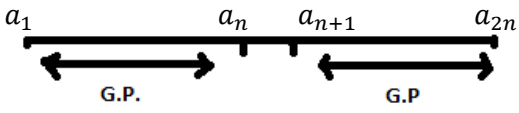


	SEQUENCE AND SERIES Class XI	
Q.1)	If f is a function satisfying $f(x+y) = f(x) \cdot f(y)$ such that $f(1) = 3$ and $\sum_{x=1}^n f(x) = 120$. Find value of n .	
Sol.1)	We have, $f(x+y) = f(x)f(y)$ $f(1) = 3$ $\sum_{x=1}^n f(x) = 120$ Now $\sum_{x=1}^n f(x) = f(1) + f(2) + f(3) + \dots n \text{ terms} = 120$ Now $f(2) = f(1+1) = f(1) \cdot f(1) = (3)(3) = 9$ $f(3) = f(1+2) = f(1) \cdot f(2) = (3)(9) = 27$ $f(4) = f(1+3) = f(1) \cdot f(3) = (3)(27) = 81$ $\therefore$ series becomes $3 + 9 + 27 + 81 + \dots n \text{ terms} = 120$ Clearly it is a G.P. with $a = 3$ & $r = 3$ $3 \left[\frac{3^{n-1} - 1}{3 - 1} \right] = 120$ $\Rightarrow 3 \frac{(3^n - 1)}{2} = 120$ $\Rightarrow 3^n - 1 = \frac{2 \times 120}{3}$ $\Rightarrow 3^n - 1 = 80$ $\Rightarrow 3^n = 81$ $\Rightarrow 3^n = 3^4$ $\Rightarrow n = 4$ ans.	
Q.2)	If a, b, c are in G.P. & $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$. Prove that x, y, z are in A.P.	
Sol.2)	Given: a, b, c are in G.P. $\Rightarrow b^2 = ac$ Given: $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ Let $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$ $\Rightarrow a^{\frac{1}{x}} = k \quad \Rightarrow b^{\frac{1}{y}} = k \quad \Rightarrow c^{\frac{1}{z}} = k$ $\Rightarrow a = k^x \quad \Rightarrow b = k^y \quad \Rightarrow c = k^z$ We have, $b^2 = ac$ $\Rightarrow (k^y)^2 = (k^x) \cdot (k^z)$ $\Rightarrow k^{2y} = k^{x+z}$ $\Rightarrow 2y = x + z$ $\therefore x, y, z$ are in G.P. ans.	
Q.3)	Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ is the G.M. between a and b .	
Sol.3)	We have, $\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = G.M. = \sqrt{ab}$ $\Rightarrow \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = a^{\frac{1}{2}} \cdot b^{\frac{1}{2}}$ $\Rightarrow a^{n+1} + b^{n+1} = a^{\frac{1}{2}} \cdot b^{\frac{1}{2}} (a^n + b^n)$ $\Rightarrow a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}} \cdot b^{\frac{1}{2}} + b^{n+\frac{1}{2}} \cdot a^{\frac{1}{2}}$ $\Rightarrow a^{n+1} - a^{n+\frac{1}{2}} \cdot b^{\frac{1}{2}} = b^{n+\frac{1}{2}} \cdot a^{\frac{1}{2}} - b^{n+1}$	

	$\Rightarrow a^{n+\frac{1}{2}} \left(a^{\frac{1}{2}} - b^{\frac{1}{2}} \right) - b^{n+\frac{1}{2}} \left(a^{\frac{1}{2}} - b^{\frac{1}{2}} \right)$ $\Rightarrow a^{n+\frac{1}{2}} = b^{n+\frac{1}{2}}$ $\Rightarrow \left(\frac{a}{b} \right)^{n+\frac{1}{2}} = 1$ $\Rightarrow \left(\frac{a}{b} \right)^{n+\frac{1}{2}} = \left(\frac{a}{b} \right)^0$ $\Rightarrow n + \frac{1}{2} = 0$ $\Rightarrow n = -\frac{1}{2} \text{ ans.}$	
Q.4)	Insert three numbers between 3 and 243 so that the resulting sequence is an G.P.	
