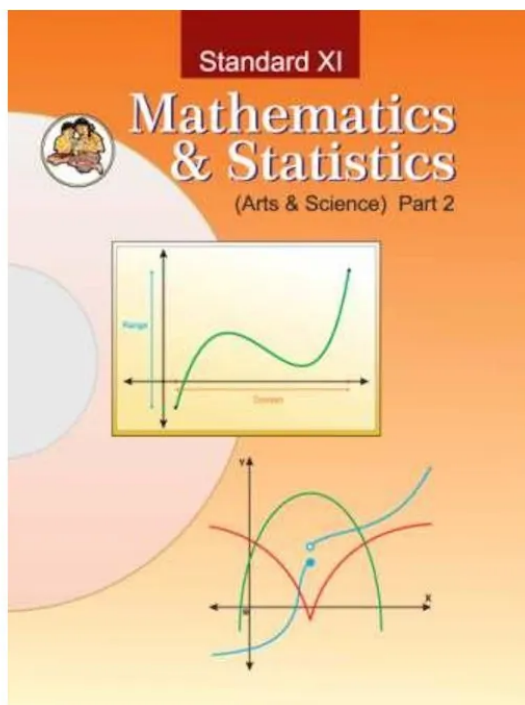


Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 1- Complex Numbers

Class 11 - Chapter 1 Complex Numbers



For any clarifications or questions you can write to info@indcareer.com

Postal Address

IndCareer.com, 52, Shilpa Nagar, Somalwada Nagpur - 440015
Maharashtra, India

WhatsApp: +91 9561 204 888, **Website:** <https://www.indcareer.com>

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Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 1- Complex Numbers

Class 11: Maths Chapter 1 solutions. Complete Class 11 Maths Chapter 1 Notes.

Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 1- Complex Numbers

Maharashtra Board 11th Maths Chapter 1, Class 11 Maths Chapter 1 solutions

Ex 1.1

Question 1.

Simplify:

(i) $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$

Solution:

$$\begin{aligned} & \sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625} \\ &= \sqrt{16 \times -1} + 3\sqrt{25 \times -1} + \sqrt{36 \times -1} - \sqrt{625 \times -1} \end{aligned}$$

$$= 4i + 3(5i) + 6i - 25i$$

$$= 25i - 25i$$

$$= 0$$

(ii) $4\sqrt{-4} + 5\sqrt{-9} - 3\sqrt{-16}$

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Solution:

$$\begin{aligned} & 4\sqrt{-4} + 5\sqrt{-9} - 3\sqrt{-16} \\ &= 4\sqrt{4 \times -1} + 5\sqrt{9 \times -1} - 3\sqrt{16 \times -1} \\ &= 4(2i) + 5(3i) - 3(4i) \\ &= 8i + 15i - 12i \\ &= 11i \end{aligned}$$

Question 2.

Write the conjugates of the following complex numbers

(i) $3 + i$

Solution:

Conjugate of $(3 + i)$ is $(3 - i)$.

(ii) $3 - i$

Solution:

Conjugate of $(3 - i)$ is $(3 + i)$.

(iii) $\sqrt{-5} - \sqrt{7} i$

Solution:

Conjugate of $(\sqrt{-5} - \sqrt{7} i)$ is $(\sqrt{-5} + \sqrt{7} i)$.

(iv) $-\sqrt{-5}$

Solution:

$$-\sqrt{-5} = -\sqrt{5} \times \sqrt{-1} = -\sqrt{5} i$$

Conjugate of $(-\sqrt{-5})$ is $\sqrt{5} i$

(v) $5i$

Solution:

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Conjugate of $(5i)$ is $(-5i)$.

(vi) $\sqrt{5} - i$

Solution:

Conjugate of $(\sqrt{5} - i)$ is $(\sqrt{5} + i)$.

(vii) $\sqrt{2} + \sqrt{3} i$

Solution:

Conjugate of $(\sqrt{2} + \sqrt{3} i)$ is $(\sqrt{2} - \sqrt{3} i)$

(viii) $\cos \theta + i \sin \theta$

Solution:

Conjugate of $(\cos \theta + i \sin \theta)$ is $(\cos \theta - i \sin \theta)$

Question 3.

Find a and b if

(i) $a + 2b + 2ai = 4 + 6i$

Solution:

$$a + 2b + 2ai = 4 + 6i$$

Equating real and imaginary parts, we get

$$a + 2b = 4 \dots\dots(i)$$

$$2a = 6 \dots\dots(ii)$$

$$\therefore a = 3$$

Substituting, $a = 3$ in (i), we get

$$3 + 2b = 4$$

$$\therefore b = 12$$

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$$\therefore a = 3 \text{ and } b = 12$$

Check:

For $a = 3$ and $b = 12$

Consider, L.H.S. $= a + 2b + 2ai$

$$= 3 + 2(12) + 2(3)i$$

$$= 4 + 6i$$

$$= \text{R.H.S.}$$

$$\text{(ii) } (a - b) + (a + b)i = a + 5i$$

Solution:

$$(a - b) + (a + b)i = a + 5i$$

Equating real and imaginary parts, we get

$$a - b = a \dots\dots\text{(i)}$$

$$a + b = 5 \dots\dots\text{(ii)}$$

From (i), $b = 0$

Substituting $b = 0$ in (ii), we get

$$a + 0 = 5$$

$$\therefore a = 5$$

$$\therefore a = 5 \text{ and } b = 0$$

$$\text{(iii) } (a + b)(2 + i) = b + 1 + (10 + 2a)i$$

Solution:

$$(a + b)(2 + i) = b + 1 + (10 + 2a)i$$

$$2(a + b) + (a + b)i = (b + 1) + (10 + 2a)i$$

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Equating real and imaginary parts, we get

$$2(a + b) = b + 1$$

$$\therefore 2a + b = 1 \dots\dots(i)$$

$$\text{and } a + b = 10 + 2a$$

$$-a + b = 10 \dots\dots(ii)$$

Subtracting equation (ii) from (i), we get

$$3a = -9$$

$$\therefore a = -3$$

Substituting $a = -3$ in (ii), we get

$$-(-3) + b = 10$$

$$\therefore b = 7$$

$$\therefore a = -3 \text{ and } b = 7$$

$$(iv) \text{ } abi = 3a - b + 12i$$

Solution:

$$abi = 3a - b + 12i$$

$$\therefore 0 + abi = (3a - b) + 12i$$

Equating real and imaginary parts, we get

$$3a - b = 0$$

$$\therefore 3a = b \dots\dots(i)$$

$$\text{and } ab = 12$$

$$\therefore 3a = b \dots\dots(i)$$

$$\text{and } ab = 12$$

$$\therefore b = \frac{12}{a} \dots\dots(ii)$$

Substituting $b = \frac{12}{a}$ in (i), we get

$$3a = \frac{12}{a}$$

$$3a^2 = 12$$

$$a^2 = 4$$

$$a = \pm 2$$

$$\text{When } a = 2, b = \frac{12}{a} = \frac{12}{2} = 6$$

$$\text{When } a = -2, b = \frac{12}{a} = \frac{12}{-2} = -6$$

$$\therefore a = 2 \text{ and } b = 6 \text{ or } a = -2 \text{ and } b = -6$$

$$(v) \frac{1}{a+ib} = 3 - 2i$$

Solution:

$$\frac{1}{a+ib} = 3 - 2i$$

$$\therefore a + ib = \frac{1}{3 - 2i}$$

$$\therefore a + ib = \frac{1}{3 - 2i} \times \frac{3 + 2i}{3 + 2i}$$

$$\therefore a + ib = \frac{3 + 2i}{3^2 - 2^2 i^2}$$

$$\therefore a + ib = \frac{3 + 2i}{9 - 4(-1)} \quad \dots[\because i^2 = -1]$$

$$\therefore a + ib = \frac{3 + 2i}{13}$$

$$\therefore a + ib = \frac{3}{13} + \frac{2}{13}i$$

Equating real and imaginary parts, we get

$$\therefore a = \frac{3}{13} \text{ and } b = \frac{2}{13}$$

(vi) $(a + ib)(1 + i) = 2 + i$

Solution:

$$(a + ib)(1 + i) = 2 + i$$

$$a + ai + bi + bi^2 = 2 + i$$

$$a + (a + b)i + b(-1) = 2 + i \dots\dots (\because i^2 = -1)$$

$$(a - b) + (a + b)i = 2 + i$$

Equating real and imaginary parts, we get

$$a - b = 2 \dots\dots (i)$$

$$a + b = 1 \dots\dots (ii)$$

Adding equations (i) and (ii), we get

$$2a = 3$$

$$\therefore a = \frac{3}{2}$$

Substituting $a = \frac{3}{2}$ in (ii), we get

$$\frac{3}{2} + b = 1$$

$$\therefore b = 1 - \frac{3}{2} = \frac{-1}{2}$$

$$\therefore a = \frac{3}{2} \text{ and } b = \frac{-1}{2}$$

Question 4.

Express the following in the form of $a + ib$, $a, b \in \mathbb{R}$, $i = \sqrt{-1}$. State the values of a and b :

(i) $(1 + 2i)(-2 + i)$

Solution:

$$(1 + 2i)(-2 + i) = -2 + i - 4i + 2i^2$$

$$= -2 - 3i + 2(-1) \dots [\because i^2 = -1]$$

$$\therefore (1 + 2i)(-2 + i) = -4 - 3i$$

$$\therefore a = -4 \text{ and } b = -3$$

(ii) $(1 + i)(1 - i)^{-1}$

Solution:

$$\begin{aligned} (1 + i)(1 - i)^{-1} &= \frac{1+i}{1-i} \\ &= \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i+i^2}{1-i^2} \\ &= \frac{1+2i-1}{1-(-1)} \quad \dots [\because i^2 = -1] \\ &= \frac{2i}{2} \end{aligned}$$

$$\therefore (1+i)(1-i)^{-1} = 0+i$$

$$\therefore a = 0 \text{ and } b = 1$$

$$\text{(iii)} \quad \frac{i(4+3i)}{1-i}$$

Solution:

$$\begin{aligned} \text{iii.} \quad \frac{i(4+3i)}{1-i} &= \frac{4i+3i^2}{1-i} \\ &= \frac{-3+4i}{1-i} && \dots[\because i^2 = -1] \\ &= \frac{(-3+4i)(1+i)}{(1-i)(1+i)} \\ &= \frac{-3-3i+4i+4i^2}{1-i^2} \\ &= \frac{-3+i+4(-1)}{1-(-1)} && \dots[\because i^2 = -1] \\ &= \frac{-7+i}{2} \end{aligned}$$

$$\therefore \frac{i(4+3i)}{1-i} = \frac{-7}{2} + \frac{1}{2}i$$

$$\therefore a = \frac{-7}{2} \text{ and } b = \frac{1}{2}$$

(iv) $\frac{(2+i)}{(3-i)(1+2i)}$

Solution:

$$\begin{aligned}\frac{2+i}{(3-i)(1+2i)} &= \frac{2+i}{3+6i-i-2i^2} \\ &= \frac{2+i}{3+5i-2(-1)} \quad \dots [\because i^2 = -1] \\ &= \frac{2+i}{5+5i} \\ &= \frac{2+i}{5(1+i)} \\ &= \frac{(2+i)(1-i)}{5(1+i)(1-i)} \\ &= \frac{2-2i+i-i^2}{5(1-i^2)} \\ &= \frac{2-i-(-1)}{5[1-(-1)]} \quad \dots [\because i^2 = -1] \\ &= \frac{3-i}{10}\end{aligned}$$

$$\therefore \frac{2+i}{(3-i)(1+2i)} = \frac{3}{10} - \frac{1}{10}i$$

$$\therefore a = \frac{3}{10} \text{ and } b = \frac{-1}{10}$$

(v) $\left(\frac{1+i}{1-i}\right)^2$

Solution:

$$\begin{aligned}\left(\frac{1+i}{1-i}\right)^2 &= \frac{1+2i+i^2}{1-2i+i^2} \\ &= \frac{1+2i-1}{1-2i-1} \quad \dots[\because i^2 = -1] \\ &= \frac{2i}{-2i} \\ &= -1\end{aligned}$$

$$\therefore \left(\frac{1+i}{1-i}\right)^2 = -1 + 0i$$

$$\therefore a = -1 \text{ and } b = 0$$

(vi) $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$

Solution:

$$\begin{aligned}&\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i} \\ &= \frac{(3+2i)(2+5i) + (2-5i)(3-2i)}{(2-5i)(2+5i)} \\ &= \frac{6+15i+4i+10i^2 + 6-4i-15i+10i^2}{4-25i^2}\end{aligned}$$

$$\begin{aligned}
 &= \frac{12 + 20i^2}{4 - 25i^2} \\
 &= \frac{12 + 20(-1)}{4 - 25(-1)} \quad \dots [\because i^2 = -1] \\
 &= \frac{-8}{29} \\
 \therefore \quad \frac{3+2i}{2-5i} + \frac{3-2i}{2+5i} &= \frac{-8}{29} + 0i \\
 \therefore \quad a &= \frac{-8}{29} \text{ and } b = 0
 \end{aligned}$$

