

- **Distance Formula** : The distance between A and B for  $A(x_1, y_1)$  and  $B(x_2, y_2)$  in  $R^2$  is  $AB = d(A, B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ .
- **Circumcentre of a Triangle** : The point of concurrence of the perpendicular bisectors of all the sides of a triangle is called its circumcentre. Circumcentre of a triangle is equidistant from all the vertices. If P is the circumcentre of  $\triangle ABC$ , then  $AP = BP = CP$ . For a right triangle the midpoint of the hypotenuse is circumcentre.
- **Centroid of a Triangle** : The point of concurrence of all the medians of a triangle is called the centroid of the triangle.
- For a triangle with vertices  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  and  $C(x_3, y_3)$ ,  
centroid  $G = \left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$
- **Incentre of a Triangle** : The point of concurrence of internal bisectors of all the angles of a triangle is called the incentre of the triangle.
- Let  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  and  $C(x_3, y_3)$  be the vertices of  $\triangle ABC$  and let  $BC = a$ ,  $CA = b$ ,  $AB = c$ . Then the incentre of the triangle  $I = \left( \frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$ .
- **Orthocentre of a Triangle** : The point of concurrence of all the altitudes of the triangle is called the orthocentre of the triangle. It is denoted by H.  
The centroid G of a triangle divides  $\overline{HP}$  in ratio 2:1 from H.
- **Division of a Line Segment** : Let A and B be two distinct points in a plane. We know that two distinct points determine a unique line. Thus, a unique line  $\overleftrightarrow{AB}$  is determined by A and B. If  $P \in \overleftrightarrow{AB}$ ,  $P \neq A$ ,  $P \neq B$ , then  $A-P-B$  or  $P-A-B$  or  $A-B-P$ . We will see that if  $P \in \overline{AB}$  (i.e.  $A-P-B$ ) or  $P \notin \overline{AB}$  (i.e.  $P-A-B$  or  $A-B-P$ ) is known and the value of the ratio  $\frac{AP}{PB}$  is known, then we can determine the coordinates of P.
- **Internal Division** : If  $A-P-B$ , then P is said to divide  $\overline{AB}$  in the ratio  $\frac{AP}{PB}$  from A. Since we have  $A-P-B$ , in this case this division is called internal division.
- **External Division** : If  $P-A-B$  or  $A-B-P$  then P divides  $\overline{AB}$  in the ratio  $-\frac{AP}{PB}$  from A. Since we have  $P-A-B$  or  $A-B-P$ , in this case this division is called external division.  
Thus, for P dividing  $\overline{AB}$  in the ratio  $\lambda$  ( $\lambda \neq 0, -1$ ), from A,  
(i) If  $A-P-B$ , then  $\lambda > 0$  and the point P divides  $\overline{AB}$  internally.  
(ii) If  $P-A-B$ , then  $-1 < \lambda < 0$  and the point P divides  $\overline{AB}$  externally.  
(iii) If  $A-B-P$ , then  $\lambda < -1$  and the point P divides  $\overline{AB}$  externally.

● **Coordinates of a Point Dividing a Line Segment :**

The coordinates of a point dividing  $\overline{AB}$  formed by joining the points  $A(x_1, y_1), B(x_2, y_2)$  are  $\left(\frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1}\right)$ , where  $\lambda \in R - \{0, -1\}$ . (Here division is assumed to be from A.)

Taking  $\lambda = \frac{m}{n}$ , the coordinates of the point of division are  $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$  (where  $m+n \neq 0, m \neq 0$ )

● **Necessary and Sufficient Condition for Three Distinct Points of  $R^2$  to be collinear :**

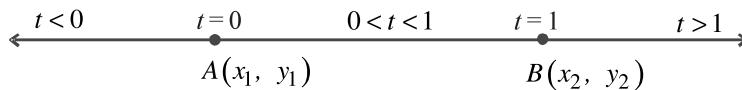
The necessary and sufficient condition for three distinct points  $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$  in

$R^2$  to be collinear is  $\Delta = 0$ , that is  $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$ .

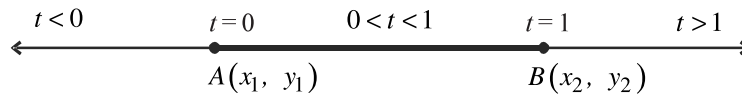
● **Parametric Equations of a Line :** Let  $A(x_1, y_1)$  and  $B(x_2, y_2) \in R^2$  be distinct points in  $R^2$ .  $x = tx_2 + (1-t)x_1, y = ty_2 + (1-t)y_1, t \in R$  are called the parametric equations of  $\overleftrightarrow{AB}$  and  $t$  is called a parameter.

● **Some Subsets of  $\overleftrightarrow{AB}$  :**

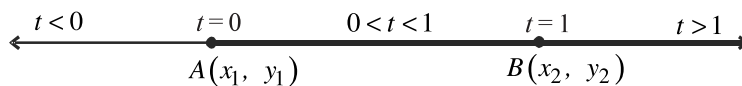
$$(1) \quad \overleftrightarrow{AB} = \left\{ (x, y) \mid \begin{array}{l} x = tx_2 + (1-t)x_1 \\ y = ty_2 + (1-t)y_1 \end{array} ; t \in R \right\}$$



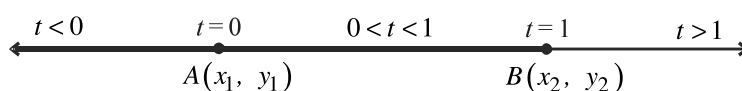
$$(2) \quad \overline{AB} = \left\{ (x, y) \mid \begin{array}{l} x = tx_2 + (1-t)x_1 \\ y = ty_2 + (1-t)y_1 \end{array} ; t \in [0, 1] \right\}$$



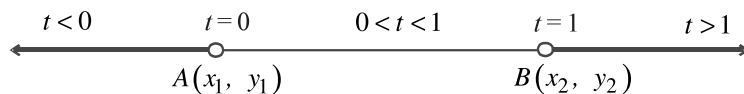
$$(3) \quad \overrightarrow{AB} = \left\{ (x, y) \mid \begin{array}{l} x = tx_2 + (1-t)x_1 \\ y = ty_2 + (1-t)y_1 \end{array} ; t \geq 0 \right\}$$



$$(4) \quad \overrightarrow{BA} = \left\{ (x, y) \mid \begin{array}{l} x = tx_2 + (1-t)x_1 \\ y = ty_2 + (1-t)y_1 \end{array} ; t \leq 1 \right\}$$



$$(5) \quad \overleftrightarrow{AB} - \overline{AB} = \left\{ (x, y) \mid \begin{cases} x = tx_2 + (1-t)x_1 \\ y = ty_2 + (1-t)y_1 \end{cases} ; t \in \mathbb{R} - [0, 1] \right\}$$



● **Cartesian Equation of a Line :**

Let  $A(x_1, y_1), B(x_2, y_2)$  be two distinct points of  $\mathbb{R}^2$ .

$$\therefore \text{ Cartesian equation of } \overleftrightarrow{AB} \text{ is } \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

The equation of a line perpendicular to X-axis is  $x = a$ , where  $a$  is a constant.

The equation of a line perpendicular to Y-axis is  $y = b$ , where  $b$  is a constant.

● **Slope of a Line :**

Let  $A(x_1, y_1), B(x_2, y_2)$  be two distinct points of  $\mathbb{R}^2$ , where  $x_1 \neq x_2$ , so that the  $\overleftrightarrow{AB}$  joining them is not perpendicular to X-axis. The slope of  $\overline{AB}$  is defined by  $\frac{y_2 - y_1}{x_2 - x_1}$ .

- For a line not perpendicular to X-axis and making an angle of measure  $\theta$  with the positive direction of X-axis,  $\tan\theta$  is called the slope of the line. It is denoted by  $m$ .

$$\therefore m = \tan \theta \quad 0 < \theta < \pi, \quad \theta \neq \frac{\pi}{2}$$

- If  $0 < \theta < \frac{\pi}{2}$ , then the slope of  $\overleftrightarrow{AB}$  is positive.
- If  $\frac{\pi}{2} < \theta < \pi$ , then the slope of  $\overleftrightarrow{AB}$  is negative.
- The slope of  $\overleftrightarrow{AB}$  is not defined, if  $\overleftrightarrow{AB}$  is perpendicular to X-axis.
- If  $\overleftrightarrow{AB}$  is perpendicular to Y-axis, its slope is zero.
- Necessary and sufficient condition for two distinct lines to be parallel is  $m_1 = m_2$ .
- Necessary and sufficient condition for two distinct lines to be perpendicular is  $m_1 \cdot m_2 = -1$ .
- **Angle Between Two Intersecting Lines :**

If two lines are not perpendicular to each other, then two pairs of congruent vertically opposite angles are formed at their point of intersection. One of these pairs is a pair of congruent obtuse angles and the other is a pair of congruent acute angles. The radian measure of this acute angle is called the measure of the angle between the two lines.

- (1) If one of the lines is perpendicular to X-axis and the slope of the second line is  $m$ , then the measure of the angle between these two lines is  $\alpha = \left| \frac{\pi}{2} - \theta \right|$ , where  $\theta$  is the measure of angle made by the second line with positive X-axis.

(2) If none of the two intersecting lines is perpendicular to X-axis; and if the measure of angle between these lines is  $\alpha$  then it is obtained from  $\tan \alpha = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$ , where  $0 < \alpha < \frac{\pi}{2}$ .

Here  $m_1$  and  $m_2$  are slopes of the lines.

- **Slope of the line  $ax + by + c = 0$ ,  $a^2 + b^2 \neq 0$ ,  $(a, b, c \in R)$  :**

Suppose the line is neither horizontal nor vertical. Hence  $a \neq 0$ ,  $b \neq 0$ .

$\therefore$  The slope of the line  $ax + by + c = 0$  is  $-\frac{a}{b}$ .

If  $a = 0$ , then the line is horizontal. Hence the slope of the line is 0.

If  $b = 0$ , then the line is vertical. Hence slope is not defined.

If  $c = 0$ , then the line passes through the origin.

- **Intercepts of a line on the axes :**

**X-intercept :** If a line intersects X-axis in one and only one point  $(a, 0)$ , then the line has X-intercept  $a$  (where  $a \neq 0$ ). There is no X-intercept for a line perpendicular to Y-axis.

**Y-intercept :** If a line intersects Y-axis in one and only one point  $(0, b)$ , then the line has Y-intercept  $b$  (where  $b \neq 0$ ). There is no Y-intercept for a line perpendicular to X-axis.

- **Intercepts of the line  $ax + by + c = 0$ ,  $a \neq 0$ ,  $b \neq 0$  :**

X-intercept  $= -\frac{c}{a}$ , Y-intercept  $= -\frac{c}{b}$ .

**Note :** Both the intercepts of a line passing through the origin are zero.

- **Perpendicular distance of an outside point from a line :**

The perpendicular distance of a point  $P(x_1, y_1)$  from the line  $ax + by + c = 0$ ,  $a^2 + b^2 \neq 0$ ,

$$p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

- **Perpendicular distance between two parallel lines :**

The perpendicular distance between two parallel lines  $ax + by + c = 0$  and  $ax + by + c' = 0$ ,

$$a^2 + b^2 \neq 0 \text{ is } p = \frac{|c - c'|}{\sqrt{a^2 + b^2}}. \text{ (where } c \neq c')$$

- If the lines  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$ ,  $(a_i^2 + b_i^2 \neq 0, i = 1, 2)$  intersect in a unique point, then  $l(a_1x + b_1y + c_1) + m(a_2x + b_2y + c_2) = 0$  is the equation of a line passing through the point of intersection of the lines,  $l^2 + m^2 \neq 0, l, m \in R$ .

- **Condition for two lines to be coincident, parallel, perpendicular or intersecting lines :**

Lines  $l_1 : a_1x + b_1y + c_1 = 0$ ,  $l_2 : a_2x + b_2y + c_2 = 0$  are given.

(1)  $l_1$  and  $l_2$  are coincident if  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ .

(2)  $l_1 \parallel l_2$  if  $\frac{a_1}{a_2} = \frac{b_1}{b_2}$  and  $\frac{a_1}{a_2} \neq \frac{c_1}{c_2}$ .

(3)  $l_1 \perp l_2$  if  $a_1a_2 + b_1b_2 = 0$ .

(4)  $l_1$  and  $l_2$  intersect each other if  $a_1b_2 \neq a_2b_1$  and coordinates of the point of intersection are

$$\left( \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \right).$$

• **Equations of the bisectors of the angle between two lines :**

Let the lines  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  be intersecting lines. Then the equations

of the bisectors of the angle between them are  $\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$ .

• **Concurrent lines :**

More than two distinct lines are said to be concurrent, if they intersect in a unique point. The common point of intersection is called the point of concurrence.

**The necessary and sufficient conditions for the lines**

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0 \quad a_i^2 + b_i^2 \neq 0, \quad i = 1, 2, 3 \quad \text{to be concurrent are,}$$

$$a_3x + b_3y + c_3 = 0$$

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0, \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} \neq 0, \begin{vmatrix} a_3 & b_3 \\ a_1 & b_1 \end{vmatrix} \neq 0, \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

**: Multiple Choice Questions :**

(1) If two lines out of the lines formed by  $ax^3 + bx^2y + cxy^2 + dy^3 = 0$  are mutually perpendicular,

then  $a^2 + d^2 + ac + bd = \dots\dots$

(A) 0

(B) 1

(C) -1

(D)  $ab + cd$

**Sol. :** Suppose  $ax^3 + bx^2y + cxy^2 + dy^3 = 0$  (1)

represents lines  $y = m_1x$ ,  $y = m_2x$  and  $y = m_3x$ , so that  $m_1m_2 = -1$ . (2)

Dividing (1) by  $x^3$  we get,  $d\left(\frac{y}{x}\right)^3 + c\left(\frac{y}{x}\right)^2 + b\left(\frac{y}{x}\right) + a = 0$  (3)

$\therefore$  The product of the roots of (3) is,  $m_1m_2m_3 = -\frac{a}{d}$

$\therefore (-1)m_3 = -\frac{a}{d}$  So,  $m_3 = \frac{a}{d}$  (From (2))

But  $m_3$  is a root of (3),

$$\therefore d\left(\frac{a}{d}\right)^3 + c\left(\frac{a}{d}\right)^2 + b\left(\frac{a}{d}\right) + a = 0$$

$$\therefore a^2 + ca + bd + d^2 = 0$$

$$\therefore a^2 + d^2 + ac + bd = 0$$

**Ans. : (A)**

- (2) If two adjacent sides of a rhombus in the first quadrant with origin as a vertex are given by  $y^2 - 6xy + 5x^2 = 0$ , then the value of the larger slope of the diagonal is .....

- (A)  $\frac{2+\sqrt{13}}{3}$  (B)  $\frac{2-\sqrt{13}}{3}$  (C)  $\frac{2\pm\sqrt{13}}{5}$  (D)  $\frac{2\pm\sqrt{13}}{9}$

**Sol. :**

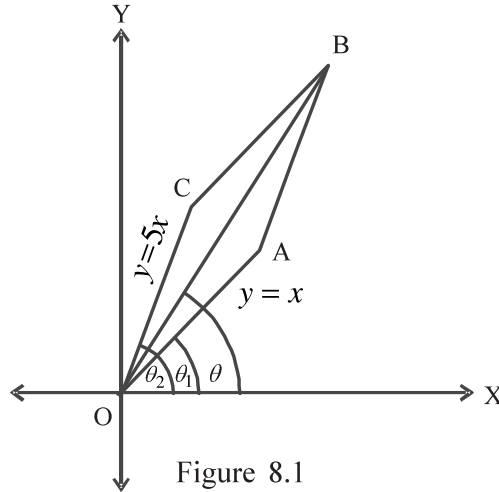


Figure 8.1

$$y^2 - 6xy + 5x^2 = (y - x)(y - 5x) = 0$$

$\therefore$  Adjacent sides are  $y = 5x$  and  $y = x$ .

The diagonals bisect the angle at the vertex in a rhombus.

$\therefore \theta - \theta_1 = \theta_2 - \theta$  as in the figure.

$$\therefore 2\theta = \theta_1 + \theta_2$$

$$\therefore \tan 2\theta = \tan(\theta_1 + \theta_2)$$

$$\therefore \frac{2\tan\theta}{1-\tan^2\theta} = \frac{\tan\theta_1 + \tan\theta_2}{1-\tan\theta_1 \tan\theta_2} = \frac{5+1}{1-5} \quad (\tan\theta_1 = m_1 = 5, \tan\theta_2 = m_2 = 1)$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{6}{-4} = \frac{-3}{2}$$

$$\therefore 4\tan\theta = -3 + 3\tan^2\theta$$

$$\therefore 3\tan^2\theta - 4\tan\theta - 3 = 0$$

$$\therefore \tan\theta = \frac{4 \pm \sqrt{16-4(3)(-3)}}{6} = \frac{4 \pm \sqrt{52}}{6} = \frac{2 \pm \sqrt{13}}{3}$$

$$\therefore \tan\theta = \frac{2+\sqrt{13}}{3} \quad (P(\theta) \text{ is in first quadrant})$$

Slope of the second diagonal will be negative.

**Ans. : (A)**

- (3) If the point  $P(x_1 + t(x_2 - x_1), y_1 + t(y_2 - y_1))$  divides the line segment joining  $A(x_1, y_1)$  and  $B(x_2, y_2)$  internally, then .....

- (A)  $t < 0$  (B)  $0 < t < 1$  (C)  $t > 1$  (D)  $t = 1$

**Sol. :**  $P(x_1 + t(x_2 - x_1), y_1 + t(y_2 - y_1)) = P(tx_2 + (1-t)x_1, ty_2 + (1-t)y_1)$

$$P \in \overleftrightarrow{AB}$$

But P divides the line segment connecting  $A(x_1, y_1)$  and  $B(x_2, y_2)$  internally,

$$\therefore P \in \overline{AB}$$

$$\therefore 0 < t < 1$$

**Ans. : (B)**

(4) The circumcentre of the triangle formed by the lines  $x + y = 0$ ,  $x - y = 0$  and  $x - 7 = 0$  is .....

- (A)  $\left(\frac{7}{2}, \frac{7}{2}\right)$  (B)  $(0, 7)$  (C)  $\left(\frac{7}{2}, 0\right)$  (D)  $(7, 0)$

**Sol. :** Lines  $x + y = 0$  and  $x - y = 0$  are perpendicular to each other.

The triangle formed by the lines  $x + y = 0$ ,  $x - y = 0$  and  $x - 7 = 0$  is a right triangle.

Solving  $x + y = 0$ ,  $x - y = 0$  and  $x - 7 = 0$ , we get the vertices of the triangle as

$O(0, 0)$ ,  $A(7, 7)$  and  $B(7, -7)$ . Also we get  $m\angle AOB = \frac{\pi}{2}$ .

The circumcentre of  $\triangle OAB$  = The midpoint of  $\overline{AB} = (7, 0)$

**Ans. : (D)**

(5) For a rectangle  $ABCD$ , the side  $\overline{AB}$  is parallel to the line  $y = x$ ; and the vertex A is on the line  $y = 1$ , the vertex B is on the line  $x = 2$  and the vertex D is on the line  $x = -2$ , then the locus of the point C is .....