Sol.4)	Here, $a = 3$, $b = 243$ and $n = 3$ Let G.M.S. are G_1, G_2, G_3 $\Rightarrow r = \left(\frac{b}{a} \right)^{\frac{1}{n+1}} = \left(\frac{243}{3} \right)^{\frac{1}{3+1}} = (81)^{\frac{1}{4}} = 3$ $\therefore r = 3$ Now, $G_1 = ar^1 = (3)(3) = 9$ $G_2 = ar^2 = (3)(9) = 27$ $G_3 = ar^3 = (3)(27) = 81$ $\therefore$ required no.s are 3, 27, 81 ans.	
Q.5)	If the first and the n^{th} term of a G.P. are a and b respectively and if P is the product of n terms. Prove that $P^2 = (ab)^n$.	
Sol.5)	Given, $a_1 = a$ $\Rightarrow a_n = b$ $\Rightarrow ar^{n-1} = b$ $\Rightarrow r^{n-1} = \frac{b}{a}$ $\Rightarrow r = \left(\frac{b}{a} \right)^{\frac{1}{n-1}}$ Now, $P \rightarrow$ product of n terms $\Rightarrow P = a \cdot ar \cdot ar^2 \cdot ar^3 \cdot \dots \dots \dots ar^{n-1}$ $\Rightarrow P = a^n \cdot r^{1+2+3+\dots+(n-1)}$ $\Rightarrow P = a^n \cdot r^{\frac{n(n-1)}{2}}$ Putting the value of r $\Rightarrow P = a^n \left[\left(\frac{b}{a} \right)^{\frac{1}{n-1}} \right]^{\frac{n(n-1)}{2}}$ $\Rightarrow P = a^n \left(\frac{b}{a} \right)^{\frac{n}{2}}$ $\Rightarrow P = a^n \cdot \frac{b^{\frac{n}{2}}}{a^{\frac{n}{2}}}$ $\Rightarrow P = a^{n-\frac{n}{2}} \cdot b^{\frac{n}{2}}$ $\Rightarrow P = a^{\frac{n}{2}} \cdot b^{\frac{n}{2}}$ $\Rightarrow P = (ab)^{\frac{n}{2}}$ Squaring $P^2 = (ab)^n \text{ (proved)}$	
Q.6)	If the p^{th} , q^{th} and r^{th} terms of a G.P. are a , b and c respectively.	

	Prove that $a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$	
Sol.6)	Let 1st term = A Common ratio = R Given, $a_p = a = AR^{p-1}$ $a_q = b = AR^{q-1}$ $a_r = c = AR^{r-1}$ Taking L.H.S $a^{q-r} \cdot b^{r-p} \cdot c^{p-q}$ Substitute the value of a, b, c in L.H.S. $= [AR^{p-1}]^{q-r} \cdot [AR^{q-1}]^{r-p} \cdot [AR^{r-1}]^{p-q}$ $= A^{q-r} \cdot R^{(p-1)(q-r)} \cdot A^{r-p} \cdot R^{(q-1)(r-p)} \cdot A^{p-q} \cdot R^{(r-1)(p-q)}$ $= A^{q-r+r-p+p-q} \cdot R^{pq-pr-q+r+qr-pq-r+p+rq-rq-p+q}$ $= A^0 \cdot R^0$ $= (1)(1)$ $= 1$ R.H.S. (proved)	
Q.7)	If A and G be A.M. and G.M. respectively between two +ve numbers. Prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.	
Sol.7)	Let the numbers are a and b Then $A = \frac{a+b}{2}$ and $G = \sqrt{ab}$ Consider, $A + \sqrt{(A+G)(A-G)}$ $= A + \sqrt{A^2 - G^2}$ Put value of A & G $= \frac{a+b}{2} + \sqrt{\left(\frac{a+b}{2}\right)^2 - (\sqrt{ab})^2}$ $= \frac{a+b}{2} + \sqrt{\frac{a^2+b^2+2ab}{4} - ab}$ $= \frac{a+b}{2} + \sqrt{\frac{a^2+b^2-2ab}{4}}$ $= \frac{a+b}{2} + \sqrt{\left(\frac{a-b}{2}\right)^2}$ $= \frac{a+b}{2} + \frac{a-b}{2}$ $= 2a$ $\therefore A + \sqrt{(A+G)(A-G)} = a$ Similarly, $A + \sqrt{(A+G)(A-G)} = b$ $\therefore$ The numbers are $A \pm \sqrt{(A+G)(A-G)}$ ans.	
