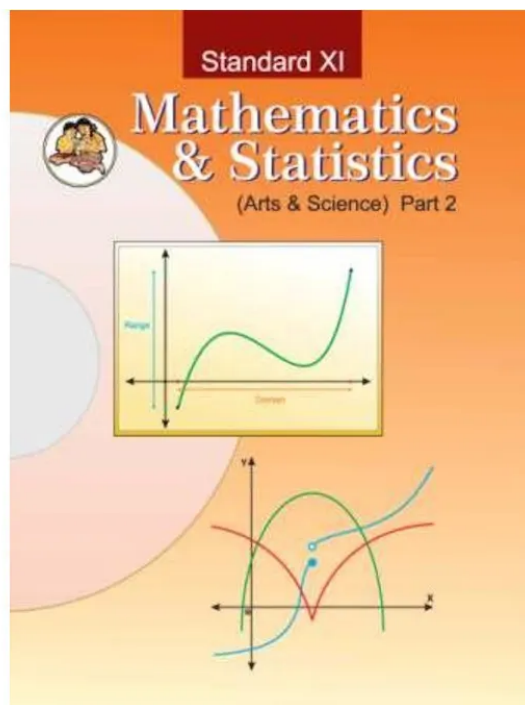


Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 7- Limits

Class 11 - Chapter 7 Limits



For any clarifications or questions you can write to info@indcareer.com

Postal Address

IndCareer.com, 52, Shilpa Nagar, Somalwada Nagpur - 440015
Maharashtra, India

WhatsApp: +91 9561 204 888, **Website:** <https://www.indcareer.com>

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Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 7- Limits

Class 11: Maths Chapter 7 solutions. Complete Class 11 Maths Chapter 7 Notes.

Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 7- Limits

Maharashtra Board 11th Maths Chapter 7, Class 11 Maths Chapter 7 solutions

Ex 7.1

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-7-limits/>

I. Evaluate the following limits:

Question 1.

$$\lim_{z \rightarrow -3} \left[\frac{\sqrt{z+6}}{z} \right]$$

Solution:

$$\begin{aligned} \lim_{z \rightarrow -3} \left[\frac{\sqrt{z+6}}{z} \right] &= \frac{\lim_{z \rightarrow -3} \sqrt{z+6}}{\lim_{z \rightarrow -3} z} \\ &= \frac{\sqrt{-3+6}}{-3} \\ &= \frac{\sqrt{3}}{-3} \\ &= \frac{-1}{\sqrt{3}} \end{aligned}$$

Question 2.

$$\lim_{y \rightarrow -3} \left[\frac{y^5 + 243}{y^3 + 27} \right]$$

Solution:

$$\lim_{y \rightarrow -3} \frac{y^5 + 243}{y^3 + 27}$$

$$\begin{aligned}
 &= \lim_{y \rightarrow -3} \frac{\left(\frac{y^5 + 243}{y + 3} \right)}{\left(\frac{y^3 + 27}{y + 3} \right)} \quad \dots \left[\because y \rightarrow -3, y \neq -3, \right. \\
 &\quad \left. \because y + 3 \neq 0 \right] \\
 &= \frac{\lim_{y \rightarrow -3} \left[\frac{y^5 - (-3)^5}{y - (-3)} \right]}{\lim_{y \rightarrow -3} \left[\frac{y^3 - (-3)^3}{y - (-3)} \right]} \\
 &= \frac{5(-3)^4}{3(-3)^2} \quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \right] \\
 &= \frac{5}{3} \times 9 \\
 &= 15
 \end{aligned}$$

Question 3.

$$\lim_{z \rightarrow -5} \left[\frac{\left(\frac{1}{z} + \frac{1}{5} \right)}{z + 5} \right]$$

Solution:

$$\lim_{z \rightarrow -5} \frac{\frac{1}{z} + \frac{1}{5}}{z + 5}$$

$$\begin{aligned} &= \lim_{z \rightarrow -5} \frac{5+z}{5z} \\ &= \lim_{z \rightarrow -5} \left[\frac{z+5}{5z(z+5)} \right] \\ &= \lim_{z \rightarrow -5} \frac{1}{5z} \quad \dots \left[\begin{array}{l} \because z \rightarrow -5, \quad z \neq -5, \\ \therefore z+5 \neq 0 \end{array} \right] \\ &= \frac{1}{5(-5)} \\ &= \frac{-1}{25} \end{aligned}$$

II. Evaluate the following limits:

Question 1.

$$\lim_{x \rightarrow 3} \left[\frac{\sqrt{2x+6}}{x} \right]$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{\sqrt{2x+6}}{x} &= \frac{\lim_{x \rightarrow 3} \sqrt{2x+6}}{\lim_{x \rightarrow 3} x} \\ &= \frac{\sqrt{2(3)+6}}{3} \\ &= \frac{\sqrt{12}}{3} = \frac{2\sqrt{3}}{3} \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow 2} \left[\frac{x^{-3} - 2^{-3}}{x - 2} \right]$$

Solution:

$$\lim_{x \rightarrow 2} \frac{x^{-3} - 2^{-3}}{x - 2} = (-3) \cdot (2)^{-4}$$

$$\dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \right]$$

$$= -3 \times \frac{1}{2^4}$$

$$= \frac{-3}{16}$$

Question 3.

$$\lim_{x \rightarrow 5} \left[\frac{x^3 - 125}{x^5 - 3125} \right]$$

Solution:

$$\lim_{x \rightarrow 5} \frac{x^3 - 125}{x^5 - 3125}$$

$$= \lim_{x \rightarrow 5} \frac{\left(\frac{x^3 - 5^3}{x - 5} \right)}{\left(\frac{x^5 - 5^5}{x - 5} \right)} \quad \dots \left[\begin{array}{l} \because x \rightarrow 5, x \neq 5, \\ \therefore x - 5 \neq 0 \end{array} \right]$$

$$= \frac{\lim_{x \rightarrow 5} \frac{x^3 - 5^3}{x - 5}}{\lim_{x \rightarrow 5} \frac{x^5 - 5^5}{x - 5}}$$

$$= \frac{\lim_{x \rightarrow 5} \frac{x^3 - 5^3}{x - 5}}{\lim_{x \rightarrow 5} \frac{x^5 - 5^5}{x - 5}}$$

$$= \frac{3(5)^2}{5(5)^4} \quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \right]$$

$$= \frac{3}{(5)^3}$$

$$= \frac{3}{125}$$

Question 4.

If $\lim_{x \rightarrow 1} \left[\frac{x^4 - 1}{x - 1} \right] = \lim_{x \rightarrow a} \left[\frac{x^3 - a^3}{x - a} \right]$, find all possible values of a .

Solution:

$$\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow a} \frac{x^3 - a^3}{x - a}$$

$$\therefore \lim_{x \rightarrow 1} \frac{x^4 - (1)^4}{x - 1} = \lim_{x \rightarrow a} \frac{x^3 - a^3}{x - a}$$

$$\therefore 4(1)^3 = 3a^2 \quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \right]$$

$$\therefore 3a^2 = 4$$

$$\therefore a^2 = \frac{4}{3}$$

$$\therefore a = \pm \frac{2}{\sqrt{3}}$$

III. Evaluate the following limits:

Question 1.

$$\lim_{x \rightarrow 1} \left[\frac{x + x^2 + x^3 + \dots + x^n - n}{x - 1} \right]$$

Solution:

$$\lim_{x \rightarrow 1} \left[\frac{x + x^2 + x^3 + \dots + x^n - n}{x - 1} \right]$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 1} \left[\frac{x + x^2 + x^3 + \dots + x^n - (1 + 1 + 1 + \dots \text{ n times})}{x - 1} \right] \\
 &= \lim_{x \rightarrow 1} \frac{(x-1) + (x^2-1) + (x^3-1) + \dots + (x^n-1)}{x-1} \\
 &= \lim_{x \rightarrow 1} \left[\frac{x^1-1^1}{x-1} + \frac{x^2-1^2}{x-1} + \frac{x^3-1^3}{x-1} + \dots + \frac{x^n-1^n}{x-1} \right] \\
 &= 1(1)^0 + 2(1)^1 + 3(1)^2 + 4(1)^3 + \dots + n(1)^{n-1} \\
 &\quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \right] \\
 &= 1 + 2 + 3 + 4 + \dots + n \\
 &= \frac{n(n+1)}{2}
 \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow 7} \left[\frac{(\sqrt[3]{x} - \sqrt[3]{7})(\sqrt[3]{x} + \sqrt[3]{7})}{x - 7} \right]$$

Solution:

$$\begin{aligned}
 &\lim_{x \rightarrow 7} \left[\frac{(\sqrt[3]{x} - \sqrt[3]{7})(\sqrt[3]{x} + \sqrt[3]{7})}{x - 7} \right] \\
 &= \lim_{x \rightarrow 7} \left[\frac{(x^{\frac{1}{3}} - 7^{\frac{1}{3}})(x^{\frac{1}{3}} + 7^{\frac{1}{3}})}{x - 7} \right]
 \end{aligned}$$

$$\begin{aligned} &= \lim_{x \rightarrow 7} \left[\frac{x^{\frac{2}{3}} - 7^{\frac{2}{3}}}{x - 7} \right] \dots [\because (a - b)(a + b) = a^2 - b^2] \\ &= \frac{2}{3} (7)^{\frac{-1}{3}} \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \right] \\ &= \frac{2}{3} \cdot \frac{1}{7^{\frac{1}{3}}} \\ &= \frac{2}{3 \sqrt[3]{7}} \end{aligned}$$

Question 3.

If $\lim_{x \rightarrow 5} \left[\frac{x^k - 5^k}{x - 5} \right] = 500$, find all possible values of k .

Solution:

$$\begin{aligned} \lim_{x \rightarrow 5} \frac{x^k - 5^k}{x - 5} &= 500 \\ \therefore k(5)^{k-1} &= 500 \quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \right] \\ \therefore k(5)^{k-1} &= 4 \times 125 \\ \therefore k(5)^{k-1} &= 4 \times (5)^3 \\ \therefore k(5)^{k-1} &= 4 \times (5)^{4-1} \\ \text{Comparing both sides, we get} \\ k &= 4 \end{aligned}$$

Question 4.

$$\lim_{x \rightarrow 0} \left[\frac{(1-x)^8 - 1}{(1-x)^2 - 1} \right]$$

Solution:

Put $1 - x = y$

As $x \rightarrow 0, y \rightarrow 1$

$$\therefore \lim_{x \rightarrow 0} \frac{(1-x)^8 - 1}{(1-x)^2 - 1}$$

$$= \lim_{y \rightarrow 1} \frac{y^8 - 1^8}{y^2 - 1^2}$$

$$= \lim_{y \rightarrow 1} \frac{\frac{y^8 - 1^8}{y - 1}}{\frac{y^2 - 1^2}{y - 1}} \quad \dots \left[\begin{array}{l} \because y \rightarrow 1, y \neq 1, \\ \therefore y - 1 \neq 0 \end{array} \right]$$

$$= \frac{\lim_{y \rightarrow 1} \frac{y^8 - 1^8}{y - 1}}{\lim_{y \rightarrow 1} \frac{y^2 - 1^2}{y - 1}}$$

$$= \frac{8(1)^7}{2(1)^1} \quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \right]$$

$$= 4$$

Question 5.

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt[3]{1+x} - \sqrt{1+x}}{x} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - \sqrt{1+x}}{x} \\ &= \lim_{x \rightarrow 0} \frac{(1+x)^{\frac{1}{3}} - (1+x)^{\frac{1}{2}}}{x} \end{aligned}$$

Put $1+x = y$

As $x \rightarrow 0, y \rightarrow 1$

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{(1+x)^{\frac{1}{3}} - (1+x)^{\frac{1}{2}}}{x} \\ &= \lim_{y \rightarrow 1} \frac{y^{\frac{1}{3}} - y^{\frac{1}{2}}}{y-1} \\ &= \lim_{y \rightarrow 1} \frac{\left(y^{\frac{1}{3}} - 1 \right) - \left(y^{\frac{1}{2}} - 1 \right)}{y-1} \\ &= \lim_{y \rightarrow 1} \left(\frac{y^{\frac{1}{3}} - 1}{y-1} - \frac{y^{\frac{1}{2}} - 1}{y-1} \right) \\ &= \lim_{y \rightarrow 1} \frac{y^{\frac{1}{3}} - 1^{\frac{1}{3}}}{y-1} - \lim_{y \rightarrow 1} \frac{y^{\frac{1}{2}} - 1^{\frac{1}{2}}}{y-1} \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{3}(1)^{\frac{-2}{3}} - \frac{1}{2}(1)^{\frac{-1}{2}} \quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \right] \\
 &= \frac{1}{3} - \frac{1}{2} \\
 &= \frac{2-3}{6} \\
 &= -\frac{1}{6}
 \end{aligned}$$

Question 6.

$$\lim_{y \rightarrow 1} \left[\frac{2y-2}{\sqrt[3]{7+y}-2} \right]$$

Solution:

$$\begin{aligned}
 &\lim_{y \rightarrow 1} \frac{2y-2}{\sqrt[3]{7+y}-2} \\
 &= \lim_{y \rightarrow 1} \frac{2(y-1)}{(7+y)^{\frac{1}{3}} - 8^{\frac{1}{3}}} \quad \dots \left[\because 2 = (2^3)^{\frac{1}{3}} = 8^{\frac{1}{3}} \right] \\
 &= \lim_{y \rightarrow 1} \frac{2}{\frac{(y+7)^{\frac{1}{3}} - 8^{\frac{1}{3}}}{y-1}} \\
 &= \frac{\lim_{y \rightarrow 1} 2}{\lim_{y \rightarrow 1} \frac{(y+7)^{\frac{1}{3}} - 8^{\frac{1}{3}}}{(y+7) - 8}}
 \end{aligned}$$

$$= \frac{2}{\frac{1}{3}(8)^{\frac{-2}{3}}}$$

$$= 2(3) \cdot (8)^{\frac{2}{3}}$$

$$= 6(2^3)^{\frac{2}{3}}$$

$$= 6 \times (2)^2$$

$$= 24$$

$$\dots \left[\begin{array}{l} \because y \rightarrow 1, y + 7 \rightarrow 8 \\ \text{and } \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \end{array} \right]$$

Question 7.

$$\lim_{z \rightarrow a} \left[\frac{(z+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{z-a} \right]$$

Solution:

$$\lim_{z \rightarrow a} \frac{(z+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{z-a}$$

$$= \lim_{z \rightarrow a} \frac{(z+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{(z+2) - (a+2)}$$

Put $z+2 = y$ and $a+2 = b$

As $z \rightarrow a$, $z+2 \rightarrow a+2$, i.e., $y \rightarrow b$

$$\therefore \lim_{z \rightarrow a} \frac{(z+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{z-a}$$

$$= \lim_{y \rightarrow b} \frac{y^{\frac{3}{2}} - b^{\frac{3}{2}}}{y-b}$$

$$= \frac{3}{2} \cdot b^{\frac{1}{2}} \quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x-a} = n \cdot a^{n-1} \right]$$

$$= \frac{3}{2} \sqrt{a+2} \quad \dots [\because b = a+2]$$

Question 8.

$$\lim_{x \rightarrow 7} \left[\frac{x^3 - 343}{\sqrt{x} - \sqrt{7}} \right]$$

Solution:

$$\begin{aligned}
 & \lim_{x \rightarrow 7} \frac{x^3 - 343}{\sqrt{x} - \sqrt{7}} \\
 &= \lim_{x \rightarrow 7} \frac{x^3 - 7^3}{\frac{1}{x^2} - \frac{1}{7^2}} \\
 &= \lim_{x \rightarrow 7} \frac{x^3 - 7^3}{\frac{1}{x^2 - 7^2}} \quad \dots \left[\begin{array}{l} \because x \rightarrow 7, x \neq 7, \\ \therefore x^2 - 7^2 \neq 0 \end{array} \right] \\
 &= \lim_{x \rightarrow 7} \frac{x^3 - 7^3}{x - 7} \\
 &= \frac{3(7)^2}{\frac{1}{2}(7)^{-\frac{1}{2}}} \\
 &= 6(49) \times 7^{\frac{1}{2}} \\
 &= 294\sqrt{7}
 \end{aligned}$$

Question 9.

$$\lim_{x \rightarrow 1} \left(\frac{x + x^3 + x^5 + \dots + x^{2n-1} - n}{x - 1} \right)$$

Solution:

$$\lim_{x \rightarrow 1} \left[\frac{x + x^3 + x^5 + \dots + x^{2n-1} - n}{x - 1} \right]$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 1} \left[\frac{x + x^3 + x^5 + \dots + x^{2n-1} - (1 + 1 + 1 + \dots n \text{ times})}{x-1} \right] \\
 &= \lim_{x \rightarrow 1} \left[\frac{(x-1) + (x^3-1) + (x^5-1) + \dots + (x^{2n-1}-1) \dots (n \text{ brackets})}{x-1} \right] \\
 &\quad \dots [\because 1, 3, 5, \dots, 2n-1 \text{ are the first } n \text{ odd numbers}] \\
 &= \lim_{x \rightarrow 1} \left[\frac{x^1-1^1}{x-1} + \frac{x^3-1^3}{x-1} + \frac{x^5-1^5}{x-1} + \dots + \frac{x^{2n-1}-1^{2n-1}}{x-1} \right] \\
 &= 1(1)^0 + 3(1)^2 + 5(1)^4 + \dots + (2n-1)(1)^{2n-2} \\
 &= 1 + 3 + 5 + \dots + (2n-1) \\
 &= \sum_{r=1}^n (2r-1) \\
 &= 2 \sum_{r=1}^n r - \sum_{r=1}^n 1 \\
 &= 2 \cdot \frac{n(n+1)}{2} - n \\
 &= n(n+1) - n \\
 &= n^2 + n - n \\
 &= n^2
 \end{aligned}$$

IV. In the following examples, given $\epsilon > 0$, find a $\delta > 0$ such that whenever, $|x - a| < \delta$, we must have $|f(x) - l| < \epsilon$.