(vii) $(1 + i)^{-3}$

Solution:

$$\begin{aligned}
 (1 + i)^{-3} &= \frac{1}{(1+i)^3} \\
 &= \frac{1}{1 + 3i + 3i^2 + i^3} \\
 &\quad \dots [\because (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3] \\
 &= \frac{1}{1 + 3i - 3 - i} \quad \dots [\because i^2 = -1, i^3 = -i] \\
 &= \frac{1}{-2 + 2i} \\
 &= \frac{1}{2(-1 + i)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \times \frac{-1-i}{(-1+i)(-1-i)} \\
 &= \frac{1}{2} \times \frac{(-1-i)}{1-i^2} \\
 &= \frac{-(1+i)}{2[1-(-1)]} \quad \dots [\because i^2 = -1] \\
 &= \frac{-1}{4} (1+i) \\
 \therefore (1+i)^{-3} &= \frac{-1}{4} - \frac{1}{4}i \\
 \therefore a = b &= \frac{-1}{4}
 \end{aligned}$$

(viii) $\frac{2+\sqrt{-3}}{4+\sqrt{-3}}$

Solution:

$$\begin{aligned}
 \frac{2+\sqrt{-3}}{4+\sqrt{-3}} &= \frac{2+\sqrt{3}i}{4+\sqrt{3}i} \\
 &= \frac{(2+\sqrt{3}i)(4-\sqrt{3}i)}{(4+\sqrt{3}i)(4-\sqrt{3}i)} \\
 &= \frac{8-2\sqrt{3}i+4\sqrt{3}i-3i^2}{16-3i^2} \\
 &= \frac{8+2\sqrt{3}i-3(-1)}{16-3(-1)} \quad \dots [\because i^2 = -1]
 \end{aligned}$$

$$= \frac{11 + 2\sqrt{3}i}{19}$$
$$\therefore \frac{2 + \sqrt{-3}}{4 + \sqrt{-3}} = \frac{11}{19} + \frac{2\sqrt{3}}{19}i$$
$$\therefore a = \frac{11}{19} \text{ and } b = \frac{2\sqrt{3}}{19}$$

(ix) $(-\sqrt{5} + 2\sqrt{-4}) + (1 - \sqrt{-9}) + (2 + 3i)(2 - 3i)$

Solution:

$$\begin{aligned} & (-\sqrt{5} + 2\sqrt{-4}) + (1 - \sqrt{-9}) + (2 + 3i)(2 - 3i) \\ &= (-\sqrt{5} + 2\sqrt{4} \cdot \sqrt{-1}) + (1 - \sqrt{9} \cdot \sqrt{-1}) + 4 - 9i^2 \\ &= [-\sqrt{5} + 2(2)i] + (1 - 3i) + 4 - 9i^2 \\ &= -\sqrt{5} + 4i + 1 - 3i + 4 - 9(-1) \dots\dots [\because i^2 = -1] \\ &= (14 - \sqrt{5}) + i \\ &\therefore a = 14 - \sqrt{5} \text{ and } b = 1 \end{aligned}$$

(x) $(2 + 3i)(2 - 3i)$

Solution:

$$\begin{aligned} & (2 + 3i)(2 - 3i) \\ &= 4 - 9i^2 \\ &= 4 - 9(-1) \dots [\because i^2 = -1] \\ &= 4 + 9 \end{aligned}$$

$$= 13$$

$$\therefore (2 + 3i)(2 - 3i) = 13 + 0i$$

$$\therefore a = 13 \text{ and } b = 0$$

$$(xi) \frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2}$$

Solution:

$$\frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2} = \frac{4(i^4)^2 - 3(i^4)^2 \cdot i + 3}{3(i^4)^2 \cdot i^3 - 4(i^4)^2 \cdot i^2 - 2}$$

$$\therefore i^2 = -1, i^3 = -i \text{ and } i^4 = 1$$

$$\begin{aligned} \therefore \frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2} &= \frac{4(1)^2 - 3(1)^2 \cdot i + 3}{3(1)^2(-i) - 4(1)^2(-1) - 2} \\ &= \frac{4 - 3i + 3}{-3i + 4 - 2} \\ &= \frac{7 - 3i}{2 - 3i} \\ &= \frac{(7 - 3i)(2 + 3i)}{(2 - 3i)(2 + 3i)} \\ &= \frac{14 + 21i - 6i - 9i^2}{4 - 9i^2} \\ &= \frac{14 + 15i - 9(-1)}{4 - 9(-1)} \\ &= \frac{23 + 15i}{13} \end{aligned}$$

$$\therefore \frac{4i^8 - 3i^9 + 3}{3i^{11} - 4i^{10} - 2} = \frac{23}{13} + \frac{15}{13}i$$

$$\therefore a = \frac{23}{13} \text{ and } b = \frac{15}{13}$$

Question 5.

Show that $(-1 + \sqrt{3}i)^3$ is a real number.

Solution:

$$\begin{aligned}(-1 + \sqrt{3}i)^3 &= (-1)^3 + 3(-1)^2(\sqrt{3}i) + 3(-1)(\sqrt{3}i)^2 \\ &\quad + (\sqrt{3}i)^3 \\ &\dots[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3] \\ &= -1 + 3\sqrt{3}i - 3(3i^2) + 3\sqrt{3}i^3 \\ &= -1 + 3\sqrt{3}i - 3(-3) - 3\sqrt{3}i \\ &\dots[\because i^2 = -1, i^3 = -i] \\ &= -1 + 9 \\ &= 8, \text{ which is a real number.}\end{aligned}$$

Question 6.

Find the value of $(3 + \frac{2}{i})(i^6 - i^7)(1 + i^{11})$.

Solution:

$$\begin{aligned}&\left(3 + \frac{2}{i}\right)(i^6 - i^7)(1 + i^{11}) \\ &= \left(3 + \frac{2}{i}\right)\left[(i^2)^3 - i^4 \cdot i^3\right]\left[1 + (i^4)^2 \cdot i^3\right] \\ &= \left(3 + \frac{2}{i}\right)\left[(-1)^3 - (1)(-i)\right]\left[1 + (1)^2(-i)\right] \\ &\dots[\because i^2 = -1, i^3 = -i \text{ and } i^4 = 1]\end{aligned}$$

$$\begin{aligned}&= \left(3 + \frac{2}{i}\right)(-1 + i)(1 - i) \\&= \left(3 + \frac{2i}{i^2}\right)(-1 + i + i - i^2) \\&= \left(3 + \frac{2i}{-1}\right)[-1 + 2i - (-1)] \\&= (3 - 2i)(2i) \\&= 6i - 4i^2 \\&= 6i - 4(-1) \\&= 4 + 6i\end{aligned}$$

Question 7.

Evaluate the following:

(i) i^{35}

(ii) i^{888}

(iii) i^{93}

(iv) i^{116}

(v) i^{403}

(vi) $\frac{1}{i^{58}}$

(vii) i^{-888}

(viii) $i^{30} + i^{40} + i^{50} + i^{60}$

Solution:

We know that, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

- i. $i^{35} = (i^4)^8 (i^2)i = (1)^8 (-1)i = -i$
- ii. $i^{888} = (i^4)^{222} = (1)^{222} = 1$
- iii. $i^{93} = (i^4)^{23} \cdot i = (1)^{23} \cdot i = i$
- iv. $i^{116} = (i^4)^{29} = (1)^{29} = 1$
- v. $i^{403} = (i^4)^{100} (i^2)i = (1)^{100} (-1)i = -i$
- vi. $\frac{1}{i^{58}} = \frac{1}{(i^4)^{14} i^2} = \frac{1}{(1)^{14} (-1)} = -1$
- vii. $i^{-888} = (i^4)^{-222} = (1)^{-222} = \frac{1}{(1)^{222}} = 1$
- viii. $i^{30} + i^{40} + i^{50} + i^{60}$
 $= (i^4)^7 i^2 + (i^4)^{10} + (i^4)^{12} i^2 + (i^4)^{15}$
 $= (1)^7 (-1) + (1)^{10} + (1)^{12} (-1) + (1)^{15}$
 $= -1 + 1 - 1 + 1 = 0$

Question 8.

Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number.

Solution:

$$\begin{aligned} & 1 + i^{10} + i^{20} + i^{30} \\ &= 1 + (i^4)^2 \cdot i^2 + (i^4)^5 + (i^4)^7 \cdot i^2 \\ &= 1 + (1)^2(-1) + (1)^5 + (1)^7(-1) \quad \dots[\because i^4 = 1, i^2 = -1] \\ &= 1 - 1 + 1 - 1 \\ &= 0, \text{ which is a real number.} \end{aligned}$$

Question 9.

Find the value of

(i) $i^{49} + i^{68} + i^{89} + i^{110}$

(ii) $i + i^2 + i^3 + i^4$

Solution:

$$\begin{aligned} \text{i.} \quad & i^{49} + i^{68} + i^{89} + i^{110} \\ &= (i^4)^{12} \cdot i + (i^4)^{17} + (i^4)^{22} \cdot i + (i^4)^{27} \cdot i^2 \\ &= (1)^{12} \cdot i + (1)^{17} + (1)^{22} \cdot i + (1)^{27}(-1) \quad \dots[\because i^4 = 1, i^2 = -1] \\ &= i + 1 + i - 1 = 2i \\ \text{ii.} \quad & i + i^2 + i^3 + i^4 = i + i^2 + i^2 \cdot i + i^4 \\ &= i - 1 - i + 1 \quad \dots[\because i^2 = -1, i^4 = 1] \end{aligned}$$

Question 8.

Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number.

Solution:

$$\begin{aligned} & 1 + i^{10} + i^{20} + i^{30} \\ &= 1 + (i^4)^2 \cdot i^2 + (i^4)^5 + (i^4)^7 \cdot i^2 \\ &= 1 + (1)^2(-1) + (1)^5 + (1)^7(-1) \quad \dots[\because i^4 = 1, i^2 = -1] \\ &= 1 - 1 + 1 - 1 \\ &= 0, \text{ which is a real number.} \end{aligned}$$

Question 9.