Q.8)	Let s be the sum, P be the product and R be the sum of reciprocal of n terms in G.P. Prove that $P^2 R^n = S^n$.	
Sol.8)	$S = a + ar + ar^2 + \dots \dots \dots ar^{n-1}$ $\Rightarrow S = a \left(\frac{r^n - 1}{r - 1} \right); r > 1$ $P = a + ar + ar^2 + \dots \dots \dots ar^{n-1}$ $P = a^n + r^{1+2+\dots \dots (n-1)}$ $P = a^n \cdot r^{\frac{n(n-1)}{2}}$ $R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \dots \dots \dots \frac{1}{ar^{n-1}}$ It is also a G.P. with 1st term $\frac{1}{a}$ and common ratio $\frac{1}{r}$ ($\because r > 1 \therefore \frac{1}{r} < 1$)	

	$\therefore R = \frac{1}{a} \left[\frac{1 - \frac{1}{r^n}}{1 - \frac{1}{r}} \right]$ $R = \frac{1}{a} \left[\frac{r^n - 1}{r - 1} \right] \cdot \frac{r}{r^n}$ Taking L.H.S. $P^2 \cdot R^n$ Put value of P and R in L.H.S. $\therefore \text{L.H.S} \left[a^n \cdot r^{\frac{n(n-1)}{2}} \right] \left[\frac{1}{a} \left(\frac{r^n - 1}{r - 1} \right) \cdot \frac{r}{r^n} \right]^n$ $= a^{2n} \cdot r^{n(n-1)} \cdot \frac{1}{a^n} \left(\frac{r^n - 1}{r - 1} \right)^n \cdot \frac{r}{r^{n^2}}$ $= a^{2n-n} \cdot r^{n^2-n+n-n^2} \cdot \left(\frac{r^n - 1}{r - 1} \right)^n$ $= a^n \cdot r^0 \cdot \left(\frac{r^n - 1}{r - 1} \right)^n$ $= a^n \cdot \left(\frac{r^n - 1}{r - 1} \right)^n$ $= \left[a \cdot \left(\frac{r^n - 1}{r - 1} \right)^n \right]$ $= S^n \text{ RHS ans.}$	
Q.9)	Show that the ratio of the sum of 1 st n terms of a G.P. to the sum of terms from $(n + 1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.	
Sol.9)	Here G.P. consist of $(2n)$ no. of terms  1st terms = $a_1 = a$ Ratio = r Terms = n Sum = S_n $S_n \rightarrow$ sum of 1st n terms $S'_n \rightarrow$ sum of terms from $(n + 1)^{th}$ to $(2n)$ terms $S_n = a \left(\frac{r^n - 1}{r - 1} \right)$ $S'_n = ar^n \left[\frac{r^n - 1}{r - 1} \right]$ Now, $\frac{S_n}{S'_n} = \frac{a \left(\frac{r^n - 1}{r - 1} \right)}{ar^n \left(\frac{r^n - 1}{r - 1} \right)} = \frac{1}{r^n}$ (proved) 1st term = $a_{n+1} = ar^n$ Ratio = r Terms = n Sum = S'_n	
Q.10)	If a, b, c, d and p are real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$ then show that a, b, c & d are in G.P.	
Sol.10)	To show that, a, b, c & d are in G.P. We have to prove $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ Given, $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$ $\Rightarrow a^2p^2 + b^2p^2 + c^2p^2 - 2abp - 2bcp - 2cdp + b^2 + c^2 + d^2 \leq 0$ $\Rightarrow (a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2) + (c^2p^2 - 2cdp + d^2) \leq 0$ $\Rightarrow (ap - b)^2 + (bp - c)^2 + (cp - d)^2 \leq 0$ But $(ap - b)^2 + (bp - c)^2 + (cp - d)^2$ cannot be less than 0 $\therefore$ sum of square can never be negative}	

	$\therefore (ap - b)^2 + (bp - c)^2 + (cp - d)^2 = 0$ This is possible only when <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;"> $ap - b = 0$ $ap = b$ $\frac{b}{a} = p$ </td><td style="padding: 5px;"> $bp - c = 0$ $bp = c$ $\frac{c}{b} = p$ </td><td style="padding: 5px;"> $cp - d = 0$ $cp = d$ $\frac{d}{c} = p$ </td></tr> </table> $\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ $\Rightarrow a, b, c, d$ are in G.P. ans.	$ap - b = 0$ $ap = b$ $\frac{b}{a} = p$	$bp - c = 0$ $bp = c$ $\frac{c}{b} = p$	$cp - d = 0$ $cp = d$ $\frac{d}{c} = p$	
$ap - b = 0$ $ap = b$ $\frac{b}{a} = p$	$bp - c = 0$ $bp = c$ $\frac{c}{b} = p$	$cp - d = 0$ $cp = d$ $\frac{d}{c} = p$			
Q.11)	If p, q, r are in G.P. and the equations $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P.				