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Question 1.

$$\lim_{x \rightarrow 2} (2x + 3) = 7$$

Solution:

We have to find some δ so that

$$\lim_{x \rightarrow 2} (2x + 3) = 7$$

Here $a = 2$, $l = 7$ and $f(x) = 2x + 3$

Consider $\epsilon > 0$ and $|f(x) - l| < \epsilon$

$$\therefore |(2x + 3) - 7| < \epsilon$$

$$\therefore |2x + 3 - 7| < \epsilon$$

$$\therefore |2(x - 2)| < \epsilon$$

$$\therefore |x - 2| < \frac{\epsilon}{2}$$

$$\therefore \delta \leq \frac{\epsilon}{2} \text{ such that}$$

$$|2x + 3 - 7| < \delta \Rightarrow |f(x) - 7| < \epsilon$$

Question 2.

$$\lim_{x \rightarrow -3} (3x + 2) = -7$$

Solution:

We have to find some δ so that

$$\lim_{x \rightarrow -3} (3x + 2) = -7$$

Here $a = -3$, $l = -7$ and $f(x) = 3x + 2$

Consider $\epsilon > 0$ and $|f(x) - l| < \epsilon$

$$\therefore |3x + 2 - (-7)| < \epsilon$$

$$\therefore |3x + 2 + 7| < \epsilon$$

$$\therefore |3(x + 3)| < \epsilon$$

$$\therefore |x + 3| < \frac{\epsilon}{3}$$

$$\therefore \delta < \frac{\epsilon}{3} \text{ such that}$$

$$|x + 3| \leq \delta \Rightarrow |f(x) + 7| < \epsilon$$

Question 3.

$$\lim_{x \rightarrow 2} (x^2 - 1) = 3$$

Solution:

We have to find some $\delta > 0$ such that

$$\lim_{x \rightarrow 2} (x^2 - 1) = 3$$

Here, $a = 2$, $l = 3$ and $f(x) = x^2 - 1$

Consider $\epsilon > 0$ and $|f(x) - l| < \epsilon$

$$\therefore |(x^2 - 1) - 3| < \epsilon$$

$$\therefore |x^2 - 4| < \epsilon$$

$$\therefore |(x + 2)(x - 2)| < \epsilon \dots\dots(i)$$

We have to get rid of the factor $|x + 2|$

$$\text{As } |x - 2| < \delta$$

$$-\delta < x - 2 < \delta$$

$$\therefore 2 - \delta < x < 2 + \delta$$

Since δ can be assumed as very small, let us choose $\delta < 1$

$$\therefore 1 < x < 3$$

$$\therefore 3 < x + 2 < 5 \dots\dots(\text{Adding 2 throughout})$$

$$\therefore |x + 2| < 5$$

$$\therefore |(x + 2)(x - 2)| < 5|x - 2| \dots\dots(ii)$$

From (i) and (ii), we get

$$5|x - 2| < \epsilon$$

$$\therefore x - 2 < \frac{\epsilon}{5}$$

$$\text{If } \delta = \frac{\epsilon}{5}, |x - 2| < \delta \Rightarrow |x^2 - 4| < \epsilon$$

$$\therefore \text{We choose } \delta = \min\{\frac{\epsilon}{5}, 1\} \text{ then}$$

$$|x - 2| < \delta \Rightarrow |f(x) - 3| < \epsilon$$

Question 4.

$$\lim_{x \rightarrow 1} (x^2 + x + 1) = 3$$

Solution:

We have to find some $\delta > 0$ such that

$$\lim_{x \rightarrow 1} (x^2 + x + 1) = 3$$

Here $a = 1$, $l = 3$ and $f(x) = x^2 + x + 1$

Consider $\epsilon > 0$ and $|f(x) - l| < \epsilon$

$$\therefore |x^2 + x + 1 - 3| < \epsilon$$

$$\therefore |x^2 + x - 2| < \epsilon$$

$$\therefore |(x + 2)(x - 1)| < \epsilon \dots\dots(i)$$

We have to get rid of the factor $|x + 2|$

$$\text{As } |x - 1| < \delta$$

$$\text{As } |x - 1| < \delta$$

$$-\delta < x - 1 < \delta$$

$$\therefore 1 - \delta < x < 1 + \delta$$

Since δ can be assumed as very small, let us choose $\delta < 1$

$$\therefore 0 < x < 2$$

$$\therefore 2 < x + 2 < 4$$

$$\therefore |x + 2| < 4$$

$$\therefore |(x + 2)(x - 1)| < 4 |x - 1| \dots\dots(ii)$$

From (i) and (ii), we get

$$4|x - 1| < \epsilon$$

$$\therefore |x - 1| < \frac{\epsilon}{4}$$

$$\text{If } \delta = \frac{\epsilon}{4},$$

$$|x - 1| < \delta \Rightarrow x^2 + x - 2 < \epsilon$$

$$\therefore \text{We choose } \delta = \min\left\{\frac{\epsilon}{4}, 1\right\} \text{ then}$$

$$|x - 1| < \delta \Rightarrow |f(x) - 3| < \epsilon$$

Ex 7.2

I. Evaluate the following limits:

Question 1.

$$\lim_{z \rightarrow 2} \left[\frac{z^2 - 5z + 6}{z^2 - 4} \right]$$

Solution:

$$\begin{aligned} & \lim_{z \rightarrow 2} \frac{z^2 - 5z + 6}{z^2 - 4} \\ &= \lim_{z \rightarrow 2} \frac{(z-3)(z-2)}{(z+2)(z-2)} \\ &= \lim_{z \rightarrow 2} \frac{z-3}{z+2} \quad \dots \left[\begin{array}{l} \because z \rightarrow 2, z \neq 2, \\ \therefore z-2 \neq 0 \end{array} \right] \\ &= \frac{\lim_{z \rightarrow 2} (z-3)}{\lim_{z \rightarrow 2} (z+2)} \\ &= \frac{2-3}{2+2} \\ &= -\frac{1}{4} \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow -3} \left[\frac{x+3}{x^2+4x+3} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow -3} \left[\frac{x+3}{x^2+4x+3} \right] \\ &= \lim_{x \rightarrow -3} \frac{x+3}{(x+3)(x+1)} \\ &= \lim_{x \rightarrow -3} \frac{1}{x+1} \quad \dots \left[\begin{array}{l} \because x \rightarrow -3, x \neq -3, \\ \therefore x+3 \neq 0 \end{array} \right] \\ &= \frac{1}{-3+1} \\ &= -\frac{1}{2} \end{aligned}$$

Question 3.

$$\lim_{y \rightarrow 0} \left[\frac{5y^3+8y^2}{3y^4-16y^2} \right]$$

Solution:

$$\begin{aligned} & \lim_{y \rightarrow 0} \left[\frac{5y^3+8y^2}{3y^4-16y^2} \right] \\ &= \lim_{y \rightarrow 0} \frac{y^2(5y+8)}{y^2(3y^2-16)} \\ &= \lim_{y \rightarrow 0} \frac{5y+8}{3y^2-16} \quad \dots \left[\begin{array}{l} \because y \rightarrow 0, y \neq 0, \\ \therefore y^2 \neq 0 \end{array} \right] \end{aligned}$$

$$\begin{aligned} &= \lim_{y \rightarrow 0} \frac{(5y + 8)}{(3y^2 - 16)} \\ &= \frac{5(0) + 8}{3(0)^2 - 16} \\ &= \frac{8}{-16} \\ &= -\frac{1}{2} \end{aligned}$$

Question 4.

$$\lim_{x \rightarrow -2} \left[\frac{-2x - 4}{x^3 + 2x^2} \right]$$

Solution:

$$\begin{aligned} &\lim_{x \rightarrow -2} \left[\frac{-2x - 4}{x^3 + 2x^2} \right] \\ &= \lim_{x \rightarrow -2} \frac{-2(x + 2)}{x^2(x + 2)} \\ &= \lim_{x \rightarrow -2} \frac{-2}{x^2} \quad \dots \left[\begin{array}{l} \because x \rightarrow -2, x \neq -2, \\ \therefore x + 2 \neq 0 \end{array} \right] \\ &= \frac{\lim_{x \rightarrow -2} (-2)}{\lim_{x \rightarrow -2} (x^2)} \\ &= \frac{(-2)}{(-2)^2} \end{aligned}$$

Question 5.

$$\lim_{x \rightarrow 3} \left[\frac{x^2 + 2x - 15}{x^2 - 5x + 6} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 3} \left[\frac{x^2 + 2x - 15}{x^2 - 5x + 6} \right] \\ &= \lim_{x \rightarrow 3} \frac{(x+5)(x-3)}{(x-2)(x-3)} \\ &= \lim_{x \rightarrow 3} \frac{x+5}{x-2} \quad \dots \left[\begin{array}{l} \because x \rightarrow 3, x \neq 3, \\ \therefore x-3 \neq 0 \end{array} \right] \\ &= \frac{\lim_{x \rightarrow 3} (x+5)}{\lim_{x \rightarrow 3} (x-2)} = \frac{3+5}{3-2} \\ &= 8 \end{aligned}$$

II. Evaluate the following limits:

Question 1.

$$\lim_{u \rightarrow 1} \left[\frac{u^4 - 1}{u^3 - 1} \right]$$

Solution:

$$\begin{aligned} & \lim_{u \rightarrow 1} \left[\frac{u^4 - 1}{u^3 - 1} \right] \\ &= \lim_{u \rightarrow 1} \frac{u^4 - 1^4}{u^3 - 1^3} \quad \dots \left[\begin{array}{l} \because u \rightarrow 1, u \neq 1, \\ \therefore u-1 \neq 0 \end{array} \right] \end{aligned}$$

$$\begin{aligned} &= \frac{\lim_{u \rightarrow 1} \left(\frac{u^4 - 1^4}{u - 1} \right)}{\lim_{u \rightarrow 1} \left(\frac{u^3 - 1^3}{u - 1} \right)} \\ &= \frac{4(1)^3}{3(1)^2} \quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right] \\ &= \frac{4}{3} \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow 3} \left[\frac{1}{x-3} - \frac{9x}{x^3-27} \right]$$

Solution:

$$\begin{aligned} &\lim_{x \rightarrow 3} \left[\frac{1}{x-3} - \frac{9x}{x^3-27} \right] \\ &= \lim_{x \rightarrow 3} \left[\frac{1}{x-3} - \frac{9x}{x^3-3^3} \right] \\ &= \lim_{x \rightarrow 3} \left[\frac{1}{x-3} - \frac{9x}{(x-3)(x^2+3x+9)} \right] \\ &= \lim_{x \rightarrow 3} \left[\frac{x^2+3x+9-9x}{(x-3)(x^2+3x+9)} \right] \\ &= \lim_{x \rightarrow 3} \left[\frac{x^2-6x+9}{(x-3)(x^2+3x+9)} \right] \end{aligned}$$

$$\begin{aligned} &= \lim_{x \rightarrow 3} \left[\frac{(x-3)^2}{(x-3)(x^2+3x+9)} \right] \\ &= \lim_{x \rightarrow 3} \left[\frac{x-3}{x^2+3x+9} \right] \quad \dots \left[\begin{array}{l} \because x \rightarrow 3, x \neq 3, \\ \therefore x-3 \neq 0 \end{array} \right] \\ &= \frac{\lim_{x \rightarrow 3} (x-3)}{\lim_{x \rightarrow 3} (x^2+3x+9)} \\ &= \frac{3-3}{(3)^2+3(3)+9} \\ &= \frac{0}{27} = 0 \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow 2} \left[\frac{x^3 - 4x^2 + 4x}{x^2 - 1} \right]$$

Solution:

$$\begin{aligned} &\lim_{x \rightarrow 2} \left[\frac{x^3 - 4x^2 + 4x}{x^2 - 1} \right] \\ &= \frac{\lim_{x \rightarrow 2} (x^3 - 4x^2 + 4x)}{\lim_{x \rightarrow 2} (x^2 - 1)} \end{aligned}$$

$$= \frac{2^3 - 4(2)^2 + 4(2)}{2^2 - 1}$$
$$= \frac{0}{3} = 0$$

Question 4.

$$\lim_{\Delta x \rightarrow 0} \left[\frac{(x + \Delta x)^2 - 2(x + \Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x} \right]$$

Solution:

$$\begin{aligned} & \lim_{\Delta x \rightarrow 0} \left[\frac{(x + \Delta x)^2 - 2(x + \Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x} \right] \\ &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x \cdot \Delta x + (\Delta x)^2 - 2x - 2\Delta x + 1 - x^2 + 2x - 1}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x \cdot \Delta x + (\Delta x)^2 - 2\Delta x}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(2x + \Delta x - 2)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} (2x + \Delta x - 2) \quad \dots [\because \Delta x \rightarrow 0, \Delta x \neq 0] \\ &= 2x + 0 - 2 \\ &= 2x - 2 \end{aligned}$$

Question 5.

$$\lim_{x \rightarrow \sqrt{2}} \left[\frac{x^2 + x\sqrt{2} - 4}{x^2 - 3x\sqrt{2} + 4} \right]$$

Solution:

$$\lim_{x \rightarrow \sqrt{2}} \left[\frac{x^2 + x\sqrt{2} - 4}{x^2 - 3x\sqrt{2} + 4} \right]$$

$$\text{Consider, } x^2 + x\sqrt{2} - 4 = x^2 + \sqrt{2}x - 4$$

$$= x^2 + 2\sqrt{2}x - \sqrt{2}x - 4$$

$$= x(x + 2\sqrt{2}) - \sqrt{2}(x + 2\sqrt{2})$$

$$= (x + 2\sqrt{2})(x - \sqrt{2})$$

$$x^2 - 3x\sqrt{2} + 4 = x^2 - 3\sqrt{2}x + 4$$

$$= x^2 - 2\sqrt{2}x - \sqrt{2}x + 4$$

$$= x(x - 2\sqrt{2}) - \sqrt{2}(x - 2\sqrt{2})$$

$$= (x - 2\sqrt{2})(x - \sqrt{2})$$

$$\text{Now, } \lim_{x \rightarrow \sqrt{2}} \left[\frac{x^2 + x\sqrt{2} - 4}{x^2 - 3x\sqrt{2} + 4} \right]$$

$$= \lim_{x \rightarrow \sqrt{2}} \frac{(x + 2\sqrt{2})(x - \sqrt{2})}{(x - 2\sqrt{2})(x - \sqrt{2})}$$

$$= \lim_{x \rightarrow \sqrt{2}} \frac{x + 2\sqrt{2}}{x - 2\sqrt{2}} \quad \dots \left[\begin{array}{l} \because x \rightarrow \sqrt{2}, x \neq \sqrt{2}, \\ \therefore x - \sqrt{2} \neq 0 \end{array} \right]$$

$$\begin{aligned}
 &= \frac{\lim_{x \rightarrow \sqrt{2}} (x + 2\sqrt{2})}{\lim_{x \rightarrow \sqrt{2}} (x - 2\sqrt{2})} \\
 &= \frac{\sqrt{2} + 2\sqrt{2}}{\sqrt{2} - 2\sqrt{2}} = \frac{3\sqrt{2}}{-\sqrt{2}} = -3
 \end{aligned}$$

Question 6.

$$\lim_{x \rightarrow 2} \left[\frac{x^3 - 7x + 6}{x^3 - 7x^2 + 16x - 12} \right]$$

Solution:

$$\lim_{x \rightarrow 2} \left[\frac{x^3 - 7x + 6}{x^3 - 7x^2 + 16x - 12} \right]$$

As $x \rightarrow 2$, numerator and denominator both tend to zero

$\therefore x - 2$ is a factor of both.

To find the other factor for both of them, by synthetic division

Consider, Numerator = $x^3 + 0x^2 - 7x + 6$

2	1	0	-7	6
	↓	2	4	-6
	1	2	-3	0

$$\therefore \text{Numerator} = (x - 2)(x^2 + 2x - 3)$$

$$\text{Now, denominator} = x^3 - 7x^2 + 16x - 12$$

$$\begin{array}{r|rrrr}
 2 & 1 & -7 & 16 & -12 \\
 & \downarrow & & & \\
 & 1 & -5 & 6 & 0
 \end{array}$$

$\therefore$ denominator = $(x-2)(x^2-5x+6)$

$$\begin{aligned}
 & \lim_{x \rightarrow 2} \left[\frac{x^3-7x+6}{x^3-7x^2+16x-12} \right] \\
 &= \lim_{x \rightarrow 2} \frac{(x-2)(x^2+2x-3)}{(x-2)(x^2-5x+6)} \\
 &= \lim_{x \rightarrow 2} \frac{x^2+2x-3}{x^2-5x+6} \quad \dots \left[\begin{array}{l} \because x \rightarrow 2, x \neq 2 \\ \therefore x-2 \neq 0 \end{array} \right] \\
 &= \frac{\lim_{x \rightarrow 2} (x^2+2x-3)}{\lim_{x \rightarrow 2} (x^2-5x+6)} \\
 &= \frac{(2)^2+2(2)-3}{(2)^2-5(2)+6} \\
 &= \frac{4+4-3}{4-10+6} = \frac{5}{0}
 \end{aligned}$$

$\therefore$ The limit does not exist

III. Evaluate the following limits:

Question 1.

$$\lim_{y \rightarrow \frac{1}{2}} \left[\frac{1-8y^3}{y-4y^3} \right]$$

Solution:

$$\begin{aligned} & \lim_{y \rightarrow \frac{1}{2}} \left[\frac{1-8y^3}{y-4y^3} \right] \\ &= \lim_{y \rightarrow \frac{1}{2}} \frac{1-8y^3}{y(1-4y^2)} \\ &= \lim_{y \rightarrow \frac{1}{2}} \frac{(1)^3 - (2y)^3}{y[(1)^2 - (2y)^2]} \\ &= \lim_{y \rightarrow \frac{1}{2}} \frac{(1-2y)(1+2y+4y^2)}{y(1-2y)(1+2y)} \\ &= \lim_{y \rightarrow \frac{1}{2}} \frac{1+2y+4y^2}{y(1+2y)} \quad \dots \left[\begin{array}{l} \because y \rightarrow \frac{1}{2}, y \neq \frac{1}{2} \\ \therefore 2y \neq 1, \therefore 2y-1 \neq 0 \\ \therefore 1-2y \neq 0 \end{array} \right] \\ &= \frac{\lim_{y \rightarrow \frac{1}{2}} (1+2y+4y^2)}{\lim_{y \rightarrow \frac{1}{2}} [y(1+2y)]} \\ &= \frac{1+2\left(\frac{1}{2}\right)+4\left(\frac{1}{2}\right)^2}{\frac{1}{2}\left[1+2\left(\frac{1}{2}\right)\right]} \end{aligned}$$

$$= \frac{1+1+1}{\frac{1}{2}(2)} = 3$$

Question 2.

$$\lim_{x \rightarrow 1} \left[\frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 1} \left[\frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x} \right] \\ &= \lim_{x \rightarrow 1} \left[\frac{x-2}{x^2-x} - \frac{1}{x(x^2-3x+2)} \right] \\ &= \lim_{x \rightarrow 1} \left[\frac{x-2}{x(x-1)} - \frac{1}{x(x-2)(x-1)} \right] \\ &= \lim_{x \rightarrow 1} \left[\frac{(x-2)^2-1}{x(x-2)(x-1)} \right] \\ &= \lim_{x \rightarrow 1} \frac{x^2-4x+4-1}{x(x-2)(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{x^2-4x+3}{x(x-2)(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{(x-3)(x-1)}{x(x-2)(x-1)} \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 1} \frac{x-3}{x(x-2)} \quad \dots \left[\begin{array}{l} \because x \rightarrow 1, x \neq 1, \\ \therefore x-1 \neq 0 \end{array} \right] \\
 &= \frac{\lim_{x \rightarrow 1} (x-3)}{\lim_{x \rightarrow 1} x(x-2)} \\
 &= \frac{1-3}{1(1-2)} \\
 &= \frac{-2}{-1} = 2
 \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow 1} \left[\frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1} \right]$$

Solution:

$$\lim_{x \rightarrow 1} \left[\frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1} \right]$$

As $x \rightarrow 1$, numerator and denominator both tend to zero

$\therefore x - 1$ is a factor of both.