Find the value of

(i) $i^{49} + i^{68} + i^{89} + i^{110}$

(ii) $i + i^2 + i^3 + i^4$

Solution:

$$\begin{aligned} \text{i.} \quad & i^{49} + i^{68} + i^{89} + i^{110} \\ &= (i^4)^{12} \cdot i + (i^4)^{17} + (i^4)^{22} \cdot i + (i^4)^{27} \cdot i^2 \\ &= (1)^{12} \cdot i + (1)^{17} + (1)^{22} \cdot i + (1)^{27}(-1) \quad \dots[\because i^4 = 1, i^2 = -1] \\ &= i + 1 + i - 1 = 2i \\ \text{ii.} \quad & i + i^2 + i^3 + i^4 = i + i^2 + i^2 \cdot i + i^4 \\ &= i - 1 - i + 1 \quad \dots[\because i^2 = -1, i^4 = 1] \end{aligned}$$

$$= 0$$

Question 10.

Simplify: $\frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}}$

Solution:

$$\begin{aligned} & \frac{i^{592} + i^{590} + i^{588} + i^{586} + i^{584}}{i^{582} + i^{580} + i^{578} + i^{576} + i^{574}} \\ &= \frac{i^{10} (i^{582} + i^{580} + i^{578} + i^{576} + i^{574})}{(i^{582} + i^{580} + i^{578} + i^{576} + i^{574})} \\ &= i^{10} = (i^4)^2 \cdot i^2 \\ &= (1)^2 (-1) \quad \dots [\because i^4 = 1, i^2 = -1] \\ &= -1 \end{aligned}$$

Question 11.

Find the value of $1 + i^2 + i^4 + i^6 + i^8 + \dots + i^{20}$.

Solution:

$$\begin{aligned} & 1 + i^2 + i^4 + i^6 + i^8 + \dots + i^{20} \\ &= 1 + (i^2 + i^4) + (i^6 + i^8) + (i^{10} + i^{12}) \\ & \quad + (i^{14} + i^{16}) + (i^{18} + i^{20}) \\ &= 1 + [i^2 + (i^2)^2] + [(i^2)^3 + (i^2)^4] \\ & \quad + [(i^2)^5 + (i^2)^6] + [(i^2)^7 + (i^2)^8] + [(i^2)^9 + (i^2)^{10}] \\ &= 1 + [-1 + (-1)^2] + [(-1)^3 + (-1)^4] \\ &= 1 + [-1 + (-1)^2] + [(-1)^3 + (-1)^4] \\ & \quad + [(-1)^5 + (-1)^6] + [(-1)^7 + (-1)^8] \\ & \quad + [(-1)^9 + (-1)^{10}] \\ & \quad \dots [\because i^2 = -1] \\ &= 1 + (-1 + 1) + (-1 + 1) + (-1 + 1) \\ & \quad + (-1 + 1) + (-1 + 1) \\ &= 1 + 0 + 0 + 0 + 0 + 0 = 1 \end{aligned}$$

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Question 12.

Show that $1 + i^{10} + i^{100} - i^{1000} = 0$.

Solution:

$$\begin{aligned} & 1 + i^{10} + i^{100} - i^{1000} \\ &= 1 + (i^4)^2 \cdot i^2 + (i^4)^{25} - (i^4)^{250} \\ &= 1 + (1)^2 \cdot (-1) + (1)^{25} - (1)^{250} \\ & \quad \dots [\because i^2 = -1, i^4 = 1] \\ &= 1 - 1 + 1 - 1 = 0 \end{aligned}$$

Question 13.

Is $(1 + i^{14} + i^{18} + i^{22})$ a real number? Justify your answer.

Solution:

$$\begin{aligned} & 1 + i^{14} + i^{18} + i^{22} \\ &= 1 + (i^4)^3 \cdot i^2 + (i^4)^4 \cdot i^2 + (i^4)^5 \cdot i^2 \\ &= 1 + (1)^3(-1) + (1)^4(-1) + (1)^5(-1) \\ & \quad \dots [\because i^2 = -1, i^4 = 1] \\ &= 1 - 1 - 1 - 1 \\ &= -2, \text{ which is a real number.} \end{aligned}$$

Question 14.

Evaluate: $\left(i^{37} + \frac{1}{i^{67}}\right)$

Solution:

$$\begin{aligned} i^{37} + \frac{1}{i^{67}} &= (i^4)^9 i + \frac{1}{(i^4)^{16} i^3} \\ &= (1)^9 i + \frac{1}{(1)^{16} \cdot (-i)} \quad \dots [\because i^3 = -i, i^4 = 1] \\ &= i - \frac{1}{i} \\ &= i + \frac{i^2}{i} \quad \dots [-1 = i^2] \\ &= i + i \\ &= 2i \end{aligned}$$

Question 15.

Prove that: $(1+i)^4 \times \left(1 + \frac{1}{i}\right)^4 = 16$

Solution:

$$\begin{aligned} &(1+i)^4 \times \left(1 + \frac{1}{i}\right)^4 \\ &= \left[(1+i)\left(1 + \frac{1}{i}\right)\right]^4 = \left[(1+i)\frac{(1+i)}{i}\right]^4 = \left[\frac{(1+i)^2}{i}\right]^4 \\ &= \frac{(1+2i+i^2)^4}{i^4} \\ &= \frac{(1+2i-1)^4}{1} \quad \dots [\because i^2 = -1] \\ &= 16i^4 \end{aligned}$$

$$= 16$$

$$\dots [\because i^4 = 1]$$

Question 16.

Find the value of $\frac{i^6 + i^7 + i^8 + i^9}{i^2 + i^3}$

Solution:

$$\begin{aligned} \frac{i^6 + i^7 + i^8 + i^9}{i^2 + i^3} &= \frac{i^4 \cdot i^2 + i^4 \cdot i^3 + (i^4)^2 + (i^4)^2 \cdot i}{i^2 + i^3} \\ &= \frac{(1) \cdot (-1) + (1) \cdot (-i) + (1)^2 + (1)^2 \cdot i}{-1 - i} \\ &\quad \dots [\because i^2 = -1, i^3 = -i, i^4 = 1] \\ &= \frac{-1 - i + 1 + i}{-1 - i} = 0 \end{aligned}$$

Question 17.

If $a = \frac{-1 + \sqrt{3}i}{2}$, $b = \frac{-1 - \sqrt{3}i}{2}$, then show that $a^2 = b$ and $b^2 = a$.

Solution:

$$\begin{aligned} a &= \frac{-1 + \sqrt{3}i}{2}, b = \frac{-1 - \sqrt{3}i}{2} \\ a^2 &= \left(\frac{-1 + \sqrt{3}i}{2} \right)^2 = \frac{1 - 2\sqrt{3}i + 3i^2}{4} \end{aligned}$$

$$= \frac{1 - 2\sqrt{3}i + 3(-1)}{4}$$

$$\dots [\because i^2 = -1]$$

$$= \frac{-2 - 2\sqrt{3}i}{4}$$

$$= \frac{-1 - \sqrt{3}i}{2} = b$$

$$\text{and } b^2 = \left(\frac{-1 - \sqrt{3}i}{2} \right)^2 = \frac{1 + 2\sqrt{3}i + 3i^2}{4}$$

$$= \frac{1 + 2\sqrt{3}i + 3(-1)}{4}$$

$$\dots [\because i^2 = -1]$$

$$= \frac{-2 + 2\sqrt{3}i}{4}$$

$$= \frac{-1 + \sqrt{3}i}{2} = a$$

Question 18.

If $x + iy = (a + ib)^3$, show that $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$

Solution:

$$x + iy = (a + ib)^3$$

$$x + iy = a^3 + 3a^2bi + 3ab^2i^2 + b^3i^3$$

$$x + iy = a^3 + 3a^2bi - 3ab^2 - b^3i \dots\dots[\because i^2 = -1, i^3 = -i]$$

$$x + iy = (a^3 - 3ab^2) + (3a^2b - b^3)i$$

Equating real and imaginary parts, we get

$$x = a^3 - 3ab^2 \text{ and } y = 3a^2b - b^3$$

$$\frac{x}{a} = a^2 - 3b^2 \text{ and } \frac{y}{b} = 3a^2 - b^2$$

$$\frac{x}{a} + \frac{y}{b} = a^2 - 3b^2 + 3a^2 - b^2$$

$$\frac{x}{a} + \frac{y}{b} = 4a^2 - 4b^2$$

$$\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$$

Alternate Method:

$$x + iy = (a + ib)^3$$

$$x + iy = a^3 + 3a^2bi + 3ab^2i^2 + b^3i^3$$

$$x + iy = a^3 + 3a^2bi - 3ab^2 - b^3i \dots\dots[\because i^2 = -1, i^3 = -i]$$

$$x + iy = (a^3 - 3ab^2) + (3a^2b - b^3)i$$

Equating real and imaginary parts, we get

$$x = a^3 - 3ab^2 \text{ and } y = 3a^2b - b^3$$

Consider

$$\begin{aligned} \text{L.H.S.} &= \frac{x}{a} + \frac{y}{b} \\ &= \frac{a^3 - 3ab^2}{a} + \frac{3a^2b - b^3}{b} \end{aligned}$$

$$\begin{aligned} &= \frac{a(a^2 - 3b^2)}{a} + \frac{b(3a^2 - b^2)}{b} \\ &= a^2 - 3b^2 + 3a^2 - b^2 \\ &= 4a^2 - 4b^2 = 4(a^2 - b^2) = \text{R.H.S.} \\ \frac{x}{a} + \frac{y}{b} &= 4(a^2 - b^2) \end{aligned}$$

Question 19.

If $\frac{a+3i}{2+ib} = 1 - i$, show that $(5a - 7b) = 0$.

Solution:

$$\frac{a+3i}{2+ib} = 1 - i$$

$$a + 3i = (1 - i)(2 + ib)$$

$$= 2 + bi - 2i - bi^2$$

$$= 2 + (b - 2)i - b(-1) \dots\dots [\because i^2 = -1]$$

$$a + 3i = (2 + b) + (b - 2)i$$

Equating real and imaginary parts, we get

$$a = 2 + b \text{ and } 3 = b - 2$$

$$a = 2 + b \text{ and } b = 5$$

$$a = 2 + 5 = 7$$

$$5a - 7b = 5(7) - 7(5) = 35 - 35 = 0$$

Question 20.

If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

Solution:

$$x + iy = \sqrt{\frac{a+ib}{c+id}}$$

$$\therefore (x + iy)^2 = \frac{a+bi}{c+di}$$

$$\therefore x^2 + 2xyi + y^2i^2 = \frac{(a+bi)(c-di)}{(c+di)(c-di)}$$

$$\therefore x^2 + 2xyi + y^2(-1) = \frac{ac - adi + bci - bdi^2}{c^2 - d^2i^2}$$

...[$\because i^2 = -1$]

$$\therefore (x^2 - y^2) + 2xyi = \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$$

$$\therefore (x^2 - y^2) + 2xyi = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2} i$$

Equating real and imaginary parts, we get

$$x^2 - y^2 = \frac{ac + bd}{c^2 + d^2} \quad \text{and} \quad 2xy = \frac{bc - ad}{c^2 + d^2}$$

$$\begin{aligned} \therefore (x^2 + y^2)^2 &= (x^2 - y^2)^2 + (2xy)^2 \\ &= \left(\frac{ac + bd}{c^2 + d^2} \right)^2 + \left(\frac{bc - ad}{c^2 + d^2} \right)^2 \\ &= \frac{a^2c^2 + b^2d^2 + 2abcd + b^2c^2 + a^2d^2 - 2abcd}{(c^2 + d^2)^2} \end{aligned}$$

$$\begin{aligned} &= \frac{a^2c^2 + b^2c^2 + a^2d^2 + b^2d^2}{(c^2 + d^2)^2} \\ &= \frac{c^2(a^2 + b^2) + d^2(a^2 + b^2)}{(c^2 + d^2)^2} \\ &= \frac{(a^2 + b^2)(c^2 + d^2)}{(c^2 + d^2)^2} \\ \therefore (x^2 + y^2)^2 &= \frac{a^2 + b^2}{c^2 + d^2} \end{aligned}$$

Question 21.