- (A)  $y = 5$  (B) Y-axis (C)  $x = 5$  (D) X-axis

**Sol. :**

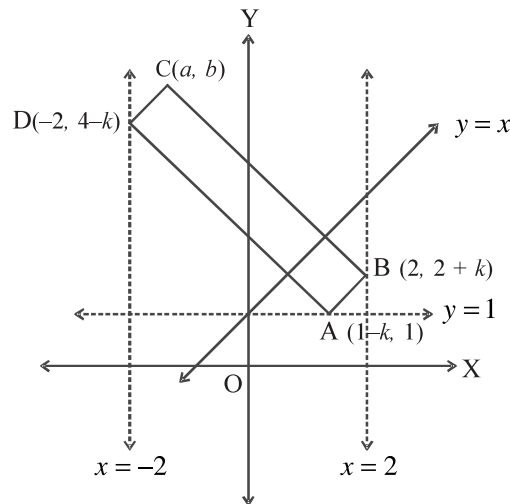


Figure 8.2

Here  $\overleftrightarrow{AB}$  is parallel to  $y = x$ . Hence let the equation of  $\overleftrightarrow{AB}$  be  $x - y + k = 0$ .

The point A is on the line  $y = 1$  and the point B is on the line  $x = 2$ .

Hence, we get  $A(1-k, 1)$ ,  $B(2, 2+k)$ .

$\overleftrightarrow{AD}$  is perpendicular to  $\overleftrightarrow{AB} : x - y + k = 0$ . So, the slope of  $\overleftrightarrow{AD} = -1$ .

$\therefore$  The equation of  $\overleftrightarrow{AD}$  is  $y - 1 = -1(x - 1 + k)$  i.e.  $y = -x + 2 - k$

But  $x$ -coordinate of  $D$  is  $-2$ . Hence  $y = 4 - k$

The coordinate of  $D$  are  $(-2, 4 - k)$ .

Suppose the coordinates of  $C$  are  $(a, b)$ .

Now midpoint of  $\overline{AC}$  = midpoint of  $\overline{BD}$

$$\therefore a + 1 - k = 2 - 2 \text{ and } b + 1 = 2 + k + 4 - k$$

$$\therefore a = k - 1 \text{ and } b = 5$$

$\therefore$  The required locus is the line  $y = 5$ .

**Ans. : (A)**

(6) The quadrilateral formed by pairs of lines  $xy + x + y + 1 = 0$  and  $xy + 3x + 3y + 9 = 0$  is .....

- (A) A rectangle (B) A square (C) A parallelogram (D) A rhombus

**Sol. :**

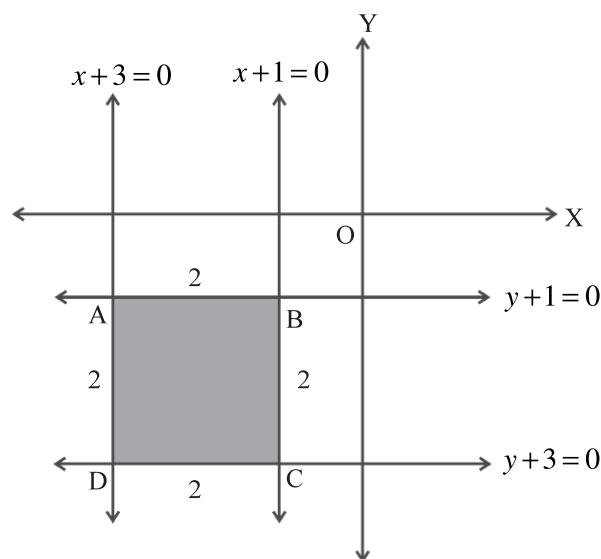


Figure 8.3

$$xy + x + y + 1 = 0$$

$$\therefore (x+1)(y+1) = 0$$

$$\therefore x+1=0 \text{ or } y+1=0$$

$$\text{Now, } xy + 3x + 3y + 9 = 0$$

$$\therefore (x+3)(y+3) = 0$$

$$\therefore x+3=0 \text{ or } y+3=0$$

The sides of the quadrilateral are  $x+1=0$ ,  $x+3=0$ ,  $y+1=0$ ,  $y+3=0$ .

$\therefore AB = BC = CD = DA = 2$  and sides are perpendicular in pairs.

$\therefore \square ABCD$  is a square.

**Ans. : (B)**

(7) The coordinates of the incentre of the triangle formed by the lines  $x=0$ ,  $y=0$  and  $3x+4y-12=0$  are .....

- (A)  $(\frac{1}{2}, \frac{1}{2})$  (B)  $(\frac{1}{2}, 1)$  (C)  $(1, \frac{1}{2})$  (D)  $(1, 1)$

Sol. :

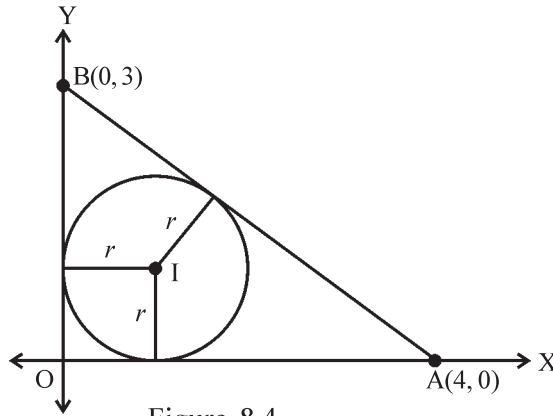


Figure 8.4

Let I be the incentre of the triangle formed by  $x=0$ ,  $y=0$  and  $3x+4y=12$ . From the figure, it is clear that the coordinates of I are  $(r, r)$ .

Now, the perpendicular distance of the centre  $(r, r)$  from the line  $3x+4y-12=0$  is equal to the radius  $r$ ,

$$\therefore r = \frac{|3r+4r-12|}{\sqrt{9+16}}$$

$$\therefore 7r-12=5r \quad \text{or} \quad 7r-12=-5r$$

$$\therefore 2r=12 \quad \text{or} \quad 7r+5r=12$$

$$\therefore r=6 \quad \text{or} \quad r=1$$

But  $r=6$  is not possible.

$((r, r)$  lies inside the triangle)

$$\therefore r=1$$

$\therefore$  The coordinates of the incentre =  $(1, 1)$

**Ans. : (D)**

**Second method :**  $OB = a = 3$ ,  $OA = b = 4$ ,  $AB = c = 5$

$$A(x_1, y_1) = (4, 0), B(x_2, y_2) = (0, 3), O(x_3, y_3) = (0, 0)$$

$$I = \left( \frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right) = \left( \frac{12}{12}, \frac{12}{12} \right) = (1, 1)$$

**Third method :** Incentre is  $(r, r)$ .

$$r = \frac{\Delta}{s} = \frac{6}{6} = 1 \quad \text{So, centre is } (1, 1).$$

**Fourth method :**  $x=0$  and  $y=0$  are mutually perpendicular. Hence a right triangle will be formed.

From the figure, the inradius of the incircle of a right angle triangle is  $r = \frac{OA + OB - AB}{2}$ .

Here  $OA = 3$ ,  $OB = 4$ ,  $AB = 5$ .

$$\therefore r = \frac{3+4-5}{2} = 1$$

$\therefore$  The coordinates of the incentre =  $(1, 1)$ .

- (8) If two vertices of a triangle are  $(5, -1)$  and  $(-2, 3)$  and the orthocentre is the origin, then the coordinates of the third vertex are .....

(A)  $(4, 7)$  (B)  $(-4, -7)$  (C)  $(2, -3)$  (D)  $(5, -1)$

**Sol. :**

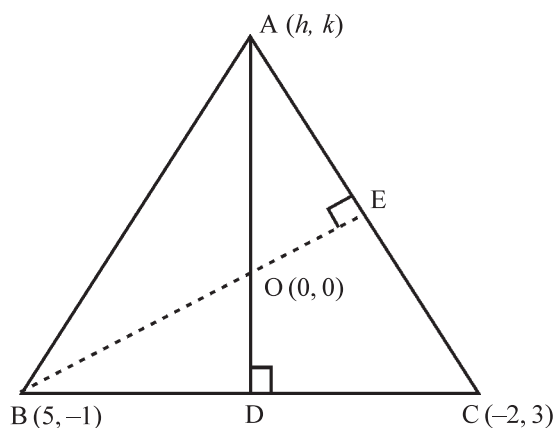


Figure 8.5

Suppose the third vertex is  $A(h, k)$ . Now,  $\overline{OA} \perp \overline{BC}$

$$\therefore \left( \frac{k-0}{h-0} \right) \left( \frac{4}{-7} \right) = -1 \quad \text{(Product of the slopes)}$$

$$\therefore 7h = 4k \quad (1)$$

$$\text{Also, } \overline{OB} \perp \overline{AC}. \quad \text{Hence } \left( \frac{k-3}{h+2} \right) \left( \frac{-1}{5} \right) = -1$$

$$\therefore 5h - k + 13 = 0 \quad (2)$$

Solving (1) and (2), we get  $h = -4$ ,  $k = -7$

$\therefore$  The coordinates of the third vertex are  $(-4, -7)$ . **Ans. : (B)**

- (9) If  $H\left(\frac{13}{2}, \frac{11}{2}\right)$  is the orthocentre and  $P\left(\frac{\sqrt{7}}{2}, \frac{\sqrt{5}}{2}\right)$  is the circumcentre of a triangle, then coordinates of the centroid are .....

(A)  $(13+2\sqrt{7}, 11)$  (B)  $(13, 11+2\sqrt{5})$

(C)  $\left(\frac{13+2\sqrt{7}}{6}, \frac{11+2\sqrt{5}}{6}\right)$  (D) None of these

**Sol. :** Here, circumcentre  $P\left(\frac{\sqrt{7}}{2}, \frac{\sqrt{5}}{2}\right)$  and orthocentre  $H\left(\frac{13}{2}, \frac{11}{2}\right)$ . Let the centroid be  $G(x, y)$ .

Centroid G divides  $\overline{PH}$  from P in the ratio 1:2.

$$\therefore \text{Coordinates G are} = \left( \frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1} \right)$$

$$= \left( \frac{\frac{1}{2} \left( \frac{13}{2} \right) + \frac{\sqrt{7}}{2}}{\frac{3}{2}}, \frac{\frac{1}{2} \left( \frac{11}{2} \right) + \frac{\sqrt{5}}{2}}{\frac{3}{2}} \right) = \left( \frac{13+2\sqrt{7}}{6}, \frac{11+2\sqrt{5}}{6} \right) \quad \text{Ans. : (C)}$$

(10) The distance of  $(1, 2)$  from the line  $3x - y - 5 = 0$  in the direction of the line  $x + 2y = 15$  is .....

- (A)  $2\sqrt{5}$  (B)  $4\sqrt{5}$  (C)  $\frac{2\sqrt{5}}{7}$  (D)  $\frac{4\sqrt{5}}{7}$

**Sol. :** The line in the direction of  $x + 2y - 15 = 0$  is the line parallel to it with general equation as  $x + 2y + k = 0$ .

Required line passes through  $(1, 2)$ . So,  $1 + 4 + k = 0$ . So,  $k = -5$ .

$\therefore$  Its equation is  $x + 2y - 5 = 0$ .

Now coordinates of the point of intersection of  $3x - y - 5 = 0$  and  $x + 2y - 5 = 0$  are  $\left(\frac{15}{7}, \frac{10}{7}\right)$ .

$\therefore$  Required distance is the distance between the points  $\left(\frac{15}{7}, \frac{10}{7}\right)$  and  $(1, 2)$ .

$$\text{It is} = \sqrt{\left(\frac{15}{7} - 1\right)^2 + \left(\frac{10}{7} - 2\right)^2} = \sqrt{\frac{64}{49} + \frac{16}{49}} = \frac{\sqrt{80}}{7} = \frac{4\sqrt{5}}{7} \quad \text{Ans. : (D)}$$

(11) Three lines which are represented by  $y^3 - 4x^2y = 0$ , .....

- (A) form an isosceles triangle (B) form an equilateral triangle  
(C) form a right triangle (D) do not form a triangle

**Sol. :**  $y^3 - 4x^2y = 0 \Rightarrow y(y^2 - 4x^2) = 0$

$$\therefore y(y - 2x)(y + 2x) = 0$$

$$\therefore y = 0, y - 2x = 0, y + 2x = 0$$

$\therefore y = 0, y = 2x, y = -2x$  are concurrent lines and their point of concurrence is  $(0, 0)$ .

$\therefore$  They do not form a triangle.

**Note :** A homogeneous cubic equation represents three lines passing through the origin. **Ans. : (D)**

(12) If  $\triangle ABC$  has circumcentre  $P(-3, 5)$  and orthocentre  $H(7, 11)$  and vertex A is  $(-1, 10)$ , then the coordinates of the midpoint of  $\overline{BC}$  are .....

- (A)  $\left(7, \frac{11}{2}\right)$  (B)  $\left(1, \frac{11}{2}\right)$  (C)  $\left(5, \frac{11}{2}\right)$  (D)  $\left(-3, \frac{11}{2}\right)$

**Sol. :**

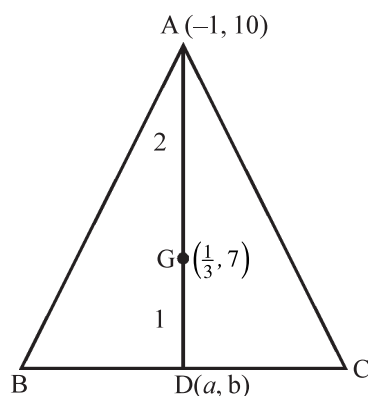


Figure 8.6

Given that circumcentre is  $P(-3, 5)$  and orthocentre is  $H(7, 11)$

Suppose the coordinates of the centroid  $G$  are  $(x, y)$ .

Now, the centroid  $G$  of  $\triangle ABC$ , divides  $\overline{PH}$  from  $P$  in the ratio 1:2.

$$\begin{aligned}\therefore \text{Coordinates of } G &= \left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right) \\ &= \left( \frac{(1)(7) + 2(-3)}{1+2}, \frac{(1)(11) + 2(5)}{1+2} \right) = \left( \frac{1}{3}, 7 \right)\end{aligned}$$

As shown in the figure if  $\overline{AD}$  is the median, then  $AG:GD = 2:1$ .

$\therefore$  G divides  $\overline{AD}$  from A in the ratio 2:1. Let  $D(a, b)$

$$\therefore \text{Coordinates of } G = \left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right),$$

$$\left( \frac{1}{3}, 7 \right) = \left( \frac{2a + 1(-1)}{2+1}, \frac{2(b) + 1(10)}{2+1} \right)$$

$$\therefore 2a - 1 = 1, 2b + 10 = 21 \quad \text{So, } a = 1, b = \frac{11}{2}. \quad \text{Hence } D\left(1, \frac{11}{2}\right) \quad \text{Ans. : (B)}$$

- (13)  $\overline{PQ}$  with negative slope passes through the point (2, 8) and intersects positive coordinate axes at points P and Q. The minimum value of  $OP + OQ$  as  $\overline{PQ}$  varies, is ..... (O is the origin.)

- (A) 10 (B) 18 (C) 2 (D) 8

Sol. :

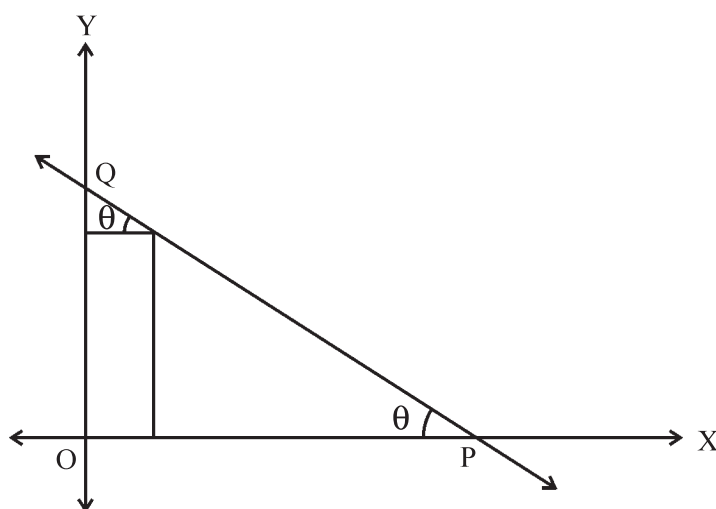


Figure 8.7

Let  $m\angle OPQ = \theta$

Then,  $OP = 2 + 8\cot\theta$  and  $OQ = 8 + 2\tan\theta$

$$\therefore OP + OQ = 10 + (8\cot\theta + 2\tan\theta)$$

$$\geq 10 + 8$$

(Using A.M.  $\geq$  G.M.)

$\therefore$  The minimum value of  $OP + OQ$  is 18.

**Note :** Here slope of  $\overline{PQ}$  is negative. So,  $\tan\theta > 0$  and  $\cot\theta > 0$ .

Ans. : (C)

- (14) If  $m(2x+3y+6) + n(3x+2y+9) = 0$  represents a family of lines, then the equation of the line from this family which is at minimum distance from the point  $P(2, 3)$  is .....

- (A)  $5x+3y-15=0$  (B)  $5x-3y+15=0$  (C)  $5x+3y+15=0$  (D)  $5x-3y-15=0$

**Sol. :** Solving  $2x+3y+6=0$  and  $3x+2y+9=0$  we get  $(-3, 0)$ .

$\therefore$  The point of intersection of these two lines is  $(-3, 0)$ .

$\therefore$  The equation of the line passing through  $(2, 3)$  and  $(-3, 0)$  is,

$$\begin{vmatrix} x & y & 1 \\ 2 & 3 & 1 \\ -3 & 0 & 1 \end{vmatrix} = 0$$

$$\therefore 3x-5y+9=0$$

$\therefore$  The line at minimum distance from  $P(2, 3)$  is the line through  $(-3, 0)$  and perpendicular to  $3x-5y+9=0$ .

$\therefore$  The equation of the line perpendicular to  $3x-5y+9=0$  is  $5x+3y+k=0$ . Also the required line passes through  $(-3, 0)$  hence  $k=15$ .

$\therefore$  The equation of the required line is  $5x+3y+15=0$ .

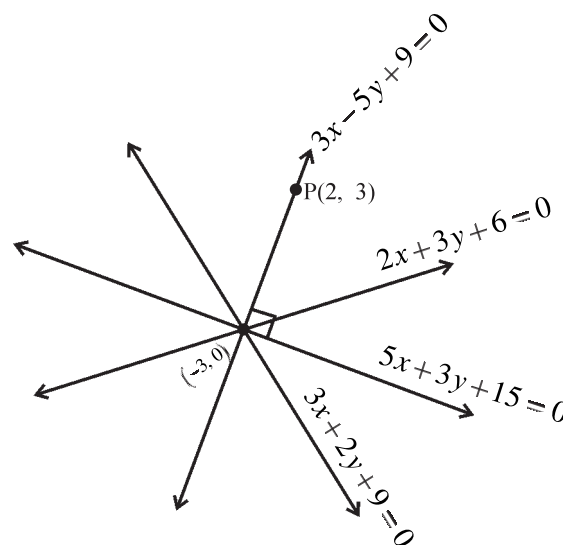


Figure 8.8

**Ans. : (C)**

- (15) If the equation of the base of an equilateral triangle is  $x+y=2$  and a vertex is  $(2, -1)$ , then the measure of a side of the triangle is .....