To find the factor of numerator and denominator by synthetic division

Consider, numerator $= x^4 + 0x^3 - 3x^2 + 0x + 2$

1	1	0	-3	0	2
		1	1	-2	-2
	1	1	-2	-2	0

$$\therefore \text{numerator} = (x-1)(x^3 + x^2 - 2x - 2)$$

$$\text{Now, denominator} = x^3 - 5x^2 + 3x + 1$$

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 3 & 1 \\ & & 1 & -4 & -1 \\ \hline & 1 & -4 & -1 & \boxed{0} \end{array}$$

$$\therefore \text{denominator} = (x-1)(x^2 - 4x - 1)$$

$$\therefore \lim_{x \rightarrow 1} \frac{x^4 - 3x^2 + 2}{x^3 - 5x^2 + 3x + 1}$$

$$= \lim_{x \rightarrow 1} \frac{(x-1)(x^3 + x^2 - 2x - 2)}{(x-1)(x^2 - 4x - 1)}$$

$$= \lim_{x \rightarrow 1} \frac{x^3 + x^2 - 2x - 2}{x^2 - 4x - 1} \quad \dots \left[\begin{array}{l} \because x \rightarrow 1, x \neq 1 \\ \because x - 1 \neq 0 \end{array} \right]$$

$$= \frac{\lim_{x \rightarrow 1} (x^3 + x^2 - 2x - 2)}{\lim_{x \rightarrow 1} (x^2 - 4x - 1)}$$

$$= \frac{1^3 + 1^2 - 2(1) - 2}{1^2 - 4(1) - 1}$$

$$= \frac{1 + 1 - 2 - 2}{1 - 4 - 1} = \frac{-2}{-4}$$

$$= \frac{1}{2}$$

Question 4.

Question 4.

$$\lim_{x \rightarrow 1} \left[\frac{x+2}{x^2-5x+4} + \frac{x-4}{3(x^2-3x+2)} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 1} \left[\frac{x+2}{x^2-5x+4} + \frac{x-4}{3(x^2-3x+2)} \right] \\ &= \lim_{x \rightarrow 1} \left[\frac{x+2}{(x-4)(x-1)} + \frac{x-4}{3(x-2)(x-1)} \right] \\ &= \lim_{x \rightarrow 1} \frac{3(x-2)(x+2) + (x-4)^2}{3(x-4)(x-2)(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{3(x^2-4) + (x-4)^2}{3(x-4)(x-2)(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{3x^2 - 12 + x^2 - 8x + 16}{3(x-4)(x-2)(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{4x^2 - 8x + 4}{3(x-4)(x-2)(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{4(x^2 - 2x + 1)}{3(x-4)(x-2)(x-1)} \\ &= \lim_{x \rightarrow 1} \frac{4(x-1)^2}{3(x-4)(x-2)(x-1)} \\ & \quad \dots [\because a^2 - 2ab + b^2 = (a-b)^2] \\ &= \lim_{x \rightarrow 1} \frac{4(x-1)}{3(x-4)(x-2)} \quad \dots \begin{bmatrix} \because x \rightarrow 1, x \neq 1 \\ \therefore x-1 \neq 0 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} &= \frac{\lim_{x \rightarrow 1} 4(x-1)}{\lim_{x \rightarrow 1} 3(x-4)(x-2)} \\ &= \frac{4(1-1)}{3(1-4)(1-2)} = 0 \end{aligned}$$

Question 5.

$$\lim_{x \rightarrow a} \left[\frac{1}{x^2 - 3ax + 2a^2} + \frac{1}{2x^2 - 3ax + a^2} \right]$$

Solution:

$$\lim_{x \rightarrow a} \left[\frac{1}{x^2 - 3ax + 2a^2} + \frac{1}{2x^2 - 3ax + a^2} \right]$$

Consider,

$$\begin{aligned} x^2 - 3ax + 2a^2 &= x^2 - 2ax - ax + 2a^2 \\ &= x(x - 2a) - a(x - 2a) \\ &= (x - 2a)(x - a) \\ 2x^2 - 3ax + a^2 &= 2x^2 - 2ax - ax + a^2 \\ &= 2x(x - a) - a(x - a) \\ &= (x - a)(2x - a) \end{aligned}$$

$$\therefore \lim_{x \rightarrow a} \left[\frac{1}{x^2 - 3ax + 2a^2} + \frac{1}{2x^2 - 3ax + a^2} \right]$$

$$\begin{aligned}&= \lim_{x \rightarrow a} \left[\frac{1}{(x-2a)(x-a)} + \frac{1}{(x-a)(2x-a)} \right] \\&= \lim_{x \rightarrow a} \frac{(2x-a) + (x-2a)}{(x-2a)(x-a)(2x-a)} \\&= \lim_{x \rightarrow a} \frac{3x-3a}{(x-2a)(x-a)(2x-a)} \\&= \lim_{x \rightarrow a} \frac{3(x-a)}{(x-2a)(x-a)(2x-a)} \\&= \lim_{x \rightarrow a} \frac{3}{(x-2a)(2x-a)} \quad \dots \left[\begin{array}{l} \because x \rightarrow a, x \neq a \\ \therefore x-a \neq 0 \end{array} \right] \\&= \frac{3}{(a-2a)(2a-a)} \\&= \frac{3}{(-a)(a)} = \frac{-3}{a^2}\end{aligned}$$

Ex 7.3

Question 1.

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{6+x+x^2} - \sqrt{6}}{x} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \left[\frac{\sqrt{6+x+x^2} - \sqrt{6}}{x} \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{\sqrt{6+x+x^2} - \sqrt{6}}{x} \times \frac{\sqrt{6+x+x^2} + \sqrt{6}}{\sqrt{6+x+x^2} + \sqrt{6}} \right] \\ & \quad \dots [\text{By rationalization}] \\ &= \lim_{x \rightarrow 0} \frac{(6+x+x^2) - 6}{x(\sqrt{6+x+x^2} + \sqrt{6})} \\ &= \lim_{x \rightarrow 0} \frac{x+x^2}{x(\sqrt{6+x+x^2} + \sqrt{6})} \\ &= \lim_{x \rightarrow 0} \frac{x(1+x)}{x(\sqrt{6+x+x^2} + \sqrt{6})} \\ &= \lim_{x \rightarrow 0} \frac{1+x}{\sqrt{6+x+x^2} + \sqrt{6}} \quad \dots [\because x \rightarrow 0, x \neq 0] \\ &= \frac{\lim_{x \rightarrow 0} (1+x)}{\lim_{x \rightarrow 0} (\sqrt{6+x+x^2} + \sqrt{6})} \\ &= \frac{(1+0)}{\sqrt{6} + \sqrt{6}} \end{aligned}$$

$$= \frac{1}{2\sqrt{6}}$$

Question 2.

$$\lim_{x \rightarrow 3} \left[\frac{\sqrt{2x+3} - \sqrt{4x-3}}{x^2-9} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 3} \left[\frac{\sqrt{2x+3} - \sqrt{4x-3}}{x^2-9} \right] \\ &= \lim_{x \rightarrow 3} \left[\frac{\sqrt{2x+3} - \sqrt{4x-3}}{x^2-9} \times \frac{\sqrt{2x+3} + \sqrt{4x-3}}{\sqrt{2x+3} + \sqrt{4x-3}} \right] \end{aligned}$$

...[By rationalization]

$$= \lim_{x \rightarrow 3} \left[\frac{(2x+3) - (4x-3)}{(x^2-9)(\sqrt{2x+3} + \sqrt{4x-3})} \right]$$

$$= \lim_{x \rightarrow 3} \left[\frac{-2x+6}{(x^2-9)(\sqrt{2x+3} + \sqrt{4x-3})} \right]$$

$$= \lim_{x \rightarrow 3} \left[\frac{-2(x-3)}{(x+3)(x-3)(\sqrt{2x+3} + \sqrt{4x-3})} \right]$$

$$= \lim_{x \rightarrow 3} \left[\frac{-2}{(x+3)(\sqrt{2x+3} + \sqrt{4x-3})} \right]$$

$$\begin{aligned} & \dots \left[\begin{array}{l} \because x \rightarrow 3, x \neq 3 \\ \therefore x-3 \neq 0 \end{array} \right] \\ &= \frac{-2}{(3+3)(\sqrt{2(3)+3} + \sqrt{4(3)-3})} \\ &= \frac{-2}{6(3+3)} \\ &= \frac{-1}{18} \end{aligned}$$

Question 3.

$$\lim_{y \rightarrow 0} \left[\frac{\sqrt{1-y^2} - \sqrt{1+y^2}}{y^2} \right]$$

Solution:

$$\begin{aligned} & \lim_{y \rightarrow 0} \left[\frac{\sqrt{1-y^2} - \sqrt{1+y^2}}{y^2} \right] \\ &= \lim_{y \rightarrow 0} \left[\frac{\sqrt{1-y^2} - \sqrt{1+y^2}}{y^2} \times \frac{\sqrt{1-y^2} + \sqrt{1+y^2}}{\sqrt{1-y^2} + \sqrt{1+y^2}} \right] \\ & \dots [\text{By rationalization}] \end{aligned}$$

$$\begin{aligned}
 &= \lim_{y \rightarrow 0} \frac{(1-y^2)-(1+y^2)}{y^2(\sqrt{1-y^2}+\sqrt{1+y^2})} \\
 &= \lim_{y \rightarrow 0} \frac{1-y^2-1-y^2}{y^2(\sqrt{1-y^2}+\sqrt{1+y^2})} \\
 &= \lim_{y \rightarrow 0} \frac{-2y^2}{y^2(\sqrt{1-y^2}+\sqrt{1+y^2})} \\
 &= \lim_{y \rightarrow 0} \frac{-2}{\sqrt{1-y^2}+\sqrt{1+y^2}} \quad \dots \begin{bmatrix} \because y \rightarrow 0, y \neq 0, \\ \therefore y^2 \neq 0 \end{bmatrix} \\
 &= \frac{-2}{\sqrt{1-0^2}+\sqrt{1+0^2}} \\
 &= \frac{-2}{1+1} \\
 &= -1
 \end{aligned}$$

Question 4.

$$\lim_{x \rightarrow 2} \left[\frac{\sqrt{2+x}-\sqrt{6-x}}{\sqrt{x}-\sqrt{2}} \right]$$

Solution:

$$\lim_{x \rightarrow 2} \left(\frac{\sqrt{2+x}-\sqrt{6-x}}{\sqrt{x}-\sqrt{2}} \right)$$

$$= \lim_{x \rightarrow 2} \left[\frac{\sqrt{2+x} - \sqrt{6-x}}{\sqrt{x} - \sqrt{2}} \times \frac{\sqrt{2+x} + \sqrt{6-x}}{\sqrt{2+x} + \sqrt{6-x}} \times \frac{\sqrt{x} + \sqrt{2}}{\sqrt{x} + \sqrt{2}} \right]$$

... [By taking conjugates of both, the numerator as well as the denominator.]

$$= \lim_{x \rightarrow 2} \left[\frac{(2+x) - (6-x)}{(x-2)} \times \frac{\sqrt{x} + \sqrt{2}}{\sqrt{2+x} + \sqrt{6-x}} \right]$$

$$= \lim_{x \rightarrow 2} \left[\frac{-4+2x}{x-2} \times \frac{\sqrt{x} + \sqrt{2}}{\sqrt{2+x} + \sqrt{6-x}} \right]$$

$$= \lim_{x \rightarrow 2} \left[\frac{2(x-2)}{x-2} \times \frac{\sqrt{x} + \sqrt{2}}{\sqrt{2+x} + \sqrt{6-x}} \right]$$

$$= \lim_{x \rightarrow 2} \left[\frac{2(\sqrt{x} + \sqrt{2})}{\sqrt{2+x} + \sqrt{6-x}} \right]$$

... $\left[\begin{array}{l} \because x \rightarrow 2, x \neq 2, \\ \therefore x-2 \neq 0 \end{array} \right]$

$$= \frac{\lim_{x \rightarrow 2} 2(\sqrt{x} + \sqrt{2})}{\lim_{x \rightarrow 2} (\sqrt{2+x} + \sqrt{6-x})}$$

$$= \frac{2(\sqrt{2} + \sqrt{2})}{\sqrt{2+2} + \sqrt{6-2}}$$

$$= \frac{2(2\sqrt{2})}{2+2}$$

$$\begin{aligned} &= \frac{4\sqrt{2}}{4} \\ &= \sqrt{2} \end{aligned}$$

II. Evaluate the following limits:

Question 1.

$$\lim_{x \rightarrow a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

Solution:

$$\begin{aligned} &\lim_{x \rightarrow a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right] \\ &= \lim_{x \rightarrow a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right] \\ &\quad \dots \left[\begin{array}{l} \text{By taking conjugates of both, the} \\ \text{numerator as well as the denominator.} \end{array} \right] \\ &= \lim_{x \rightarrow a} \left[\frac{(a+2x) - 3x}{(3a+x) - 4x} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}} \right] \\ &= \lim_{x \rightarrow a} \left[\frac{a-x}{3a-3x} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}} \right] \\ &= \lim_{x \rightarrow a} \left[\frac{-(x-a)}{-3(x-a)} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{a+2x} + \sqrt{3x}} \right] \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow a} \left[\frac{\sqrt{3a+x} + 2\sqrt{x}}{3(\sqrt{a+2x} + \sqrt{3x})} \right] \quad \dots \left[\begin{array}{l} \because x \rightarrow a, x \neq a \\ \therefore x - a \neq 0 \end{array} \right] \\
 &= \frac{\sqrt{3a+a} + 2\sqrt{a}}{3(\sqrt{a+2a} + \sqrt{3a})} \\
 &= \frac{\sqrt{4a} + 2\sqrt{a}}{3(\sqrt{3a} + \sqrt{3a})} \\
 &= \frac{2\sqrt{a} + 2\sqrt{a}}{3(2\sqrt{3a})} \\
 &= \frac{4\sqrt{a}}{6\sqrt{3}\sqrt{a}} \\
 &= \frac{2}{3\sqrt{3}}
 \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow 2} \left[\frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} \right]$$

Solution:

$$\begin{aligned}
 &\lim_{x \rightarrow 2} \left[\frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} \right] \\
 &= \lim_{x \rightarrow 2} \left[\frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} \times \frac{\sqrt{x+2} + \sqrt{3x-2}}{\sqrt{x+2} + \sqrt{3x-2}} \right] \\
 &\quad \dots [\text{By rationalization}]
 \end{aligned}$$

$$\begin{aligned}&= \lim_{x \rightarrow 2} \frac{(x^2 - 4)(\sqrt{x+2} + \sqrt{3x-2})}{(x+2) - (3x-2)} \\&= \lim_{x \rightarrow 2} \frac{(x^2 - 4)(\sqrt{x+2} + \sqrt{3x-2})}{-2x + 4} \\&= \lim_{x \rightarrow 2} \frac{(x+2)(x-2)(\sqrt{x+2} + \sqrt{3x-2})}{-2(x-2)} \\&= \lim_{x \rightarrow 2} \frac{(x+2)(\sqrt{x+2} + \sqrt{3x-2})}{-2} \\&\quad \dots \left[\begin{array}{l} \because x \rightarrow 2, x \neq 2 \\ \therefore x-2 \neq 0 \end{array} \right] \\&= \frac{(2+2)(\sqrt{2+2} + \sqrt{3(2)-2})}{-2} \\&= \frac{4(2+2)}{-2} \\&= \frac{16}{-2} \\&= -8\end{aligned}$$

Question 3.

$$\lim_{x \rightarrow 2} \left[\frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2} \right]$$

Solution:

$$\lim_{x \rightarrow 2} \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$$

$$= \lim_{x \rightarrow 2} \left[\frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2} \times \frac{\sqrt{1+\sqrt{2+x}} + \sqrt{3}}{\sqrt{1+\sqrt{2+x}} + \sqrt{3}} \right]$$

...[By rationalization]

$$= \lim_{x \rightarrow 2} \frac{1 + \sqrt{2+x} - 3}{(x-2)(\sqrt{1+\sqrt{2+x}} + \sqrt{3})}$$

$$= \lim_{x \rightarrow 2} \frac{\sqrt{2+x} - 2}{(x-2)(\sqrt{1+\sqrt{2+x}} + \sqrt{3})} \times \frac{\sqrt{2+x} + 2}{\sqrt{2+x} + 2}$$

...[By rationalization]

$$= \lim_{x \rightarrow 2} \frac{2+x-4}{(x-2)(\sqrt{1+\sqrt{2+x}} + \sqrt{3})(\sqrt{2+x} + 2)}$$

$$= \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(\sqrt{1+\sqrt{2+x}} + \sqrt{3})(\sqrt{2+x} + 2)}$$

$$= \lim_{x \rightarrow 2} \frac{1}{(\sqrt{1+\sqrt{2+x}} + \sqrt{3})(\sqrt{2+x} + 2)}$$

...[$\because x \rightarrow 2, x \neq 2, \therefore x-2 \neq 0$]

$$\begin{aligned} &= \frac{1}{(\sqrt{1+\sqrt{2+2}}+\sqrt{3})(\sqrt{2+2}+2)} \\ &= \frac{1}{(\sqrt{1+2}+\sqrt{3})(2+2)} \\ &= \frac{1}{(2\sqrt{3})(4)} \\ &= \frac{1}{8\sqrt{3}} \end{aligned}$$

Question 4.

$$\lim_{y \rightarrow 0} \left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \right]$$

Solution:

$$\begin{aligned} &\lim_{y \rightarrow 0} \left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \right] \\ &\lim_{y \rightarrow 0} \left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}} \right] \\ &\quad \dots [\text{By rationalization}] \\ &= \lim_{y \rightarrow 0} \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})} \end{aligned}$$

$$\begin{aligned}
 &= \lim_{y \rightarrow 0} \frac{y}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})} \\
 &= \lim_{y \rightarrow 0} \frac{1}{\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})} \quad \dots \left[\begin{array}{l} \because y \rightarrow 0, \\ \therefore y \neq 0 \end{array} \right] \\
 &= \frac{1}{\sqrt{a+0}(\sqrt{a+0} + \sqrt{a})} \\
 &= \frac{1}{\sqrt{a}(2\sqrt{a})} \\
 &= \frac{1}{2a}
 \end{aligned}$$