If $(a + ib) = \frac{1+i}{1-i}$, then prove that $a^2 + b^2 = 1$.

Solution:

$$\begin{aligned} a + bi &= \frac{1+i}{1-i} = \frac{(1+i)^2}{(1-i)(1+i)} \\ a + bi &= \frac{1+2i+i^2}{1-i^2} = \frac{1+2i-1}{1-(-1)} \quad \dots [\because i^2 = -1] \\ &= \frac{2i}{2} = i \end{aligned}$$

$$\therefore a + bi = 0 + i$$

Equating real and imaginary parts, we get

$$a = 0 \text{ and } b = 1$$

$$a^2 + b^2 = 0^2 + 1^2 = 1$$

Question 22.

Show that $\left(\frac{\sqrt{7}+i\sqrt{3}}{\sqrt{7}-i\sqrt{3}} + \frac{\sqrt{7}-i\sqrt{3}}{\sqrt{7}+i\sqrt{3}} \right)$ is real.

Solution:

$$\begin{aligned} & \left(\frac{\sqrt{7}+\sqrt{3}i}{\sqrt{7}-\sqrt{3}i} + \frac{\sqrt{7}-\sqrt{3}i}{\sqrt{7}+\sqrt{3}i} \right) \\ &= \left[\frac{(\sqrt{7}+\sqrt{3}i)^2 + (\sqrt{7}-\sqrt{3}i)^2}{(\sqrt{7}+\sqrt{3}i)(\sqrt{7}-\sqrt{3}i)} \right] \\ &= \frac{7+2\sqrt{21}i+3i^2+7-2\sqrt{21}i+3i^2}{7-3i^2} = \frac{14+6i^2}{7-3i^2} \\ &= \frac{14+6(-1)}{7-3(-1)} \quad \dots [\because i^2 = -1] \\ &= \frac{8}{10} = \frac{4}{5}, \text{ which is real.} \end{aligned}$$

Question 23.

If $(x + iy)^3 = y + vi$, then show that $\frac{y}{x} + \frac{v}{y} = 4(x^2 - y^2)$.

Solution:

$$\begin{aligned} & (x + yi)^3 = y + vi \\ \therefore & x^3 + 3x^2yi + 3xy^2i^2 + y^3i^3 = y + vi \\ \therefore & x^3 + 3x^2yi + 3xy^2(-1) - y^3i = y + vi \\ & \dots [\because i^2 = -1, i^3 = -i] \\ \therefore & (x^3 - 3xy^2) + (3x^2y - y^3)i = y + vi \end{aligned}$$

Equating real and imaginary parts, we get

$$y = x^3 - 3xy^2 \quad \text{and} \quad v = 3x^2y - y^3$$

$$\therefore \frac{y}{x} = x^2 - 3y^2 \quad \text{and} \quad \frac{v}{y} = 3x^2 - y^2$$

$$\therefore \frac{y}{x} + \frac{v}{y} = x^2 - 3y^2 + 3x^2 - y^2 = 4x^2 - 4y^2$$

$$\therefore \frac{y}{x} + \frac{v}{y} = 4(x^2 - y^2)$$

Question 24.

Find the values of x and y which satisfy the following equations ($x, y \in \mathbb{R}$)

(i) $(x + 2y) + (2x - 3y)i + 4i = 5$

Solution:

$$(x + 2y) + (2x - 3y)i + 4i = 5$$

$$(x + 2y) + (2x - 3y)i = 5 - 4i$$

Equating real and imaginary parts, we get

$$x + 2y = 5 \quad \dots\dots(i)$$

$$\text{and } 2x - 3y = -4 \quad \dots\dots(ii)$$

Equation (i) $\times 2$ – equation (ii) gives

$$7y = 14$$

$$\therefore y = 2$$

Substituting $y = 2$ in (i), we get

$$x + 2(2) = 5$$

$$x + 4 = 5$$

$$\therefore x = 1$$

$$\therefore x = 1 \text{ and } y = 2$$

Check:

For $x = 1$ and $y = 2$

$$\text{Consider, L.H.S.} = (x + 2y) + (2x - 3y)i + 4i$$

$$= (1 + 4) + (2 - 6)i + 4i$$

$$= 5 - 4i + 4i$$

$$= 5$$

$$= \text{R.H.S.}$$

$$(ii) \frac{x+1}{1+i} + \frac{y-1}{1-i} = i$$

Solution:

$$\frac{x+1}{1+i} + \frac{y-1}{1-i} = i$$

$$\frac{(x+1)(1-i) + (y-1)(1+i)}{(1+i)(1-i)} = i$$

$$\frac{x - xi + 1 - i + y + yi - 1 - i}{1 - i^2} = i$$

$$\frac{(x+y) + (y-x-2)i}{1 - (-1)} = i \quad \dots [\because i^2 = -1]$$



$$(x + y) + (y - x - 2)i = 2i$$

$$(x + y) + (y - x - 2)i = 0 + 2i$$

Equating real and imaginary parts, we get

$$x + y = 0 \text{ and } y - x - 2 = 2$$

$$\therefore x + y = 0 \dots\dots(i)$$

$$\text{and } -x + y = 4 \dots\dots(ii)$$

Adding (i) and (ii), we get

$$2y = 4$$

$$\therefore y = 2$$

Substituting $y = 2$ in (i), we get

$$x + 2 = 0$$

$$\therefore x = -2$$

$$\therefore x = -2 \text{ and } y = 2$$

$$(iii) \frac{x+iy}{2+3i} + \frac{2+i}{2-3i} = \frac{9}{13}(1+i)$$

Solution:

$$\frac{x+iy}{2+3i} + \frac{2+i}{2-3i} = \frac{9}{13}(1+i)$$

$$\frac{(x+iy)(2-3i) + (2+i)(2+3i)}{(2+3i)(2-3i)} = \frac{9}{13}(1+i)$$

$$\frac{2x-3xi+2yi-3yi^2+4+6i+2i+3i^2}{4-9i^2} = \frac{9}{13}(1+i)$$

$$\frac{2x - 3xi + 2yi + 3y + 4 + 8i - 3}{4 + 9} = \frac{9 + 9i}{13}$$

...[$\because i^2 = -1$]

$$\frac{2x + 3y + 1 + 8i - 3xi + 2yi}{13} = \frac{9 + 9i}{13}$$

$$(2x + 3y + 1) + (8 - 3x + 2y)i = 9 + 9i$$

Equating real and imaginary parts, we get

$$2x + 3y + 1 = 9 \text{ and } 8 - 3x + 2y = 9$$

$$2x + 3y = 8 \dots\dots(i)$$

$$\text{and } 3x - 2y = -1 \dots\dots(ii)$$

Equation (i) $\times$ 2 + equation (ii) $\times$ 3 gives

$$13x = 13$$

$$\therefore x = 1$$

Substituting $x = 1$ in (i), we get

$$2(1) + 3y = 8$$

$$3y = 6$$

$$\therefore y = 2$$

$$\therefore x = 1 \text{ and } y = 2$$

(iv) If $x(1 + 3i) + y(2 - i) - 5 + i^3 = 0$, find $x + y$

Solution:

$$x(1 + 3i) + y(2 - i) - 5 + i^3 = 0$$

$$x + 3xi + 2y - yi - 5 - i = 0 \dots\dots[\because i^3 = -i]$$

$$(x + 2y - 5) + (3x - y - 1)i = 0 + 0i$$

Equating real and imaginary parts, we get

$$x + 2y - 5 = 0 \dots\dots(i)$$

$$\text{and } 3x - y - 1 = 0 \dots\dots(ii)$$

Equation (i) + equation (ii) $\times 2$ gives

$$7x - 7 = 0$$

$$7x = 7$$

$$\therefore x = 1$$

Substituting $x = 1$ in (i), we get

$$1 + 2y - 5 = 0$$

$$2y = 4$$

$$y = 2$$

$$\therefore x = 1 \text{ and } y = 2$$

$$\therefore x + y = 1 + 2 = 3$$

(v) If $x + 2i + 15i^6y = 7x + i^3(y + 4)$, find $x + y$

Solution:

$$x + 2i + 15i^6y = 7x + i^3(y + 4)$$

$$x + 2i + 15(i^2)^3y = 7x + i^3(y + 4)$$

$$x + 2i + 15(-1)^3y = 7x - i(y + 4) \dots\dots[\because i^2 = -1, i^3 = -i]$$

$$x + 2i - 15y - 7x + iy + 4i = 0$$

$$(-6x - 15y) + i(y + 6) = 0 + 0i$$

Equating real and imaginary parts, we get

$$-6x - 15y = 0 \text{ and } y + 6 = 0$$

$$-6x - 15y = 0 \text{ and } y = -6$$

$$-6x - 15(-6) = 0$$

$$-6x + 90 = 0$$

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-1-complex-numbers/>

$$\therefore x = 15$$

$$\therefore x + y = 15 - 6 = 9$$

Ex 1.2

Question 1.

Find the square root of the following complex numbers:

(i) $-8 - 6i$

Solution:

Let $\sqrt{-8 - 6i} = a + bi$, where $a, b \in \mathbb{R}$.

Squaring on both sides, we get

$$-8 - 6i = (a + bi)^2$$

$$-8 - 6i = a^2 + b^2i^2 + 2abi$$

$$-8 - 6i = (a^2 - b^2) + 2abi \dots [\because i^2 = -1]$$

Equating real and imaginary parts, we get

$$a^2 - b^2 = -8 \text{ and } 2ab = -6$$

$$a^2 - b^2 = -8 \text{ and } b = \frac{-3}{a}$$

$$a^2 - \left(\frac{-3}{a}\right)^2 = -8$$

$$a^2 - \frac{9}{a^2} = -8$$

$$a^4 - 9 = -8a^2$$

$$a^4 + 8a^2 - 9 = 0$$

$$(a^2 + 9)(a^2 - 1) = 0$$

$$a^2 = -9 \text{ or } a^2 = 1$$

But $a \in \mathbb{R}$

$$a^2 \neq -9$$

$$a^2 = 1$$

$$a = \pm 1$$

$$\text{When } a = 1, b = \frac{-3}{1} = -3$$

$$\text{When } a = -1, b = \frac{-3}{-1} = 3$$

$$\sqrt{-8 - 6i} = \pm (1 - 3i)$$

(ii) $7 + 24i$

Solution:

Let $\sqrt{7 + 24i} = a + bi$, where $a, b \in \mathbb{R}$.

Squaring on both sides, we get

$$7 + 24i = (a + bi)^2$$

$$7 + 24i = a^2 + b^2i^2 + 2abi$$

$$7 + 24i = (a^2 - b^2) + 2abi \dots [\because i^2 = -1]$$

Equating real and imaginary parts, we get

$$a^2 - b^2 = 7 \text{ and } 2ab = 24$$

$$a^2 - b^2 = 7 \text{ and } b = \frac{12}{a}$$

$$a^2 - \left(\frac{12}{a}\right)^2 = 7$$

$$a^2 - \frac{144}{a^2} = 7$$

$$a^4 - 144 = 7a^2$$

$$a^4 - 7a^2 - 144 = 0$$

$$(a^2 - 16)(a^2 + 9) = 0$$

$$a^2 = 16 \text{ or } a^2 = -9$$

But $a \in \mathbb{R}$

$$a^2 \neq -9$$

$$a^2 = 16$$

$$a = \pm 4$$

$$\text{When } a = 4, b = \frac{12}{4} = 3$$

$$\text{When } a = -4, b = \frac{12}{-4} = -3$$

$$\sqrt{7+24i} = \pm (4 + 3i)$$

(iii) $1 + 4\sqrt{3}i$

Solution:

Let $\sqrt{1 + 4\sqrt{3}i} = a + bi$, where $a, b \in \mathbb{R}$.