(A)  $\sqrt{\frac{1}{3}}$

(B)  $\sqrt{3}$

(C)  $\sqrt{\frac{2}{3}}$

(D)  $\sqrt{\frac{3}{2}}$

**Sol. :**

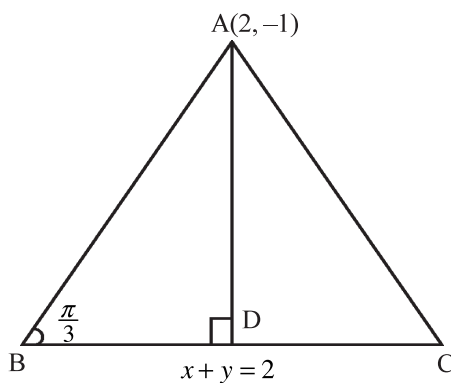


Figure 8.9

Given triangle is an equilateral triangle.  $(2, -1)$  does not lie on  $x+y=2$

$\therefore$  Altitude coincides with a median and  $m\angle ABC = \frac{\pi}{3}$

$$\text{Now, } AD = \frac{|2-1-2|}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

$\therefore \angle ADB$  is right angle.  $\overline{AD}$  is the median.

$$\text{So, } \frac{AD}{AB} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\therefore AB = AD \left( \frac{2}{\sqrt{3}} \right) = \frac{1}{\sqrt{2}} \left( \frac{2}{\sqrt{3}} \right) = \sqrt{\frac{2}{3}}$$

So, the length of a side of the equilateral triangle is  $\sqrt{\frac{2}{3}}$ .

**Ans. : (C)**

- (16) The equation of a diagonal of the rectangle formed by  $x^2 - 7x + 6 = 0$  and  $y^2 - 14y + 40 = 0$  is .....

(A)  $5x - 6y = 0$

(B)  $5x + 6y = 0$

(C)  $6x - 5y - 14 = 0$

(D)  $6x - 5y + 14 = 0$

**Sol. :**

$$x^2 - 7x + 6 = 0 \quad \text{Hence } (x-6)(x-1) = 0$$

$$\therefore x-6=0 \text{ and } x-1=0 \quad (1)$$

$$y^2 - 14y + 40 = 0 \quad \text{Hence } (y-4)(y-10) = 0$$

$$\therefore y-4=0 \text{ and } y-10=0 \quad (2)$$

Solving the equations in (1) and (2) the vertices are obtained as  $A(1, 4)$ ,  $B(6, 4)$ ,  $C(6, 10)$  and  $D(1, 10)$ .

The equation of the diagonal  $\overleftrightarrow{AC}$  is

$$y-4 = \frac{10-4}{6-1} (x-1) \text{ i.e. } 5y-20 = 6x-6$$

$$\therefore 6x-5y+14=0$$

$$\overleftrightarrow{BD}: y-4 = \frac{-6}{5} (x-6) \text{ i.e. } 6x+5y-56=0$$

**Ans. : (D)**

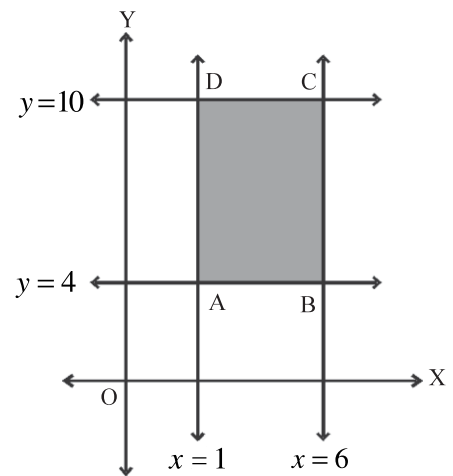


Figure 8.10

- (17) Area of the pentagon having vertices  $(4, 1)$ ,  $(3, 6)$ ,  $(-5, 1)$ ,  $(-3, -3)$  and  $(0, -3)$  is .....

(A) 56.5 sq. units

(B) 30 sq. units

(C) 93 sq. units

(D) 46.5 sq. units

**Sol. :**

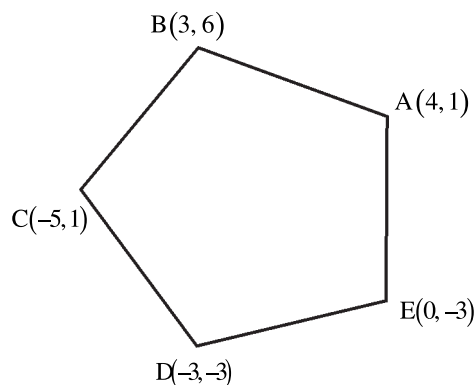


Figure 8.11

Area of the pentagon is given by

$$= \frac{1}{2} \left| \left\{ \begin{vmatrix} 4 & 1 \\ 3 & 6 \end{vmatrix} + \begin{vmatrix} 3 & 6 \\ -5 & 1 \end{vmatrix} + \begin{vmatrix} -5 & 1 \\ -3 & -3 \end{vmatrix} + \begin{vmatrix} -3 & -3 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} 0 & -3 \\ 4 & 1 \end{vmatrix} \right\} \right|$$

$$= \left| \frac{1}{2} \{ (24-3) + (3+30) + (15+3) + (9-0) + (0+12) \} \right|$$

$$= \left| \frac{1}{2} (21+33+18+9+12) \right| = \left| \frac{1}{2} (93) \right| = 46.5$$

$\therefore$  Area of pentagon = 46.5 sq. units

**Ans. : (D)**

(18) Points  $P(2, 3)$ ,  $Q(3, 5)$ ,  $R(7, 7)$  and  $S(4, 5)$  satisfy .....

- (A)  $P$ ,  $Q$ ,  $R$  and  $S$  are collinear (B)  $PQRS$  is a parallelopiped  
(C)  $S$  lies on  $\Delta PQR$  (D)  $S$  lies inside  $\Delta PQR$

**Sol. :** Centroid of  $\Delta PQR = \left( \frac{2+3+7}{3}, \frac{3+5+7}{3} \right) = (4, 5) = S$

Hence,  $S$  lies inside the  $\Delta PQR$ .  $P$ ,  $Q$ ,  $R$  are non-collinear.

**Ans. : (D)**

(19) If the angle between the lines  $\frac{x}{a} + \frac{y}{b} = 1$ ,  $\frac{x}{a} - \frac{y}{b} = 1$  has measure  $\alpha$ , then  $\tan \alpha = \dots$

- (A)  $\left| \frac{a^2+b^2}{a^2-b^2} \right|$  (B)  $\frac{2ab}{a^2+b^2}$  (C)  $\left| \frac{a^2-b^2}{a^2+b^2} \right|$  (D)  $\left| \frac{2ab}{a^2-b^2} \right|$

**Sol. :** Slope of the line  $\frac{x}{a} + \frac{y}{b} = 1$  is  $m_1 = \frac{-\frac{1}{a}}{\frac{1}{b}} = \frac{-b}{a}$  and slope of the line  $\frac{x}{a} - \frac{y}{b} = 1$  is  $m_2 = -\frac{\frac{1}{a}}{-\frac{1}{b}} = \frac{b}{a}$

$$\tan \alpha = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{\frac{-b}{a} - \frac{b}{a}}{1 + \left(\frac{-b}{a}\right)\left(\frac{b}{a}\right)} \right| = \left| \frac{2ab}{a^2 - b^2} \right|$$

**Ans. : (D)**

(20) The measure of the angle between lines  $x + y = 0$  and  $y = [\pi]$  is = .....

- (A)  $\frac{\pi}{4}$  (B)  $\frac{\pi}{3}$  (C)  $\frac{\pi}{2}$  (D) 0

**Sol. :** Slope of  $x + y = 0$  is  $m_1 = \frac{-1}{1} = -1$  and slope of  $y = [\pi] = 3$  is  $m_2 = 0$ .

$\therefore$  If the measure of the angle between the lines is  $\alpha$ , then

$$\tan \alpha = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{-1-0}{1 + (-1)0} \right| = 1. \text{ So, } \alpha = \frac{\pi}{4}.$$

**Ans. : (A)**

(21) If the distance of the point of intersection of a line drawn from  $(1, 2)$  and the line  $x + y + 3 = 0$  from the point  $(1, 2)$  is  $3\sqrt{2}$ , then the slope of the line drawn = .....

- (A)  $\frac{1}{\sqrt{3}}$  (B)  $\sqrt{3}$  (C) 1 (D)  $\frac{\sqrt{3}-1}{2}$

Sol. :

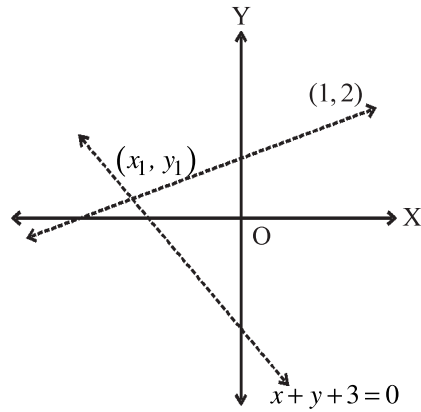


Figure 8.12

The equation of a line passing through  $(1, 2)$  is

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r. \quad (0 < \theta < 2\pi) \quad \text{i.e.} \quad \frac{x - 1}{\cos\theta} = \frac{y - 2}{\sin\theta} = r.$$

If the point at a distance  $3\sqrt{2}$  units from  $(1, 2)$  lies on this line, then the point,

$(1 + 3\sqrt{2} \cos\theta, 2 + 3\sqrt{2} \sin\theta)$  will be on the line  $x + y + 3 = 0$ .

$$\therefore \text{Then, } 1 + 3\sqrt{2} \cos\theta + 2 + 3\sqrt{2} \sin\theta + 3 = 0$$

$$\therefore \cos\theta + \sin\theta = -\sqrt{2}$$

$$\cos^2\theta + \sin^2\theta + 2\sin\theta \cos\theta = 2$$

$$\therefore \sin 2\theta = 1. \quad \text{So, } 2\theta = \frac{\pi}{2} \quad \text{or} \quad \frac{5\pi}{2}$$

$$\therefore \theta = \frac{\pi}{4} \quad \text{or} \quad \frac{5\pi}{4}$$

$$\text{Since } \cos\theta + \sin\theta = -\sqrt{2}, \quad \theta = \frac{5\pi}{4}$$

$$\text{The slope of the required line} = \tan\theta = \tan \frac{5\pi}{4} = 1$$

**Ans. : (C)**

**Second method :**

$$\cos\theta + \sin\theta = -\sqrt{2}$$

$$1 + \tan\theta = -\sqrt{2} \sec\theta$$

$$1 + 2\tan\theta + \tan^2\theta = 2(1 + \tan^2\theta)$$

$$\tan^2\theta - 2\tan\theta + 1 = 0$$

$$(\tan\theta - 1)^2 = 0$$

$$\tan\theta = 1 = m$$

**Third method :**

$$\cos\theta + \sin\theta = -\sqrt{2}$$

$$\cos\theta \cdot \frac{1}{\sqrt{2}} + \sin\theta \cdot \frac{1}{\sqrt{2}} = -1$$

$$\cos\left(\theta - \frac{\pi}{4}\right) = -1$$

$$\cos\left(\theta - \frac{\pi}{4}\right) = \cos\pi$$

$$\theta - \frac{\pi}{4} = (2n + 1)\pi$$

$$\theta - \frac{\pi}{4} = -\pi \quad (-\pi < \theta \leq \pi)$$

$$\theta = -\frac{3\pi}{4}. \quad \text{So, } m = \tan\left(-\frac{3\pi}{4}\right) = 1$$

(22) If the lengths of the perpendiculars drawn from origin to  $x \sec \theta + y \operatorname{cosec} \theta = a$  and  $x \cos \theta - y \sin \theta = a \cos 2\theta$  are  $p$  and  $p'$  respectively, then  $4p^2 + p'^2 = \dots$

- (A)  $a^2$  (B)  $3a^2$  (C)  $2a^2$  (D)  $4a^2$

**Sol. :**  $p = \frac{|0+0-a|}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}} = |a \sin \theta \cos \theta|$

$$p' = \frac{|0-0-a \cos 2\theta|}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = |a \cos 2\theta| \quad (1)$$

$$\therefore p = |a \sin \theta \cos \theta| = \left| \frac{a}{2} \sin 2\theta \right|. \quad \text{So, } 2p = |a \sin 2\theta|. \quad (2)$$

Adding squares of (1) and (2), we get  $4p^2 + (p')^2 = a^2$ . **Ans. : (A)**

(23) If the lines  $x + (a-1)y + 1 = 0$  and  $2x + a^2y - 1 = 0$  are mutually perpendicular, then .....

- (A)  $|a| = 2$  (B)  $0 < a < 1$  (C)  $-1 < a < 0$  (D)  $a = -1$

**Sol. :** Slopes of the given lines are  $-\frac{1}{a-1}$  and  $-\frac{2}{a^2}$ .

They are mutually perpendicular.

$$\therefore \left(-\frac{1}{a-1}\right) \left(-\frac{2}{a^2}\right) = -1$$

$$\therefore 2 = -(a-1)a^2 = -a^3 + a^2$$

$$\therefore a^3 - a^2 + 2 = 0$$

$$\therefore (a+1)(a^2 - 2a + 2) = 0$$

For any  $a \in R$ ,  $a^2 - 2a + 2 \neq 0$

$$\therefore a = -1$$

**Ans. : (D)**

**Note :** Verify that  $a = 0, 1$  are not possible.

(24) If the equations of the sides of a triangle are the lines  $x + y - 6 = 0$ ,  $7x + y - 6 = 0$  and  $x - 7y = 6$ , then coordinates of the incentre are .....

- (A)  $\left(\frac{12}{5}, \frac{6}{5}\right)$  (B)  $\left(\frac{3}{4}, -\frac{1}{7}\right)$  (C)  $(-1, 5)$  (D) None of these

**Sol. :** Perpendicular distance of  $\left(\frac{12}{5}, \frac{6}{5}\right)$  from each of the given lines is  $\frac{12}{5\sqrt{2}}$ .

$$\therefore \text{Incentre} = \left(\frac{12}{5}, \frac{6}{5}\right).$$

**Ans. : (A)**

(25) All the lines  $ax+by+c=0$  ( $a^2+b^2 \neq 0$ ,  $a, b, c \in \mathbb{R}$ ) for which  $a+b+c=0$  pass through a fixed point .....

- (A) (1,1) (B) (1,0) (C) (0,0) (D) (0,1)

**Sol. :** The equations of the lines are  $ax+by+c=0$ .

Using  $c=-a-b$ , we get  $ax+by-a-b=0$

$$\therefore a(x-1)+b(y-1)=0, \text{ where } a^2+b^2 \neq 0$$

$\therefore$  All the lines  $ax+by+c=0$  pass through the point (1,1). **Ans. : (A)**

**Note :**  $ax+by+c = a \cdot 1 + b \cdot 1 + c = 0$  passes through  $C(1,1)$  which is a fixed point.

(26) If the line  $(a^2-a-2)x+(a+1)y+a=0$  is parallel to X-axis, then  $a = \dots$

- (A) -1 (B) 2 (C) 3 (D) 0

**Sol. :**  $(a^2-a-2)x+(a+1)y+a=0$  is parallel to X-axis.

$\therefore$  The coefficient of  $x = 0$ , the coefficient of  $y = a+1 \neq 0$

$$\therefore a^2-a-2=0, a+1 \neq 0$$

$$\therefore (a-2)(a+1)=0, a+1 \neq 0$$

$\therefore a=2$  **Ans. : (B)**

(27) If the points (1,2) and (3,4) are on the same side of the line  $3x-5y+a=0$ , then .....

- (A)  $7 < a < 11$  (B)  $a < 0$  (C)  $a = 0$  (D)  $a \in \mathbb{R} - [7, 11]$

**Sol. :** Given that the points (1,2) and (3,4) are on the same side of the line  $3x-5y+a=0$ .

$\therefore 3-10+a$  and  $9-20+a$  must have same sign.

$\therefore a-7$  and  $a-11$  must have same sign.

$\therefore a-7 > 0$  and  $a-11 > 0$  or  $a-7 < 0$  and  $a-11 < 0$

i.e.  $a > 7, a > 11$  or  $a < 7, a < 11$

$\therefore a > 11$  or  $a < 7$ . **Ans. : (D)**

(28) If the distance between the lines  $5x+12y-1=0$  and  $10x+24y+k=0$  is 2, then  $k = \dots$

- (A) -54 (B) 50 (C) -54, 50 (D) 53

**Sol. :**  $5x+12y-1=0$  and  $5x+12y+\frac{k}{2}=0$  are parallel lines.

The distance between them is  $\frac{|\frac{k}{2}+1|}{\sqrt{25+144}}=2$ .

$\therefore |k+2|=52$ . So,  $k=50, -54$  **Ans. : (A), (B) (C)**

- (29) If the lines  $x + 2ay + a = 0$ ,  $x + 3by + b = 0$  and  $x + 4cy + c = 0$  are concurrent then  $a, b, c$  are in ..... . ( $a + c \neq 0$ )
- (A) Arithmetic progression (B) Geometric progression  
(C) Harmonic progression (D) None of these

**Sol. :** Given lines are concurrent.

$$\therefore \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$$\therefore -bc + 2a(b - c) + a(4c - 3b) = 0$$

$$\therefore -bc + 2ac + (-ab) = 0$$

$$\therefore 2ac = b(a + c)$$

$$\therefore b = \frac{2ac}{a + c}$$

$\therefore a, b, c$  are in harmonic progression.

**Ans. : (C)**

- (30) The equation of the line which is parallel to X-axis and intersects the curve  $y = \sqrt{x}$  at an angle of measure  $\frac{\pi}{4}$  is .....

- (A)  $x = \frac{1}{4}$  (B)  $y = \frac{1}{4}$  (C)  $y = \frac{1}{2}$  (D)  $y = 1$

**Sol. :** The equation of the line parallel to X-axis is  $y = k$ . ( $k > 0$  because  $\sqrt{x} > 0$ )

This line intersects the curve  $y = \sqrt{x}$ . So,  $x = k^2$

The coordinates of the point of intersection are  $(k^2, k)$ .

Now,  $y = \sqrt{x}$ . So,  $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

At  $(k^2, k)$ ,  $\frac{dy}{dx} = \frac{1}{2k} = \tan \frac{\pi}{4} = 1$ . So,  $k = \frac{1}{2}$ .

[line intersects the curve at an angle of measure  $\frac{\pi}{4}$ ]

$\therefore$  The equation of required line is  $y = \frac{1}{2}$ .