Question 5.

$$\lim_{x \rightarrow 0} \left(\frac{\sqrt{x^2+9} - \sqrt{2x^2+9}}{\sqrt{3x^2+4} - \sqrt{2x^2+4}} \right)$$

Solution:

$$\begin{aligned}
 &\lim_{x \rightarrow 0} \left[\frac{\sqrt{x^2+9} - \sqrt{2x^2+9}}{\sqrt{3x^2+4} - \sqrt{2x^2+4}} \right] \\
 &= \lim_{x \rightarrow 0} \left[\frac{\sqrt{x^2+9} - \sqrt{2x^2+9}}{\sqrt{3x^2+4} - \sqrt{2x^2+4}} \times \frac{\sqrt{x^2+9} + \sqrt{2x^2+9}}{\sqrt{x^2+9} + \sqrt{2x^2+9}} \right. \\
 &\quad \left. \times \frac{\sqrt{3x^2+4} + \sqrt{2x^2+4}}{\sqrt{3x^2+4} + \sqrt{2x^2+4}} \right]
 \end{aligned}$$

$$\begin{aligned}
 & \dots \left[\begin{array}{l} \text{By taking conjugates of both, the} \\ \text{numerator as well as the denominator.} \end{array} \right] \\
 &= \lim_{x \rightarrow 0} \left[\frac{(x^2 + 9) - (2x^2 + 9)}{(3x^2 + 4) - (2x^2 + 4)} \times \frac{\sqrt{3x^2 + 4} + \sqrt{2x^2 + 4}}{\sqrt{x^2 + 9} + \sqrt{2x^2 + 9}} \right] \\
 &= \lim_{x \rightarrow 0} \left[\frac{-x^2(\sqrt{3x^2 + 4} + \sqrt{2x^2 + 4})}{x^2(\sqrt{x^2 + 9} + \sqrt{2x^2 + 9})} \right] \\
 &= -\lim_{x \rightarrow 0} \left[\frac{\sqrt{3x^2 + 4} + \sqrt{2x^2 + 4}}{\sqrt{x^2 + 9} + \sqrt{2x^2 + 9}} \right] \\
 & \dots \left[\begin{array}{l} \because x \rightarrow 0, x \neq 0 \\ \therefore x^2 \neq 0 \end{array} \right] \\
 &= \frac{-\lim_{x \rightarrow 0} (\sqrt{3x^2 + 4} + \sqrt{2x^2 + 4})}{\lim_{x \rightarrow 0} (\sqrt{x^2 + 9} + \sqrt{2x^2 + 9})} \\
 &= \frac{-(\sqrt{0 + 4} + \sqrt{0 + 4})}{\sqrt{0 + 9} + \sqrt{0 + 9}} \\
 &= \frac{-(2 + 2)}{3 + 3} \\
 &= \frac{-2}{3}
 \end{aligned}$$

III. Evaluate the following limits:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-7-limits/>

Question 1.

$$\lim_{x \rightarrow 1} \left[\frac{x^2 + x\sqrt{x} - 2}{x - 1} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 1} \left[\frac{x^2 + x\sqrt{x} - 2}{x - 1} \right] \\ &= \lim_{x \rightarrow 1} \left[\frac{(x^2 - 1) + (x\sqrt{x} - 1)}{x - 1} \right] \\ &= \lim_{x \rightarrow 1} \left[\frac{x^2 - 1}{x - 1} + \frac{x^{\frac{3}{2}} - 1}{x - 1} \right] \quad \dots \left[\begin{array}{l} \because x\sqrt{x} = x^1 \cdot x^{\frac{1}{2}} \\ = x^{1 + \frac{1}{2}} = x^{\frac{3}{2}} \end{array} \right] \\ &= \lim_{x \rightarrow 1} \left(\frac{x^2 - 1^2}{x - 1} \right) + \lim_{x \rightarrow 1} \left(\frac{x^{\frac{3}{2}} - 1^{\frac{3}{2}}}{x - 1} \right) \\ &= 2(1)^1 + \frac{3}{2}(1)^{\frac{1}{2}} \quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right] \\ &= 2 + \frac{3}{2} = \frac{7}{2} \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow 0} \left[\frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1+x}}{\sqrt{1+x^3} - \sqrt{1+x}} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{1+x^2} - \sqrt{1+x})}{(\sqrt{1+x^3} - \sqrt{1+x})} \times \frac{(\sqrt{1+x^2} + \sqrt{1+x})}{(\sqrt{1+x^3} + \sqrt{1+x})} \\ & \quad \times \frac{(\sqrt{1+x^3} + \sqrt{1+x})}{(\sqrt{1+x^2} + \sqrt{1+x})} \end{aligned}$$

... [By taking conjugates of both, the numerator as well as the denominator.]

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{[1+x^2 - (1+x)](\sqrt{1+x^3} + \sqrt{1+x})}{[1+x^3 - (1+x)](\sqrt{1+x^2} + \sqrt{1+x})} \\ &= \lim_{x \rightarrow 0} \frac{(x^2 - x)(\sqrt{1+x^3} + \sqrt{1+x})}{(x^3 - x)(\sqrt{1+x^2} + \sqrt{1+x})} \\ &= \lim_{x \rightarrow 0} \frac{x(x-1)(\sqrt{1+x^3} + \sqrt{1+x})}{x(x^2 - 1)(\sqrt{1+x^2} + \sqrt{1+x})} \\ &= \lim_{x \rightarrow 0} \frac{x(x-1)(\sqrt{1+x^3} + \sqrt{1+x})}{x(x-1)(x+1)(\sqrt{1+x^2} + \sqrt{1+x})} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{1+x^3} + \sqrt{1+x}}{(x+1)(\sqrt{1+x^2} + \sqrt{1+x})} \dots [\because x \rightarrow 0, x \neq 0] \end{aligned}$$

$$\begin{aligned} &= \frac{\lim_{x \rightarrow 0} (\sqrt{1+x^3} + \sqrt{1+x})}{\lim_{x \rightarrow 0} (x+1) \cdot \lim_{x \rightarrow 0} (\sqrt{1+x^2} + \sqrt{1+x})} \\ &= \frac{\sqrt{1+0^3} + \sqrt{1+0}}{(0+1)(\sqrt{1+0^2} + \sqrt{1+0})} \\ &= \frac{1+1}{1(1+1)} = 1 \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow 4} \left[\frac{x^2 + x - 20}{\sqrt{3x+4} - 4} \right]$$

Solution:

$$\begin{aligned} &\lim_{x \rightarrow 4} \left[\frac{x^2 + x - 20}{\sqrt{3x+4} - 4} \right] \\ &= \lim_{x \rightarrow 4} \left[\frac{(x+5)(x-4)}{\sqrt{3x+4} - 4} \times \frac{\sqrt{3x+4} + 4}{\sqrt{3x+4} + 4} \right] \\ &\quad \dots [\text{By rationalization}] \\ &= \lim_{x \rightarrow 4} \frac{(x+5)(x-4)(\sqrt{3x+4} + 4)}{(3x+4) - 16} \\ &= \lim_{x \rightarrow 4} \frac{(x+5)(x-4)(\sqrt{3x+4} + 4)}{3x - 12} \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 4} \frac{(x+5)(x-4)(\sqrt{3x+4}+4)}{3(x-4)} \\
 &= \lim_{x \rightarrow 4} \frac{(x+5)(\sqrt{3x+4}+4)}{3} \quad \dots \left[\begin{array}{l} \because x \rightarrow 4, x \neq 4 \\ \therefore x-4 \neq 0 \end{array} \right] \\
 &= \frac{(4+5)(\sqrt{3(4)+4}+4)}{3} \\
 &= \frac{9(4+4)}{3} \\
 &= 24
 \end{aligned}$$

Question 4.

$$\lim_{z \rightarrow 4} \left[\frac{3 - \sqrt{5+z}}{1 - \sqrt{5-z}} \right]$$

Solution:

$$\begin{aligned}
 &\lim_{z \rightarrow 4} \left[\frac{3 - \sqrt{5+z}}{1 - \sqrt{5-z}} \right] \\
 &= \lim_{z \rightarrow 4} \left[\frac{3 - \sqrt{5+z}}{1 - \sqrt{5-z}} \times \frac{3 + \sqrt{5+z}}{3 + \sqrt{5+z}} \times \frac{1 + \sqrt{5-z}}{1 + \sqrt{5-z}} \right] \\
 &\quad \dots \left[\begin{array}{l} \text{By taking conjugates of both, the} \\ \text{numerator as well as the denominator.} \end{array} \right] \\
 &= \lim_{z \rightarrow 4} \left[\frac{9 - (5+z)}{1 - (5-z)} \times \frac{1 + \sqrt{5-z}}{3 + \sqrt{5+z}} \right]
 \end{aligned}$$

$$\begin{aligned} &= \lim_{z \rightarrow 4} \left[\frac{4-z}{-4+z} \times \frac{1+\sqrt{5-z}}{3+\sqrt{5+z}} \right] \\ &= \lim_{z \rightarrow 4} \left[\frac{-(z-4)}{z-4} \times \frac{1+\sqrt{5-z}}{3+\sqrt{5+z}} \right] \\ &= \lim_{z \rightarrow 4} \left[\frac{-(1+\sqrt{5-z})}{3+\sqrt{5+z}} \right] \quad \dots \left[\begin{array}{l} \because z \rightarrow 4, z \neq 4, \\ \therefore z-4 \neq 0 \end{array} \right] \\ &= \frac{-\lim_{z \rightarrow 4} (1+\sqrt{5-z})}{\lim_{z \rightarrow 4} (3+\sqrt{5+z})} \\ &= \frac{-(1+\sqrt{5-4})}{3+\sqrt{5+4}} \\ &= \frac{-(1+1)}{3+3} \\ &= \frac{-2}{6} \\ &= -\frac{1}{3} \end{aligned}$$

Question 5.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-7-limits/>

$$\lim_{x \rightarrow 0} \left(\frac{3}{x\sqrt{9-x}} - \frac{1}{x} \right)$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \left(\frac{3}{x\sqrt{9-x}} - \frac{1}{x} \right) \\ &= \lim_{x \rightarrow 0} \left[\frac{3 - \sqrt{9-x}}{x\sqrt{9-x}} \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{3 - \sqrt{9-x}}{x\sqrt{9-x}} \times \frac{3 + \sqrt{9-x}}{3 + \sqrt{9-x}} \right] \\ & \quad \dots [\text{By rationalization}] \\ &= \lim_{x \rightarrow 0} \left[\frac{9 - (9-x)}{x\sqrt{9-x}(3 + \sqrt{9-x})} \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{x}{x\sqrt{9-x}(3 + \sqrt{9-x})} \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{1}{\sqrt{9-x}(3 + \sqrt{9-x})} \right] \dots [\because x \rightarrow 0, x \neq 0] \\ &= \frac{1}{\sqrt{9-0}(3 + \sqrt{9-0})} \\ &= \frac{1}{3(3+3)} \\ &= \frac{1}{18} \end{aligned}$$

Ex 7.4

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-7-limits/>

I. Evaluate the following limits:

Question 1.

$$\lim_{\theta \rightarrow 0} \left[\frac{\sin(m\theta)}{\tan(n\theta)} \right]$$

Solution:

$$\begin{aligned} & \lim_{\theta \rightarrow 0} \left[\frac{\sin(m\theta)}{\tan(n\theta)} \right] \\ &= \lim_{\theta \rightarrow 0} \left[\frac{\frac{\sin(m\theta)}{\theta}}{\frac{\tan(n\theta)}{\theta}} \right] \\ & \quad \dots \left[\begin{array}{l} \text{Divide Numerator and Denominator by } \theta \\ \because \theta \rightarrow 0, \theta \neq 0 \end{array} \right] \\ &= \frac{\lim_{\theta \rightarrow 0} \left[\frac{\sin(m\theta)}{\theta} \right]}{\lim_{\theta \rightarrow 0} \left[\frac{\tan(n\theta)}{\theta} \right]} \\ &= \frac{\lim_{\theta \rightarrow 0} \left[\frac{\sin(m\theta)}{m\theta} \times m \right]}{\lim_{\theta \rightarrow 0} \left[\frac{\tan(n\theta)}{n\theta} \times n \right]} \\ &= \frac{m}{n} \frac{\lim_{\theta \rightarrow 0} \frac{\sin(m\theta)}{m\theta}}{\lim_{\theta \rightarrow 0} \frac{\tan(n\theta)}{n\theta}} \end{aligned}$$

$$= \frac{m}{n} \frac{(1)}{(1)} \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin p\theta}{p\theta} = 1, \lim_{\theta \rightarrow 0} \frac{\tan p\theta}{p\theta} = 1 \right]$$
$$= \frac{m}{n}$$

Question 2.

$$\lim_{\theta \rightarrow 0} \left[\frac{1 - \cos 2\theta}{\theta^2} \right]$$

Solution:

$$\begin{aligned} \lim_{\theta \rightarrow 0} \frac{1 - \cos 2\theta}{\theta^2} &= \lim_{\theta \rightarrow 0} \frac{2\sin^2 \theta}{\theta^2} \\ &= 2 \lim_{\theta \rightarrow 0} \left(\frac{\sin \theta}{\theta} \right)^2 = 2(1)^2 = 2 \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow 0} \left[\frac{x \cdot \tan x}{1 - \cos x} \right]$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x \cdot \tan x}{1 - \cos x} \\ = \lim_{x \rightarrow 0} \left(\frac{x \cdot \tan x}{1 - \cos x} \times \frac{1 + \cos x}{1 + \cos x} \right) \end{aligned}$$

$$= \lim_{x \rightarrow 0} \frac{x \cdot \tan x (1 + \cos x)}{1 - \cos^2 x}$$

$$= \lim_{x \rightarrow 0} \frac{x \cdot \tan x (1 + \cos x)}{\sin^2 x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{x \cdot \tan x}{x^2} (1 + \cos x)}{\frac{\sin^2 x}{x^2}}$$

... $\left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x^2 \\ \because x \rightarrow 0, x \neq 0 \therefore x^2 \neq 0 \end{array} \right]$

$$= \frac{\lim_{x \rightarrow 0} \left[\frac{\tan x}{x} (1 + \cos x) \right]}{\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2}$$

$$= \frac{\lim_{x \rightarrow 0} \frac{\tan x}{x} \times \lim_{x \rightarrow 0} (1 + \cos x)}{\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2}$$

$$= \frac{1 \times (1 + 1)}{(1)^2} \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1, \lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1 \right]$$

$$= 2$$

Question 4.

Question 4.

$$\lim_{x \rightarrow 0} \left(\frac{\sec x - 1}{x^2} \right)$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\sec x - 1}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{(\sec x - 1)(\sec x + 1)}{x^2(\sec x + 1)} \\ &= \lim_{x \rightarrow 0} \frac{\sec^2 x - 1}{x^2(\sec x + 1)} = \lim_{x \rightarrow 0} \frac{\tan^2 x}{x^2(\sec x + 1)} \\ &= \lim_{x \rightarrow 0} \left[\left(\frac{\tan x}{x} \right)^2 \times \frac{1}{\sec x + 1} \right] \\ &= \lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^2 \times \lim_{x \rightarrow 0} \frac{1}{\sec x + 1} \\ &= (1)^2 \times \frac{1}{1+1} \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1 \right] \\ &= \frac{1}{2} \end{aligned}$$

II. Evaluate the following limits:

Question 1.

$$\lim_{x \rightarrow 0} \left[\frac{1 - \cos(nx)}{1 - \cos(mx)} \right]$$

Solution:

$$\begin{aligned}
 \lim_{x \rightarrow 0} \left[\frac{1 - \cos(nx)}{1 - \cos(mx)} \right] &= \lim_{x \rightarrow 0} \left[\frac{2\sin^2\left(\frac{nx}{2}\right)}{2\sin^2\left(\frac{mx}{2}\right)} \right] \\
 &= \lim_{x \rightarrow 0} \left[\frac{\left(\sin \frac{nx}{2}\right)^2}{x^2} \cdot \frac{x^2}{\left(\sin \frac{mx}{2}\right)^2} \right] \\
 &\quad \dots \left[\begin{array}{l} \text{Divide numerator and denominator by } x^2 \\ \because x \rightarrow 0, x \neq 0 \therefore x^2 \neq 0 \end{array} \right] \\
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{\sin \frac{nx}{2}}{x} \right)^2}{\lim_{x \rightarrow 0} \left(\frac{\sin \frac{mx}{2}}{x} \right)^2} \\
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{\sin \frac{nx}{2}}{\frac{nx}{2}} \right)^2 \times \left(\frac{n}{2} \right)^2}{\lim_{x \rightarrow 0} \left(\frac{\sin \frac{mx}{2}}{\frac{mx}{2}} \right)^2 \times \left(\frac{m}{2} \right)^2}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(1)^2 \times \frac{n^2}{4}}{(1)^2 \times \frac{m^2}{4}} \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin p\theta}{p\theta} = 1 \right] \\
 &= \frac{n^2}{m^2}
 \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{2 - \operatorname{cosec} x}{\cot^2 x - 3} \right]$$

Solution:

$$\begin{aligned}
 \lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{2 - \operatorname{cosec} x}{\cot^2 x - 3} \right] &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 - \operatorname{cosec} x}{\operatorname{cosec}^2 x - 1 - 3} \\
 &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 - \operatorname{cosec} x}{\operatorname{cosec}^2 x - 4} \\
 &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{-(\operatorname{cosec} x - 2)}{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)} \\
 &= \lim_{x \rightarrow \frac{\pi}{6}} \frac{-1}{(\operatorname{cosec} x + 2)} \\
 &\quad \dots \left[\because x \rightarrow \frac{\pi}{6}, x \neq \frac{\pi}{6}, \therefore \operatorname{cosec} x \neq \operatorname{cosec} \frac{\pi}{6}, \right. \\
 &\quad \left. \therefore \operatorname{cosec} x \neq 2, \therefore \operatorname{cosec} x - 2 \neq 0 \right]
 \end{aligned}$$

$$\begin{aligned} &= \frac{-1}{\operatorname{cosec}\left(\frac{\pi}{6}\right) + 2} \\ &= \frac{-1}{2+2} = \frac{-1}{4} \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow \frac{\pi}{4}} \left[\frac{\cos x - \sin x}{\cos 2x} \right]$$