Sol.11)	Given, p, q, r are in G.P. $\therefore q^2 = pr$ To prove, $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in G.P. i.e., $\frac{2e}{q} = \frac{d}{p} + \frac{f}{r}$ consider the equation, $px^2 + 2qx + r = 0$ by quadratic formula, $x = \frac{-2q \pm \sqrt{4q^2 - 4pr}}{2p}$ $\Rightarrow x = \frac{-2q \pm \sqrt{4pr - 4pr}}{2p}$ $\{\because q^2 = pr\}$ $\Rightarrow x = \frac{-2q}{2p}$ $\Rightarrow x = \frac{-q}{p}$ This is also the root of the equation $dx^2 + 2ex + f = 0$ $\therefore d\left(\frac{-q}{p}\right)^2 + 2e\left(\frac{-q}{p}\right) + f = 0$ $\Rightarrow \frac{dq^2}{p^2} - \frac{2eq}{p} + f = 0$ $\Rightarrow \frac{dpr}{p^2} - \frac{2eq}{p} + f = 0$ $\{\because q^2 = pr\}$ $\Rightarrow \frac{dr}{p} - \frac{2eq}{p} + f = 0$ $\Rightarrow dr - 2eq + fp = 0$ $\Rightarrow 2eq = dr + fp$ Divide by q^2 $\Rightarrow \frac{2e}{q} = \frac{dr}{q^2} + \frac{fp}{q^2}$ $\Rightarrow \frac{2e}{q} = \frac{dr}{pr} + \frac{fp}{pr}$ $\Rightarrow \frac{2e}{q} = \frac{d}{p} + \frac{f}{r}$ $\therefore \frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P. (proved)				
Q.12)	Fid the sum of the products of the corresponding terms of the sequences 2,4,8,16,32 and 128,32,8,2, $\frac{1}{2}$.				
Sol.12)	1 st sequence 2, 4, 8, 16, 32 2 nd sequence 128, 32, 8, 2, $\frac{1}{2}$ New sequence (products of corresponding terms) = 256, 128, 64, 32, 16 Now, $\frac{a_2}{a_1} = \frac{128}{256} = \frac{1}{2}$				

	$\frac{a_3}{a_2} = \frac{64}{128} = \frac{1}{2}$ Clearly it is a G.P. with $a = 256, r = \frac{1}{2}$ and no. of term = 5 $\therefore \text{sum} = S_n = a \left(\frac{1-r^n}{1-r} \right)$ $= S_5 = 256 \left(\frac{1 - \left(\frac{1}{2}\right)^5}{1 - \frac{1}{2}} \right)$ $= 256 \left(\frac{1 - \frac{1}{32}}{\frac{1}{2}} \right)$ $= 2 \times 256 \left(\frac{31}{32} \right)$ $= 16 \times 31$ $= 496 \text{ ans.}$	
Q.13)	Find four numbers forming a G.P. in which the third term is greater than the first term by 9 & the second term is greater than fourth term by 18.	