**Ans. : (C)**

- (31) If two vertices of an equilateral triangle are  $A(-a, 0)$  and  $B(a, 0)$ ,  $a > 0$ , and the third vertex  $C$  lies above x-axis, then the equation of the circumcircle of  $\triangle ABC$  is ..... . [Online April 22, 2013]

- (A)  $3x^2 + 3y^2 - 2\sqrt{3}ay = 3a^2$  (B)  $3x^2 + 3y^2 - 2ay = 3a^2$   
(C)  $x^2 + y^2 - 2ay = a^2$  (D)  $x^2 + y^2 - \sqrt{3}ay = a^2$

**Sol. :** Let  $C$  be  $(x, y)$ .

Now,  $CA^2 = CB^2 = AB^2$

$$\Rightarrow (x + a)^2 + y^2 = (x - a)^2 + y^2 = (2a)^2$$

$$\Rightarrow x^2 + 2ax + a^2 + y^2 = 4a^2 \quad \dots(i)$$

$$\text{and } x^2 - 2ax + a^2 + y^2 = 4a^2 \quad \dots(ii)$$

From (i) and (ii),  $x = 0$  and  $y = \pm\sqrt{3}a$

Since point  $C(x, y)$  lies above the x-axis and  $a > 0$ .

Hence  $y = \sqrt{3}a$

$\therefore C$  is  $(0, \sqrt{3}a)$ .

Let the equation of circumcircle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Since points  $A(-a, 0)$ ,  $B(a, 0)$  and  $C(0, \sqrt{3}a)$  lie on the circle, therefore,

$$a^2 - 2ga + c = 0 \quad \dots(\text{iii})$$

$$a^2 + 2ga + c = 0 \quad \dots(\text{iv})$$

$$\text{and } 3a^2 + 2\sqrt{3}a + c = 0 \quad \dots(\text{v})$$

From (iii), (iv) and (v)

$$g = 0, \quad c = -a^2, \quad f = -\frac{a}{\sqrt{3}}$$

Hence the equation of the circumcircle is

$$x^2 + y^2 - \frac{2a}{\sqrt{3}}y - a^2 = 0$$

$$\Rightarrow x^2 + y^2 - \frac{2\sqrt{3}ay}{3} - a^2 = 0$$

$$\Rightarrow 3x^2 + 3y^2 - 2\sqrt{3}ay = 3a^2$$

**Ans. : (A)**

(32) If  $A(1, 3)$  and  $C(7, 5)$  are opposite vertices of a square, then the equation of the side passing through A is .....

(A)  $x + 2y - 7 = 0$  or  $2x - y + 1 = 0$

(B)  $x - 2y + 5 = 0$

(C)  $2x + y - 5 = 0$

(D) None of these

**Sol. :**

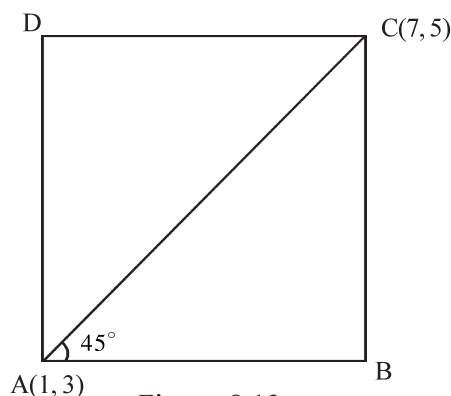


Figure 8.13

Let the slope of the side of the square passing through A be  $m$ .

$$\text{Slope of } \overleftrightarrow{AC} = \frac{5-3}{7-1} = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \tan \frac{\pi}{4} = \left| \frac{m - \frac{1}{3}}{1 + \frac{m}{3}} \right|. \quad \text{Hence } 1 = \left| \frac{3m-1}{m+3} \right|$$

$$\therefore 3m-1 = m+3 \quad \text{or} \quad 3m-1 = -m-3$$

$$\therefore 2m = 4 \quad \text{or} \quad 4m = -2$$

$$\therefore m = 2 \quad \text{or} \quad m = -\frac{1}{2}$$

$$\therefore \text{The equation of } \overleftrightarrow{AD} \text{ is } y-3 = 2(x-1). \quad \text{Hence } 2x-y+1=0$$

$$\text{The equation of } \overleftrightarrow{AB} \text{ is } y-3 = -\frac{1}{2}(x-1). \quad \text{Hence } x+2y-7=0 \quad \text{Ans. : (A)}$$

(33) If  $p$  and  $p'$  are the lengths of perpendiculars drawn from the origin to lines  $x \sec \theta + y \csc \theta = 2a$

and  $x \cos \theta + y \sin \theta = a \cos 2\theta$  respectively, then  $\left( \frac{p}{p'} + \frac{p'}{p} \right)^2 = \dots\dots$

(A)  $4 \cos^2 4\theta$

(B)  $4 \csc^2 4\theta$

(C)  $4 \sin^2 4\theta$

(D)  $4 \cot^2 4\theta$

**Sol. :** Here,  $p = \frac{|0+0-2a|}{\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta}} = |2a \sin \theta \cos \theta| = |a \sin 2\theta|$

and  $p' = \frac{|0+0-a \cos 2\theta|}{\sqrt{\cos^2 \theta + \sin^2 \theta}} = |a \cos 2\theta|$

$$\left(\frac{p}{p'} + \frac{p'}{p}\right)^2 = (\tan 2\theta + \cot 2\theta)^2 = \tan^2 2\theta + \cot^2 2\theta + 2$$

$$= \frac{\sin^2 2\theta}{\cos^2 2\theta} + \frac{\cos^2 2\theta}{\sin^2 2\theta} + 2 = \frac{1}{\sin^2 2\theta \cos^2 2\theta} = \frac{4}{\sin^2 4\theta} = 4 \operatorname{cosec}^2 4\theta$$

**Ans. : (B)**

(34) For which of the following value of  $\alpha = \dots$ , the slope of the line  $x \cos \alpha + y \sin \alpha = p$  (where  $p \neq 0$ ) is  $\sqrt{3}$ .

- (A)  $-\frac{\pi}{3}$  (B)  $\frac{\pi}{3}$  (C)  $-\frac{5\pi}{6}$  (D)  $\frac{5\pi}{6}$

**Sol. :** The slope of the line  $x \cos \alpha + y \sin \alpha = p$  is  $-\frac{\cos \alpha}{\sin \alpha} = \sqrt{3}$ .

$$\therefore \tan \alpha = -\frac{1}{\sqrt{3}}$$

$$\therefore \tan \alpha = \tan\left(-\frac{\pi}{6}\right) = \tan\left(\pi - \left(\frac{\pi}{6}\right)\right)$$

$$\therefore \alpha = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

**Ans. : (D)**

(35) To which point should the origin be shifted so that the coordinates of the point  $(6, -1)$  become  $(8, -4)$ ?

- (A)  $(2, -3)$  (B)  $(-3, 2)$  (C)  $(3, -2)$  (D)  $(-2, 3)$

**Sol. :** Suppose by shifting the origin  $O(0, 0)$  to  $O'(h, k)$  the coordinates  $(6, -1)$  transform to  $(8, -4)$ .

$$\text{Thus, } (x, y) = (6, -1), (x', y') = (8, -4)$$

$$\text{Now, } x = x' + h, y = y' + k$$

$$\therefore 6 = 8 + h \text{ and } -1 = -4 + k$$

$$\therefore h = -2 \text{ and } k = 3$$

$$\therefore \text{The required point is } (h, k) = (-2, 3)$$

**Ans. : (D)**

(36) If we shift the origin to  $(-2, 3)$  then ..... will become  $(3, -2)$ .

- (A)  $(-1, 1)$  (B)  $(1, 1)$  (C)  $(1, -1)$  (D)  $(-1, -1)$

**Sol. :** Here  $(h, k) = (-2, 3), (x', y') = (3, -2)$

$$\text{Now } x = x' + h, y = y' + k$$

$$\therefore x = 3 - 2 = 1, y = -2 + 3 = 1$$

$$\therefore \text{Coordinates of the required point are } (1, 1).$$

**Ans. : (B)**

(37) The slopes of the lines making an angle of measure  $\frac{\pi}{6}$  with Y-axis are .....

- (A)  $\pm \frac{1}{\sqrt{3}}$  (B)  $\pm \sqrt{3}$  (C)  $-\sqrt{3}$  (D)  $\sqrt{3}$

**Sol. :** Given that the line makes angle of measure  $\frac{\pi}{6}$  with Y-axis. Hence the measure of the angle with positive X-axis is  $\frac{\pi}{3}$  or  $\frac{2\pi}{3}$ .

$\therefore$  The slope is  $\tan \frac{\pi}{3} = \sqrt{3}$  or  $\tan \frac{2\pi}{3} = -\sqrt{3}$ .

$\therefore$  The slopes are  $\pm \sqrt{3}$

**Ans. : (B), (C), (D)**

(38) The measure of the angle between the lines  $\{(x, 0) | x \in R\}$  and  $\{(0, y) | y \in R\}$  is .....

- (A)  $\frac{\pi}{2}$  (B)  $-\frac{\pi}{2}$  (C) 0 (D) 3

**Sol. :**  $\{(x, 0) | x \in R\} = \text{X-axis.}$

$\{(0, y) | y \in R\} = \text{Y-axis.}$

The angle between the given lines has measure  $\frac{\pi}{2}$ .

**Ans. : (A)**

(39) The measure of the angle between the lines  $x = y$  and  $y = 0$  is .....

- (A)  $\frac{\pi}{3}$  (B)  $\frac{\pi}{6}$  (C)  $\frac{\pi}{2}$  (D)  $\frac{\pi}{4}$

**Sol. :** Slope of the line  $x = y$  is  $m_1 = 1$  and the slope of the line  $y = 0$  is  $m_2 = 0$ .

$\therefore \tan \alpha = 1$

$\therefore \alpha = \frac{\pi}{4}$

**Ans. : (D)**

(40) If the midpoint of the line segment cut off on the axes is  $\left(\frac{3}{2}, \frac{5}{2}\right)$ , then the equation of the line is .....

- (A)  $3x + 5y - 15 = 0$  (B)  $5x + 3y - 15 = 0$  (C)  $3x + 5y + 15 = 0$  (D)  $5x + 3y + 15 = 0$

**Sol. :**

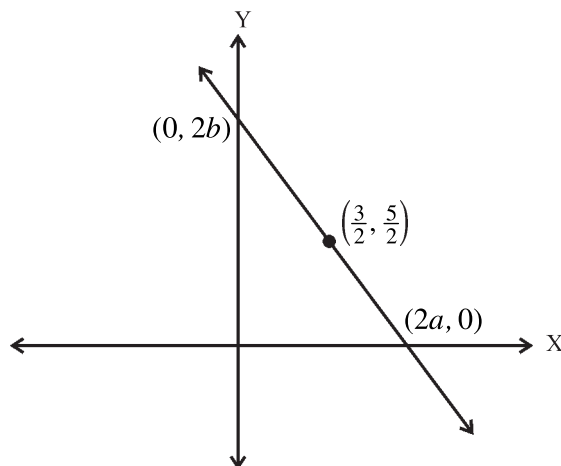


Figure 8.14

If the midpoint of the line segment joining  $(2a, 0)$  and  $(0, 2b)$  is  $(a, b)$ , then the equation of the line is  $\frac{x}{2a} + \frac{y}{2b} = 1$ .

$$\therefore bx + ay - 2ab = 0$$

$$\therefore \text{The equation of required line is } \frac{5}{2}x + \frac{3}{2}y - 2\left(\frac{5}{2}\right)\left(\frac{3}{2}\right) = 0.$$

$$\therefore \text{The equation of required line is } 5x + 3y - 15 = 0.$$

**Ans. : (B)**

- (41) The orthocentre of the triangle formed by the lines  $4x - 7y + 10 = 0$ ,  $x + y = 5$ ,  $7x + 4y = 15$  is .....

- (A)  $(-1, -2)$  (B)  $(-1, 2)$  (C)  $(1, 2)$  (D)  $(1, -2)$

**Sol. :** The sides of the triangle are

$$4x - 7y + 10 = 0, x + y = 5 \text{ and } 7x + 4y = 15.$$

Here the lines  $4x - 7y + 10 = 0$  and  $7x + 4y - 15 = 0$  are mutually perpendicular.

$\therefore$  The triangle is a right triangle.

$\therefore$  The orthocentre is the vertex at which right angle is formed. Here it is the point of intersection of  $7x + 4y - 15 = 0$  and  $4x - 7y + 10 = 0$ .

Solving these equations we get orthocentre =  $(1, 2)$ .

**Ans. : (C)**

- (42) If the equation of the line containing one side of an equilateral triangle is  $\sqrt{3}x + y = 2$  and  $(0, -1)$  is a vertex, then the measure of a side of the triangle is .....

- (A)  $\frac{\sqrt{3}}{2}$  (B)  $\frac{2}{\sqrt{3}}$  (C)  $\sqrt{3}$  (D)  $2\sqrt{3}$

**Sol. :**

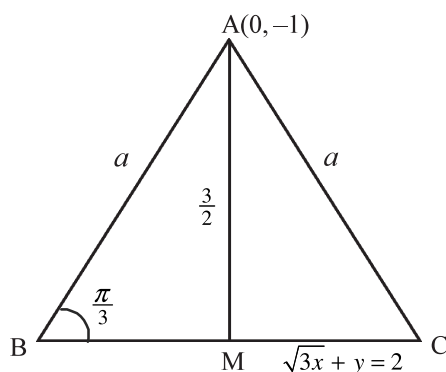


Figure 8.15

Clearly  $(0, -1)$  is not on the line  $\sqrt{3}x + y = 2$ .

Length of the perpendicular drawn from  $(0, -1)$  to  $\sqrt{3}x + y = 2$  is  $\frac{|\sqrt{3}(0) + (-1) - 2|}{\sqrt{3+1}} = \frac{3}{2}$

If the measure of a side is  $a$ , then

$$\therefore \left(\frac{a}{2}\right)^2 + \left(\frac{3}{2}\right)^2 = a^2$$

$$\therefore \frac{a^2}{4} + \frac{9}{4} = a^2$$

$$\therefore a^2 = 3$$

$$\therefore a = \sqrt{3}.$$

**Second method :**

$$a \sin \frac{\pi}{3} = \frac{3}{2}$$

$$\therefore \frac{\sqrt{3} \cdot a}{2} = \frac{3}{2}$$

$$\therefore a = \sqrt{3}$$

**Ans. : (C)**

- (43) Y-intercept of the perpendicular bisector of the line segment joining  $P(1, 4)$  and  $Q(k, 3)$  is  $-4$ , then a possible value of  $k$  is .....

(A)  $-4$  (B)  $1$  (C)  $2$  (D)  $-2$

**Sol. :** Let  $\overleftrightarrow{AB}$  be the perpendicular bisector of  $\overline{PQ}$ .

$$\therefore \text{The slope of } \overline{PQ} \text{ is } \frac{3-4}{k-1} = \frac{-1}{k-1}$$

$$\therefore \text{The slope of } \overleftrightarrow{AB} = k - 1$$

If R is the midpoint of  $\overline{PQ}$ , then its

coordinates are  $\left(\frac{k+1}{2}, \frac{3+4}{2}\right)$  i.e.  $\left(\frac{k+1}{2}, \frac{7}{2}\right)$ .

$$\therefore \text{The equation of } \overleftrightarrow{AB} \text{ is } y - \frac{7}{2} = (k-1)\left(x - \frac{k+1}{2}\right)$$

Since  $(0, -4)$  is on  $\overleftrightarrow{AB}$ , we get

$$\therefore -4 - \frac{7}{2} = (k-1)\left(0 - \frac{k+1}{2}\right)$$

$$\therefore \frac{-15}{2} = -\frac{k^2-1}{2}. \quad \text{So, } k^2 - 1 = 15$$

$$\therefore k^2 = 16. \quad \text{So, } k = \pm 4$$

**Ans. : (A)**

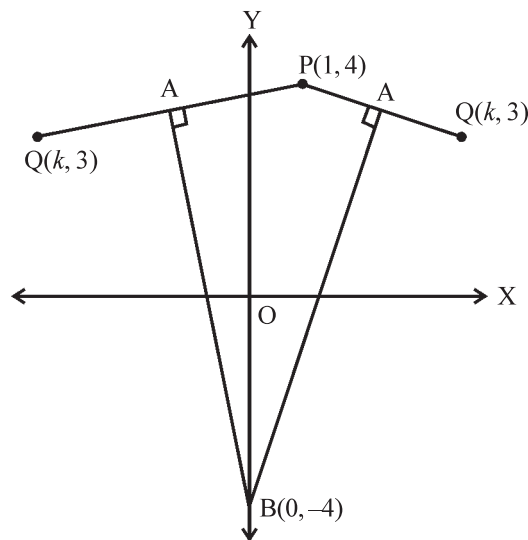


Figure 8.16

- (44) A square of side  $a$  lies in the upper half plane of X-axis and one of its vertices is at the origin. If the line containing a side passing through the origin makes an angle of measure  $\alpha$  with positive X-axis, then the equation of the diagonal not passing through the origin is ..... .  $\left(0 < \alpha < \frac{\pi}{4}\right)$
- (A)  $y(\cos \alpha + \sin \alpha) + x(\sin \alpha - \cos \alpha) = a$  (B)  $y(\cos \alpha + \sin \alpha) + x(\sin \alpha + \cos \alpha) = a$
- (C)  $y(\cos \alpha + \sin \alpha) + x(\cos \alpha - \sin \alpha) = a$  (D)  $y(\cos \alpha - \sin \alpha) - x(\sin \alpha - \cos \alpha) = a$

**Sol. :**

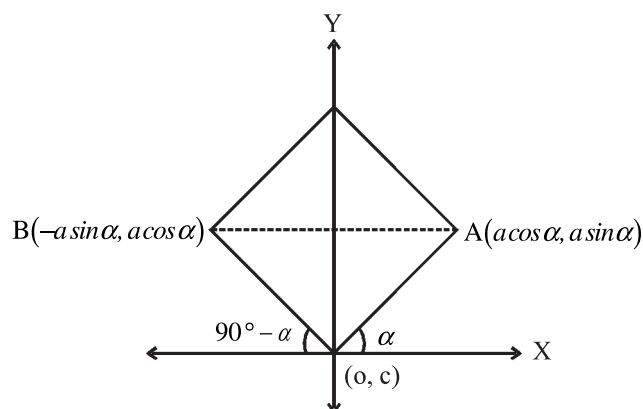


Figure 8.17

The coordinates of A and B are shown in the figure.

∴ The equation of the diagonal  $\overleftrightarrow{AB}$  is,

$$y - a \sin \alpha = \frac{a \cos \alpha - a \sin \alpha}{-a \sin \alpha - a \cos \alpha} (x - a \cos \alpha)$$

$$\therefore y - a \sin \alpha = -\frac{\cos \alpha - \sin \alpha}{\cos \alpha + \sin \alpha} (x - a \cos \alpha)$$

$$\therefore (y - a \sin \alpha)(\cos \alpha + \sin \alpha) = -(\cos \alpha - \sin \alpha)x + a \cos \alpha (\cos \alpha - \sin \alpha)$$

$$\therefore x(\cos \alpha - \sin \alpha) + y(\cos \alpha + \sin \alpha) = a \quad \text{Ans. : (C)}$$

(45) A ray passing through  $(-2, 3)$  is reflected from a point on X-axis. The reflected ray passes through  $(4, 2)$ . The coordinates of the point on X-axis are .....

