

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Answer the following questions:

1x10=10

- An R module M is.....if it is a direct sum of simple sub modules.
- Every module is a homomorphic image of a free module.
- α is.....if and only if $0 \rightarrow M \xrightarrow{\alpha} N$ is exact.
-is a basis of R -module(0).
- An R -module M is called noetherian if.....holds form M .
- A homomorphism $f: M \rightarrow N$ OF R -module M and N is called an automorphism if.....
- Minimal monic polynomial in $\theta(x)$ satisfied by $\sqrt{3}$ is.....
- The degree of complex field C over the field of real number is.....
- The number of elements in finite field is.....
- Every finite extension of a finite field is normal.

2. Answer the following short type questions:

2x5=10

- Find the degree of $\theta(\sqrt{3})$ over θ
- Show by an example that A noetherion module need not be artiniun.
- Write maximum condition for module M
- Define perfect field.
- Define rank of Linear mapping.

Section-B

12x5=60

Answer the following questions:

3. Let M be a finitely generated free module over a commutative ring R then all bases of M have the same number of element.

OR

State and prove Rank Nullity theorem.

- Let E be the splitting field of $x^n - a \in F(x)$. Then show that $G\left(\frac{E}{F}\right)$ is a solvable group.
- Show that the Galois group of $x^3 - 2 \in \theta(x)$ is the group of symmetries of the triangle.

OR

- If the finite field F has P^m elements show that every $a \in F$ satisfies $a^{P^m} = a$
- Let $F \leq E \leq K$ be fields. If K is a finite extension of a finite field is normal

5. State and prove Hilbert basis theorem.

OR

Show that the ring of real-valued continuous function on the closed interval $[0, 1]$ is neither artinian nor noetherian.

6. State and prove fundamental theorem of Galois Theory.

OR

Prove that $a \in K$ is algebraic over F if and only if $[F(a): F]$ is finite.

- Show that field generated by a root of $x^3 - x - 1 \in \theta[x]$ is not normal over θ
- Let E be a finite separable extension of F . then E is normal extension of F if F is the fixed field of $G(E/F)$.

OR

State and prove Artin theorem.