Squaring on both sides, we get

$$1 + 4\sqrt{3}i = (a + bi)^2$$

$$1 + 4\sqrt{3}i = a^2 + b^2i^2 + 2abi$$

$$1 + 4\sqrt{3}i = (a^2 - b^2) + 2abi \dots [\because i^2 = -1]$$

Equating real and imaginary parts, we get

$$a^2 - b^2 = 1 \text{ and } 2ab = 4\sqrt{3}$$

$$a^2 - b^2 = 1 \text{ and } b = \frac{2\sqrt{3}}{a}$$

$$a^2 - \left(\frac{2\sqrt{3}}{a}\right)^2 = 1$$

$$a^2 - \frac{12}{a^2} = 1$$

$$a^4 - 12 = a^2$$

$$a^4 - a^2 - 12 = 0$$

$$(a^2 - 4)(a^2 + 3) = 0$$

$$a^2 = 4 \text{ or } a^2 = -3$$

But $a \in \mathbb{R}$

$$a^2 \neq -3$$

$$a^2 = 4$$

$$a = \pm 2$$

$$\text{When } a = 2, b = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\text{When } a = -2, b = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$$

$$\sqrt{1+4\sqrt{3}i} = \pm (2 + \sqrt{3}i)$$

(iv) $3 + 2\sqrt{10}i$

Solution:

Let $\sqrt{3 + 2\sqrt{10}i} = a + bi$, where $a, b \in \mathbb{R}$.

Squaring on both sides, we get

$$3 + 2\sqrt{10}i = a^2 + b^2i^2 + 2abi$$

$$3 + 2\sqrt{10}i = (a^2 - b^2) + 2abi \dots\dots [\because i^2 = -1]$$

Equating real and imaginary parts, we get

$$a^2 - b^2 = 3 \text{ and } 2ab = 2\sqrt{10}$$

$$a^2 - b^2 = 3 \text{ and } b = \frac{\sqrt{10}}{a}$$

$$a^2 - \left(\frac{\sqrt{10}}{a}\right)^2 = 3$$

$$a^2 - \frac{10}{a^2} = 3$$

$$a^4 - 10 = 3a^2$$

$$a^4 - 3a^2 - 10 = 0$$

$$(a^2 - 5)(a^2 + 2) = 0$$

$$a^2 = 5 \text{ or } a^2 = -2$$

But $a \in \mathbb{R}$

$$a^2 \neq -2$$

$$a^2 = 5$$

$$a = \pm\sqrt{5}$$

$$\text{When } a = \sqrt{5}, b = \frac{\sqrt{10}}{\sqrt{5}} = \sqrt{2}$$

$$\text{When } a = -\sqrt{5}, b = \frac{\sqrt{10}}{-\sqrt{5}} = -\sqrt{2}$$

$$\sqrt{3+2\sqrt{10}i} = \pm(\sqrt{5} + \sqrt{2}i)$$

$$a^2 - \left(-\frac{\sqrt{3}}{a}\right)^2 = 2$$

$$a^2 - \frac{3}{a^2} = 2$$

$$a^4 - 3 = 2a^2$$

$$a^4 - 2a^2 - 3 = 0$$

$$(a^2 - 3)(a^2 + 1) = 0$$

$$a^2 = 3 \text{ or } a^2 = -1$$

But $a \in \mathbb{R}$

$$a^2 \neq -1$$

$$a^2 = 3$$

$$a = \pm \sqrt{3}$$

$$\text{When } a = \sqrt{3}, b = \frac{-\sqrt{3}}{\sqrt{3}} = -1$$

$$\text{When } a = -\sqrt{3}, b = \frac{-\sqrt{3}}{-\sqrt{3}} = 1$$

$$\sqrt{2(1-\sqrt{3}i)} = \pm (\sqrt{3}-i)$$

$$(v) 2(1-\sqrt{3}i)$$

Solution:

$$\text{Let } \sqrt{2(1-\sqrt{3}i)} = a + bi, \text{ where } a, b \in \mathbb{R}.$$

Squaring on both sides, we get

$$2(1-\sqrt{3}i) = a^2 + b^2i^2 + 2abi$$

$$2 - 2\sqrt{3}i = (a^2 - b^2) + 2abi \dots [\because i^2 = -1]$$

Equating real and imaginary parts, we get

$$a^2 - b^2 = 2 \text{ and } 2ab = -2\sqrt{3}$$

$$a^2 - b^2 = 2 \text{ and } b = -\frac{\sqrt{3}}{a}$$

$$a^2 - \left(-\frac{\sqrt{3}}{a}\right)^2 = 2$$

$$a^2 - \frac{3}{a^2} = 2$$

$$a^4 - 3 = 2a^2$$

$$a^4 - 2a^2 - 3 = 0$$

$$(a^2 - 3)(a^2 + 1) = 0$$

$$a^2 = 3 \text{ or } a^2 = -1$$

But $a \in \mathbb{R}$

$$a^2 \neq -1$$

$$a^2 = 3$$

$$a = \pm \sqrt{3}$$

$$\text{When } a = \sqrt{3}, b = \frac{-\sqrt{3}}{\sqrt{3}} = -1$$

When $a = -\sqrt{3}$, $b = \frac{-\sqrt{3}}{-\sqrt{3}} = 1$

$$\sqrt{2(1-\sqrt{3}i)} = \pm (\sqrt{3}-i)$$

Question 2.

Solve the following quadratic equations:

(i) $8x^2 + 2x + 1 = 0$

Solution:

Given equation is $8x^2 + 2x + 1 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 8$, $b = 2$, $c = 1$

Discriminant $= b^2 - 4ac$

$= (2)^2 - 4 \times 8 \times 1$

$= 4 - 32$

$= -28 < 0$

So, the given equation has complex roots.

These roots are given by

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{-28}}{2(8)} = \frac{-2 \pm 2\sqrt{7}i}{16} \end{aligned}$$

$$x = \frac{-1 \pm \sqrt{7}i}{8}$$

The roots of the given equation are

$$\frac{-1 + \sqrt{7}i}{8} \text{ and } \frac{-1 - \sqrt{7}i}{8}.$$

(ii) $2x^2 - \sqrt{3}x + 1 = 0$

Solution:

Given equation is $2x^2 - \sqrt{3}x + 1 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -\sqrt{3}, c = 1$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= (-\sqrt{3})^2 - 4 \times 2 \times 1$$

$$= 3 - 8$$

$$= -5 < 0$$

So, the given equation has complex roots.

These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-\sqrt{3}) \pm \sqrt{-5}}{2(2)}$$

$$x = \frac{\sqrt{3} \pm \sqrt{5}i}{4}$$

(iii) $3x^2 - 7x + 5 = 0$

Solution:

Given equation is $3x^2 - 7x + 5 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 3, b = -7, c = 5$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= (-7)^2 - 4 \times 3 \times 5$$

$$= 49 - 60$$

$$= -11 < 0$$

So, the given equation has complex roots.

These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{-11}}{2(3)}$$

$$x = \frac{7 \pm \sqrt{11}i}{6}$$

The roots of the given equation are $\frac{7 + \sqrt{11}i}{6}$ and $\frac{7 - \sqrt{11}i}{6}$

(iv) $x^2 - 4x + 13 = 0$

Solution:

Given equation is $x^2 - 4x + 13 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -4, c = 13$$

$$\text{Discriminant} = b^2 - 4ac$$

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$$= (-4)^2 - 4 \times 1 \times 13$$

$$= 16 - 52$$

$$= -36 < 0$$

So, the given equation has complex roots.

These roots are given by

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-(-4) \pm \sqrt{-36}}{2(1)} \\&= \frac{4 \pm 6i}{2} = 2 \pm 3i\end{aligned}$$

$\therefore$ The roots of the given equation are $2 + 3i$ and $2 - 3i$.

Question 3.

Solve the following quadratic equations:

(i) $x^2 + 3ix + 10 = 0$

Solution:

Given equation is $x^2 + 3ix + 10 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 3i, c = 10$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= (3i)^2 - 4 \times 1 \times 10$$

$$= 9i^2 - 40$$

$$= -9 - 40 \dots\dots [\because i^2 = -1]$$

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$$= -49 < 0$$

So, the given equation has complex roots.

These roots are given by

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-3i \pm \sqrt{-49}}{2(1)} \\x &= \frac{-3i \pm 7i}{2} \\x &= \frac{-3i + 7i}{2} \text{ or } x = \frac{-3i - 7i}{2}\end{aligned}$$

$$\therefore x = 2i \text{ or } x = -5i$$

$\therefore$ The roots of the given equation are $2i$ and $-5i$.

(ii) $2x^2 + 3ix + 2 = 0$

Solution:

Given equation is $2x^2 + 3ix + 2 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 2, b = 3i, c = 2$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= (3i)^2 - 4 \times 2 \times 2$$

$$= 9i^2 - 16$$

$$= -9 - 16 \dots [\because i^2 = -1]$$

$$= -25 < 0$$

So, the given equation has complex roots.

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These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3i \pm \sqrt{-25}}{2(2)}$$

$$x = \frac{-3i \pm 5i}{4}$$

$$x = \frac{-3i + 5i}{4} \text{ or } x = \frac{-3i - 5i}{4}$$

$$x = \frac{1}{2}i \text{ or } x = -2i$$

∴ The roots of the given equation are $\frac{1}{2}i$ and $-2i$.

(iii) $x^2 + 4ix - 4 = 0$

Solution:

Given equation is $x^2 + 4ix - 4 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 4i, c = -4$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= (4i)^2 - 4 \times 1 \times (-4)$$

$$= 16i^2 + 16$$

$$= -16 + 16 \dots\dots [\because i^2 = -1]$$

$$= 0$$

So, the given equation has equal roots.

These roots are given by

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$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-4i \pm \sqrt{0}}{2(1)} = \frac{-4i}{2} \\x &= -2i\end{aligned}$$

$$\therefore x = -2i$$

$\therefore$ The root of the given equation is $-2i$.

$$\text{(iv) } ix^2 - 4x - 4i = 0$$

Solution:

$$ix^2 - 4x - 4i = 0$$

Multiplying throughout by i , we get

$$i2x^2 - 4ix - 4i^2 = 0$$

$$-x^2 - 4ix + 4 = 0 \dots [\because i^2 = -1]$$

$$x^2 + 4ix - 4 = 0$$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = 4i, c = -4$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= (4i)^2 - 4 \times 1 \times (-4)$$

$$= 16i^2 + 16$$

$$= -16 + 16$$

$$= 0$$

So, the given equation has equal roots.

These roots are given by

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$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-4i \pm \sqrt{0}}{2(1)} \\&= \frac{-4i}{2} \\x &= -2i\end{aligned}$$

∴ The root of the given equation is $-2i$

Question 4.

Solve the following quadratic equations:

(i) $x^2 - (2 + i)x - (1 - 7i) = 0$

Solution:

Given equation is $x^2 - (2 + i)x - (1 - 7i) = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -(2 + i), c = -(1 - 7i)$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= [-(2 + i)]^2 - 4 \times 1 \times -(1 - 7i)$$

$$= 4 + 4i + i^2 + 4 - 28i$$

$$= 4 + 4i - 1 + 4 - 28i \dots [\because i^2 = -1]$$

$$= 7 - 24i$$

So, the given equation has complex roots.