- (A)  $(-\frac{3}{5}, 0)$       (B)  $(-\frac{8}{5}, 0)$       (C)  $(\frac{13}{5}, 0)$       (D)  $(\frac{8}{5}, 0)$

Sol. :

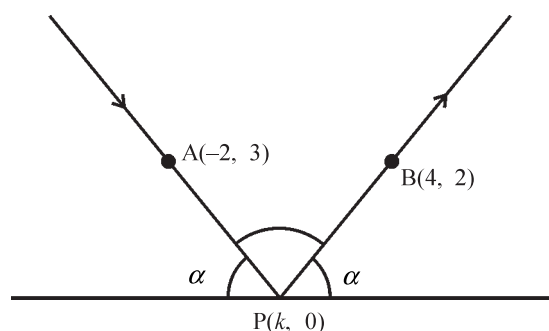


Figure 8.18

Suppose  $A(-2, 3)$ ,  $B(4, 2)$  and the point on X-axis is  $P(k, 0)$ .

If  $\overline{PB}$  makes an angle of measure  $\alpha$  with positive X-axis, then the slope of  $\overline{PB} = \tan \alpha$ .

$$\therefore \tan \alpha = \text{Slope of } \overleftrightarrow{PB} = \frac{2-0}{4-k} \quad (1)$$

Clearly, the measure of the angle made by  $\overline{AP}$  with the positive X-axis is  $\pi - \alpha$ . Hence the slope of  $\overline{AP} = \tan(\pi - \alpha)$ .

$$\therefore \tan(\pi - \alpha) = \text{Slope of } \overline{AP} = \frac{3-0}{-2-k} = \frac{-3}{k+2}$$

$$\therefore -\tan \alpha = \frac{-3}{k+2} \quad (2)$$

∴ From (1) and (2),

$$\frac{-2}{4-k} = \frac{-3}{k+2}$$

$$\therefore 2k + 4 = 12 - 3k$$

$$\therefore 5k = 8. \quad \text{So, } k = \frac{8}{5}$$

$$\therefore \text{The required point on X-axis is } P\left(\frac{8}{5}, 0\right). \quad \text{Ans. : (D)}$$

- (46) The measure of the angle between the lines  $x \cos \beta + y \sin \beta = \frac{5\pi}{2}$  and  $x \cos \alpha + y \sin \alpha = \frac{\pi}{2}$  is .....

(A)  $-\alpha - \beta$  (B)  $\alpha + \beta$  (C)  $|\alpha + \beta|$  (D)  $|\beta - \alpha|$

**Sol. :** Slope  $m_1$  of the line  $x \cos \beta + y \sin \beta = \frac{5\pi}{2}$  is  $-\cot \beta$ .

Slope  $m_2$  of the line  $x \cos \alpha + y \sin \alpha = \frac{\pi}{2}$  is  $-\cot \alpha$ .

If the measure of the angle between the two lines is  $\theta$ , then

$$\begin{aligned} \tan \theta &= \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \\ &= \left| \frac{-\cot \beta + \cot \alpha}{1 + \cot \alpha \cot \beta} \right| \\ &= \left| \frac{\tan \beta - \tan \alpha}{1 + \tan \alpha \tan \beta} \right| \end{aligned}$$

$$\tan \theta = \pm \tan(\beta - \alpha)$$

$$\therefore \theta = \pm(\beta - \alpha). \quad \text{So, } \theta = |\beta - \alpha| \quad \text{Ans. : (D)}$$

- (47) If the parametric coordinates of a point are  $(\tan \theta - \sin \theta, \tan \theta + \sin \theta)$ , then the locus of the point is .....

(A)  $x^2 - y^2 = 16xy$  (B)  $(x^2 - y^2)^2 = 16xy$   
(C)  $(x^2 - y^2)^2 = 12xy$  (D)  $x^2 - y^2 = 12xy$

**Sol. :** If the coordinates of the point are  $(x, y)$ , then

$$\therefore (x, y) = (\tan \theta - \sin \theta, \tan \theta + \sin \theta)$$

$$\therefore x = \tan \theta - \sin \theta, \quad y = \tan \theta + \sin \theta$$

$$\therefore \tan \theta = \frac{x + y}{2}, \quad \sin \theta = \frac{y - x}{2}$$

$$\cot \theta = \frac{2}{x + y}, \quad \operatorname{cosec} \theta = \frac{2}{y - x}$$

$$\operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$\therefore \left( \frac{2}{y - x} \right)^2 - \left( \frac{2}{x + y} \right)^2 = 1$$

$$\therefore 4(x + y)^2 - 4(y - x)^2 = (y - x)^2 (x + y)^2$$

$$\therefore 4[x^2 + 2xy + y^2 - y^2 + 2xy - x^2] = (x^2 - y^2)^2$$

$$\therefore 16xy = (x^2 - y^2)^2 \quad \text{Ans. : (B)}$$

(48) If the reflection of a point  $(3\sqrt{3}, 5)$  relative to a line is  $(5\sqrt{3}, 7)$ , then the equation of line is .....

(A)  $3\sqrt{3}x + y + 18 = 0$

(B)  $\sqrt{3}x + y + 18 = 0$

(C)  $\sqrt{3}x + y - 18 = 0$

(D)  $3\sqrt{3}x + y - 18 = 0$

**Sol. :**

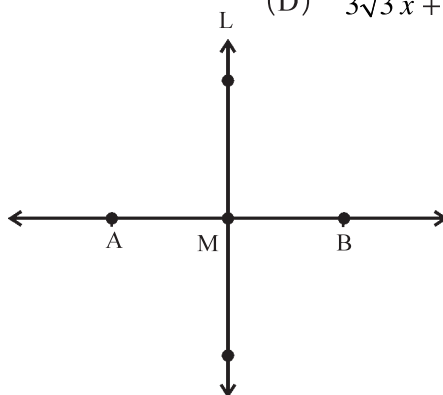


Figure 8.19

Let the reflection of  $A(3\sqrt{3}, 5)$  be  $B(5\sqrt{3}, 7)$  and M be the midpoint of  $\overline{AB}$ .

$\therefore$  The coordinates of M  $= \left( \frac{3\sqrt{3} + 5\sqrt{3}}{2}, \frac{5 + 7}{2} \right) = (4\sqrt{3}, 6)$

Now the slope of  $\overline{AB} = \frac{7 - 5}{5\sqrt{3} - 3\sqrt{3}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore$  The required line L is perpendicular to  $\overline{AB}$  and it passes through M. Thus, slope of L is  $-\sqrt{3}$ . and L passes through  $M(4\sqrt{3}, 6)$ . Hence the equation of L is,

$$y - 6 = -\sqrt{3}(x - 4\sqrt{3})$$

$\therefore y - 6 = -\sqrt{3}x + 12$

$\therefore \sqrt{3}x + y - 18 = 0$

**Ans. : (C)**

(49) The coordinates of incentre of  $\triangle ABC$ , where  $A(-3, -2)$ ,  $B(5, 0)$  and  $C(-\frac{1}{3}, \frac{13}{3})$  are .....

(A)  $(1, \frac{1}{2})$

(B)  $(\frac{1}{2}, 1)$

(C)  $(\frac{5}{3}, \frac{7}{3})$

(D)  $(2, \frac{3}{2})$

**Sol. :** Let  $A(-3, -2)$ ,  $B(5, 0)$  and  $C(-\frac{1}{3}, \frac{13}{3})$ .

$$\therefore a = BC = \sqrt{\left(5 + \frac{1}{3}\right)^2 + \left(0 - \frac{13}{3}\right)^2} = \sqrt{\frac{425}{9}} = \frac{5}{3}\sqrt{17}$$

$$b = CA = \sqrt{\left(-\frac{1}{3} + 3\right)^2 + \left(\frac{13}{3} + 2\right)^2} = \sqrt{\frac{425}{9}} = \frac{5}{3}\sqrt{17}$$

$$c = AB = \sqrt{(-3-5)^2 + (-2-0)^2} = \sqrt{68} = 2\sqrt{17}$$

If the incentre of  $\triangle ABC$  is I,

$$\begin{aligned} \text{The coordinates of I} &= \left( \frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c} \right) \\ &= \left( \frac{\frac{5}{3}\sqrt{17}(-3) + \frac{5}{3}\sqrt{17}(5) + 2\sqrt{17}\left(-\frac{1}{3}\right)}{\frac{5}{3}\sqrt{17} + \frac{5}{3}\sqrt{17} + 2\sqrt{17}}, \frac{\frac{5}{3}\sqrt{17}(-2) + \frac{5}{3}\sqrt{17}(0) + 2\sqrt{17}\left(\frac{13}{3}\right)}{\frac{5}{3}\sqrt{17} + \frac{5}{3}\sqrt{17} + 2\sqrt{17}} \right) \\ &= \left( \frac{\frac{5}{3}(-3) + \frac{5}{3}(5) - \frac{2}{3}}{\frac{5}{3} + \frac{5}{3} + 2}, \frac{\frac{5}{3}(-2) + \frac{5}{3}(0) + 2\left(\frac{13}{3}\right)}{\frac{5}{3} + \frac{5}{3} + 2} \right) = \left( \frac{-15 + 25 - 2}{5 + 5 + 6}, \frac{-10 + 26}{5 + 5 + 6} \right) = \left( \frac{8}{16}, \frac{16}{16} \right) \end{aligned}$$

$\therefore$  The coordinates of the incentre =  $\left(\frac{1}{2}, 1\right)$  **Ans. : (B)**

(50) If Y-intercept of a line is 8 more than its X-intercept and it passes through  $(5, -3)$ , then the equation of the line is .....

(A)  $\frac{x}{10} + \frac{y}{2} = -1$  and  $\frac{x}{4} + \frac{y}{-12} = \frac{3}{2}$  (B)  $\frac{x}{-10} + \frac{y}{-2} = 1$  and  $\frac{x}{-4} - \frac{y}{12} = -1$

(C)  $\frac{x}{10} + \frac{y}{2} = -1$  and  $\frac{x}{4} + \frac{y}{12} = 1$  (D)  $\frac{x}{10} + \frac{y}{-3} = \frac{3}{2}$  and  $\frac{x}{4} + \frac{2y}{12} = \frac{3}{4}$

**Sol. :** Let X-intercept be  $a$  and Y-intercept be  $b$ .

$$\therefore b = a + 8$$

$$\therefore \text{The equation of the required straight line is } \frac{x}{a} + \frac{y}{a+8} = 1.$$

It passes through  $(5, -3)$ .

$$\therefore \frac{5}{a} - \frac{3}{a+8} = 1$$

$$\therefore 5a + 40 - 3a = a^2 + 8a$$

$$\therefore a^2 + 6a - 40 = 0$$

$$\therefore (a+10)(a-4) = 0$$

$$\therefore a = -10 \text{ or } a = 4$$

$$\therefore \text{If } a = -10, b = -2. \text{ So, the equation of the line is } \frac{x}{-10} + \frac{y}{-2} = 1$$

$$\therefore \frac{x}{10} + \frac{y}{2} = -1$$

$$\therefore \text{If } a = 4, b = 12. \text{ So, the equation of the line is } \frac{x}{4} + \frac{y}{12} = 1. \quad \text{Ans. : (C)}$$

(51) Given that  $A(-4, 6)$ ,  $B(2, -2)$  and  $C$  is a point on the line  $2x - y + 1 = 0$ . Locus of the centroid of  $\triangle ABC$  is .....

- (A)  $2x + y + 3 = 0$  (B)  $2x - y - 3 = 0$  (C)  $2x - y + 3 = 0$  (D)  $2x + y - 3 = 0$

**Sol. :** Given that  $A(-4, 6)$ ,  $B(2, -2)$  and  $C$  is on the line  $2x - y + 1 = 0$ . Let  $C$  be  $(x, 2x + 1)$ .

$\therefore$  If the coordinates of the centroid are  $(\alpha, \beta)$ , then

$$\alpha = \frac{-4+2+x}{3}, \beta = \frac{6-2+2x+1}{3}$$

$$\therefore x = 3\alpha + 2, 2x = 3\beta - 5$$

$$\therefore 2(3\alpha + 2) = 3\beta - 5$$

$$\therefore 6\alpha - 3\beta + 9 = 0$$

$$\therefore 2\alpha - \beta + 3 = 0$$

In general the locus is  $2x - y + 3 = 0$ .

**Ans. : (C)**

(52) If the medians  $\overline{CD}$ ,  $\overline{BE}$  of  $\triangle ABC$  whose vertices are  $A(a, 0)$ ,  $B(0, 0)$  and  $C(0, c)$  are mutually perpendicular, then .....

- (A)  $c^2 = 4a^2$  (B)  $c^2 = 2a^2$  (C)  $a^2 = 4c^2$  (D)  $a^2 = 2c^2$

**Sol. :**  $\overline{CD}$  and  $\overline{BE}$  are medians.

$\therefore$  D is the midpoint of  $\overline{AB}$  and E is the midpoint of  $\overline{AC}$ .

$\therefore$  The coordinates of D =  $\left(\frac{a}{2}, 0\right)$ , the coordinates of E =  $\left(\frac{a}{2}, \frac{c}{2}\right)$

$\overline{CD} \perp \overline{BE}$ . Hence (Slope of  $\overline{CD}$ ) (Slope of  $\overline{BE}$ ) = -1

$$\therefore \left(\frac{c-0}{0-\frac{a}{2}}\right)\left(\frac{0-\frac{c}{2}}{0-\frac{a}{2}}\right) = -1$$

$$\therefore \frac{\frac{-c^2}{2}}{\frac{a^2}{4}} = -1. \quad \text{So, } \frac{-2c^2}{a^2} = -1$$

$$\therefore 2c^2 = a^2$$

**Ans. : (D)**

(53)  $A(1, 2)$ ,  $B(3, 4)$  and  $C(-2, 5)$  are the vertices of  $\triangle ABC$ . The coordinates of the point of intersection of  $\overline{BC}$  and the line containing the bisector of  $\angle A$  are .....

- (A)  $\left(\frac{2}{5}, \frac{11}{5}\right)$  (B)  $\left(1, \frac{22}{5}\right)$  (C)  $\left(\frac{2}{5}, \frac{22}{5}\right)$  (D)  $\left(\frac{22}{5}, \frac{2}{5}\right)$

**Sol. :**

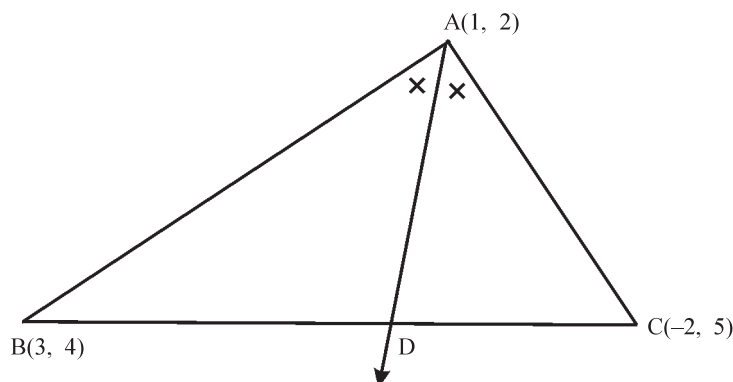


Figure 8.20

Here,  $AB^2 = (1-3)^2 + (2-4)^2 = 4 + 4 = 8$ . So,  $AB = 2\sqrt{2}$

$AC^2 = (1+2)^2 + (2-5)^2 = 9 + 9 = 18$ . So,  $AC = 3\sqrt{2}$

$\therefore$  D divides  $\overline{BC}$  from B in the ratio  $BD : DC = AB : AC = 2:3$ .

$\therefore$  The coordinates of D =  $\left( \frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1} \right)$

$$= \left( \frac{\frac{2}{3}(-2) + 3}{\frac{2}{3} + 1}, \frac{\frac{2}{3}(5) + 4}{\frac{2}{3} + 1} \right) = \left( 1, \frac{22}{5} \right)$$

**Ans. : (B)**

(54) If  $\alpha$  is the measure of the angle between two non-perpendicular lines  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$  then,  $\alpha = \dots\dots\dots$  . ( $b_1 \neq 0, b_2 \neq 0$ )

(A)  $\tan^{-1} \left( \frac{a_1b_2 + a_2b_1}{a_1b_2 - a_2b_1} \right)$

(B)  $\tan^{-1} \left| \frac{a_1b_2 - a_2b_1}{a_1a_2 - b_1b_2} \right|$

(C)  $\tan^{-1} \left( \left| \frac{a_1a_2 + b_1b_2}{a_1b_2 - a_2b_1} \right| \right)$

(D)  $\tan^{-1} \left( \left| \frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2} \right| \right)$

**Sol. :** The slope of the line  $a_1x + b_1y + c_1 = 0$  is  $m_1 = -\frac{a_1}{b_1}$ .

The slope of the line  $a_2x + b_2y + c_2 = 0$  is  $m_2 = -\frac{a_2}{b_2}$ .

If  $\alpha$  be the measure of angle between them, then

$$\tan \alpha = \left| \frac{m_1 - m_2}{1 + m_1m_2} \right|$$

$$= \frac{\left| -\frac{a_1}{b_1} + \frac{a_2}{b_2} \right|}{\left| 1 + \left( -\frac{a_1}{b_1} \right) \left( -\frac{a_2}{b_2} \right) \right|}$$

$$\therefore \tan \alpha = \left| \frac{a_1 b_2 - a_2 b_1}{a_1 a_2 + b_1 b_2} \right|$$

$$\therefore \alpha = \tan^{-1} \left( \left| \frac{a_1 b_2 - a_2 b_1}{a_1 a_2 + b_1 b_2} \right| \right)$$

**Ans. : (D)**

(55) The image of the origin with respect to the line  $ax + by + c = 0$  is ..... . ( $c \neq 0$ )

(A)  $\left( \frac{2ac}{a^2 + b^2}, \frac{2bc}{a^2 + b^2} \right)$

(B)  $\left( \frac{-2ac}{a^2 + b^2}, \frac{2bc}{a^2 + b^2} \right)$

(C)  $\left( \frac{-2ac}{a^2 + b^2}, \frac{-2bc}{a^2 + b^2} \right)$

(D) None of these

**Sol. :**

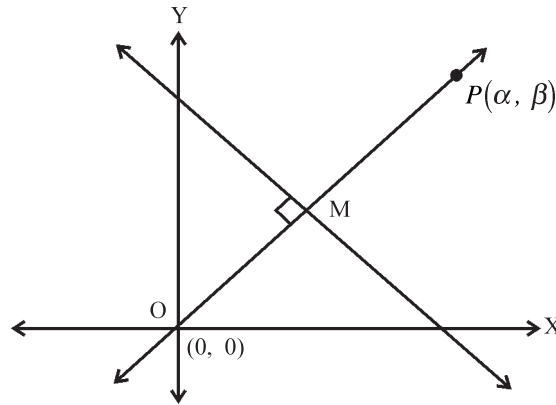


Figure 8.21

Suppose the image of  $O(0, 0)$  is  $P(\alpha, \beta)$ .

