline{\hspace{2cm}}$ convex.
- j. The space l_p for $p \neq 2$ $\underline{\hspace{2cm}}$ inner-product space.

2- Answer the following questions

2x5=10

- a- Give example of a nls.
- b- State Riesz lemma.
- c- Define weak sequential compactness.
- d- Write Parsevals identity.
- e- Define self adjoint operator.

Section-B

Answer all questions:-

12x5=60

3- Prove that the operation of addition and scalar multiplication in a normed linear space $(X, \|\cdot\|)$ are jointly continuous.

Or

Prove that a nls X is complete iff every absolutely convergent series in X is convergent.

4- Prove that on a finite dimensional linear space all norms are equivalent.

Or

If M is a closed linear subspace of a nls X and $x_0 \in M$, then prove that there exists a functional $g \in X^*$ such that $g(M) = 0$ and $g(x_0) \neq 0$.

5- In a nls prove that strong convergence implies weak convergence but the converse need not be true.

Or

State and prove polarization identity.

6- State and prove uniform boundedness theorem.

Or

State and prove projection theorem.

7- State and prove Riesz representation theorem.

Or

If $\{e_k\}$ is an orthonormal sequence in an inner product space X , and $x \in X$, show that x is orthogonal to the subspace $Y_n = \text{span} \{e_1, \dots, e_n\}$ given by $y = \sum_{k=1}^{\infty} \alpha_k e_k$, $\alpha_k = \langle x, e_k \rangle$.