Solution:

$$\begin{aligned} &\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos 2x} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos^2 x - \sin^2 x} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{(\cos x + \sin x)(\cos x - \sin x)} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{1}{\cos x + \sin x} \\ &\quad \dots \left[\begin{array}{l} \because x \rightarrow \frac{\pi}{4}, \cos x \rightarrow \frac{1}{\sqrt{2}} \text{ and } \sin x \rightarrow \frac{1}{\sqrt{2}} \\ \therefore \cos x - \sin x \rightarrow 0, \therefore \cos x - \sin x \neq 0 \end{array} \right] \end{aligned}$$

$$\begin{aligned} &= \frac{1}{\cos \frac{\pi}{4} + \sin \frac{\pi}{4}} \\ &= \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \frac{1}{\frac{2}{\sqrt{2}}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \end{aligned}$$

III. Evaluate the following limits:

Question 1.

$$\lim_{x \rightarrow 0} \left[\frac{\cos(ax) - \cos(bx)}{\cos(cx) - 1} \right]$$

Solution:

$$\begin{aligned} &\lim_{x \rightarrow 0} \frac{\cos(ax) - \cos(bx)}{\cos(cx) - 1} \\ &= \lim_{x \rightarrow 0} \frac{-\cos(ax) + \cos(bx)}{1 - \cos(cx)} \\ &= \lim_{x \rightarrow 0} \frac{1 - \cos ax - 1 + \cos bx}{1 - \cos cx} \\ &= \lim_{x \rightarrow 0} \frac{(1 - \cos ax) - (1 - \cos bx)}{1 - \cos cx} \\ &= \lim_{x \rightarrow 0} \frac{2\sin^2\left(\frac{ax}{2}\right) - 2\sin^2\frac{bx}{2}}{2\sin^2\frac{cx}{2}} \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{\frac{\sin^2\left(\frac{ax}{2}\right) - \sin^2\left(\frac{bx}{2}\right)}{x^2}}{\frac{\sin^2\left(\frac{cx}{2}\right)}{x^2}} \\
 &\quad \dots \left[\begin{array}{l} \text{Divide numerator and denominator by } x^2 \\ \because x \rightarrow 0, x \neq 0, \therefore x^2 \neq 0 \end{array} \right] \\
 &= \lim_{x \rightarrow 0} \left[\frac{\sin^2\left(\frac{ax}{2}\right)}{x^2} - \frac{\sin^2\left(\frac{bx}{2}\right)}{x^2} \right] \\
 &= \frac{\lim_{x \rightarrow 0} \left[\frac{\sin^2\left(\frac{ax}{2}\right)}{x^2} - \frac{\sin^2\left(\frac{bx}{2}\right)}{x^2} \right]}{\lim_{x \rightarrow 0} \frac{\sin^2\left(\frac{cx}{2}\right)}{x^2}} \\
 &= \frac{\lim_{x \rightarrow 0} \left[\left(\frac{\sin \frac{ax}{2}}{x} \right)^2 - \left(\frac{\sin \frac{bx}{2}}{x} \right)^2 \right]}{\lim_{x \rightarrow 0} \left(\frac{\sin \frac{cx}{2}}{x} \right)^2}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{\sin \frac{ax}{2}}{\frac{ax}{2}} \right)^2 \cdot \left(\frac{a}{2} \right)^2 - \lim_{x \rightarrow 0} \left(\frac{\sin \frac{bx}{2}}{\frac{bx}{2}} \right)^2 \cdot \left(\frac{b}{2} \right)^2}{\lim_{x \rightarrow 0} \left(\frac{\sin \frac{cx}{2}}{\frac{cx}{2}} \right)^2 \cdot \left(\frac{c}{2} \right)^2} \\
 &= \frac{(1)^2 \cdot \frac{a^2}{4} - (1)^2 \cdot \frac{b^2}{4}}{(1)^2 \cdot \frac{c^2}{4}} = \frac{\frac{a^2}{4} - \frac{b^2}{4}}{\frac{c^2}{4}} \\
 &= \frac{a^2 - b^2}{c^2}
 \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow \pi} \left[\frac{\sqrt{1 - \cos x} - \sqrt{2}}{\sin^2 x} \right]$$

Solution:

$$\begin{aligned}
 &\lim_{x \rightarrow \pi} \frac{\sqrt{1 - \cos x} - \sqrt{2}}{\sin^2 x} \\
 &= \lim_{x \rightarrow \pi} \frac{\sqrt{1 - \cos x} - \sqrt{2}}{\sin^2 x} \times \frac{\sqrt{1 - \cos x} + \sqrt{2}}{\sqrt{1 - \cos x} + \sqrt{2}} \\
 &\quad \dots [\text{By rationalization}] \\
 &= \lim_{x \rightarrow \pi} \frac{1 - \cos x - 2}{\sin^2 x} \times \frac{1}{\sqrt{1 - \cos x} + \sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow \pi} \frac{-\cos x - 1}{1 - \cos^2 x} \times \frac{1}{\sqrt{1 - \cos x} + \sqrt{2}} \\
 &= \lim_{x \rightarrow \pi} \frac{-(1 + \cos x)}{(1 - \cos x)(1 + \cos x)} \times \frac{1}{\sqrt{1 - \cos x} + \sqrt{2}} \\
 &= \lim_{x \rightarrow \pi} \frac{-1}{(1 - \cos x)[\sqrt{1 - \cos x} + \sqrt{2}]} \\
 &\quad \dots \left[\begin{array}{l} \because x \rightarrow \pi, \cos x \neq \cos \pi \\ \therefore \cos x \neq -1, \therefore 1 + \cos x \neq 0 \end{array} \right] \\
 &= \frac{-1}{\lim_{x \rightarrow \pi} (1 - \cos x) \cdot \lim_{x \rightarrow \pi} (\sqrt{1 - \cos x} + \sqrt{2})} \\
 &= \frac{-1}{(1 - \cos \pi)[\sqrt{1 - \cos \pi} + \sqrt{2}]} \\
 &= \frac{-1}{[1 - (-1)][\sqrt{1 - (-1)} + \sqrt{2}]} \\
 &= \frac{-1}{(2)(2\sqrt{2})} = \frac{-1}{4\sqrt{2}}
 \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow \frac{\pi}{4}} \left[\frac{\tan^2 x - \cot^2 x}{\sec x - \operatorname{cosec} x} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan^2 x - \cot^2 x}{\sec x - \operatorname{cosec} x} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\sec^2 x - 1) - (\operatorname{cosec}^2 x - 1)}{\sec x - \operatorname{cosec} x} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec^2 x - \operatorname{cosec}^2 x}{\sec x - \operatorname{cosec} x} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\sec x - \operatorname{cosec} x)(\sec x + \operatorname{cosec} x)}{(\sec x - \operatorname{cosec} x)} \\ &= \lim_{x \rightarrow \frac{\pi}{4}} (\sec x + \operatorname{cosec} x) \\ & \quad \dots \left[\begin{array}{l} \because x \rightarrow \frac{\pi}{4}, \sec x \rightarrow \sqrt{2} \text{ and } \operatorname{cosec} x \rightarrow \sqrt{2} \\ \therefore \sec x - \operatorname{cosec} x \rightarrow 0, \therefore \sec x - \operatorname{cosec} x \neq 0 \end{array} \right] \\ &= \lim_{x \rightarrow \frac{\pi}{4}} (\sec x) + \lim_{x \rightarrow \frac{\pi}{4}} (\operatorname{cosec} x) \\ &= \sec \frac{\pi}{4} + \operatorname{cosec} \frac{\pi}{4} \\ &= \sqrt{2} + \sqrt{2} = 2\sqrt{2} \end{aligned}$$

Question 4.

$$\lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1} \right]$$

Solution:

$$\lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1} \right]$$

$$\begin{aligned} \text{Consider, } 2 \sin^2 x + \sin x - 1 \\ = 2 \sin^2 x + 2 \sin x - \sin x - 1 \\ = 2 \sin x (\sin x + 1) - 1 (\sin x + 1) \\ = (\sin x + 1)(2 \sin x - 1) \end{aligned}$$

Now,

$$\begin{aligned} 2 \sin^2 x - 3 \sin x + 1 &= 2 \sin^2 x - 2 \sin x - \sin x + 1 \\ &= 2 \sin x (\sin x - 1) - 1 (\sin x - 1) \\ &= (\sin x - 1)(2 \sin x - 1) \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{2 \sin^2 x + \sin x - 1}{2 \sin^2 x - 3 \sin x + 1} \right] \\ = \lim_{x \rightarrow \frac{\pi}{6}} \frac{(\sin x + 1)(2 \sin x - 1)}{(\sin x - 1)(2 \sin x - 1)} \end{aligned}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin x + 1}{\sin x - 1} \quad \dots \left[\begin{array}{l} \because x \rightarrow \frac{\pi}{6}, \sin x \rightarrow \frac{1}{2} \\ \therefore \sin x \neq \frac{1}{2} \therefore 2 \sin x \neq 1 \\ \therefore 2 \sin x - 1 \neq 0 \end{array} \right]$$

$$= \frac{\lim_{x \rightarrow \frac{\pi}{6}} (\sin x + 1)}{\lim_{x \rightarrow \frac{\pi}{6}} (\sin x - 1)}$$

$$\begin{aligned} x \rightarrow \frac{\pi}{6} \\ = \frac{\sin \frac{\pi}{6} + 1}{\sin \frac{\pi}{6} - 1} = \frac{\frac{1}{2} + 1}{\frac{1}{2} - 1} = \frac{\frac{3}{2}}{\frac{-1}{2}} = -3 \end{aligned}$$

Ex 7.5

I. Evaluate the following:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-7-limits/>

Question 1.

$$\lim_{x \rightarrow \frac{\pi}{2}} \left[\frac{\operatorname{cosec} x - 1}{\left(\frac{\pi}{2} - x\right)^2} \right]$$

Solution:

$$\text{Put } \frac{\pi}{2} - x = h,$$

$$\therefore x = \frac{\pi}{2} - h$$

$$\text{As } x \rightarrow \frac{\pi}{2}, h \rightarrow 0$$

$$\begin{aligned} \therefore \lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{cosec} x - 1}{\left(\frac{\pi}{2} - x\right)^2} &= \lim_{h \rightarrow 0} \frac{\operatorname{cosec}\left(\frac{\pi}{2} - h\right) - 1}{(h)^2} \\ &= \lim_{h \rightarrow 0} \frac{\operatorname{sech} - 1}{h^2} \quad \dots \left[\because \operatorname{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec \theta \right] \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{\cosh} - 1}{h^2} \\ &= \lim_{h \rightarrow 0} \frac{1 - \cosh}{h^2 \cdot \cosh} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{h}{2} \cdot \frac{1}{\cosh}}{h^2} \end{aligned}$$

$$\begin{aligned}
 &= 2 \lim_{h \rightarrow 0} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right)^2 \cdot \frac{1}{4} \cdot \frac{1}{\cosh} \\
 &= \frac{1}{2} \lim_{h \rightarrow 0} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \right)^2 \lim_{h \rightarrow 0} \frac{1}{\cosh} \\
 &= \frac{1}{2} (1)^2 \times \frac{1}{\cos 0} \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin p\theta}{p\theta} = 1 \right] \\
 &= \frac{1}{2} \times 1 \\
 &= \frac{1}{2}
 \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt[5]{x} - \sqrt[5]{a}}$$

Solution:

$$\begin{aligned}
 &\lim_{x \rightarrow a} \frac{\sin x - \sin a}{\sqrt[5]{x} - \sqrt[5]{a}} \\
 &= \lim_{x \rightarrow a} \frac{2 \cos \left(\frac{x+a}{2} \right) \cdot \sin \left(\frac{x-a}{2} \right)}{x^{\frac{1}{5}} - a^{\frac{1}{5}}}
 \end{aligned}$$

$$= \lim_{x \rightarrow a} \frac{2 \cos\left(\frac{x+a}{2}\right) \cdot \frac{\sin\left(\frac{x-a}{2}\right)}{x-a}}{\frac{x^{\frac{1}{5}} - a^{\frac{1}{5}}}{x-a}}$$

... $\left[\begin{array}{l} \text{Divide numerator and denominator by } x-a. \\ \because x \rightarrow a, x \neq a, \therefore x-a \neq 0 \end{array} \right]$

$$= \frac{\lim_{x \rightarrow a} \cos\left(\frac{x+a}{2}\right) \cdot \lim_{x \rightarrow a} \left[\frac{\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2}} \right]}{\lim_{x \rightarrow a} \frac{x^{\frac{1}{5}} - a^{\frac{1}{5}}}{x-a}}$$

$$= \frac{\cos\left(\frac{a+a}{2}\right) \times 1}{\frac{1}{5} \cdot a^{-\frac{4}{5}}}$$

$$\dots \left[\begin{array}{l} \because x \rightarrow a, x-a \rightarrow 0 \\ \therefore \frac{x-a}{2} \rightarrow 0; \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \\ \text{and } \lim_{x \rightarrow a} \frac{x^n - a^n}{x-a} = na^{n-1} \end{array} \right]$$

$$= 5a^{\frac{4}{5}} \cdot \cos a$$

Question 3.

$$\lim_{x \rightarrow \pi} \left[\frac{\sqrt{5 + \cos x} - 2}{(\pi - x)^2} \right]$$

Solution:

$$\lim_{x \rightarrow \pi} \left[\frac{\sqrt{5 + \cos x} - 2}{(\pi - x)^2} \right]$$

Put $\pi - x = h$

$$\therefore x = \pi - h$$

As $x \rightarrow \pi$, $h \rightarrow 0$

$$\therefore \lim_{x \rightarrow \pi} \left[\frac{\sqrt{5 + \cos x} - 2}{(\pi - x)^2} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\sqrt{5 + \cos(\pi - h)} - 2}{(h)^2} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\sqrt{5 - \cos h} - 2}{h^2} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\sqrt{5 - \cos h} - 2}{h^2} \times \frac{\sqrt{5 - \cos h} + 2}{\sqrt{5 - \cos h} + 2} \right]$$

...[By rationalization]

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{5 - \cosh h - 4}{h^2 (\sqrt{5 - \cosh h} + 2)} \\
 &= \lim_{h \rightarrow 0} \frac{1 - \cosh h}{h^2 (\sqrt{5 - \cosh h} + 2)} \\
 &= \lim_{h \rightarrow 0} \frac{1 - \cosh h}{h^2 (\sqrt{5 - \cosh h} + 2)} \times \frac{1 + \cosh h}{1 + \cosh h} \\
 &= \lim_{h \rightarrow 0} \frac{\sin^2 h}{h^2 (\sqrt{5 - \cosh h} + 2)(1 + \cosh h)} \\
 &= \lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \right)^2 \times \frac{1}{(\sqrt{5 - \cosh h} + 2)(1 + \cosh h)} \\
 &= \lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \right)^2 \times \lim_{h \rightarrow 0} \frac{1}{(\sqrt{5 - \cosh h} + 2)(1 + \cosh h)} \\
 &= (1)^2 \times \frac{1}{(\sqrt{5 - 1} + 2)(1 + 1)} \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\
 &= \frac{1}{(\sqrt{4} + 2)(2)} = \frac{1}{(2 + 2)(2)} \\
 &= \frac{1}{8}
 \end{aligned}$$

Question 4.

$$\lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{\cos x - \sqrt{3} \sin x}{\pi - 6x} \right]$$

Solution:

$$\lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{\cos x - \sqrt{3} \sin x}{\pi - 6x} \right]$$

$$\text{Put } \frac{\pi}{6} - x = h$$

$$\therefore x = \frac{\pi}{6} - h$$

$$\text{As } x \rightarrow \frac{\pi}{6}, h \rightarrow 0$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{6}} \frac{\cos x - \sqrt{3} \sin x}{\pi - 6x}$$

$$= \lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{6} - h\right) - \sqrt{3} \sin\left(\frac{\pi}{6} - h\right)}{\pi - 6\left(\frac{\pi}{6} - h\right)}$$

$$= \lim_{h \rightarrow 0} \frac{\left(\cos \frac{\pi}{6} \cdot \cos h + \sin \frac{\pi}{6} \cdot \sin h \right) - \sqrt{3} \left(\sin \frac{\pi}{6} \cdot \cos h - \cos \frac{\pi}{6} \cdot \sin h \right)}{\pi - \pi + 6h}$$

$$= \lim_{h \rightarrow 0} \frac{\left(\frac{\sqrt{3}}{2} \cos h + \frac{1}{2} \sin h \right) - \sqrt{3} \left(\frac{1}{2} \cos h - \frac{\sqrt{3}}{2} \sin h \right)}{6h}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{\frac{\sqrt{3}}{2} \cosh + \frac{1}{2} \sinh - \frac{\sqrt{3}}{2} \cosh + \frac{3}{2} \sinh}{6h} \\
 &= \lim_{h \rightarrow 0} \frac{2\sinh}{6h} \\
 &= \frac{1}{3} \lim_{h \rightarrow 0} \frac{\sinh}{h} \\
 &= \frac{1}{3} (1) \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\
 &= \frac{1}{3}
 \end{aligned}$$

Question 5.

$$\lim_{x \rightarrow 1} \left[\frac{1-x^2}{\sin \pi x} \right]$$

Solution:

$$\lim_{x \rightarrow 1} \frac{1-x^2}{\sin \pi x} = \lim_{x \rightarrow 1} \frac{(1-x)(1+x)}{\sin \pi x}$$

Put $1-x = h$,

$$\therefore x = 1-h$$

As $x \rightarrow 1$, $h \rightarrow 0$

$$\therefore \lim_{x \rightarrow 1} \frac{(1-x)(1+x)}{\sin \pi x}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{(h)(1+1-h)}{\sin \pi (1-h)} \\
 &= \lim_{h \rightarrow 0} \frac{(h)(2-h)}{\sin (\pi - \pi h)} \\
 &= \lim_{h \rightarrow 0} \frac{(h)(2-h)}{\sin \pi h} \quad \dots [\because \sin (\pi - \theta) = \sin \theta] \\
 &= \lim_{h \rightarrow 0} \frac{(2-h)}{\left(\frac{\sin \pi h}{\pi h} \right) \cdot \pi} \\
 &= \frac{1}{\pi} \cdot \frac{\lim_{h \rightarrow 0} (2-h)}{\lim_{h \rightarrow 0} \left(\frac{\sin \pi h}{\pi h} \right)} \\
 &= \frac{1}{\pi} \cdot \frac{(2-0)}{1} \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin p\theta}{p\theta} = 1 \right] \\
 &= \frac{2}{\pi}
 \end{aligned}$$

II. Evaluate the following:

Question 1.

$$\lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{2 \sin x - 1}{\pi - 6x} \right]$$