M is the midpoint of  $\overline{OP}$ . Hence M is  $\left( \frac{\alpha}{2}, \frac{\beta}{2} \right)$ .

Now, M is on the line  $ax + by + c = 0$ .

$$\therefore a \left( \frac{\alpha}{2} \right) + b \left( \frac{\beta}{2} \right) + c = 0$$

$$\therefore a\alpha + b\beta + 2c = 0 \quad (1)$$

Slope of the given line  $ax + by + c = 0$  is  $m_1 = -\frac{a}{b}$

Slope of  $\overline{OP}$  is  $m_2 = \frac{\beta - 0}{\alpha - 0} = \frac{\beta}{\alpha}$

$$\therefore m_1 m_2 = -1$$

$$\therefore \left(-\frac{a}{b}\right)\left(\frac{\beta}{\alpha}\right) = -1$$

$$\therefore -\frac{a\beta}{\alpha b} = -1. \quad \text{So, } \alpha = \frac{a}{b}\beta \quad (2)$$

$$\text{From (1) and (2), } a\left(\frac{a\beta}{b}\right) + b\beta + 2c = 0$$

$$\therefore a^2\beta + b^2\beta = -2bc. \quad \text{So, } \beta = \frac{-2bc}{a^2+b^2} \text{ and } \alpha = \frac{-2ac}{a^2+b^2}$$

$$\text{Thus the image of the origin relative to the line } ax+by+c=0 \text{ is } \left(\frac{-2ac}{a^2+b^2}, \frac{-2bc}{a^2+b^2}\right).$$

**Ans. : (C)**

(56) The lines  $p(p^2+1)x - y + q = 0$  and  $(p^2+1)^2x + (p^2+1)y + 2q = 0$  have a common perpendicular for ..... .. [AIEEE : 2009]

(A) no value of  $p$

(B) one and only one value of  $p$

(C) only two values of  $p$

(D) more than two values of  $p$

**Sol. :** Two lines have common perpendicular.

$\therefore$  They are parallel

$\therefore m_1 = m_2$  (They are coplanar)

$$\therefore p(p^2+1) = -\frac{(p^2+1)^2}{p^2+1}$$

$$\therefore p(p^2+1) = -(p^2+1)$$

$$\therefore p = -1 \quad \left[ (p^2+1) \neq 0 \right] \quad \text{Ans. : (B)}$$

(57) If the line  $L: \frac{x}{5} + \frac{y}{b} = 1$  passes through  $(13, 32)$  and line  $K: \frac{x}{c} + \frac{y}{3} = 1$  is parallel to  $L$ , then the distance between  $L$  and  $K$  = ..... .. [AIEEE : 2010]

(A)  $\frac{23}{\sqrt{15}}$

(B)  $\sqrt{17}$

(C)  $\frac{17}{\sqrt{15}}$

(D)  $\frac{23}{\sqrt{17}}$

**Sol. :** Line  $L: \frac{x}{5} + \frac{y}{b} = 1$  passes through  $(13, 32)$ .

$$\therefore \frac{13}{5} + \frac{32}{b} = 1. \quad \text{So, } \frac{32}{b} = 1 - \frac{13}{5} = -\frac{8}{5}$$

$$\therefore b = \frac{32 \times 5}{-8} = -20$$

$$\therefore \text{The equation of the line } L \text{ is } \frac{x}{5} + \frac{y}{-20} = 1 \text{ i.e. } 4x - y = 20. \quad (1)$$

Line L is parallel to K i.e. it is parallel to  $\frac{x}{c} + \frac{y}{3} = 1$ .

Hence, their slopes are equal.

$$\therefore 4 = -\frac{1}{\frac{c}{3}} \text{ hence } 4 = \frac{-3}{c}$$

$$\therefore c = \frac{-3}{4}$$

$$\therefore \text{Line K is } \frac{x}{\frac{-3}{4}} + \frac{y}{3} = 1 \text{ i.e. } -4x + y = 3$$

$$\text{The distance between } 4x - y + 3 = 0 \text{ and } 4x - y - 20 = 0 \text{ is } \frac{|23|}{\sqrt{16+1}} = \frac{23}{\sqrt{17}}.$$

**Ans. : (D)**

(58) A line L passes through  $(3, -2)$  and it makes an angle of measure  $\frac{\pi}{3}$  with the line  $\sqrt{3}x + y = 1$ .

If L intersects X-axis, then the equation of the line L is .....

[IIT : 2011]

(A)  $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$

(B)  $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(C)  $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$

(D)  $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

**Sol. :** The equation of any line passing through  $(3, -2)$  is  $y + 2 = m(x - 3)$  (1)

This line makes an angle of measure  $\frac{\pi}{3}$  with the line  $\sqrt{3}x + y = 1$ .

The slope of the line  $\sqrt{3}x + y = 1$  is  $-\sqrt{3}$ .

$$\therefore \tan \frac{\pi}{3} = \left| \frac{m + \sqrt{3}}{1 - \sqrt{3}m} \right|. \quad \text{So, } \sqrt{3} = \pm \frac{m + \sqrt{3}}{1 - \sqrt{3}m}$$

$$\therefore m = \sqrt{3} \text{ or } 0.$$

Since the required line intersects X-axis,  $m \neq 0$ . Hence,  $m = \sqrt{3}$ .

Using this value in (1) we get,

$$y + 2 = \sqrt{3}(x - 3) \text{ i.e. } y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$$

**Ans. : (B)**

(Can L be vertical ?)

(59) Lines  $x + y = |a|$  and  $ax - y = 1$  intersect each other in the first quadrant. The interval of all possible values of  $a$  is ..... . [AIEEE : 2010]

(A)  $(0, \infty)$

(B)  $(1, \infty)$

(C)  $(-1, \infty)$

(D)  $(-1, 1)$

**Sol. :** Suppose  $a > 0$ .

$x + y = a$  and  $ax - y = 1$  are given lines. Let us get the point of intersection.

Adding the equations of given lines we get,  $(a + 1)x = a + 1$ . So,  $x = 1$ . ( $a \neq -1, a > 0$ )

$$\therefore 1+y=a. \quad \text{So, } y=a-1$$

The point  $(1, a-1)$  is in first quadrant. Hence,  $a-1>0$ . So,  $a>1$

$$\therefore a \in (1, \infty)$$

Suppose  $a < 0$  and  $a \neq -1$ . Then given lines are  $x+y=-a$  and  $ax-y=1$ . ( $a \neq -1$ )

$$\text{Their sum gives } (1+a)x=1-a. \quad \text{Hence } x=\frac{1-a}{1+a}$$

$$\therefore y=-a-\frac{1-a}{1+a}=\frac{-a-a^2-1+a}{1+a}=-\frac{a^2+1}{1+a}$$

Now,  $\left(\frac{1-a}{1+a}, -\frac{a^2+1}{1+a}\right)$  is in the first quadrant.

$$\therefore \frac{1-a}{1+a}>0 \quad \text{and} \quad a+1<0$$

$$\therefore \frac{a-1}{a+1}<0 \quad \text{and} \quad a+1<0$$

$$\therefore a-1>0 \Rightarrow a>1$$

$$\therefore a+1<0. \quad a<-1. \quad \text{Also, } a-1>0 \Rightarrow a>1 \text{ which is not possible.}$$

$$a \in (1, \infty)$$

**Note :**  $a=-1$  gives  $x+y=1$  and  $-x-y=1$  which are parallel lines.

**Ans. : (B)**

(60) If the line  $2x+y=k$  passes through a point which divides the line segment  $\overline{AB}$  joining  $A(1,1)$  and  $B(2,4)$  in ratio 3:2 from A, then  $k = \dots\dots$  [AIEEE : 2012]

(A)  $\frac{29}{5}$

(B) 5

(C) 6

(D)  $\frac{11}{5}$

**Sol. :**

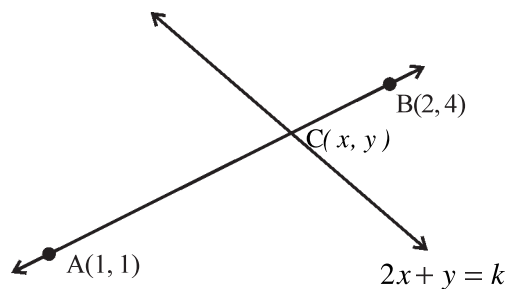


Figure 8.22

If  $C(x, y)$  divides  $\overline{AB}$  in the ratio 3:2 from A then  $(x, y) = \left(\frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1}\right)$

$$x = \frac{3(2)+2(1)}{3+2} = \frac{8}{5}, \quad y = \frac{3(4)+2(1)}{3+2} = \frac{14}{5}$$

Now  $C\left(\frac{8}{5}, \frac{14}{5}\right)$  lies on  $2x + y = k$ .

$$\therefore \frac{16}{5} + \frac{14}{5} = k. \quad \text{So, } k = \frac{30}{5} = 6$$

**Ans. : (C)**

(61) The equation of the line passing through  $(4, 3)$  and having the sum of intercepts  $-1$  is = .....

[AIEEE : 2004]

$$(A) \quad \frac{x}{2} + \frac{y}{3} = -1 \quad \text{and} \quad \frac{x}{-2} + \frac{y}{1} = -1 \quad (B) \quad \frac{x}{2} - \frac{y}{3} = -1 \quad \text{and} \quad \frac{x}{-2} + \frac{y}{1} = -1$$

$$(C) \quad \frac{x}{2} + \frac{y}{3} = 1 \quad \text{and} \quad \frac{x}{2} + \frac{y}{1} = 1 \quad (D) \quad \frac{x}{2} - \frac{y}{3} = 1 \quad \text{and} \quad \frac{x}{-2} + \frac{y}{1} = 1$$

**Sol. :** Let the intercepts be  $a$  and  $b$ .

$$\text{The equation of the line in the intercept form is } \frac{x}{a} + \frac{y}{b} = 1. \quad (1)$$

Line (1) passes through  $(4, 3)$ .

$$\therefore \frac{4}{a} + \frac{3}{b} = 1 \quad (2)$$

$$\text{Given that } a + b = -1 \quad (3)$$

$$\text{Using } b = -1 - a \text{ in (2), } \frac{4}{a} + \frac{3}{-1-a} = 1 \quad (4)$$

$$\therefore -4 - 4a + 3a = -a - a^2$$

$$\therefore a^2 = 4. \quad \text{i.e. } a = \pm 2$$

$$\text{If } a = 2, \text{ then } b = -1 - 2 = -3. \quad \text{If } a = -2, \text{ then } b = -1 + 2 = 1$$

$$\therefore \text{The lines are } \frac{x}{2} + \frac{y}{-3} = 1 \text{ and } \frac{x}{-2} + \frac{y}{1} = 1 \text{ i.e. } \frac{x}{2} - \frac{y}{3} = 1 \text{ and } \frac{x}{-2} + \frac{y}{1} = 1. \quad \text{Ans. : (D)}$$

**Note :**  $(4, 3)$  satisfies only option (D).

(62) If  $A(2, -3)$  and  $B(-2, 1)$  are two vertices of a triangle and the third vertex lies on the line  $2x + 3y = 9$ , then the locus of the centroid is ..... [AIEEE : 2011]

$$(A) \quad x - y = 1 \quad (B) \quad 2x + 3y = 1 \quad (C) \quad 2x + 3y = 3 \quad (D) \quad 2x - 3y = 1$$

**Sol. :**

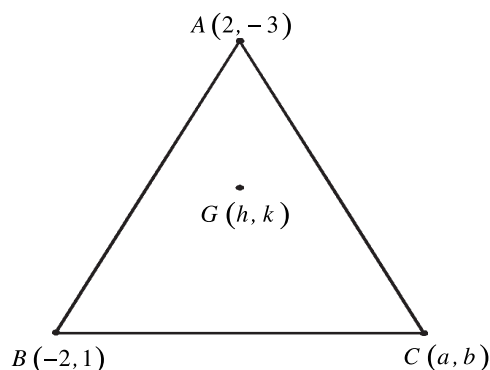


Figure 8.23

Suppose the third vertex is  $C(a, b)$  and centroid is  $G(h, k)$ .

$$\text{Now, } h = \frac{2-2+a}{3} \text{ and } k = \frac{-3+1+b}{3}$$

$$\therefore a = 3h \text{ and } b = 3k + 2 \quad (1)$$

$C(a, b)$  lies on the line  $2x + 3y = 9$ .

$$\therefore 2a + 3b = 9$$

$$2(3h) + 3(3k + 2) = 9 \quad (\text{From (1)})$$

$$\therefore 2h + 3k + 2 = 3$$

$$\therefore 2h + 3k - 1 = 0$$

The locus of  $G(h, k)$  is  $2x + 3y - 1 = 0$ .

**Ans. : (B)**

- (63) If  $(1, 1)$  is a vertex of a triangle and the midpoints of sides passing through this vertex are  $(-1, 2)$  and  $(3, 2)$ , then the centroid of the triangle is ..... . [AIEEE : 2005]

- (A)  $\left(-\frac{1}{3}, \frac{7}{3}\right)$  (B)  $\left(-1, \frac{7}{3}\right)$  (C)  $\left(\frac{1}{3}, \frac{7}{3}\right)$  (D)  $\left(1, \frac{7}{3}\right)$

**Sol. :**

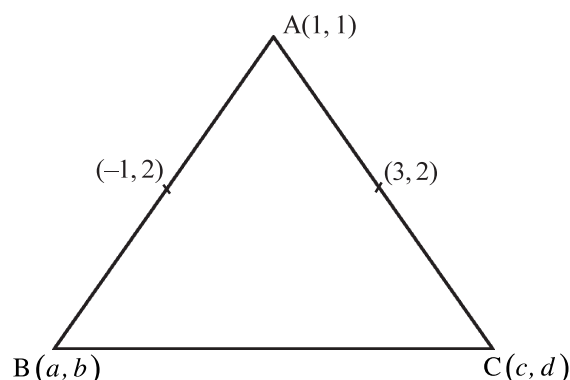


Figure 8.24

$$\text{Here, } a+1 = 2(-1), b+1 = 2(2) \quad \text{So, } a = -3, b = 3.$$

$$\therefore B \text{ is } (-3, 3).$$

$$\text{Also, } c+1 = 6, d+1 = 4 \quad \text{So, } c = 5, d = 3$$

$$\therefore C \text{ is } (5, 3).$$

$$\therefore \text{The centroid} = \left(\frac{1-3+5}{3}, \frac{1+3+3}{3}\right) = \left(1, \frac{7}{3}\right).$$

**Ans. : (D)**

- (64) A line is drawn from the point  $(1, 2)$  intersecting the axes in P and Q and forming a  $\Delta OPQ$ , where O is the origin. If the area of  $\Delta OPQ$  is minimum, then the slope of  $\overleftrightarrow{PQ}$  is ..... [AIEEE : 2012]

- (A)  $-\frac{1}{4}$  (B)  $-4$  (C)  $-2$  (D)  $-\frac{1}{2}$

Sol. :

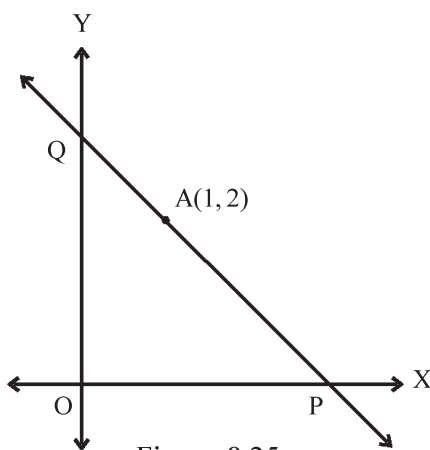


Figure 8.25

The equation of a line passing through  $A(1, 2)$  is  $y - 2 = m(x - 1)$ .

For the point where the line intersects X-axis,  $y = 0$ .

$$\therefore -2 = m(x - 1)$$

$$\therefore OP = 1 - \frac{2}{m}$$

The point at which the line intersects Y-axis,  $x = 0$ .

$$y - 2 = -m. \quad \text{So, } OQ = 2 - m.$$

$$\text{Area of } (\triangle OPQ) = \frac{1}{2}(OP)(OQ)$$

$$= \frac{1}{2}\left(1 - \frac{2}{m}\right)(2 - m) = \frac{1}{2}\left(2 - m - \frac{4}{m} + 2\right)$$

$$= \frac{1}{2}\left(4 - \left(m + \frac{4}{m}\right)\right)$$

$$\text{Area of } (\triangle OPQ) = \frac{4m - m^2 - 4}{2m}$$

$$\text{Let } f(m) = \frac{4m - m^2 - 4}{2m}. \quad \text{Thus, } f'(m) = -\frac{1}{2} + \frac{2}{m^2}$$

$\therefore$  The area of  $(\triangle OPQ)$  is minimum. So,  $f'(m) = 0$ .

$$\therefore \frac{-1}{2} + \frac{2}{m^2} = 0. \quad \text{Hence } m^2 = 4$$

$$\therefore m = \pm 2. \quad \text{So, } m = -2. \quad (\text{For } m = 2, OP = 0)$$

(Note :  $f''(m) = \frac{-4}{m^3}$ . So,  $m = -2$  makes  $f''(m) > 0$ . The minimum area is  $f(m)$ .)

**Ans. : (C)**

(65) If  $x_1, x_2, x_3$  and  $y_1, y_2, y_3$  are in geometric sequence with equal ratios, then the points

$(x_1, y_1), (x_2, y_2)$  and  $(x_3, y_3)$  are .....

[IIT : 1999]

(A) collinear

(B) on an ellipse

(C) concyclic

(D) vertices of a triangle

**Sol. :**  $x_2 = x_1 r$ ,  $x_3 = x_1 r^2$  and  $y_2 = y_1 r$ ,  $y_3 = y_1 r^2$ .

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_1 r & y_1 r & 1 \\ x_1 r^2 & y_1 r^2 & 1 \end{vmatrix}$$

$$= \frac{1}{2} x_1 y_1 \begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix} = 0$$

$\therefore$  The points are collinear.