Sol.13)	Let the four numbers are a, ar, ar^2, ar^3 We have, $a_3 = a_1 + 9$ $\Rightarrow ar^2 = a + 9$ $\Rightarrow ar^2 - a = 9$ $\Rightarrow a(r^2 - 1) = 9 \dots\dots\dots (i)$ And $a_2 = a_4 + 18$ $\Rightarrow ar = ar^3 + 18$ $\Rightarrow ar - ar^3 = 18$ $\Rightarrow ar(r^2 - 1) = 18$ $\Rightarrow -ar(r^2 - 1) = 18 \dots\dots\dots (ii)$ Divide (ii) and (i) $\frac{-ar(r^2 - 1)}{a(r^2 - 1)} = \frac{18}{9}$ $\therefore -r = 2$ $\Rightarrow r = -2$ Put in (i) $9(4 - 1) = 9$ $\Rightarrow 3a = 9$ $\Rightarrow a = 3$ $\therefore$ the no.s are 3, -6, 12, 24 ans.	
Q.14)	Evaluate $\sum_{k=1}^{11} (2 + 3^k)$	
Sol.14)	$\sum_{k=1}^{11} (2 + 3^k) = (2 + 3^1) + (2 + 3^2) + (2 + 3^3) + \dots\dots\dots (2 + 3^{11})$ $= (2 + 2 + 2 + \dots\dots\dots 11 \text{ terms}) + (3^1 + 3^2 + 3^3 \dots\dots\dots 3^{11})$ G.P. $a = 3, r = 3, n = 11$ $= 22 + 3 \left(\frac{3^{11} - 1}{3 - 1} \right)$ $= 22 + \frac{3^{12} - 3}{2}$ $= \frac{44 + 3^{12} - 3}{2}$ $= \frac{41 + 3^{12}}{2} \text{ ans.}$	
Q.15)	If p^{th}, q^{th}, r^{th} and s^{th} terms of an A.P. are in G.P., then show that $(p - q), (q - r), (r - s)$ are also in G.P.	

Sol.15)	$\Rightarrow a_p = a + (p - 1)d \dots\dots\dots \{ \because a_p, a_q, a_r, a_s \text{ terms of an A. P. } \}$ $\Rightarrow a_q = a + (q - 1)d$ $\Rightarrow a_r = a + (r - 1)d$ $\Rightarrow a_s = a + (s - 1)d$ Given that a_p, a_q, a_r, a_s are in G.P. $\therefore \frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_s}{a_r} \dots\dots\dots (i)$ Consider $\frac{a_q}{a_p} = \frac{a_r}{a_q}$ $\Rightarrow \frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_q - a_r}{a_p - a_q} \dots\dots\dots \left\{ \text{if } \frac{a}{b} = \frac{c}{d} \text{ then } \frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-a} \right\}$ $\Rightarrow \frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{[a+(q-1)d] - [a+(r-1)d]}{[a+(p-1)d] - [a+(q-1)d]}$ $\Rightarrow \frac{d(q-1) - d(r-1)}{d(p-1) - d(q-1)}$ $\therefore \frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{q-r}{p-q} \dots\dots\dots (ii)$ Now, consider $\frac{a_r}{a_q} = \frac{a_s}{a_r} = \frac{a_r - a_s}{a_q - a_r}$ $\Rightarrow \frac{a_r}{a_q} = \frac{a_s}{a_r} = \frac{[a+(r-1)d] - [a+(s-1)d]}{[a+(q-1)d] - [a+(r-1)d]}$ $\Rightarrow \frac{d(r-1-s+1)}{d(q-1-r+1)}$ $\therefore \frac{a_r}{a_q} = \frac{a_s}{a_r} = \frac{r-s}{q-r} \dots\dots\dots (iii)$ From (i), (ii) & (iii) $\Rightarrow \frac{q-r}{p-q} = \frac{r-q}{q-r}$ $\Rightarrow (q-r)^2 = (p-q)(r-s)$ $\Rightarrow (p-q), (q-r), (r-s)$ are in G.P.	
Q.16)	If the 4 th , 10 th and 16 th term of a G.P. are x, y, z respectively. Prove that x, y, z are in G.P.	
Sol.16)	$\Rightarrow a_4 = x \Rightarrow ar^3 = x$ $\Rightarrow a_{10} = y \Rightarrow ar^9 = y$ $\Rightarrow a_{16} = z \Rightarrow ar^{15} = z$ To prove, x, y, z are in G.P. i.e., to prove $y^2 = xz$ L.H.S. $y^2 = (ar^9)^2 = a^2r^{18}$ R.H.S. $xz = (ar^3)(ar^{15}) = a^2r^{18}$ Clearly $y^2 = xz$ $\therefore x, y, z$ are in G.P.	