These roots are given by

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-[-(2+i)] \pm \sqrt{7-24i}}{2(1)} = \frac{(2+i) \pm \sqrt{7-24i}}{2}$$

Let $\sqrt{7-24i} = a + bi$, where $a, b \in \mathbb{R}$

Squaring on both sides, we get

$$7 - 24i = a^2 + i^2b^2 + 2abi$$

$$7 - 24i = a^2 - b^2 + 2abi$$

Equating real and imaginary parts, we get

$$a^2 - b^2 = 7 \text{ and } 2ab = -24$$

$$a^2 - b^2 = 7 \text{ and } b = \frac{-12}{a}$$

$$a^2 - \left(\frac{-12}{a}\right)^2 = 7$$

$$a^2 - \frac{144}{a^2} = 7$$

$$a^4 - 144 = 7a^2$$

$$a^4 - 7a^2 - 144 = 0$$

$$(a^2 - 16)(a^2 + 9) = 0$$

$$a^2 = 16 \text{ or } a^2 = -9$$

But $a \in \mathbb{R}$

$$a^2 \neq -9$$

$$a^2 = 16$$

$$a = \pm 4$$

$$\text{When } a = 4, b = \frac{-12}{4} = -3$$

$$\text{When } a = -4, b = \frac{-12}{-4} = 3$$

$$\sqrt{7-24i} = \pm(4-3i)$$

$$x = \frac{(2+i) \pm (4-3i)}{2}$$

$$x = \frac{(2+i) + (4-3i)}{2} \text{ or } x = \frac{(2+i) - (4-3i)}{2}$$

$$x = 3 - i \text{ or } x = -1 + 2i$$

$$(ii) x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$$

Solution:

Given equation is $x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -(3\sqrt{2} + 2i), c = 6\sqrt{2}i$$

$$\text{Discriminant} = b^2 - 4ac$$

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$$= [-(3\sqrt{2} + 2i)]^2 - 4 \times 1 \times 6\sqrt{2} i$$

$$= 18 + 12\sqrt{2}i + 4i^2 - 24\sqrt{2} i$$

$$= 18 - 12\sqrt{2} i - 4 \dots\dots [\because i^2 = -1]$$

$$= 14 - 12\sqrt{2} i$$

So, the given equation has complex roots.

These roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-[-(3\sqrt{2} + 2i)] \pm \sqrt{14 - 12\sqrt{2}i}}{2(1)}$$

$$= \frac{(3\sqrt{2} + 2i) \pm \sqrt{14 - 12\sqrt{2}i}}{2}$$

Let $\sqrt{14 - 12\sqrt{2}i} = a + bi$, where $a, b \in \mathbb{R}$

Squaring on both sides, we get

$$14 - 12\sqrt{2}i = a^2 + i^2b^2 + 2abi$$

$$14 - 12\sqrt{2}i = a^2 - b^2 + 2abi$$

Equating real and imaginary parts, we get

$$a^2 - b^2 = 14 \text{ and } 2ab = -12\sqrt{2}$$

$$a^2 - b^2 = 14 \text{ and } b = \frac{-6\sqrt{2}}{a}$$

$$a^2 - \left(\frac{-6\sqrt{2}}{a}\right)^2 = 14$$

$$a^2 - \frac{72}{a^2} = 14$$

$$a^4 - 72 = 14a^2$$

$$a^4 - 14a^2 - 72 = 0$$

$$(a^2 - 18)(a^2 + 4) = 0$$

$$a^2 = 18 \text{ or } a^2 = -4$$

But $a \in \mathbb{R}$

$$a^2 \neq -4$$

$$a^2 = 18$$

$$a = \pm 3\sqrt{2}$$

$$\text{When } a = 3\sqrt{2}, b = \frac{-6\sqrt{2}}{3\sqrt{2}} = -2$$

$$\text{When } a = -3\sqrt{2}, b = \frac{-6\sqrt{2}}{-3\sqrt{2}} = 2$$

$$\sqrt{14 - 12\sqrt{2}i} = \pm (3\sqrt{2} - 2i)$$

$$x = \frac{(3\sqrt{2} + 2i) \pm (3\sqrt{2} - 2i)}{2}$$

$$x = \frac{(3\sqrt{2} + 2i) + (3\sqrt{2} - 2i)}{2}$$

$$\text{or } x = \frac{(3\sqrt{2} + 2i) - (3\sqrt{2} - 2i)}{2}$$

$$x = 3\sqrt{2} \text{ or } x = 2i$$

$$(iii) x^2 - (5 - i)x + (18 + i) = 0$$

Solution:

$$\text{Given equation is } x^2 - (5 - i)x + (18 + i) = 0$$

Comparing with $ax^2 + bx + c = 0$, we get

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$$a = 1, b = -(5 - i), c = 18 + i$$

$$\text{Discriminant} = b^2 - 4ac$$

$$= [-(5 - i)]^2 - 4 \times 1 \times (18 + i)$$

$$= 25 - 10i + i^2 - 72 - 4i$$

$$= 25 - 10i - 1 - 72 - 4i \dots\dots[\because i^2 = -1]$$

$$= -48 - 14i$$

So, the given equation has complex roots.

These roots are given by

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-[-(5-i)] \pm \sqrt{-48-14i}}{2(1)} = \frac{(5-i) \pm \sqrt{-48-14i}}{2}\end{aligned}$$

Let $\sqrt{-48-14i} = a + bi$, where $a, b \in \mathbb{R}$

Squaring on both sides, we get

$$-48 - 14i = a^2 + b^2 i^2 + 2abi$$

$$-48 - 14i = a^2 - b^2 + 2abi$$

Equating real and imaginary parts, we get

$$a^2 - b^2 = -48 \text{ and } 2ab = -14$$

$$\therefore a^2 - b^2 = -48 \text{ and } b = \frac{-7}{a}$$

$$\therefore a^2 - \left(\frac{-7}{a}\right)^2 = -48$$

$$\therefore a^2 - \frac{49}{a^2} = -48$$

$$\therefore a^4 - 49 = -48a^2$$

$$\therefore a^4 + 48a^2 - 49 = 0$$

$$\therefore (a^2 + 49)(a^2 - 1) = 0$$

$$\therefore a^2 = -49 \text{ or } a^2 = 1$$

But $a \in \mathbb{R}$

$$\therefore a^2 \neq -49$$

$$\therefore a^2 = 1$$

$$\therefore a = \pm 1$$

$$\text{When } a = 1, b = \frac{-7}{1} = -7$$

$$\text{When } a = -1, b = \frac{-7}{-1} = 7$$

$$\therefore \sqrt{-48 - 14i} = \pm (1 - 7i)$$

$$\therefore x = \frac{(5-i) \pm (1-7i)}{2}$$

$$\therefore x = \frac{5-i+1-7i}{2} \text{ or } x = \frac{5-i-1+7i}{2}$$

$$\therefore x = 3 - 4i \text{ or } x = 2 + 3i$$

$$(iv) (2 + i)x^2 - (5 - i)x + 2(1 - i) = 0$$

Solution:

Given equation is $(2 + i)x^2 - (5 - i)x + 2(1 - i) = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 2 + i, b = -(5 - i), c = 2(1 - i)$$

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$$\text{Discriminant} = b^2 - 4ac$$

$$= [-(5-i)]^2 - 4 \times (2+i) \times 2(1-i)$$

$$= 25 - 10i + i^2 - 8(2+i)(1-i)$$

$$= 25 - 10i + i^2 - 8(2 - 2i + i - i^2)$$

$$= 25 - 10i - 1 - 8(2 - i + 1) \dots [\because i^2 = -1]$$

$$= 25 - 10i - 1 - 16 + 8i - 8$$

$$= -2i$$

So, the given equation has complex roots.

These roots are given by

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-[-(5-i)] \pm \sqrt{-2i}}{2(2+i)} = \frac{(5-i) \pm \sqrt{-2i}}{2(2+i)} \end{aligned}$$

Let $\sqrt{-2i} = a + bi$, where $a, b \in \mathbb{R}$

Squaring on both sides, we get

$$-2i = a^2 + b^2i^2 + 2abi$$

$$-2i = a^2 - b^2 + 2abi$$

Equating real and imaginary parts, we get

$$a^2 - b^2 = 0 \text{ and } 2ab = -2$$

$$a^2 - b^2 = 0 \text{ and } b = -1a$$

$$a^2 - \left(\frac{-1}{a}\right)^2 = 0$$

$$a^2 - \frac{1}{a^2} = 0$$

$$a^4 - 1 = 0$$

$$(a^2 - 1)(a^2 + 1) = 0$$

$$a^2 = 1 \text{ or } a^2 = -1$$

But $a \in \mathbb{R}$

$$a^2 \neq -1$$

$$a^2 = 1$$

$$a = \pm 1$$

When $a = 1, b = -1$

When $a = -1, b = 1$

$$\sqrt{-2i} = \pm (1 - i)$$

$$x = \frac{(5-i) \pm (1-i)}{2(2+i)}$$

$$x = \frac{5-i+1-i}{2(2+i)} \text{ or } x = \frac{5-i-1+i}{2(2+i)}$$

$$x = \frac{6-2i}{2(2+i)} \text{ or } x = \frac{4}{2(2+i)}$$

$$x = \frac{2(3-i)}{2(2+i)} \text{ or } x = \frac{2}{2+i}$$

$$x = \frac{3-i}{2+i} \text{ or } x = \frac{2(2-i)}{(2+i)(2-i)}$$

$$x = \frac{(3-i)(2-i)}{(2+i)(2-i)} \text{ or } x = \frac{2(2-i)}{4-i^2}$$

$$x = \frac{6-5i+i^2}{4-i^2} \text{ or } x = \frac{4-2i}{4-i^2}$$

$$x = \frac{5-5i}{5} \text{ or } x = \frac{4-2i}{5} \quad \dots [\because i^2 = -1]$$

$$x = 1 - i \text{ or } x = \frac{4}{5} - \frac{2i}{5}$$

Question 5.