**Ans. : (A)**

(66) For an isosceles triangle  $PQR$ , the angle at  $P(2, 1)$  is a right angle. If the equation of  $\overleftrightarrow{QR}$

is  $2x + y = 3$ , then the combined equation of  $\overleftrightarrow{PQ}$  and  $\overleftrightarrow{PR}$  is ..... [IIT : 1999]

(A)  $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$  (B)  $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

(C)  $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$  (D)  $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

**Sol. :**

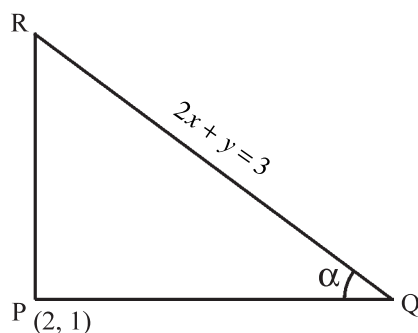


Figure 8.26

For  $2x + y = 3$ ,  $m = -2$ ,  $\alpha = \frac{\pi}{4}$ ,  $t = \tan \alpha = 1$

$$m_1 = \frac{m-t}{1+mt} = \frac{-2-1}{1-2} = 3 = \text{Slope of } \overleftrightarrow{PQ} \text{ or } \overleftrightarrow{PR}$$

$\overleftrightarrow{PQ}$  and  $\overleftrightarrow{PR}$  are mutually perpendicular, so  $m_2 = \frac{-1}{3}$

$$\therefore y-1 = 3(x-2) \text{ and } y-1 = -\left(\frac{1}{3}\right)(x-2)$$

$\therefore$  The combined equation of the lines is  $(y-3x+5)(3y+x-5) = 0$

$$\therefore 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$$

**Ans. : (B)**

(67) If three of the vertices of a parallelogram  $PQRS$  are  $P(1, 2)$ ,  $Q(4, 6)$ ,  $R(5, 7)$  and  $S(a, b)$ , then ..... [IIT : 1998]

- (A)  $a = 2, b = 4$       (B)  $a = 3, b = 4$       (C)  $a = 2, b = 3$       (D)  $a = 2, b = 5$

**Sol. :** Diagonals bisect each other,

$$\therefore \left( \frac{1+5}{2}, \frac{7+2}{2} \right) = \left( \frac{a+4}{2}, \frac{b+6}{2} \right)$$

$$\therefore a = 2 \text{ and } b = 3$$

**Ans. : (C)**

(68) A triangle is formed in the first quadrant by a tangent to the curve  $f(x) = x^2 + bx - b$  at  $(1, 1)$  and coordinate axes. If the area of the triangle is 2, then  $b = \dots\dots$  [IIT : 2001]

- (A)  $-1$       (B)  $3$       (C)  $-3$       (D)  $1$

**Sol. :**  $f(x) = x^2 + bx - b$

$$\therefore f'(x) = 2x + b \quad \text{So, } (f'(x))_{(1,1)} = 2(1) + b = 2 + b$$

$\therefore$  The slope of the tangent at  $(1, 1)$  is  $2 + b$ .

$\therefore$  The equation of the tangent at  $(1, 1)$  is  $y - 1 = (2 + b)(x - 1)$ .

$$\therefore x - \frac{y}{2+b} = 1 - \frac{1}{2+b} = \frac{1+b}{2+b}$$

$$\therefore \frac{x}{\frac{(1+b)}{2+b}} - \frac{y}{(2+b)\frac{(1+b)}{2+b}} = 1$$

$$\therefore \frac{x}{\frac{(1+b)}{2+b}} - \frac{y}{1+b} = 1$$

This is the intercept form of the equation of the tangent.

$$\text{Now, } OA = \frac{1+b}{2+b}, OB = -(1+b)$$

$\therefore$  The area of the triangle OAB,

$$= \frac{1}{2} OA \cdot OB = \frac{1}{2} \left( \frac{1+b}{2+b} \right) (-1)(1+b) = 2 \quad (\text{Given})$$

$$\therefore (1+b)^2 = -4(2+b)$$

$$\therefore 1 + b^2 + 2b = -8 - 4b$$

$$\therefore b^2 + 6b + 9 = 0 \text{ i.e. } (b + 3)^2 = 0$$

$$\therefore b = -3.$$

**Ans. : (C)**

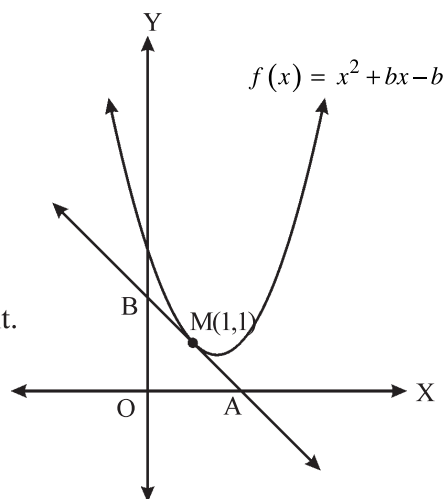


Figure 8.27

(69) The incentre of the triangle with vertices  $(1, \sqrt{3})$ ,  $(0, 0)$  and  $(2, 0)$  is ..... [AIEEE : 2002]

- (A)  $\left(1, \frac{\sqrt{3}}{2}\right)$  (B)  $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$  (C)  $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$  (D)  $\left(1, \frac{1}{\sqrt{3}}\right)$

**Sol. :**  $A(1, \sqrt{3})$ ,  $B(0, 0)$ ,  $C(2, 0)$  are vertices of the given triangle.

$$\text{Now, } AB = \sqrt{(0-1)^2 + (0-\sqrt{3})^2} = \sqrt{1+3} = 2$$

$$BC = \sqrt{(2-0)^2 + (0-0)^2} = \sqrt{4+0} = 2$$

$$\text{and } CA = \sqrt{(2-1)^2 + (0-\sqrt{3})^2} = \sqrt{1+3} = 2$$

$\therefore \Delta ABC$  is an equilateral triangle.

$\therefore$  Incentre and centroid coincide.

$$\text{The centroid of } \Delta ABC \text{ is } \left(\frac{1+0+2}{3}, \frac{\sqrt{3}+0+0}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right).$$

This point is incentre also.

**Ans. : (D)**

(70) The triangle formed by vertices  $(4, 0)$ ,  $(-1, -1)$ ,  $(3, 5)$  is ..... [AIEEE : 2002]

- (A) Isosceles and right triangle (B) Isosceles but not right triangle  
(C) Right triangle but not isosceles (D) neither right triangle nor isosceles

**Sol. :**  $A(4, 0)$ ,  $B(-1, -1)$  and  $C(3, 5)$  are given points,

$$\text{then } AB = \sqrt{(-1-4)^2 + (-1-0)^2} = \sqrt{25+1} = \sqrt{26},$$

$$BC = \sqrt{(3+1)^2 + (5+1)^2} = \sqrt{16+36} = \sqrt{52}$$

$$\text{and } CA = \sqrt{(4-3)^2 + (0-5)^2} = \sqrt{1+25} = \sqrt{26}$$

Since  $AB = CA$ , the triangle is isosceles

$$\text{and also } BC^2 = AB^2 + CA^2$$

$\therefore$  The triangle is a right triangle.

**Ans. : (A)**

(71) Area of the quadrilateral formed by the lines  $y = mx$ ,  $y = mx + 1$ ,  $y = nx$  and  $y = nx + 1$  is = ..... [IIT : 2001]

- (A)  $\frac{|m+n|}{(m-n)^2}$  (B)  $\frac{2}{|m+n|}$  (C)  $\frac{1}{(m+n)}$  (D)  $\frac{1}{|m-n|}$

**Sol. :**

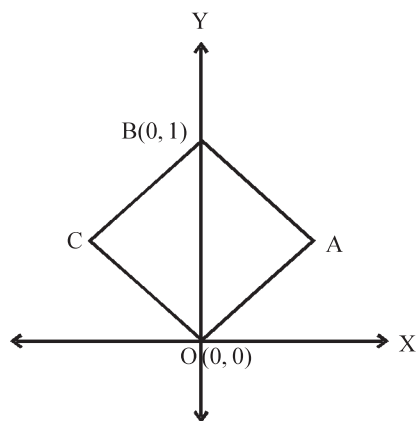


Figure 8.28

Solving the equations four vertices are obtained as

$$O(0, 0), A\left(\frac{1}{m-n}, \frac{1}{m-n}\right), B(0, 1), C\left(\frac{1}{n-m}, \frac{1}{n-m}\right)$$

$$\begin{aligned} \text{Area of } OACB &= 2(\text{Area of } OAB) \\ &= 2\left(\frac{1}{2} \times \text{base} \times \text{height}\right) \\ &= (1-0) \left| \frac{1}{m-n} \right| = \frac{1}{|m-n|} \end{aligned}$$

(Two vertices O and B are on Y-axis and  $\Delta = \frac{1}{2} \times \text{base} \times \text{altitude}$ ; where altitude = height = x-coordinate of point A.)

**Ans. : (D)**

(72)  $m$  has ..... integer values for the integer values of x-coordinate of the point of intersection of the lines  $3x+4y=9$  and  $y=mx+1$ . [IIT : 2001]

- (A) 2 (B) 0 (C) 4 (D) 1

**Sol. :** x-coordinate of the point of intersection is,

$$x = \frac{5}{3+4m} = \frac{5}{1} \text{ or } \frac{5}{-1} \text{ or } \frac{5}{5} \text{ or } \frac{5}{-5} \text{ because } x \text{ is an integer.}$$

$$\therefore 3+4m=1, -1, 5 \text{ or } -5$$

$$\therefore 4m = -2, -4, 2, -8$$

$$\therefore m = -\frac{1}{2}, -1, \frac{1}{2}, -2 \text{ are four values.}$$

Only two values of  $m$  out of these, namely  $-1$  and  $-2$  are integers.

**Ans. : (A)**

(73) If the vertices of a triangle are  $P(2, 2)$ ,  $Q(6, -1)$  and  $R(7, 3)$  and  $\overline{PS}$  is a median, then equation

of line parallel to  $\overleftrightarrow{PS}$  and passing through  $(1, -1)$  is ..... [IIT : 2000]

- (A)  $2x-9y-7=0$  (B)  $2x-9y-11=0$  (C)  $2x+9y-11=0$  (D)  $2x+9y+7=0$

**Sol. :**

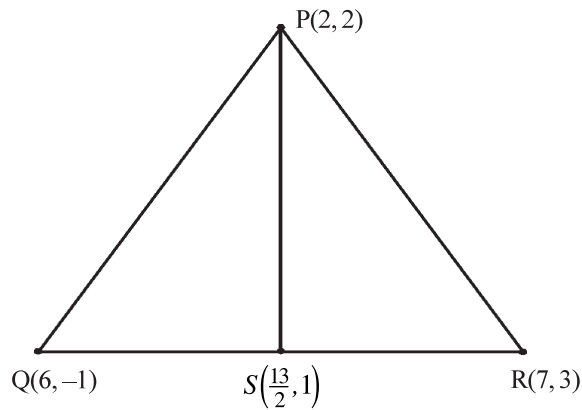


Figure 8.29

$$\text{Slope of } \overleftrightarrow{PS} = \frac{1-2}{\frac{13}{2}-2} = -\frac{2}{9} \text{ as } S = \left(\frac{13}{2}, 1\right)$$

$\therefore$  The equation of the straight line with slope  $-\frac{2}{9}$  and passing through  $(1, -1)$  is

$$y+1 = -\frac{2}{9}(x-1) \text{ i.e. } 2x+9y+7=0.$$

**Ans. : (D)**

- (74) If a line passing through the origin is perpendicular to the parallel lines  $4x+2y=9$  and  $2x+y+6=0$ ; and it intersects the lines in points P and Q respectively, then O divides  $\overline{PQ}$  in the ratio ..... . [IIT : 2002]

(A) 1 : 2                      (B) 3 : 4                      (C) 2 : 1                      (D) 4 : 3

**Sol. :**  $p-\alpha$  form of  $4x+2y=9$  is  $\frac{4x}{\sqrt{20}} + \frac{2y}{\sqrt{20}} = \frac{9}{\sqrt{20}}$ . Here  $p = \frac{9}{\sqrt{20}}$

Also  $p-\alpha$  form of  $2x+y+6=0$  is  $-\frac{2x}{\sqrt{5}} - \frac{y}{\sqrt{5}} = \frac{6}{\sqrt{5}}$ . So,  $p' = \frac{6}{\sqrt{5}}$ .

$$\therefore \frac{PO}{QO} = \frac{\frac{9}{\sqrt{20}}}{\frac{6}{\sqrt{5}}} = \frac{3}{4}$$

**Ans. : (B)**

- (75) If  $P(-1, 0)$ ,  $Q(0, 0)$  and  $R(3, 3\sqrt{3})$  are three points, then the equation of the line containing bisector of  $\angle PQR$  is = ..... [IIT : 2002]

(A)  $\frac{\sqrt{3}}{2}x + y = 0$                       (B)  $x + \sqrt{3}y = 0$                       (C)  $\sqrt{3}x + y = 0$                       (D)  $x + \frac{\sqrt{3}}{2}y = 0$

**Sol. :**

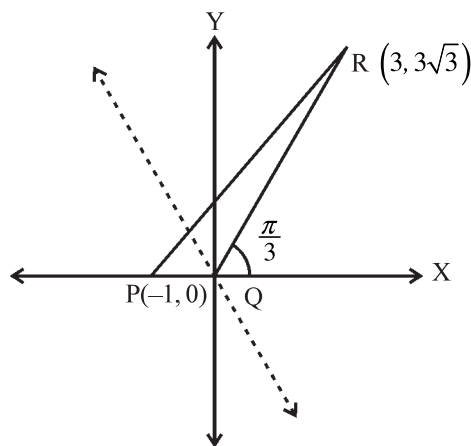


Figure 8.30

$$\text{Slope of } \overleftrightarrow{QR} = \frac{3\sqrt{3}-0}{3-0} = \sqrt{3}$$

$\therefore \overleftrightarrow{QR}$  makes an angle of measure  $\frac{\pi}{3}$  with positive X-axis.

$$\therefore m \angle RQP = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$\therefore$  The bisector of  $\angle RQP$  makes an angle of measure  $\frac{\pi}{3}$  with  $\overleftrightarrow{QR}$

and hence it makes an angle of measure  $\frac{2\pi}{3}$  with positive X-axis, also the bisector of  $\angle PQR$  passes through the origin  $(0, 0)$ .

$\therefore$  The equation of the line containing the bisector is  $y-0 = \tan \frac{2\pi}{3} (x-0)$

$$\therefore y = -\sqrt{3}x \quad \text{So, } \sqrt{3}x + y = 0. \quad \text{Ans. : (C)}$$

(76) If the set of points equidistant from  $(a_1, b_1)$  and  $(a_2, b_2)$  is  $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$ , then  $c = \dots\dots$  [AIEEE : 2003]

$$(A) \quad a_1^2 - a_2^2 + b_1^2 - b_2^2 \quad (B) \quad \frac{1}{2} (a_1^2 + a_2^2 + b_1^2 + b_2^2)$$

$$(C) \quad \sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2} \quad (D) \quad \frac{1}{2} (a_2^2 + b_2^2 - a_1^2 - b_1^2)$$

**Sol. :** Let  $P(x, y)$  be any point equidistant from A and B.

$A(a_1, b_1)$  and  $B(a_2, b_2)$  are given points.

$$PA = PB \Rightarrow PA^2 = PB^2$$

$$\therefore (x - a_1)^2 + (y - b_1)^2 = (x - a_2)^2 + (y - b_2)^2$$

$$\therefore a_1^2 + b_1^2 - 2a_1x - 2b_1y = a_2^2 + b_2^2 - 2a_2x - 2b_2y$$

$$\therefore 2(a_1 - a_2)x + 2(b_1 - b_2)y + a_2^2 + b_2^2 - a_1^2 - b_1^2 = 0$$

$$\therefore (a_1 - a_2)x + (b_1 - b_2)y + \frac{1}{2} (a_2^2 + b_2^2 - a_1^2 - b_1^2) = 0$$

Now,  $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$  is the equation of the set of all such points.

$$\therefore c = \frac{1}{2} (a_2^2 + b_2^2 - a_1^2 - b_1^2) \quad \text{Ans. : (D)}$$

(77)  $0 < \theta < \alpha < \frac{\pi}{4}$  is the measure of an angle. If  $P = (\cos \theta, \sin \theta)$  and  $Q = (\cos(\alpha - \theta), \sin(\alpha - \theta))$ , then Q can be obtained from P by ..... . [IIT : 2002]

- (A) rotating clockwise by an angle of measure  $\alpha$  relative to origin
- (B) rotating anticlockwise by an angle of measure  $\alpha$  relative to the origin
- (C) reflection of P with respect to a line with slope  $\tan \alpha$
- (D) reflection of P with respect to a line with slope  $\tan \frac{\alpha}{2}$

**Sol. :** Rotating clockwise at an angle  $\alpha$ ;  $P(\cos \theta, \sin \theta)$  will be  $(\cos(\alpha - \theta), \sin(\alpha - \theta))$  and rotating anticlockwise at an angle  $\alpha$ ; P will be  $(\cos(\alpha + \theta), \sin(\alpha + \theta))$ .

$$\begin{aligned} \text{The slope of } \overleftrightarrow{PQ} &= \frac{\sin \theta - \sin(\alpha - \theta)}{\cos \theta - \cos(\alpha - \theta)} \\ &= \frac{2 \cos \frac{\alpha}{2} \sin \frac{2\theta - \alpha}{2}}{-2 \sin \frac{\alpha}{2} \sin \frac{2\theta - \alpha}{2}} = -\cot \frac{\alpha}{2} \end{aligned}$$

$\therefore \overleftrightarrow{PQ}$  is perpendicular to the line with slope  $\tan \frac{\alpha}{2}$ .

Q can be obtained by reflection of P with respect to a line with slope  $\tan \frac{\alpha}{2}$ . **Ans. : (D)**

(78) If  $(a \cos t, a \sin t)$ ,  $(b \sin t, -b \cos t)$  and  $(1, 0)$  are vertices of a triangle, then the locus of the centroid is ..... [AIEEE : 2003]

- (A)  $(3x+1)^2 + (3y)^2 = a^2 - b^2$
- (B)  $(3x-1)^2 + (3y)^2 = a^2 - b^2$
- (C)  $(3x-1)^2 + (3y)^2 = a^2 + b^2$
- (D)  $(3x+1)^2 + (3y)^2 = a^2 + b^2$

**Sol. :** If  $(\alpha, \beta)$  is the centroid then,

$$\alpha = \frac{a \cos t + b \sin t + 1}{3} \quad \text{and} \quad \beta = \frac{a \sin t - b \cos t}{3}$$

$$\therefore a \cos t + b \sin t = 3\alpha - 1 \quad \text{and} \quad a \sin t - b \cos t = 3\beta$$

Squaring and adding, we get  $a^2 + b^2 = (3\alpha - 1)^2 + (3\beta)^2$

The equation of the locus is,  $(3x-1)^2 + (3y)^2 = a^2 + b^2$ . **Ans. : (C)**

(79) If the non-zero numbers  $a, b, c$  are in harmonic progression, then the line  $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$  always passes through a fixed point and the fixed point is ..... [A.I.E.E.E.-2005]

- (A)  $(-1, -2)$
- (B)  $(-1, 2)$
- (C)  $(1, -\frac{1}{2})$
- (D)  $(1, -2)$

**Sol. :**  $a, b, c$  are in harmonic sequence.

$$\therefore \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

Line  $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$  passes through  $(1, -2)$ .