Solution:

$$\begin{aligned}
 & \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin x - 1}{\pi - 6x} \\
 & \text{Put } \frac{\pi}{6} - h = x \\
 \therefore & \quad h = \frac{\pi}{6} - x \\
 & \text{As } x \rightarrow \frac{\pi}{6}, h \rightarrow 0 \\
 \therefore & \quad \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin x - 1}{\pi - 6x} \\
 & = \lim_{h \rightarrow 0} \frac{2 \sin \left(\frac{\pi}{6} - h \right) - 1}{\pi - 6 \left(\frac{\pi}{6} - h \right)} \\
 & = \lim_{h \rightarrow 0} \frac{2 \left[\sin \left(\frac{\pi}{6} - h \right) - \frac{1}{2} \right]}{6h} \\
 & = \frac{1}{3} \lim_{h \rightarrow 0} \frac{\sin \left(\frac{\pi}{6} - h \right) - \sin \frac{\pi}{6}}{h} \\
 & = \frac{1}{3} \lim_{h \rightarrow 0} \frac{2 \cos \left(\frac{\frac{\pi}{6} - h + \frac{\pi}{6}}{2} \right) \cdot \sin \left(\frac{\frac{\pi}{6} - h - \frac{\pi}{6}}{2} \right)}{h}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{3} \lim_{h \rightarrow 0} \frac{2 \cos\left(\frac{\pi}{6} - \frac{h}{2}\right) \cdot \sin\left(\frac{-h}{2}\right)}{h} \\
 &= \frac{-1}{3} \lim_{h \rightarrow 0} \cos\left(\frac{\pi}{6} - \frac{h}{2}\right) \cdot \frac{\sin \frac{h}{2}}{\frac{h}{2}} \\
 &= \frac{-1}{3} \lim_{h \rightarrow 0} \cos\left(\frac{\pi}{6} - \frac{h}{2}\right) \cdot \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \\
 &= \frac{-1}{3} \cdot \cos\left(\frac{\pi}{6} - 0\right) \cdot (1) \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin p\theta}{p\theta} = 1 \right] \\
 &= \frac{-1}{3} \times \frac{\sqrt{3}}{2} = \frac{-1}{2\sqrt{3}}
 \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow \frac{\pi}{4}} \left[\frac{\sqrt{2} - \cos x - \sin x}{(4x - \pi)^2} \right]$$

Solution:

$$\lim_{x \rightarrow \frac{\pi}{4}} \left[\frac{\sqrt{2} - \cos x - \sin x}{(4x - \pi)^2} \right]$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \left[\frac{\sqrt{2} - (\cos x + \sin x)}{\left[4 \left(x - \frac{\pi}{4} \right) \right]^2} \right]$$

$$\text{Put } x - \frac{\pi}{4} = h$$

$$\therefore x = \frac{\pi}{4} + h$$

$$\text{As } x \rightarrow \frac{\pi}{4}, h \rightarrow 0$$

Also,

$$\begin{aligned} \cos x + \sin x &= \cos x + \cos \left(\frac{\pi}{2} - x \right) \\ &= \cos \left(\frac{\pi}{4} + h \right) + \cos \left[\frac{\pi}{2} - \left(\frac{\pi}{4} + h \right) \right] \\ &= \cos \left(\frac{\pi}{4} + h \right) + \cos \left(\frac{\pi}{4} - h \right) \\ &= 2 \cos \frac{\pi}{4} \cdot \cosh \quad \dots (\text{By defactorisation}) \\ &= 2 \left(\frac{1}{\sqrt{2}} \right) \cosh \\ &= \sqrt{2} \cosh \end{aligned}$$

∴ Required limit

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{\sqrt{2} - \sqrt{2} \cos h}{(4h)^2} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{2} (1 - \cos h)}{16h^2} \\ &= \frac{\sqrt{2}}{16} \lim_{h \rightarrow 0} \left(\frac{1 - \cos h}{h^2} \times \frac{1 + \cos h}{1 + \cos h} \right) \\ &= \frac{1}{8\sqrt{2}} \lim_{h \rightarrow 0} \frac{\sin^2 h}{h^2 (1 + \cos h)} \\ &= \frac{1}{8\sqrt{2}} \lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \right)^2 \times \frac{1}{1 + \cos h} \\ &= \frac{1}{8\sqrt{2}} \times \lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \right)^2 \times \lim_{h \rightarrow 0} \frac{1}{1 + \cos h} \\ &= \frac{1}{8\sqrt{2}} \times (1)^2 \times \frac{1}{1+1} \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\ &= \frac{1}{8\sqrt{2}} \times \frac{1}{2} = \frac{1}{16\sqrt{2}} \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow \frac{\pi}{6}} \left[\frac{2 - \sqrt{3} \cos x - \sin x}{(6x - \pi)^2} \right]$$

Solution:

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 - \sqrt{3} \cos x - \sin x}{(6x - \pi)^2}$$
$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 - \sqrt{3} \cos x - \sin x}{36 \left(x - \frac{\pi}{6}\right)^2}$$

Put $x - \frac{\pi}{6} = h$,

$$x = \frac{\pi}{6} + h$$

As $x \rightarrow \frac{\pi}{6}$, $h \rightarrow 0$

$$\therefore \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 - \sqrt{3} \cos x - \sin x}{36 \left(x - \frac{\pi}{6}\right)^2}$$
$$= \lim_{h \rightarrow 0} \frac{2 - \sqrt{3} \cos \left(\frac{\pi}{6} + h\right) - \sin \left(\frac{\pi}{6} + h\right)}{36h^2}$$
$$= \lim_{h \rightarrow 0} \frac{2 - \sqrt{3} \left(\cos \frac{\pi}{6} \cdot \cos h - \sin \frac{\pi}{6} \cdot \sin h \right) - \left(\sin \frac{\pi}{6} \cdot \cos h + \cos \frac{\pi}{6} \cdot \sin h \right)}{36h^2}$$
$$= \lim_{h \rightarrow 0} \frac{2 - \sqrt{3} \left(\frac{\sqrt{3}}{2} \cdot \cos h - \frac{1}{2} \sin h \right) - \left(\frac{1}{2} \cos h + \frac{\sqrt{3}}{2} \cdot \sin h \right)}{36h^2}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{\left[2 - \frac{3}{2} \cosh + \frac{\sqrt{3}}{2} \sinh - \frac{1}{2} \cosh - \frac{\sqrt{3}}{2} \sinh \right]}{36h^2} \\
 &= \lim_{h \rightarrow 0} \frac{2 - 2 \cosh}{36h^2} \\
 &= \lim_{h \rightarrow 0} \frac{2(1 - \cosh)}{36h^2} \\
 &= \frac{1}{18} \lim_{h \rightarrow 0} \frac{1 - \cos h}{h^2} \times \frac{1 + \cos h}{1 + \cos h} \\
 &= \frac{1}{18} \lim_{h \rightarrow 0} \frac{1 - \cos^2 h}{h^2(1 + \cosh)} \\
 &= \frac{1}{18} \lim_{h \rightarrow 0} \frac{\sin^2 h}{h^2(1 + \cosh)} \\
 &= \frac{1}{18} \left(\lim_{h \rightarrow 0} \frac{\sin h}{h} \right)^2 \lim_{h \rightarrow 0} \frac{1}{(1 + \cosh)} \\
 &= \frac{1}{18} (1)^2 \frac{1}{(1 + \cos 0)} \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\
 &= \frac{1}{18(1+1)} \\
 &= \frac{1}{36}
 \end{aligned}$$

Question 4.

$$\lim_{x \rightarrow a} \left[\frac{\sin(\sqrt{x}) - \sin(\sqrt{a})}{x - a} \right]$$

Solution:

$$\lim_{x \rightarrow a} \left[\frac{\sin(\sqrt{x}) - \sin(\sqrt{a})}{x - a} \right]$$

Put $\sqrt{x} = y$, $\sqrt{a} = b$ and $y = b + h$.

As $x \rightarrow a$, $y \rightarrow b$ and $h \rightarrow 0$.

$$\lim_{x \rightarrow a} \frac{\sin \sqrt{x} - \sin \sqrt{a}}{x - a}$$

$$= \lim_{y \rightarrow b} \frac{\sin y - \sin b}{y^2 - b^2}$$

$$= \lim_{y \rightarrow b} \frac{\sin y - \sin b}{(y - b)(y + b)}$$

$$= \lim_{h \rightarrow 0} \frac{\sin(b + h) - \sin b}{h(b + h + b)}$$

$$= \lim_{h \rightarrow 0} \frac{2 \cos\left(\frac{b+h+b}{2}\right) \sin\left(\frac{b+h-b}{2}\right)}{h(2b+h)}$$

$$= \lim_{h \rightarrow 0} \frac{2 \cos\left(b + \frac{h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{h(2b+h)}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{\cos\left[b + \frac{h}{2}\right] \cdot \frac{\sin \frac{h}{2}}{\frac{h}{2}}}{2b + h} \\
 &= \lim_{h \rightarrow 0} \frac{\cos\left[b + \frac{h}{2}\right]}{2b + h} \cdot \left[\lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \right] \\
 &= \frac{\cos(b+0)}{2b+0} \cdot 1 \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin p\theta}{p\theta} = 1 \right] \\
 &= \frac{\cos b}{2b} \\
 &= \frac{\cos \sqrt{a}}{2\sqrt{a}}
 \end{aligned}$$

Question 5.

$$\lim_{x \rightarrow \frac{\pi}{2}} \left[\frac{\cos 3x + 3 \cos x}{(2x - \pi)^3} \right]$$

Solution:

$$\begin{aligned}
 &\lim_{x \rightarrow \frac{\pi}{2}} \frac{3 \cos x + \cos 3x}{(2x - \pi)^3} \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{3 \cos x + 4 \cos^3 x - 3 \cos x}{(2x - \pi)^3}
 \end{aligned}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{4\cos^3 x}{(2x - \pi)^3} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{4\cos^3 x}{8\left(x - \frac{\pi}{2}\right)^3}$$

Put $x - \frac{\pi}{2} = h$,

$$\therefore x = \frac{\pi}{2} + h$$

As $x \rightarrow \frac{\pi}{2}$, $h \rightarrow 0$

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}} \frac{4\cos^3 x}{8\left(x - \frac{\pi}{2}\right)^3}$$

$$= \lim_{h \rightarrow 0} \frac{4\cos^3\left(\frac{\pi}{2} + h\right)}{8h^3}$$

$$= \lim_{h \rightarrow 0} \frac{4(-\sin h)^3}{8h^3} \quad \dots \left[\because \cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta \right]$$

$$= -\frac{1}{2} \cdot \lim_{h \rightarrow 0} \left(\frac{\sin h}{h}\right)^3$$

$$= -\frac{1}{2} \cdot \left(\lim_{h \rightarrow 0} \frac{\sin h}{h}\right)^3$$

$$= -\frac{1}{2} \cdot (1)^3 \quad \dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right]$$

$$= -\frac{1}{2}$$

Ex 7.6

I. Evaluate the following limits:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-7-limits/>

Question 1.

$$\lim_{x \rightarrow 0} \left[\frac{9^x - 5^x}{4^x - 1} \right]$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{9^x - 5^x}{4^x - 1} &= \lim_{x \rightarrow 0} \frac{9^x - 1 + 1 - 5^x}{4^x - 1} \\ &= \lim_{x \rightarrow 0} \frac{(9^x - 1) - (5^x - 1)}{4^x - 1} \\ &= \lim_{x \rightarrow 0} \frac{(9^x - 1) - (5^x - 1)}{\frac{x}{x}} \\ &\quad \dots [\because x \rightarrow 0, x \neq 0] \end{aligned}$$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{\left(\frac{9^x - 1}{x} \right) - \left(\frac{5^x - 1}{x} \right)}{\left(\frac{4^x - 1}{x} \right)} \\ &= \frac{\lim_{x \rightarrow 0} \frac{9^x - 1}{x} - \lim_{x \rightarrow 0} \frac{5^x - 1}{x}}{\lim_{x \rightarrow 0} \frac{4^x - 1}{x}} \\ &= \frac{\log 9 - \log 5}{\log 4} \quad \dots \left[\because \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a \right] \\ &= \frac{1}{(\log 4)} \log \left(\frac{9}{5} \right) \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow 0} \left[\frac{5^x + 3^x - 2^x - 1}{x} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{5^x + 3^x - 2^x - 1}{x} \\ &= \lim_{x \rightarrow 0} \frac{(5^x - 1) + (3^x - 2^x)}{x} \\ &= \lim_{x \rightarrow 0} \frac{(5^x - 1) + (3^x - 1) - (2^x - 1)}{x} \\ &= \lim_{x \rightarrow 0} \left(\frac{5^x - 1}{x} + \frac{3^x - 1}{x} - \frac{2^x - 1}{x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{5^x - 1}{x} \right) + \lim_{x \rightarrow 0} \left(\frac{3^x - 1}{x} \right) - \lim_{x \rightarrow 0} \left(\frac{2^x - 1}{x} \right) \\ &= \log 5 + \log 3 - \log 2 \quad \dots \left[\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a \right] \\ &= \log \frac{5 \times 3}{2} \\ &= \log \frac{15}{2} \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x - 3}{\sin x} \right)$$

Solution:

$$\lim_{x \rightarrow 0} \frac{a^x + b^x + c^x - 3}{\sin x}$$

$$= \lim_{x \rightarrow 0} \frac{(a^x - 1) + (b^x - 1) + (c^x - 1)}{\sin x}$$

$$= \lim_{x \rightarrow 0} \frac{(a^x - 1) + (b^x - 1) + (c^x - 1)}{\frac{\sin x}{x}}$$

... $\left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x. \\ \because x \rightarrow 0, x \neq 0 \end{array} \right]$

$$= \lim_{x \rightarrow 0} \frac{\left(\frac{a^x - 1}{x} \right) + \left(\frac{b^x - 1}{x} \right) + \left(\frac{c^x - 1}{x} \right)}{\frac{\sin x}{x}}$$

$$= \frac{\left(\lim_{x \rightarrow 0} \frac{a^x - 1}{x} \right) + \left(\lim_{x \rightarrow 0} \frac{b^x - 1}{x} \right) + \left(\lim_{x \rightarrow 0} \frac{c^x - 1}{x} \right)}{\left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right)}$$

$$= \frac{\log a + \log b + \log c}{1} \quad \dots \left[\because \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a \right]$$

$$= \log(abc)$$

Question 4.

$$\lim_{x \rightarrow 0} \left(\frac{6^x + 5^x + 4^x - 3^{x+1}}{\sin x} \right)$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{6^x + 5^x + 4^x - 3^{x+1}}{\sin x} \\ &= \lim_{x \rightarrow 0} \frac{6^x + 5^x + 4^x - 3^x \cdot 3^1}{\sin x} \\ &= \lim_{x \rightarrow 0} \frac{6^x - 1 + 5^x - 1 + 4^x - 1 - 3^x(3) + 3}{\sin x} \\ &= \lim_{x \rightarrow 0} \frac{(6^x - 1) + (5^x - 1) + (4^x - 1) - 3(3^x - 1)}{\sin x} \\ &= \lim_{x \rightarrow 0} \frac{(6^x - 1) + (5^x - 1) + (4^x - 1) - 3(3^x - 1)}{\frac{\sin x}{x}} \\ & \quad \dots \left[\begin{array}{l} \text{Divide Numerator and} \\ \text{Denominator by } x. \\ \because x \rightarrow 0, x \neq 0 \end{array} \right] \\ &= \lim_{x \rightarrow 0} \frac{\left[\frac{6^x - 1}{x} + \frac{5^x - 1}{x} + \frac{4^x - 1}{x} - 3 \left(\frac{3^x - 1}{x} \right) \right]}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} \end{aligned}$$

$$\begin{aligned}
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{6^x - 1}{x} \right) + \lim_{x \rightarrow 0} \left(\frac{5^x - 1}{x} \right) + \lim_{x \rightarrow 0} \left(\frac{4^x - 1}{x} \right) - 3 \lim_{x \rightarrow 0} \left(\frac{3^x - 1}{x} \right)}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} \\
 &= \frac{\log 6 + \log 5 + \log 4 - 3 \log 3}{1} \dots \left[\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a \right] \\
 &= \log (6 \times 5 \times 4) - \log (3)^3 \\
 &= \log \frac{120}{27} \\
 &= \log \frac{40}{9}
 \end{aligned}$$

Question 5.

$$\lim_{x \rightarrow 0} \left(\frac{8^{\sin x} - 2^{\tan x}}{e^{2x} - 1} \right)$$

Solution:

$$\begin{aligned}
 &\lim_{x \rightarrow 0} \frac{8^{\sin x} - 2^{\tan x}}{e^{2x} - 1} \\
 &= \lim_{x \rightarrow 0} \frac{(8^{\sin x} - 1) - (2^{\tan x} - 1)}{e^{2x} - 1} \\
 &= \lim_{x \rightarrow 0} \frac{(8^{\sin x} - 1) - (2^{\tan x} - 1)}{\frac{e^{2x} - 1}{x}} \\
 &= \lim_{x \rightarrow 0} \frac{x}{\frac{e^{2x} - 1}{x}}
 \end{aligned}$$

... $\left[\begin{array}{l} \text{Divide Numerator and} \\ \text{Denominator by } x. \\ \because x \rightarrow 0, x \neq 0 \end{array} \right]$

$$\begin{aligned}
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{8^{\sin x} - 1}{x} - \frac{2^{\tan x} - 1}{x} \right)}{\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}} \\
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{8^{\sin x} - 1}{\sin x} \cdot \frac{\sin x}{x} - \frac{2^{\tan x} - 1}{\tan x} \cdot \frac{\tan x}{x} \right)}{\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}} \\
 &= \frac{\left(\lim_{x \rightarrow 0} \frac{8^{\sin x} - 1}{\sin x} \right) \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) - \left(\lim_{x \rightarrow 0} \frac{2^{\tan x} - 1}{\tan x} \right) \left(\lim_{x \rightarrow 0} \frac{\tan x}{x} \right)}{\left(\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{2x} \right) \times 2} =
 \end{aligned}$$

$$\frac{(\log 8)(1) - (\log 2)(1)}{(1) \times 2} \dots \left[\begin{array}{l} \because x \rightarrow 0, 2x \rightarrow 0, \\ \sin x \rightarrow 0, \tan x \rightarrow 0 \\ \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a \end{array} \right]$$

$$\begin{aligned}
 &= \frac{\log \frac{8}{2}}{2} \\
 &= \frac{\log 4}{2}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\log(2)^2}{2} \\
 &= \frac{2\log 2}{2} = \log 2
 \end{aligned}$$

II. Evaluate the following limits:

Question 1.