Find the value of

(i) $x^3 - x^2 + x + 46$, if $x = 2 + 3i$

Solution:

$$x = 2 + 3i$$

$$x - 2 = 3i$$

$$(x - 2)^2 = 9i^2$$

$$x^2 - 4x + 4 = 9(-1) \dots [\because i^2 = -1]$$

$$x^2 - 4x + 13 = 0 \dots (i)$$

$$\begin{array}{r}
 x^2 - 4x + 13 \overline{) x^3 - x^2 + x + 46} \\
 \underline{x^3 - 4x^2 + 13x} \\
 3x^2 - 12x + 46 \\
 \underline{3x^2 - 12x + 39} \\
 7
 \end{array}$$

Dividend = Divisor $\times$ Quotient + Remainder

$$\therefore x^3 - x^2 + x + 46 = (x^2 - 4x + 13)(x + 3) + 7$$

$$= 0(x + 3) + 7 \dots\dots[\text{from (i)}]$$

$$= 7$$

Alternate Method:

$$x = 2 + 3i$$

$$\alpha = 2 + 3i, \alpha^- = 2 - 3i$$

$$\alpha\alpha^- = (2 + 3i)(2 - 3i)$$

$$= 4 - 6i + 6i - 9i^2$$

$$= 4 - 9(-1)$$

$$= 4 + 9$$

$$= 13$$

$$\alpha + \alpha^- = 2 + 3i + 2 - 3i = 4$$

$\therefore$ Standard form of quadratic equation,

$$x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$$

$$x^2 - 4x + 13 = 0$$

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$$\begin{array}{r}
 x+3 \\
 x^2-4x+13 \overline{) x^3 - x^2 + x + 46} \\
 \underline{x^3 - 4x^2 + 13x} \\
 3x^2 - 12x + 46 \\
 \underline{3x^2 - 12x + 39} \\
 7
 \end{array}$$

Dividend = Divisor $\times$ Quotient + Remainder

$$\therefore x^3 - x^2 + x + 46 = (x^2 - 4x + 13)(x + 3) + 7$$

$$= 0(x + 3) + 7 \dots\dots[\text{From (i)}]$$

$$= 7$$

(ii) $2x^3 - 11x^2 + 44x + 27$, if $x = 253-4i$

Solution:

$$x = \frac{25}{3-4i}$$

$$x = \frac{25(3+4i)}{(3-4i)(3+4i)}$$

$$= \frac{25(3+4i)}{9-16i^2}$$

$$= \frac{25(3+4i)}{9-16(-1)}$$

$$= \frac{25(3+4i)}{25}$$

$$x = 3 + 4i$$

$$x - 3 = 4i$$

$$(x - 3)^2 = 16i^2$$

$$x^2 - 6x + 9 = 16(-1) \quad \dots[\because i^2 = -1]$$

$$x^2 - 6x + 25 = 0 \quad \dots(i)$$

$$\dots[\because i^2 = -1]$$

$$\begin{array}{r} x^2 - 6x + 25 \overline{) 2x^3 - 11x^2 + 44x + 27} \\ \underline{2x^3 - 12x^2 + 50x} \\ -x^2 + 27 \\ \underline{-x^2 + 6x - 25} \\ 2 \end{array}$$

Dividend = Divisor $\times$ Quotient + Remainder

$$2x^3 - 11x^2 + 44x + 27 = (x^2 - 6x + 25)(2x + 1) + 2$$

$$= 0.(2x + 1) + 2 \dots\dots[\text{From (i)}]$$

$$= 0 + 2$$

$$= 2$$

$$(iii) x^3 + x^2 - x + 22, \text{ if } x = 51-2i$$

Solution:

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$$\begin{aligned}
 x &= \frac{5}{1-2i} \\
 &= \frac{5}{1-2i} \times \frac{1+2i}{1+2i} = \frac{5(1+2i)}{1-4i^2} \\
 &= \frac{5(1+2i)}{1-4(-1)} \quad \dots [\because i^2 = -1] \\
 &= \frac{5(1+2i)}{5}
 \end{aligned}$$

$$\begin{aligned}
 x &= 1 + 2i \\
 x - 1 &= 2i \\
 (x - 1)^2 &= 4i^2 \\
 x^2 - 2x + 1 &= 4(-1) \quad \dots [\because i^2 = -1] \\
 x^2 - 2x + 1 &= -4 \\
 x^2 - 2x + 5 &= 0 \quad \dots (i)
 \end{aligned}$$

$$\begin{array}{r}
 \overline{) \begin{array}{r} x^3 + x^2 - x + 22 \\ x^3 - 2x^2 + 5x \\ \hline 3x^2 - 6x + 22 \\ \underline{3x^2 - 6x + 15} \\ 7 \end{array}}
 \end{array}$$

Dividend = Divisor $\times$ Quotient + Remainder

$$x^3 + x^2 - x + 22 = (x^2 - 2x + 5)(x + 3) + 7$$

$$= 0.(x + 3) + 7 \dots \dots [\text{From (i)}]$$

$$= 0 + 7$$

$$= 7$$

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(iv) $x^4 + 9x^3 + 35x^2 - x + 4$, if $x = -5 + \sqrt{-4}$

Solution:

$$x = -5 + \sqrt{-4}$$

$$x + 5 = \sqrt{-4}$$

$$x + 5 = \sqrt{4} \sqrt{-1}$$

$$x + 5 = 2i$$

$$(x + 5)^2 = 4i^2$$

$$x^2 + 10x + 25 = 4(-1) \dots [\because i^2 = -1]$$

$$x^2 + 10x + 29 = 0 \dots (i)$$

$$\begin{array}{r}
 x^2 - x + 16 \\
 \hline
 x^2 + 10x + 29 \overline{) x^4 + 9x^3 + 35x^2 - x + 4} \\
 \underline{x^4 + 10x^3 + 29x^2} \\
 -x^3 + 6x^2 - x + 4 \\
 \underline{-x^3 - 10x^2 - 29x} \\
 16x^2 + 28x + 4 \\
 \underline{16x^2 + 160x + 464} \\
 -132x - 460
 \end{array}$$

Dividend = Divisor $\times$ Quotient + Remainder

$$x^4 + 9x^3 + 35x^2 - x + 4 = (x^2 + 10x + 29)(x^2 - x + 16) - 132x - 460$$

$$= 0.(x^2 - x + 16) - 132x - 460 \dots [\text{From (i)}]$$

$$= -132(-5 + 2i) - 460$$

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$$= 660 - 264i - 460$$

$$= 200 - 264i$$

(v) $2x^4 + 5x^3 + 7x^2 - x + 41$, if $x = -2 - \sqrt{3}i$

Solution:

$$x = -2 - \sqrt{3}i$$

$$x + 2 = -\sqrt{3}i$$

$$(x + 2)^2 = 3i^2$$

$$x^2 + 4x + 4 = 3(-1) \quad \dots [\because i^2 = -1]$$

$$x^2 + 4x + 7 = 0 \quad \dots (i)$$

$$\begin{array}{r}
 2x^2 - 3x + 5 \\
 x^2 + 4x + 7 \overline{) 2x^4 + 5x^3 + 7x^2 - x + 41} \\
 \underline{2x^4 + 8x^3 + 14x^2} \\
 - - - \\
 \underline{-3x^3 - 7x^2 - x + 41} \\
 -3x^3 - 12x^2 - 21x \\
 \underline{+ + + } \\
 5x^2 + 20x + 41 \\
 \underline{5x^2 + 20x + 35} \\
 - - - \\
 \hline
 6
 \end{array}$$

Dividend = Divisor $\times$ Quotient + Remainder

$$2x^4 + 5x^3 + 7x^2 - x + 41 = (x^2 + 4x + 7)(2x^2 - 3x + 5) + 6$$

$$= 0(2x^2 - 3x + 5) + 6 \dots \dots [\text{From (i)}]$$

$$= 0 + 6$$

$$= 6$$

Ex 1.3

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-1-complex-numbers/>

Question 1.

Find the modulus and amplitude for each of the following complex numbers:

(i) $7 - 5i$

Solution:

Let $z = 7 - 5i$

$a = 7, b = -5$

i.e. $a > 0, b < 0$

$$|z| = \sqrt{a^2 + b^2} = \sqrt{7^2 + (-5)^2} = \sqrt{49 + 25} = \sqrt{74}$$

Here, $(7, -5)$ lies in 4th quadrant.

$$\begin{aligned}\therefore \text{amp}(z) &= -\tan^{-1}\left|\frac{b}{a}\right| \\ &= -\tan^{-1}\left|\frac{-5}{7}\right| \\ &= -\tan^{-1}\left(\frac{5}{7}\right)\end{aligned}$$

(ii) $\sqrt{3} + \sqrt{2}i$

Solution:

Let $z = \sqrt{3} + \sqrt{2}i$

$a = \sqrt{3}, b = \sqrt{2},$

i.e. $a > 0, b > 0$

$$\therefore |z| = \sqrt{a^2 + b^2} = \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2} = \sqrt{3+2} = \sqrt{5}$$

Here, $(\sqrt{3}, \sqrt{2})$ lies in 1st quadrant.

$$\begin{aligned}\therefore \text{amp}(z) &= \tan^{-1}\left(\frac{b}{a}\right) \\ &= \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{3}}\right) \\ &= \tan^{-1}\left(\sqrt{\frac{2}{3}}\right)\end{aligned}$$

(iii) $-8 + 15i$

Solution:

Let $z = -8 + 15i$

$a = -8$, $b = 15$, i.e. $a < 0$, $b > 0$

$$\begin{aligned}\therefore |z| &= \sqrt{a^2 + b^2} = \sqrt{(-8)^2 + 15^2} = \sqrt{64 + 225} \\ &= \sqrt{289} = 17\end{aligned}$$

Here, $(-8, 15)$ lies in 2nd quadrant.

$$\begin{aligned}\therefore \text{amp}(z) &= \pi - \tan^{-1}\left|\frac{b}{a}\right| \\ &= \pi - \tan^{-1}\left|\frac{15}{-8}\right| \\ &= \pi - \tan^{-1}\left(\frac{15}{8}\right)\end{aligned}$$

(iv) $-3(1 - i)$

Solution:

Let $z = -3(1 - i) = -3 + 3i$

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$a = -3, b = 3$, i.e. $a < 0, b > 0$

$$|z| = \sqrt{a^2 + b^2} = \sqrt{(-3)^2 + 3^2} = \sqrt{9 + 9} = 3\sqrt{2}$$

Here, $(-3, 3)$ lies in 2nd quadrant.

$$\text{amp}(z) = \pi - \tan^{-1} \left| \frac{b}{a} \right|$$

$$= \pi - \tan^{-1} \left| \frac{3}{-3} \right|$$

$$= \pi - \tan^{-1} |-1| = \pi - \tan^{-1} (1)$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

(v) $-4 - 4i$

Solution:

Let $z = -4 - 4i$

$a = -4, b = -4$, i.e. $a < 0, b < 0$

$$\begin{aligned} \therefore |z| &= \sqrt{a^2 + b^2} = \sqrt{(-4)^2 + (-4)^2} = \sqrt{16 + 16} \\ &= \sqrt{32} = 4\sqrt{2} \end{aligned}$$

Here, $(-4, -4)$ lies in 3rd quadrant.

$$\therefore \text{amp}(z) = \pi + \tan^{-1} \left| \frac{b}{a} \right| = \pi + \tan^{-1} \left| \frac{-4}{-4} \right|$$

$$= \pi + \tan^{-1} (1)$$

$$\therefore \text{amp}(z) = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

(vi) $\sqrt{3} - i$

Solution:

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Let $z = \sqrt{3} - i$

$a = \sqrt{3}$, $b = -1$, i.e. $a > 0$, $b < 0$

$$\therefore |z| = \sqrt{a^2 + b^2} = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$$

Here, $(\sqrt{3}, -1)$ lies in 4th quadrant.

$$\begin{aligned}\therefore \text{amp}(z) &= 2\pi - \tan^{-1} \left| \frac{b}{a} \right| \\ &= 2\pi - \tan^{-1} \left| \frac{-1}{\sqrt{3}} \right| = 2\pi - \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) \\ &= 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}\end{aligned}$$

(vii) 3

Solution:

Let $z = 3 + 0i$

$a = 3, b = 0$

z is a real number, it lies on the positive real axis.

$$|z| = \sqrt{a^2 + b^2} = \sqrt{3^2 + 0^2} = \sqrt{9 + 0} = 3$$

and $\text{amp}(z) = 0$

(viii) $1 + i$

Solution:

Let $z = 1 + i$

$a = 1, b = 1$, i.e. $a > 0, b > 0$

$$|z| = \sqrt{a^2 + b^2} = \sqrt{1^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2}$$

Here, $(1, 1)$ lies in 1st quadrant.

$$\text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

(ix) $1 + i\sqrt{3}$

Solution:

Let $z = 1 + i\sqrt{3}$

$a = 1, b = \sqrt{3}$, i.e. $a > 0, b > 0$

$$|z| = \sqrt{a^2 + b^2} = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = 2$$

Here, $(1, \sqrt{3})$ lies in 1st quadrant.

$$\text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$(x) (1 + 2i)^2 (1 - i)$$

Solution:

$$\text{Let } z = (1 + 2i)^2 (1 - i)$$

$$= (1 + 4i + 4i^2) (1 - i)$$

$$= [1 + 4i + 4(-1)] (1 - i) \dots [\because i^2 = -1]$$

$$= (-3 + 4i) (1 - i)$$

$$= -3 + 3i + 4i - 4i^2$$

$$= -3 + 7i - 4(-1)$$

$$= -3 + 7i + 4$$

$$\therefore z = 1 + 7i$$

$$\therefore a = 1, b = 7, \text{ i.e. } a > 0, b > 0$$

$$\therefore |z| = \sqrt{a^2 + b^2} = \sqrt{1^2 + 7^2} = \sqrt{1 + 49} = 5\sqrt{2}$$

Here, $(1, 7)$ lies in 1st quadrant.

$$\therefore \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}(7)$$

Question 2.