**Ans. : (D)**

(80) Orthocentre of the triangle with vertices  $(0, 0)$ ,  $(3, 4)$  and  $(4, 0)$  is ..... [I.I.T.-2003]

- (A)  $\left(3, \frac{7}{3}\right)$  (B)  $\left(3, \frac{5}{4}\right)$  (C)  $(5, -2)$  (D)  $\left(3, \frac{3}{4}\right)$

**Sol. :**

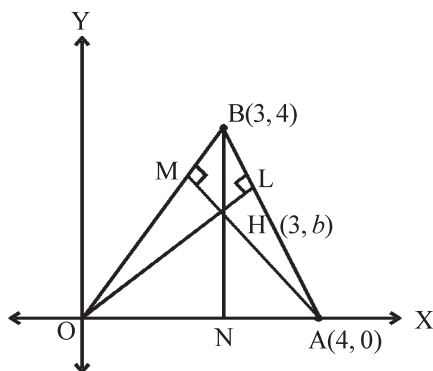


Figure 8.31

$$\text{Slope of } \overleftrightarrow{OB} = \frac{4-0}{3-0} = \frac{4}{3}$$

$$\therefore \text{The slope of } \overleftrightarrow{AM} = \frac{-3}{4}$$

$$\therefore \text{The equation of } \overleftrightarrow{AM} = y-0 = \frac{-3}{4}(x-4).$$

$$\therefore 3x+4y-12=0 \quad (1)$$

$$\text{Similarly, the equation of } \overleftrightarrow{OL} \text{ is } x-4y=0 \quad (2)$$

$$\text{Solving (1) and (2), we get orthocentre } \left(3, \frac{3}{4}\right).$$

**Second method :**

If  $\overline{BN}$  is the altitude, then its equation is  $x=3$ .

$\therefore$   $x$ -coordinate of the orthocentre is 3.

Suppose  $y$ -coordinate is  $b$ .

$$\text{Now, (Slope of } \overleftrightarrow{OH})(\text{Slope of } \overleftrightarrow{AB}) = -1$$

$$\therefore \left(\frac{b}{3}\right)(-4) = -1$$

$$\therefore b = \frac{3}{4}$$

$$\therefore \text{Orthocenter is } \left(3, \frac{3}{4}\right). \quad \text{Ans. : (D)}$$

(81) Three lines  $2x+11y-5=0$ ,  $4x-3y-2=0$  and  $24x+7y-20=0$ , ..... [AIEEE : 2002]

- (A) Form a triangle (B) are concurrent  
(C) one of them is a bisector of the angle formed by other two (D) None of these

**Sol. :** The bisectors of the angles formed by  $24x+7y-20=0$  and  $4x-3y-2=0$  are,

$$\frac{24x+7y-20}{\sqrt{576+49}} = \pm \frac{4x-3y-2}{\sqrt{16+9}}$$

$$\therefore \frac{24x+7y-20}{25} = \pm \frac{4x-3y-2}{5}$$

$$\text{Taking positive sign, we get } \frac{24x+7y-20}{25} = \frac{4x-3y-2}{5}$$

$$\therefore 24x+7y-20 = 5(4x-3y-2)$$

$$\therefore 24x+7y-20 = 20x-15y-10$$

$$\therefore 4x+22y-10=0$$

$$\therefore 2x+11y-5=0 \text{ is the third line.}$$

**Ans. : (C)**

(82) If the sum of the slopes of the lines represented by  $x^2 - 2cxy - 7y^2 = 0$  is 4 times their product then  $c = \dots\dots\dots$  [AIEEE : 2004]

- (A) 1 (B) -1 (C) 2 (D) -2

**Sol. :**  $x^2 - 2cxy - 7y^2 = 0$

$$7\left(\frac{y}{x}\right)^2 + 2c\left(\frac{y}{x}\right) - 1 = 0$$

Here,  $m_1 + m_2 = \frac{2c}{-7}$  and  $m_1 m_2 = \frac{1}{-7}$

Given that,  $m_1 + m_2 = 4 m_1 m_2$

$$\therefore \frac{-2c}{7} = \frac{-4}{7}$$

$$\therefore c = 2.$$

**Ans. : (C)**

(83)  $3x + 4y = 0$  is one of lines represented by  $6x^2 - xy + 4cy^2 = 0$ , then  $c = \dots\dots\dots$  [AIEEE : 2004]

- (A) 1 (B) -1 (C) 3 (D) -3

**Sol. :**  $6x^2 - xy + 4cy^2 = 0$

$$\therefore 6 - \frac{y}{x} + 4c \frac{y^2}{x^2} = 0 \quad (1)$$

The line  $3x + 4y = 0$  is one of the lines.

Hence,  $\frac{y}{x} = \frac{-3}{4}$  is a root of (1),

Substituting  $\frac{y}{x} = \frac{-3}{4}$  in (1),  $6 + \frac{3}{4} + 4c\left(\frac{9}{16}\right) = 0$

$$\therefore 24 + 3 + 9c = 0$$

$$\therefore c = -3$$

**Ans. : (D)**

(84) The line passing through the point of intersection of the lines  $ax + 2by + 3b = 0$  and  $bx - 2ay - 3a = 0$  and parallel to X-axis is  $\dots\dots\dots$   $(a, b) \neq (0, 0)$  [AIEEE : 2005]

- (A) below X-axis at a distance  $\frac{2}{3}$  (B) below X-axis at a distance  $\frac{3}{2}$   
(C) above X-axis at a distance  $\frac{2}{3}$  (D) above X-axis at a distance  $\frac{3}{2}$

**Sol. :** The equation of a line passing through point of the intersection of two given lines is,

$$(ax + 2by + 3b) + k(bx - 2ay - 3a) = 0 \quad (1)$$

The line is parallel to X-axis.

Hence coefficient of  $x$  is zero.

$$\therefore a + kb = 0. \quad \text{So, } k = -\frac{a}{b}.$$

$$\text{Substituting this in (1), } (ax + 2by + 3b) - \frac{a}{b} (bx - 2ay - 3a) = 0$$

$$\therefore 2(b^2 + a^2)y + 3(b^2 + a^2) = 0 \quad \text{i.e. } y = -\frac{3}{2}, \quad (a^2 + b^2 \neq 0)$$

The required line is below X-axis at distance  $\frac{3}{2}$ .

**Ans. : (B)**

- (85) Two pairs of lines  $y^2 - 14y + 45 = 0$  and  $x^2 - 8x + 12 = 0$  form a square. The centre of the incircle is .....

[IIT : 2003]

- (A) (7, 4)      (B) (4, 7)      (C) (3, 7)      (D)  $(\frac{3}{8}, 4)$

**Sol. :**

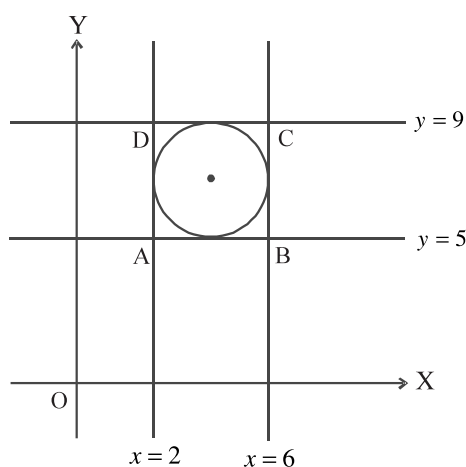


Figure 8.32

$$y^2 - 14y + 45 = 0$$

$$\therefore (y-9)(y-5) = 0. \quad \text{gives two lines } y=9 \text{ and } y=5.$$

$$\text{Similarly, } x^2 - 8x + 12 = 0 \text{ gives two lines } x=2 \text{ and } x=6.$$

$$\text{The centre of incircle in the square} = \left(\frac{2+6}{2}, \frac{5+9}{2}\right) \text{ i.e. } (4, 7).$$

**Ans. : (B)**

- (86) If one of the lines  $my^2 + (1-m^2)xy - mx^2 = 0$  is a bisector of the angle formed by the lines  $xy = 0$ , then  $m = \dots$

[AIEEE : 2007]

- (A) 2      (B) 1      (C) 2      (D)  $-\frac{1}{2}$

**Sol. :** The bisectors of the angle between  $x=0$  and  $y=0$  are  $y = \pm x$  i.e.  $y^2 - x^2 = 0$

$$\text{Comparing } y^2 - x^2 = 0 \text{ with } my^2 + (1-m^2)xy - mx^2 = 0, \text{ we have}$$

$$m = 1. \quad (\text{Why not } m = -1 ?)$$

**Ans. : (B)**

- (87) Consider the straight lines :  $L_1 : x - y = 1$ ,  $L_2 : x + y = 1$ ,  $L_3 : 2x + 2y = 5$ ,  $L_4 : 2x - 2y = 7$   
The correct statement is ..... . (Online May 26, 2012)

- (A)  $L_1 \parallel L_4$ ,  $L_2 \parallel L_3$ ,  $L_1$  intersect  $L_4$  (B)  $L_1 \perp L_2$ ,  $L_1 \parallel L_3$ ,  $L_1$  intersect  $L_2$   
(C)  $L_1 \perp L_2$ ,  $L_2 \parallel L_3$ ,  $L_1$  intersect  $L_4$  (D)  $L_1 \perp L_2$ ,  $L_1 \perp L_3$ ,  $L_2$  intersect  $L_4$

**Sol. :** Consider the lines,  $L_1 : x - y = 1$ ,  $L_2 : x + y = 1$ ,  $L_3 : 2x + 2y = 5$ ,  $L_4 : 2x - 2y = 7$

$L_1 \perp L_2$  is correct statement. (Product of their slopes = -1)

$L_1 \perp L_3$  is also correct statement. (Product of their slopes = -1)

Now,  $L_2 : x + y = 1$

$$L_4 : 2x - 2y = 7$$

$$\therefore 2x - 2(1 - x) = 7$$

$$\therefore 2x - 2 + 2x = 7$$

$$\therefore x = \frac{9}{4} \text{ and } y = \frac{-5}{4}$$

Hence,  $L_2$  intersect  $L_4$ .

**Ans. : (D)**

- (88) The equation of the locus of the midpoint of the line segment intercepted between axes by the line  $x \cos \alpha + y \sin \alpha = p$  (where  $p$  is constant) is ..... . (AIEEE : 2002)

- (A)  $x^2 + y^2 = \frac{4}{p^2}$  (B)  $x^2 + y^2 = 4p^2$  (C)  $\frac{1}{x^2} + \frac{1}{y^2} = \frac{2}{p^2}$  (D)  $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

**Sol. :** The line  $x \cos \alpha + y \sin \alpha = p$  intersects X-axis at  $A\left(\frac{p}{\cos \alpha}, 0\right)$  and Y-axis at  $B\left(0, \frac{p}{\sin \alpha}\right)$ .

Suppose midpoint of  $\overline{AB}$  is  $M(x_1, y_1)$ . Then  $(x_1, y_1) = \left(\frac{p}{2 \cos \alpha}, \frac{p}{2 \sin \alpha}\right)$

$$\therefore \cos \alpha = \frac{p}{2x_1}, \sin \alpha = \frac{p}{2y_1}$$

From  $\cos^2 \alpha + \sin^2 \alpha = 1$ , we get,  $\frac{p^2}{4x_1^2} + \frac{p^2}{4y_1^2} = 1$

$$\therefore \frac{1}{x_1^2} + \frac{1}{y_1^2} = \frac{4}{p^2}$$

$\therefore$  The equation of the locus of the midpoint of  $\overline{AB}$  is  $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$

**Ans. : (D)**

- (89)  $A(1, k)$ ,  $B(1, 1)$  and  $C(k, 1)$  are the vertices of a right triangle with hypotenuse  $\overline{AC}$  and if the area of the triangle is 2, then  $k \in$  ..... (AIEEE : 2007)

- (A)  $\{0, 2\}$  (B)  $\{-1, 3\}$  (C)  $\{-3, -2\}$  (D)  $\{1, 3\}$

Sol. :

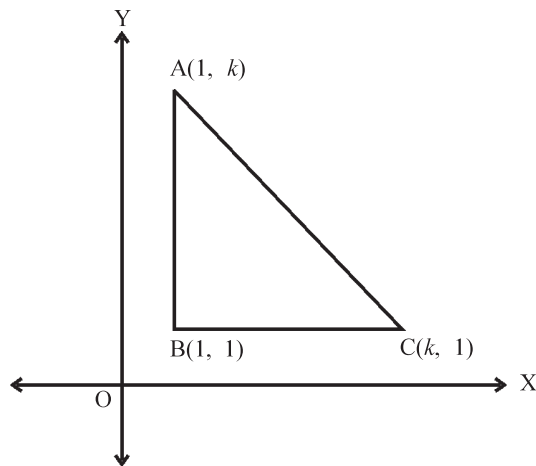


Figure 8.33

Area of  $\triangle ABC$  is given to be 2.

$$\therefore \frac{1}{2} \times AB \times BC = 2$$

$$\therefore \frac{1}{2} \times |k - 1| |k - 1| = 2$$

$$\therefore (k - 1)^2 = 4$$

$$\therefore |k - 1| = \pm 2$$

$$\therefore k = 3, -1$$

$$\therefore k \in \{-1, 3\}$$

Ans. : (B)

- (90) If the point  $(1, a)$  lies between the straight lines  $x + y = 1$  and  $2(x + y) = 3$ , then  $a$  lies in interval ..... .  
(Online May 12, 2012)

- (A)  $\left(\frac{3}{2}, \infty\right)$  (B)  $\left(1, \frac{3}{2}\right)$  (C)  $(-\infty, 0)$  (D)  $\left(0, \frac{1}{2}\right)$

Sol. :

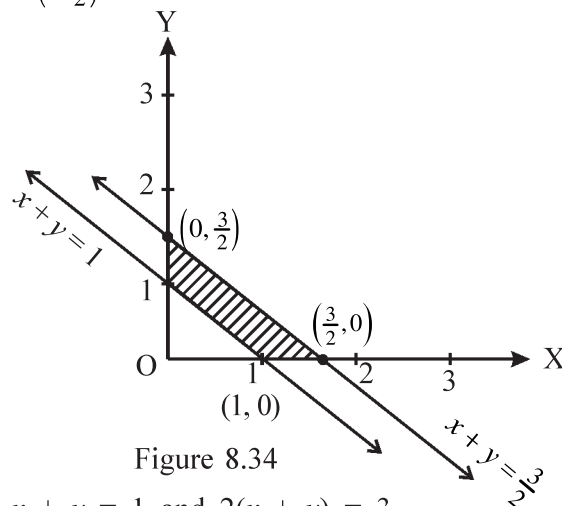


Figure 8.34

Since,  $(1, a)$  lies between  $x + y = 1$  and  $2(x + y) = 3$

$\therefore$  Put  $x = 1$  in  $2(x + y) = 3$ .

We get the range of  $y$ . Thus,

$$2(1 + y) = 3 \Rightarrow y = \frac{3}{2} - 1 = \frac{1}{2}$$

Thus, ' $a$ ' lies in  $\left(0, \frac{1}{2}\right)$ .

Sol. : (D)

**: Questions with More Than One Correct Answers :**

- (91) If lines represented by pair of lines  $3a^2x^2 + 7xy + (4a - 5)y^2 = 0$  are mutually perpendicular, then  $a = \dots\dots$

(A)  $-2 - \sqrt{19}$       (B)  $-2 + \sqrt{19}$       (C)  $\frac{-2 + \sqrt{19}}{3}$       (D)  $\frac{-2 - \sqrt{19}}{3}$

**Sol. :** If pair of lines represented by  $3a^2x^2 + 7xy + (4a - 5)y^2 = 0$  are mutually perpendicular, then Coefficient of  $x^2$  + Coefficient of  $y^2 = 0$

$$\therefore 3a^2 + 4a - 5 = 0$$

$$\therefore a = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-4 \pm \sqrt{16 - 4(3)(-5)}}{2 \times 3} = \frac{-4 \pm \sqrt{76}}{2 \times 3} = \frac{-2 \pm \sqrt{19}}{3} \quad \text{Sol. : (C), (D)}$$

- (92) If the area of the triangle is 31 and two of its vertices are (3, 4) and (-4, 6), then the equation of the locus of the third vertex is ..... .

(A)  $2x + 7y + 28 = 0$       (B)  $2x + 7y - 28 = 0$   
(C)  $2x + 7y - 96 = 0$       (D)  $2x + 7y + 96 = 0$

**Sol. :** Let  $A(3, 4)$ ,  $B(-4, 6)$  and  $C(x, y)$  be vertices of the triangle.

$$\therefore D = \begin{vmatrix} 3 & 4 & 1 \\ -4 & 6 & 1 \\ x & y & 1 \end{vmatrix}$$

Now area of  $\Delta ABC$  is 31.

$$\therefore 31 = \frac{1}{2} |D|$$

$$\therefore |-2x - 7y + 34| = 62$$

$$\therefore -2x - 7y + 34 = 62 \quad \text{or} \quad -2x - 7y + 34 = -62$$

$$\therefore 2x + 7y + 28 = 0 \quad \text{or} \quad 2x + 7y - 96 = 0$$

**Sol. : (A), (C)**

- (93) A point lies on the line  $x + y = 3$  in such a way that it is equidistant from the lines  $|x| = |y|$ . Then its coordinates are .....

(A) (0, 3)      (B) (3, 0)      (C) (-3, 0)      (D) (0, -3)

**Sol. :**

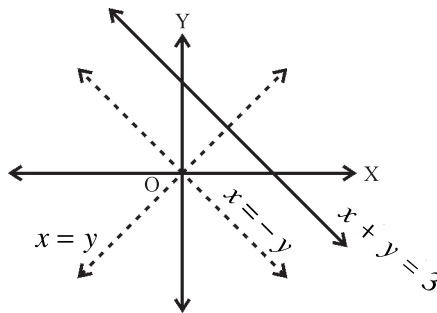


Figure 8.35

From  $|x| = |y|$ ,  $x - y = 0$  and  $x + y = 0$

Bisector of the angle between these lines are  $\frac{x-y}{\sqrt{2}} = \pm \frac{x+y}{\sqrt{2}}$

Hence they are  $y = 0$  and  $x = 0$

Taking  $y = 0$ ,  $x + y = 3$  gives  $(3, 0)$ .

Taking  $x = 0$ ,  $x + y = 3$  gives  $(0, 3)$ .

**Ans. : (A), (B)**

**Second method :**

It is obvious that the line  $x + y = 3$  intersects the coordinate axes at  $A(3, 0)$  and  $B(0, 3)$ , then  $OA = OB = 3$ .

Hence, required points are  $(3, 0)$  and  $(0, 3)$ .

- (94)  $S_1, S_2, \dots$  are squares such that for every  $n > 1$  the length of the side of  $S_n$  is equal to the length of diagonal of  $S_{n+1}$ . If the length of the side of  $S_1$  is 10, then for  $n = \dots$ , area of  $S_n$  is less than 1. [IIT : 1999]

(A) 7

(B) 8

(C) 9

(D) 10

**Sol. :** If the length of the side of a square is  $a$ , then length of its diagonal  $d = a\sqrt{2}$

Given that  $a_n = \sqrt{2} \cdot a_{n+1}$

$$\therefore a_{n+1} = \frac{a_n}{\sqrt{2}} = \frac{a_{n-1}}{(\sqrt{2})^2} = \frac{a_{n-2}}{(\sqrt{2})^3} = \dots = \frac{a_1}{(\sqrt{2})^n}.$$

Replacing  $n$  by  $n-1$ ,

$$a_n = \frac{a_1}{(\sqrt{2})^{n-1}} = \frac{10}{(2)^{\frac{n-1}{2}}}$$

Area of  $S_n < 1$ .

So,  $a_n^2 < 1$ .

$$\therefore \frac{100}{2^{n-1}} < 1.$$

So,  $200 < 2^n$

$$\therefore 2^7 = 128 < 200, 2^8 = 256 > 200$$

$$\therefore n = 8, 9, 10$$

**Ans. : (B), (C), (D)**