$$\lim_{x \rightarrow 0} \left[\frac{3^x + 3^{-x} - 2}{x \cdot \tan x} \right]$$

Solution:

$$\begin{aligned}
 &\lim_{x \rightarrow 0} \frac{3^x + 3^{-x} - 2}{x \cdot \tan x} \\
 &= \lim_{x \rightarrow 0} \frac{3^x + \frac{1}{3^x} - 2}{x \cdot \tan x} \\
 &= \lim_{x \rightarrow 0} \frac{3^x + \frac{1}{3^x} - 2}{x \cdot \tan x} \\
 &= \lim_{x \rightarrow 0} \frac{(3^x)^2 + 1 - 2(3^x)}{3^x \cdot x \tan x} \\
 &= \lim_{x \rightarrow 0} \frac{(3^x - 1)^2}{3^x \cdot x \cdot \tan x} \quad \dots [\because a^2 - 2ab + b^2 = (a - b)^2] \\
 &= \lim_{x \rightarrow 0} \frac{(3^x - 1)^2}{3^x \cdot x \cdot \tan x} \quad \dots \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x^2. \\ \because x \rightarrow 0, x \neq 0 \end{array} \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{3^x - 1}{x} \right)^2}{\lim_{x \rightarrow 0} 3^x \cdot \left(\frac{\tan x}{x} \right)} \\
 &= \frac{\left(\lim_{x \rightarrow 0} \frac{3^x - 1}{x} \right)^2}{\left(\lim_{x \rightarrow 0} 3^x \right) \left(\lim_{x \rightarrow 0} \frac{\tan x}{x} \right)} \\
 &= \frac{(\log 3)^2}{3^0 \times 1} \dots \left[\because \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a, \lim_{\theta \rightarrow 0} \frac{\tan \theta}{\theta} = 1 \right] \\
 &= (\log 3)^2
 \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow 0} \left[\frac{3+x}{3-x} \right]^{\frac{1}{x}}$$

Solution:

$$\begin{aligned}
 &\lim_{x \rightarrow 0} \left(\frac{3+x}{3-x} \right)^{\frac{1}{x}} \\
 &= \lim_{x \rightarrow 0} \left(\frac{1 + \frac{x}{3}}{1 - \frac{x}{3}} \right)^{\frac{1}{x}} \quad \dots \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by 3} \end{array} \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{\left(1 + \frac{x}{3}\right)^{\frac{1}{x}}}{\left(1 - \frac{x}{3}\right)^{\frac{1}{x}}} \\
 &= \lim_{x \rightarrow 0} \frac{\left(1 + \frac{x}{3}\right)^{\frac{3}{x} \times \frac{1}{3}}}{\left(1 - \frac{x}{3}\right)^{-\frac{3}{x} \times \frac{1}{3}}} \\
 &= \frac{\lim_{x \rightarrow 0} \left[\left(1 + \frac{x}{3}\right)^{\frac{3}{x}} \right]^{\frac{1}{3}}}{\lim_{x \rightarrow 0} \left[\left(1 - \frac{x}{3}\right)^{-\frac{3}{x}} \right]^{\frac{1}{3}}} \\
 &= \frac{e^{1/3}}{e^{-1/3}} \quad \dots \left[\begin{array}{l} \because x \rightarrow 0, \frac{x}{3} \rightarrow 0, \frac{-x}{3} \rightarrow 0 \text{ and} \\ \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e \end{array} \right] \\
 &= e^{\frac{1}{3} + \frac{1}{3}} \\
 &= e^{\frac{2}{3}}
 \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow 0} \left[\frac{5x+3}{3-2x} \right]^{\frac{2}{x}}$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \left(\frac{5x+3}{3-2x} \right)^{\frac{2}{x}} \\ &= \lim_{x \rightarrow 0} \left(\frac{3+5x}{3-2x} \right)^{\frac{2}{x}} \\ &= \lim_{x \rightarrow 0} \left(\frac{1+\frac{5}{3}x}{1-\frac{2}{3}x} \right)^{\frac{2}{x}} \quad \dots \left[\begin{array}{l} \text{divide numerator and} \\ \text{denominator by 3} \end{array} \right] \end{aligned}$$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{\left(1+\frac{5}{3}x \right)^{\frac{2}{x}}}{\left(1-\frac{2}{3}x \right)^{\frac{2}{x}}} = \frac{\lim_{x \rightarrow 0} \left[\left(1+\frac{5}{3}x \right)^{\frac{3}{5x}} \right]^{\frac{5}{3} \times 2}}{\lim_{x \rightarrow 0} \left[\left(1-\frac{2}{3}x \right)^{\frac{-3}{2x}} \right]^{\frac{-2}{3} \times 2}} \\ &= \frac{e^{\frac{10}{3}}}{e^{\frac{-4}{3}}} \quad \dots \left[\begin{array}{l} \because x \rightarrow 0, \frac{5x}{3} \rightarrow 0, \frac{-2x}{3} \rightarrow 0 \text{ and} \\ \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e \end{array} \right] \\ &= e^{\frac{14}{3}} \end{aligned}$$

Question 4.

$$\lim_{x \rightarrow 0} \left[\frac{\log(3-x) - \log(3+x)}{x} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\log(3-x) - \log(3+x)}{x} \\ &= \lim_{x \rightarrow 0} \frac{1}{x} \log \left(\frac{3-x}{3+x} \right) \\ &= \lim_{x \rightarrow 0} \log \left(\frac{3-x}{3+x} \right)^{\frac{1}{x}} \\ &= \lim_{x \rightarrow 0} \log \left(\frac{1 - \frac{x}{3}}{1 + \frac{x}{3}} \right)^{\frac{1}{x}} = \log \left[\lim_{x \rightarrow 0} \frac{\left(1 - \frac{x}{3} \right)^{\frac{1}{x}}}{\left(1 + \frac{x}{3} \right)^{\frac{1}{x}}} \right] \\ &= \log \left[\frac{\left\{ \lim_{x \rightarrow 0} \left(1 - \frac{x}{3} \right)^{\frac{-3}{x}} \right\}^{\frac{-1}{3}}}{\left\{ \lim_{x \rightarrow 0} \left(1 + \frac{x}{3} \right)^{\frac{3}{x}} \right\}^{\frac{1}{3}}} \right] \\ &= \log \left(\frac{e^{\frac{1}{3}}}{e^{\frac{1}{3}}} \right) \end{aligned}$$

$$\begin{aligned} & \dots \left[\because x \rightarrow 0, \pm \frac{x}{3} \rightarrow 0 \text{ and } \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e \right] \\ & = \log e^{-2/3} \\ & = -\frac{2}{3} \cdot \log e = -\frac{2}{3} (1) = -\frac{2}{3} \end{aligned}$$

Question 5.

$$\lim_{x \rightarrow 0} \left[\frac{4x+1}{1-4x} \right]^{\frac{1}{x}}$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \left(\frac{4x+1}{1-4x} \right)^{\frac{1}{x}} \\ & = \lim_{x \rightarrow 0} \frac{(1+4x)^{\frac{1}{x}}}{(1-4x)^{\frac{1}{x}}} = \frac{\lim_{x \rightarrow 0} \left[(1+4x)^{\frac{1}{4x}} \right]^4}{\lim_{x \rightarrow 0} \left[(1-4x)^{\frac{-1}{4x}} \right]^{-4}} \\ & = \frac{e^4}{e^{-4}} \quad \dots \left[\because x \rightarrow 0, 4x \rightarrow 0, -4x \rightarrow 0 \right] \\ & \quad \dots \left[\text{and } \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e \right] \\ & = e^8 \end{aligned}$$

Question 6.

$$\lim_{x \rightarrow 0} \left[\frac{5+7x}{5-3x} \right]^{\frac{1}{3x}}$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \left(\frac{5+7x}{5-3x} \right)^{\frac{1}{3x}} \\ &= \lim_{x \rightarrow 0} \left(\frac{1+\frac{7}{5}x}{1-\frac{3}{5}x} \right)^{\frac{1}{3x}} \quad \dots \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by 5} \end{array} \right] \\ &= \frac{\lim_{x \rightarrow 0} \left(1+\frac{7x}{5} \right)^{\frac{1}{3x}}}{\lim_{x \rightarrow 0} \left(1-\frac{3x}{5} \right)^{\frac{1}{3x}}} = \frac{\lim_{x \rightarrow 0} \left[\left(1+\frac{7x}{5} \right)^{\frac{5}{7x}} \right]^{\frac{7}{5} \times \frac{1}{3}}}{\lim_{x \rightarrow 0} \left[\left(1-\frac{3x}{5} \right)^{\frac{-5}{3x}} \right]^{\frac{-3}{5} \times \frac{1}{3}}} \\ &= \frac{e^{\frac{7}{15}}}{e^{\frac{-3}{15}}} \quad \dots \left[\begin{array}{l} \because x \rightarrow 0, \frac{7x}{5} \rightarrow 0, \frac{-3x}{5} \rightarrow 0 \\ \text{and } \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e \end{array} \right] \\ &= e^{\frac{7}{15} + \frac{3}{15}} = e^{\frac{10}{15}} \\ &= e^{\frac{2}{3}} \end{aligned}$$

III. Evaluate the following limits:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-7-limits/>

Question 1.

$$\lim_{x \rightarrow 0} \left[\frac{a^x - b^x}{\sin(4x) - \sin(2x)} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{a^x - b^x}{\sin 4x - \sin 2x} \\ &= \lim_{x \rightarrow 0} \frac{(a^x - 1) - (b^x - 1)}{2 \cos \left(\frac{4x + 2x}{2} \right) \sin \left(\frac{4x - 2x}{2} \right)} \\ &= \lim_{x \rightarrow 0} \frac{(a^x - 1) - (b^x - 1)}{2 \cos 3x \cdot \sin x} \\ &= \lim_{x \rightarrow 0} \frac{(a^x - 1) - (b^x - 1)}{\frac{x}{2 \cos 3x \cdot \sin x}} \quad \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x. \\ \because x \rightarrow 0, x \neq 0 \end{array} \right] \\ &= \frac{\lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x} - \frac{b^x - 1}{x} \right)}{\lim_{x \rightarrow 0} 2 \cos 3x \cdot \left(\frac{\sin x}{x} \right)} \\ &= \frac{\left(\lim_{x \rightarrow 0} \frac{a^x - 1}{x} \right) - \left(\lim_{x \rightarrow 0} \frac{b^x - 1}{x} \right)}{2 \lim_{x \rightarrow 0} (\cos 3x) \cdot \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)} \\ &= \frac{\log a - \log b}{2(1)(1)} \end{aligned}$$

$$\dots \left[\because \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1, \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a \right]$$

$$= \frac{1}{2} \log \left(\frac{a}{b} \right)$$

Question 2.

$$\lim_{x \rightarrow 0} \left[\frac{(2^x - 1)^3}{(3^x - 1) \cdot \sin x \cdot \log(1+x)} \right]$$

Solution:

$$\lim_{x \rightarrow 0} \frac{(2^x - 1)^3}{(3^x - 1) \cdot \sin x \cdot \log(1+x)}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{(2^x - 1)^3}{x^3}}{\frac{(3^x - 1) \cdot \sin x \cdot \log(1+x)}{x^3}}$$

$$\dots \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x^3. \\ \because x \rightarrow 0, x \neq 0 \\ \therefore x^3 \neq 0 \end{array} \right]$$

$$\begin{aligned}
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{2^x - 1}{x} \right)^3}{\lim_{x \rightarrow 0} \left(\frac{3^x - 1}{x} \right) \cdot \frac{\sin x}{x} \cdot \frac{\log(1+x)}{x}} \\
 &= \frac{\left(\lim_{x \rightarrow 0} \frac{2^x - 1}{x} \right)^3}{\left(\lim_{x \rightarrow 0} \frac{3^x - 1}{x} \right) \cdot \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right) \cdot \left(\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} \right)} \\
 &= \frac{(\log 2)^3}{(\log 3)(1)(1)} \\
 \dots \left[\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1, \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a, \lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1 \right] \\
 &= \frac{(\log 2)^3}{\log 3}
 \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow 0} \left[\frac{15^x - 5^x - 3^x + 1}{x \cdot \sin x} \right]$$

Solution:

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{15^x - 5^x - 3^x + 1}{x \sin x} &= \lim_{x \rightarrow 0} \frac{5^x \cdot 3^x - 5^x - 3^x + 1}{x \sin x} \\
 &= \lim_{x \rightarrow 0} \frac{5^x(3^x - 1) - 1(3^x - 1)}{x \sin x}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{(3^x - 1)(5^x - 1)}{x \sin x} \\
 &= \lim_{x \rightarrow 0} \frac{(3^x - 1)(5^x - 1)}{\frac{x^2}{\left(\frac{x \sin x}{x^2}\right)}} \quad \dots [\because x \rightarrow 0, x \neq 0] \\
 &= \lim_{x \rightarrow 0} \frac{\left(\frac{3^x - 1}{x}\right) \left(\frac{5^x - 1}{x}\right)}{\left(\frac{\sin x}{x}\right)} \\
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{3^x - 1}{x}\right) \cdot \lim_{x \rightarrow 0} \left(\frac{5^x - 1}{x}\right)}{\left(\lim_{x \rightarrow 0} \frac{\sin x}{x}\right)} \\
 &= \frac{(\log 3) \cdot (\log 5)}{1} \\
 &\quad \dots \left[\because \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a, \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\
 &= (\log 3) \cdot (\log 5)
 \end{aligned}$$

Question 4.

$$\lim_{x \rightarrow 0} \left[\frac{(25)^x - 2(5)^x + 1}{x \cdot \sin x} \right]$$

Solution:

$$\lim_{x \rightarrow 0} \frac{(25)^x - 2(5)^x + 1}{x \cdot \sin x}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{(5^x)^2 - 2(5^x) + 1}{x \cdot \sin x} \dots \left[(25)^x = (5^2)^x = (5^x)^2 \right] \\
 &= \lim_{x \rightarrow 0} \frac{(5^x - 1)^2}{x \cdot \sin x} \\
 &= \lim_{x \rightarrow 0} \frac{(5^x - 1)^2}{\frac{x^2}{\frac{\sin x}{x}}} \dots \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x^2. \\ \therefore x \rightarrow 0, x \neq 0 \\ \therefore x^2 \neq 0 \end{array} \right] \\
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{5^x - 1}{x} \right)^2}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} \\
 &= \frac{(\log 5)^2}{1} \dots \left[\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a, \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 \right] \\
 &= (\log 5)^2
 \end{aligned}$$

Question 5.

$$\lim_{x \rightarrow 0} \left[\frac{(49)^x - 2(35)^x + (25)^x}{\sin x \cdot \log(1+2x)} \right]$$

Solution:

$$\begin{aligned}
 & \lim_{x \rightarrow 0} \frac{(49)^x - 2(35)^x + (25)^x}{\sin x \cdot \log(1+2x)} \\
 &= \lim_{x \rightarrow 0} \frac{(7^x)^2 - 2(7^x)(5^x) + (5^x)^2}{\sin x \cdot \log(1+2x)} \\
 &= \lim_{x \rightarrow 0} \frac{(7^x - 5^x)^2}{\sin x \cdot \log(1+2x)} \\
 &= \lim_{x \rightarrow 0} \frac{\left[\frac{(7^x - 1) - (5^x - 1)}{x} \right]^2}{\frac{\sin x}{x} \cdot \frac{\log(1+2x)}{x}} \quad \left[\begin{array}{l} \text{Divide Numerator and} \\ \text{Denominator by } x^2. \\ \because x \rightarrow 0, x \neq 0 \\ \therefore x^2 \neq 0 \end{array} \right] \\
 &= \frac{\lim_{x \rightarrow 0} \left[\frac{(7^x - 1) - (5^x - 1)}{x} \right]^2}{\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{\log(1+2x)}{x}} \\
 &= \frac{\lim_{x \rightarrow 0} \left[\frac{7^x - 1}{x} - \frac{5^x - 1}{x} \right]^2}{\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{\log(1+2x)}{x}} \\
 &= \frac{\left[\lim_{x \rightarrow 0} \frac{7^x - 1}{x} - \lim_{x \rightarrow 0} \frac{5^x - 1}{x} \right]^2}{\lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{\log(1+2x)}{2x} \times 2} \\
 &= \frac{(\log 7 - \log 5)^2}{1 \times 1 \times 2}
 \end{aligned}$$

$$\begin{aligned}
 & \dots \left[\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1, \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a, \lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1 \right] \\
 &= \frac{1}{2} \left(\log \frac{7}{5} \right)^2
 \end{aligned}$$

Ex 7.7

I. Evaluate the following:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-7-limits/>

Question 1.

$$\lim_{x \rightarrow \infty} \left[\frac{ax^3 + bx^2 + cx + d}{ex^3 + fx^2 + gx + h} \right]$$

Solution:

$$\lim_{x \rightarrow \infty} \left[\frac{ax^3 + bx^2 + cx + d}{ex^3 + fx^2 + gx + h} \right]$$

$$= \lim_{x \rightarrow \infty} \left[\frac{\frac{ax^3 + bx^2 + cx + d}{x^3}}{\frac{ex^3 + fx^2 + gx + h}{x^3}} \right]$$

.. $\left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x^3 \end{array} \right]$

$$= \frac{\lim_{x \rightarrow \infty} \left(a + \frac{b}{x} + \frac{c}{x^2} + \frac{d}{x^3} \right)}{\lim_{x \rightarrow \infty} \left(e + \frac{f}{x} + \frac{g}{x^2} + \frac{h}{x^3} \right)}$$

$$= \frac{\lim_{x \rightarrow \infty} \left(a + \frac{b}{x} + \frac{c}{x^2} + \frac{d}{x^3} \right)}{\lim_{x \rightarrow \infty} \left(e + \frac{f}{x} + \frac{g}{x^2} + \frac{h}{x^3} \right)}$$

$$= \frac{\lim_{x \rightarrow \infty} a + \lim_{x \rightarrow \infty} \frac{b}{x} + \lim_{x \rightarrow \infty} \frac{c}{x^2} + \lim_{x \rightarrow \infty} \frac{d}{x^3}}{\lim_{x \rightarrow \infty} e + \lim_{x \rightarrow \infty} \frac{f}{x} + \lim_{x \rightarrow \infty} \frac{g}{x^2} + \lim_{x \rightarrow \infty} \frac{h}{x^3}}$$

$$= \frac{\lim_{x \rightarrow \infty} a + \lim_{x \rightarrow \infty} \frac{b}{x} + \lim_{x \rightarrow \infty} \frac{c}{x^2} + \lim_{x \rightarrow \infty} \frac{d}{x^3}}{\lim_{x \rightarrow \infty} e + \lim_{x \rightarrow \infty} \frac{f}{x} + \lim_{x \rightarrow \infty} \frac{g}{x^2} + \lim_{x \rightarrow \infty} \frac{h}{x^3}}$$

$$= \frac{a + 0 + 0 + 0}{e + 0 + 0 + 0} \dots \left[\lim_{x \rightarrow \infty} \frac{1}{x^k} = 0, k > 0 \right]$$

$$= \frac{a}{e}$$

Question 2.

