



Roll No.....

Total No. of Questions : 29]

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XIARKDN20
2005-C
MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 100

Section-A

(Multiple Choice Questions)

1. A relation R in the set Z of all integers defined as :

$$R = \{(x, y) : x - y \text{ is an integer}\},$$

then R is :

(A) Reflexive

(B) Transitive

(C) Symmetric

(D) None of these

2. If $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = (5 - x^6)^{1/6}$, then $f \circ f(x)$ is :

(A) 5

(B) x^6

(C) x

(D) None of these

3. If X and Y are two square matrices such that $XY = X$ and $YX = Y$, then X^2 is equal to :

(A) X

(B) Y

(C) 1

(D) None of these

4. Let \vec{a} and \vec{b} be two unit vectors and θ is the angle between them.

Then $\vec{a} + \vec{b}$ is a unit vector, if :

(A) $\theta = \frac{\pi}{4}$

(B) $\theta = \frac{\pi}{3}$

(C) $\theta = \frac{\pi}{2}$

(D) $\theta = 2\frac{\pi}{3}$

Section–B

(Very Short Answer Type Questions)

5. Find AB, if $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 0 & 2 \end{pmatrix}$

6. Prove that :

$$\int \operatorname{cosec} x \, dx = \log | \operatorname{cosec} x - \cot x | + C$$

7. Evaluate :

$$\int_0^{\pi/2} \cos^2 x \, dx$$

8. Find the order and degree of the differential equation :

$$\frac{d^3 y}{dx^3} + 2 \frac{d^2 y}{dx^2} + \frac{dy}{dx} = 0$$

9. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.
10. Define the term optimal solution.
11. If $P(B) = 0.5$ and $P(A \cap B) = 0.32$, evaluate $P(A/B)$.
12. A family has two children. What is the probability that both the children are boys given that at least one of them is a boy ?

Section–C

(Short Answer Type Questions)

13. Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$, defined as $f(x) = x^2$, is neither one–one nor onto.
14. Solve for x :

$$\tan^{-1} \left(\frac{1-x}{1+x} \right) = \frac{1}{2} \tan^{-1} x, (x > 0)$$

15. Find the inverse of $\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$

16. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long? <https://www.jkboseonline.com>

17. Using differentials find the approximate value upto 3 decimal places of $(25)^{1/3}$.

18. Find d^2y/dx^2 , if $y = x^3 \log x$.

19. Prove that :

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$$

20. Using properties of definite integrals, to evaluate :

$$\int_0^{\pi/2} \frac{\sin^{3/2} x dx}{\sin^{3/2} x + \cos^{3/2} x}$$

21. Show that $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$, for any two non–zero vectors \vec{a} and \vec{b} .

22. Find the distance of a point $(2, 5, -3)$ from the plane

$$\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$$

23. Determine graphically the minimum value of the objective function

$$Z = -50x + 20y$$

Subject to the constraints

$$2x - y \geq -5$$

$$3x + y \geq 3$$

$$2x - 3y \leq 12$$

$$x \geq 0, y \geq 0$$

Section–D

(Long Answer Type Questions)

24. Using properties of determinants, prove that :

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx)$$

OR

For the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

show that :

$$A^3 - 6A^2 + 5A + 11I = 0.$$

Hence, find A^{-1} .

25. Find the relation between a and b, so that the function f defined by :

$$f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$$

is continuous at $x = 3$.

OR

Find dy/dx of the function $xy = e^{(x-y)}$.

26. Evaluate :

$$\int \frac{x^2}{(x^2 + 1)(x^2 + 4)} dx$$

Or

Find the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

27. Show that the differential equation :

$$y dx + x \log (y/x) dy - 2x dy = 0$$

is homogeneous and solve it.

Or

Find the particular solution of the differential equation :

$$(1 + x^2) \frac{dy}{dx} + 2xy = \frac{1}{1 + x^2},$$

given that $y = 0$ when $x = 1$.

18. Find the shortest distance between the lines whose vector equations are :

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k} \quad \text{and}$$

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$.

29. Find the mean number of heads in three tosses of a fair coin.

OR

There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?