$$\lim_{x \rightarrow \infty} \left[\frac{x^3 + 3x + 2}{(x+4)(x-6)(x-3)} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow \infty} \left[\frac{x^3 + 3x + 2}{(x+4)(x-6)(x-3)} \right] \\ &= \lim_{x \rightarrow \infty} \frac{\frac{x^3 + 3x + 2}{x^3}}{\frac{(x+4)(x-6)(x-3)}{x^3}} \\ & \quad \dots \left[\begin{array}{l} \text{Divide Numerator and} \\ \text{Denominator by } x^3 \end{array} \right] \\ &= \frac{\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x^2} + \frac{2}{x^3} \right)}{\lim_{x \rightarrow \infty} \left(\frac{x+4}{x} \right) \left(\frac{x-6}{x} \right) \left(\frac{x-3}{x} \right)} \\ &= \frac{\lim_{x \rightarrow \infty} 1 + \lim_{x \rightarrow \infty} \frac{3}{x^2} + \lim_{x \rightarrow \infty} \frac{2}{x^3}}{\lim_{x \rightarrow \infty} \left(1 + \frac{4}{x} \right) \cdot \lim_{x \rightarrow \infty} \left(1 - \frac{6}{x} \right) \cdot \lim_{x \rightarrow \infty} \left(1 - \frac{3}{x} \right)} \\ &= \frac{1+0+0}{(1+0)(1-0)(1-0)} \quad \dots \left[\lim_{x \rightarrow \infty} \frac{1}{x^k} = 0, k > 0 \right] \\ &= 1 \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow \infty} \left[\frac{7x^2 + 5x - 3}{8x^2 - 2x + 7} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow \infty} \frac{7x^2 + 5x - 3}{8x^2 - 2x + 7} \\ &= \lim_{x \rightarrow \infty} \frac{\left(\frac{7x^2 + 5x - 3}{x^2} \right)}{\left(\frac{8x^2 - 2x + 7}{x^2} \right)} \\ &= \lim_{x \rightarrow \infty} \frac{\left(7 + \frac{5}{x} - \frac{3}{x^2} \right)}{\left(8 - \frac{2}{x} + \frac{7}{x^2} \right)} \\ &= \frac{\lim_{x \rightarrow \infty} 7 + \lim_{x \rightarrow \infty} \frac{5}{x^2} - \lim_{x \rightarrow \infty} \frac{3}{x^2}}{\lim_{x \rightarrow \infty} 8 - \lim_{x \rightarrow \infty} \frac{2}{x} + \lim_{x \rightarrow \infty} \frac{7}{x^2}} \\ &= \frac{7 + 0 - 0}{8 - 0 + 0} \quad \dots \left[\because \lim_{x \rightarrow \infty} \frac{1}{x^k} = 0, k > 0 \right] \\ &= \frac{7}{8} \end{aligned}$$

II. Evaluate the following:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-7-limits/>

Question 1.

$$\lim_{x \rightarrow \infty} \left[\frac{7x^2 + 2x - 3}{\sqrt{x^4 + x + 2}} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow \infty} \left[\frac{7x^2 + 2x - 3}{\sqrt{x^4 + x + 2}} \right] \\ &= \lim_{x \rightarrow \infty} \frac{\frac{7x^2 + 2x - 3}{x^2}}{\frac{\sqrt{x^4 + x + 2}}{x^2}} \quad \dots \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x^2 \end{array} \right] \\ &= \frac{\lim_{x \rightarrow \infty} \left(7 + \frac{2}{x} - \frac{3}{x^2} \right)}{\lim_{x \rightarrow \infty} \sqrt{\frac{x^4 + x + 2}{x^4}}} \\ &= \frac{\lim_{x \rightarrow \infty} 7 + \lim_{x \rightarrow \infty} \frac{2}{x} - \lim_{x \rightarrow \infty} \frac{3}{x^2}}{\lim_{x \rightarrow \infty} \sqrt{1 + \frac{1}{x^3} + \frac{2}{x^4}}} \\ &= \frac{7 + 0 - 0}{\sqrt{1 + 0 + 0}} \quad \dots \left[\lim_{x \rightarrow \infty} \frac{1}{x^k} = 0, k > 0 \right] \\ &= 7 \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow \infty} \left[\sqrt{x^2 + 4x + 16} - \sqrt{x^2 + 16} \right]$$

Solution:

$$\begin{aligned}
 & \lim_{x \rightarrow \infty} \sqrt{x^2 + 4x + 16} - \sqrt{x^2 + 16} \\
 &= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + 4x + 16} - \sqrt{x^2 + 16})(\sqrt{x^2 + 4x + 16} + \sqrt{x^2 + 16})}{(\sqrt{x^2 + 4x + 16} + \sqrt{x^2 + 16})} \\
 & \quad \dots [\text{By rationalization}] \\
 &= \lim_{x \rightarrow \infty} \frac{(x^2 + 4x + 16 - x^2 - 16)}{\sqrt{x^2 + 4x + 16} + \sqrt{x^2 + 16}} \\
 &= \lim_{x \rightarrow \infty} \frac{4x}{\sqrt{x^2 + 4x + 16} + \sqrt{x^2 + 16}} \\
 &= \lim_{x \rightarrow \infty} \frac{\frac{4x}{x}}{\frac{\sqrt{x^2 + 4x + 16}}{x} + \frac{\sqrt{x^2 + 16}}{x}} \\
 &= \lim_{x \rightarrow \infty} \frac{4}{\sqrt{\frac{x^2 + 4x + 16}{x^2}} + \sqrt{\frac{x^2 + 16}{x^2}}} \\
 &= \lim_{x \rightarrow \infty} \frac{4}{\sqrt{1 + \frac{4}{x} + \frac{16}{x^2}} + \sqrt{1 + \frac{16}{x^2}}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{4}{\lim_{x \rightarrow \infty} \sqrt{1 + \frac{4}{x} + \frac{16}{x^2}} + \lim_{x \rightarrow \infty} \sqrt{1 + \frac{16}{x^2}}} \\
 &= \frac{4}{\sqrt{1+0+0} + \sqrt{1+0}} \quad \dots \left[\because \lim_{x \rightarrow \infty} \frac{1}{x^k} = 0, k > 0 \right] \\
 &= \frac{4}{2} \\
 &= 2
 \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow \infty} [\sqrt{x^4 + 4x^2} - x^2]$$

Solution:

$$\begin{aligned}
 &\lim_{x \rightarrow \infty} [\sqrt{x^4 + 4x^2} - x^2] \\
 &= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^4 + 4x^2} - x^2)(\sqrt{x^4 + 4x^2} + x^2)}{\sqrt{x^4 + 4x^2} + x^2} \\
 &\quad \dots [\text{By rationalization}] \\
 &= \lim_{x \rightarrow \infty} \frac{x^4 + 4x^2 - x^4}{\sqrt{x^4 + 4x^2} + x^2} \\
 &= \lim_{x \rightarrow \infty} \frac{4x^2}{\sqrt{x^4 + 4x^2} + x^2}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow \infty} \frac{\frac{4x^2}{x^2}}{\frac{\sqrt{x^4 + 4x^2 + x^2}}{x^2}} \quad \dots \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x^2 \end{array} \right] \\
 &= \lim_{x \rightarrow \infty} \frac{4x^2}{\sqrt{\frac{x^4 + 4x^2}{x^4} + 1}} \\
 &= \frac{\lim_{x \rightarrow \infty} 4}{\lim_{x \rightarrow \infty} \left(\sqrt{1 + \frac{4}{x^2}} + 1 \right)} \\
 &= \frac{4}{\sqrt{1 + 0} + 1} \quad \dots \left[\lim_{x \rightarrow \infty} \frac{1}{x^k} = 0, k > 0 \right] \\
 &= \frac{4}{2} = 2
 \end{aligned}$$

III. Evaluate the following:

Question 1.

$$\lim_{x \rightarrow \infty} \left[\frac{(3x^2 + 4)(4x^2 - 6)(5x^2 + 2)}{4x^6 + 2x^4 - 1} \right]$$

Solution:

$$\begin{aligned}
 & \lim_{x \rightarrow \infty} \frac{(3x^2+4)(4x^2-6)(5x^2+2)}{4x^6+2x^4-1} \\
 &= \lim_{x \rightarrow \infty} \frac{\frac{(3x^2+4)(4x^2-6)(5x^2+2)}{x^6}}{\frac{4x^6+2x^4-1}{x^6}} \\
 & \quad \dots \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x^6 \end{array} \right] \\
 &= \lim_{x \rightarrow \infty} \frac{\left(\frac{3x^2+4}{x^2}\right)\left(\frac{4x^2-6}{x^2}\right)\left(\frac{5x^2+2}{x^2}\right)}{4+\frac{2}{x^2}-\frac{1}{x^6}} \\
 &= \frac{\lim_{x \rightarrow \infty} \left(3+\frac{4}{x^2}\right) \lim_{x \rightarrow \infty} \left(4-\frac{6}{x^2}\right) \lim_{x \rightarrow \infty} \left(5+\frac{2}{x^2}\right)}{\lim_{x \rightarrow \infty} \left(4+\frac{2}{x^2}-\frac{1}{x^6}\right)} \\
 &= \frac{\lim_{x \rightarrow \infty} \left(3+\frac{4}{x^2}\right) \cdot \lim_{x \rightarrow \infty} \left(4-\frac{6}{x^2}\right) \cdot \lim_{x \rightarrow \infty} \left(5+\frac{2}{x^2}\right)}{\lim_{x \rightarrow \infty} 4 + \lim_{x \rightarrow \infty} \frac{2}{x^2} - \lim_{x \rightarrow \infty} \frac{1}{x^6}} \\
 &= \frac{(3+0)(4-0)(5+0)}{4+0-0} \dots \left[\lim_{x \rightarrow \infty} \frac{1}{x^k} = 0, k > 0 \right] \\
 &= \frac{3 \times 4 \times 5}{4} \\
 &= 15
 \end{aligned}$$

Question 2.

$$\lim_{x \rightarrow \infty} \left[\frac{(3x-4)^3(4x+3)^4}{(3x+2)^7} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow \infty} \left[\frac{(3x-4)^3(4x+3)^4}{(3x+2)^7} \right] \\ &= \lim_{x \rightarrow \infty} \frac{\frac{(3x-4)^3}{x^3} \frac{(4x+3)^4}{x^4}}{\frac{(3x+2)^7}{x^7}} \dots \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x^7 \end{array} \right] \\ &= \frac{\lim_{x \rightarrow \infty} \left(\frac{3x-4}{x} \right)^3 \lim_{x \rightarrow \infty} \left(\frac{4x+3}{x} \right)^4}{\lim_{x \rightarrow \infty} \left(\frac{3x+2}{x} \right)^7} \\ &= \frac{\lim_{x \rightarrow \infty} \left(3 - \frac{4}{x} \right)^3 \times \lim_{x \rightarrow \infty} \left(4 + \frac{3}{x} \right)^4}{\lim_{x \rightarrow \infty} \left(3 + \frac{2}{x} \right)^7} \\ &= \frac{(3-0)^3 \times (4+0)^4}{(3+0)^7} \dots \left[\lim_{x \rightarrow \infty} \frac{1}{x^k} = 0, k > 0 \right] \\ &= \frac{(3)^3 \times (4)^4}{(3)^7} \\ &= \frac{(4)^4}{(3)^4} \end{aligned}$$

Question 3.

$$\lim_{x \rightarrow \infty} [\sqrt{x}(\sqrt{x+1} - \sqrt{x})]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow \infty} [\sqrt{x}(\sqrt{x+1} - \sqrt{x})] \\ &= \lim_{x \rightarrow \infty} \sqrt{x}(\sqrt{x+1} - \sqrt{x}) \times \frac{(\sqrt{x+1} + \sqrt{x})}{\sqrt{x+1} + \sqrt{x}} \\ & \quad \dots [\text{By rationalization}] \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{x}(x+1-x)}{\sqrt{x+1} + \sqrt{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\sqrt{x+1} + \sqrt{x}} \\ &= \lim_{x \rightarrow \infty} \frac{1}{\frac{\sqrt{x+1}}{\sqrt{x}} + 1} \quad \dots \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } \sqrt{x} \end{array} \right] \\ &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{x+1}{x}} + 1} \\ &= \frac{1}{\lim_{x \rightarrow \infty} \left(\sqrt{1 + \frac{1}{x}} + 1 \right)} \\ &= \frac{1}{\sqrt{1+0} + 1} \quad \dots \left[\lim_{x \rightarrow \infty} \frac{1}{x^k} = 0, k > 0 \right] \end{aligned}$$

$$= \frac{1}{1+1}$$
$$= \frac{1}{2}$$

Question 4.

$$\lim_{x \rightarrow \infty} \left[\frac{(2x-1)^{20}(3x-1)^{30}}{(2x+1)^{50}} \right]$$

Solution:

$$\begin{aligned} & \lim_{x \rightarrow \infty} \frac{(2x-1)^{20}(3x-1)^{30}}{(2x+1)^{50}} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{(2x-1)^{20}}{x^{20}} \cdot \frac{(3x-1)^{30}}{x^{30}}}{\frac{(2x+1)^{50}}{x^{50}}} \quad \dots \left[\begin{array}{l} \text{Divide numerator and} \\ \text{denominator by } x^{50} \end{array} \right] \\ &= \frac{\lim_{x \rightarrow \infty} \left(\frac{2x-1}{x} \right)^{20} \cdot \left(\frac{3x-1}{x} \right)^{30}}{\lim_{x \rightarrow \infty} \left(\frac{2x+1}{x} \right)^{50}} \end{aligned}$$

$$\begin{aligned}
 &= \frac{\lim_{x \rightarrow \infty} \left(2 - \frac{1}{x}\right)^{20} \cdot \lim_{x \rightarrow \infty} \left(3 - \frac{1}{x}\right)^{30}}{\lim_{x \rightarrow \infty} \left(2 + \frac{1}{x}\right)^{50}} \\
 &= \frac{(2-0)^{20} \times (3-0)^{30}}{(2+0)^{50}} \quad \dots \left[\lim_{x \rightarrow \infty} \frac{1}{x^k} = 0, k > 0 \right] \\
 &= \frac{(2)^{20} \times (3)^{30}}{(2)^{50}} \\
 &= \frac{(3)^{30}}{(2)^{30}} \\
 &= \left(\frac{3}{2}\right)^{30}
 \end{aligned}$$

Question 5.

$$\lim_{x \rightarrow \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$$

Solution:

$$\begin{aligned}
 &\lim_{x \rightarrow \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right] \\
 &= \lim_{x \rightarrow \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \right] \\
 &\quad \sim \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+5} + \sqrt{x^2-3}}
 \end{aligned}$$

... [By taking conjugates of both, the numerator as well as the denominator.]

$$= \lim_{x \rightarrow \infty} \left[\frac{(x^2+5)-(x^2-3)}{(x^2+3)-(x^2+1)} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \right]$$

$$= \lim_{x \rightarrow \infty} \frac{8(\sqrt{x^2+3}+\sqrt{x^2+1})}{2(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$= 4 \lim_{x \rightarrow \infty} \left(\frac{\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{x}}{\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{x}} \right)$$

... [Divide numerator and denominator by x]

$$= 4 \lim_{x \rightarrow \infty} \left(\frac{\frac{\sqrt{x^2+3}}{x} + \frac{\sqrt{x^2+1}}{x}}{\frac{\sqrt{x^2+5}}{x} + \frac{\sqrt{x^2-3}}{x}} \right)$$

$$= \frac{4 \lim_{x \rightarrow \infty} \left(\sqrt{\frac{x^2+3}{x^2}} + \sqrt{\frac{x^2+1}{x^2}} \right)}{\lim_{x \rightarrow \infty} \left(\sqrt{\frac{x^2+5}{x^2}} + \sqrt{\frac{x^2-3}{x^2}} \right)}$$

$$= \frac{4 \left(\lim_{x \rightarrow \infty} \sqrt{1+\frac{3}{x^2}} + \lim_{x \rightarrow \infty} \sqrt{1+\frac{1}{x^2}} \right)}{\lim_{x \rightarrow \infty} \sqrt{1+\frac{5}{x^2}} + \lim_{x \rightarrow \infty} \sqrt{1-\frac{3}{x^2}}}$$

$$= \frac{4(1+1)}{1+1}$$

$$= 4$$



Maharashtra Board Solutions

Class 11 Arts & Science Maths

(Part 2)

- Chapter 1- Complex Numbers
- Chapter 2- Sequences and Series
- Chapter 3- Permutations and Combination
- Chapter 4- Methods of Induction and Binomial Theorem
- Chapter 5- Sets and Relations
- Chapter 6- Functions
- Chapter 7- Limits
- Chapter 8- Continuity
- Chapter 9- Differentiation

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About About Maharashtra State Board (MSBSHSE)

The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

The Maharashtra State Board of Secondary & Higher Secondary Education (MSBSHSE), Pune is an independent body of the Maharashtra Government. There are more than 1.4 million students that appear in the examination every year. The Maha State Board conducts the board examination twice a year. This board conducts the examination for SSC and HSC.

The Maharashtra government established the Maharashtra State Bureau of Textbook Production and Curriculum Research, also commonly referred to as Ebalbharati, in 1967 to take up the responsibility of providing quality textbooks to students from all classes studying under the Maharashtra State Board. MSBHSE prepares and updates the curriculum to provide holistic development for students. It is designed to tackle the difficulty in understanding the concepts with simple language with simple illustrations. Every year around 10 lakh students are enrolled in schools that are affiliated with the Maharashtra State Board.

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FAQs

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As of now, the MSCERT and Balbharti are responsible for the syllabus and textbooks of Classes 1 to 8, while Classes 9 and 10 are under the Maharashtra State Board of Secondary and Higher Secondary Education (MSBSHSE).

How many state boards are there in Maharashtra?

The Maharashtra State Board of Secondary & Higher Secondary Education, conducts the HSC and SSC Examinations in the state of Maharashtra through its nine Divisional Boards located at Pune, Mumbai, Aurangabad, Nasik, Kolhapur, Amravati, Latur, Nagpur and Ratnagiri.

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For any clarifications or questions you can write to **info@indcareer.com**

Postal Address

IndCareer.com
52, Shilpa Nagar,
Somalwada
Nagpur - 440015
Maharashtra, India

WhatsApp: +91 9561 204 888

Website: <https://www.indcareer.com>

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