Find real values of θ for which $\left(\frac{4+3i \sin \theta}{1-2i \sin \theta}\right)$ is purely real.

Solution:

$$\begin{aligned} \frac{4+3i \sin \theta}{1-2i \sin \theta} &= \frac{(4+3i \sin \theta)(1+2i \sin \theta)}{(1-2i \sin \theta)(1+2i \sin \theta)} \\ &= \frac{4+8i \sin \theta+3i \sin \theta+6i^2 \sin^2 \theta}{1-4i^2 \sin^2 \theta} \end{aligned}$$

$$= \frac{4 + 11(i \sin \theta) - 6 \sin^2 \theta}{1 + 4 \sin^2 \theta} \quad \dots [\because i^2 = -1]$$

$$= \left(\frac{4 - 6 \sin^2 \theta}{1 + 4 \sin^2 \theta} \right) + \left(\frac{11 \sin \theta}{1 + 4 \sin^2 \theta} \right) i$$

Since z is purely real, $\text{Im}(z) = 0$

$$\therefore \frac{11 \sin \theta}{1 + 4 \sin^2 \theta} = 0$$

$$\therefore \sin \theta = 0$$

$$\therefore \sin \theta = \sin n\pi, \text{ where } n \in \mathbb{Z}$$

$$\therefore \theta = n\pi, \text{ where } n \in \mathbb{Z}$$

Question 3.

If $z = 3 + 5i$, then represent the z , $\bar{z}$, $-z$, $-\bar{z}$ in Argand's diagram.

Solution:

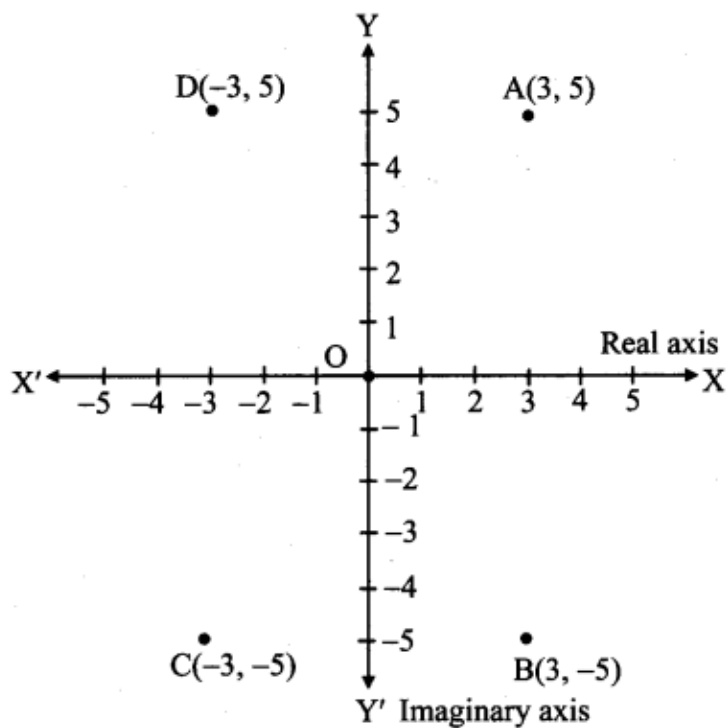
$$z = 3 + 5i$$

$$\bar{z} = 3 - 5i$$

$$-z = -3 - 5i$$

$$-\bar{z} = -3 + 5i$$

The above complex numbers will be represented by the points
A (3, 5), B (3, -5), C (-3, -5), D (-3, 5) respectively as shown below:



Question 4.

Express the following complex numbers in polar form and exponential form.

(i) $-1 + \sqrt{3}i$

Solution:

Let $z = -1 + \sqrt{3}i$

$a = -1, b = \sqrt{3}$

$$\therefore |z| = r = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$$

Here, $(-1, \sqrt{3})$ lies in 2nd quadrant.

$$\begin{aligned}\therefore \theta = \text{amp}(z) &= \pi - \tan^{-1} \left| \frac{b}{a} \right| \\ &= \pi - \tan^{-1} \left| \frac{\sqrt{3}}{-1} \right| \\ &= \pi - \tan^{-1}(\sqrt{3}) \\ &= \pi - \frac{\pi}{3} = \frac{2\pi}{3}\end{aligned}$$

$$\therefore \theta = 120^\circ = \frac{2\pi}{3}$$

$$\begin{aligned}\therefore \text{The polar form of } z &= r(\cos \theta + i \sin \theta) \\ &= 2(\cos 120^\circ + i \sin 120^\circ) \\ &= 2\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)\end{aligned}$$

The exponential form of $z = re^{i\theta} = 2e^{\frac{2\pi}{3}i}$

(ii) -i

Solution:

$$\text{Let } z = -i = 0 - i$$

$$a = 0, b = -1$$

z lies on negative imaginary Y-axis.

$$|z| = r = \sqrt{a^2 + b^2} = \sqrt{0^2 + (-1)^2} = 1 \text{ and}$$

$$\theta = \text{amp } z = 270^\circ = \frac{3\pi}{2}$$

The polar form of $z = r (\cos \theta + i \sin \theta)$

$$= 1 (\cos 270^\circ + i \sin 270^\circ)$$

$$= 1 \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$$

$$\text{The exponential form of } z = r e^{i\theta} = e^{\frac{3\pi}{2}i}$$

(iii) -1

Solution:

$$\text{Let } z = -1 = -1 + 0.i$$

$$a = -1, b = 0$$

z lies on negative real X-axis.

$$|z| = r = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + 0^2} = 1 \text{ and}$$

$$\theta = \text{amp } z = 180^\circ = \pi$$

The polar form of $z = r (\cos \theta + i \sin \theta)$

$$= 1 (\cos 180^\circ + i \sin 180^\circ)$$

$$= 1 (\cos \pi + i \sin \pi)$$

$$\text{The exponential form of } z = r e^{i\theta} = e^{\pi i}$$

(iv) $\frac{1}{1+i}$

Solution:

$$\begin{aligned}\text{Let } z &= \frac{1}{1+i} = \frac{1-i}{(1+i)(1-i)} \\ &= \frac{1-i}{1-i^2} = \frac{1-i}{1-(-1)} \quad \dots [\because i^2 = -1] \\ &= \frac{1-i}{2}\end{aligned}$$

$$\therefore z = \frac{1}{2} - \frac{1}{2}i$$

$$\therefore a = \frac{1}{2}, b = -\frac{1}{2}$$

$$\begin{aligned}\therefore |z| = r &= \sqrt{a^2 + b^2} \\ &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}}\end{aligned}$$

Here, $\left(\frac{1}{2}, -\frac{1}{2}\right)$ lies in 4th quadrant.

$$\begin{aligned}\therefore \theta = \text{amp}(z) &= 2\pi - \tan^{-1} \left| \frac{b}{a} \right| \\ &= 2\pi - \tan^{-1} \left| \frac{-1}{1} \right| \\ &= 2\pi - \tan^{-1} |-1| \\ &= 2\pi - \tan^{-1}(1)\end{aligned}$$

$$= 2\pi - \tan^{-1}(1)$$

$$= 2\pi - \frac{\pi}{4}$$

$$= \frac{7\pi}{4}$$

$$\therefore \theta = 315^\circ = \frac{7\pi}{4}$$

$\therefore$ The polar form of $z = r(\cos \theta + i \sin \theta)$

$$= \frac{1}{\sqrt{2}}(\cos 315^\circ + i \sin 315^\circ)$$

$$= \frac{1}{\sqrt{2}} \left[\cos\left(\frac{7\pi}{4}\right) + i \sin\left(\frac{7\pi}{4}\right) \right]$$

$$\text{The exponential form of } z = re^{i\theta} = \frac{1}{\sqrt{2}} e^{\frac{7\pi}{4}i}$$

(v) $\frac{1+2i}{1-3i}$

Solution:

$$\begin{aligned} \text{Let } z &= \frac{1+2i}{1-3i} = \frac{(1+2i)(1+3i)}{(1-3i)(1+3i)} \\ &= \frac{1+2i+3i+6i^2}{1-9i^2} \\ &= \frac{1+5i+6(-1)}{1-9(-1)} \quad \dots [\because i^2 = -1] \end{aligned}$$

$$= \frac{-5 + 5i}{10}$$

$$= \frac{-1}{2} + \frac{1}{2}i$$

$$\therefore a = \frac{-1}{2}, b = \frac{1}{2}$$

$$\begin{aligned}\therefore |z| = r = \sqrt{a^2 + b^2} &= \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \\ &= \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}}\end{aligned}$$

Here, $\left(\frac{-1}{2}, \frac{1}{2}\right)$ lies in 2nd quadrant.

$$\begin{aligned}\therefore \theta = \text{amp } z &= \pi - \tan^{-1} \left| \frac{b}{a} \right| \\ &= \pi - \tan^{-1} \left| \frac{\frac{1}{2}}{\frac{-1}{2}} \right| \\ &= \pi - \tan^{-1} |-1| \\ &= \pi - \tan^{-1} (1) \\ &= \pi - \frac{\pi}{4} \\ &= \frac{3\pi}{4}\end{aligned}$$

$$\therefore \theta = 135^\circ = \frac{3\pi}{4}$$

$$\begin{aligned}\therefore \text{The polar form of } z &= r (\cos \theta + i \sin \theta) \\ &= \frac{1}{\sqrt{2}} (\cos 135^\circ + i \sin 135^\circ) \\ &= \frac{1}{\sqrt{2}} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)\end{aligned}$$

$$\text{The exponential form of } z = re^{i\theta} = \frac{1}{\sqrt{2}} e^{\frac{3\pi}{4}i}$$

(vi) $\frac{1+7i}{(2-i)^2}$

Solution:

$$\begin{aligned}\text{Let } z &= \frac{1+7i}{(2-i)^2} \\ &= \frac{1+7i}{4-4i+i^2} = \frac{1+7i}{4-4i-1} \\ &= \frac{1+7i}{3-4i} = \frac{(1+7i)(3+4i)}{(3-4i)(3+4i)} \\ &= \frac{3+4i+21i+28i^2}{9-16i^2} \\ &= \frac{25i+3+28(-1)}{9-16(-1)} \\ &= \frac{25i-25}{25}\end{aligned}$$

$$\therefore z = -1 + i$$

$$\therefore a = -1, b = 1$$

$$\therefore |z| = r = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$

Here, $(-1, 1)$ lies in 2nd quadrant

$$\begin{aligned}\therefore \theta = \text{amp } z &= \pi - \tan^{-1} \left| \frac{b}{a} \right| \\ &= \pi - \tan^{-1} \left| \frac{1}{-1} \right| \\ &= \pi - \tan^{-1} |-1| = \pi - \tan^{-1} (1) \\ &= \pi - \frac{\pi}{4} \\ &= \frac{3\pi}{4}\end{aligned}$$

$$\therefore \theta = 135^\circ = \frac{3\pi}{4}$$

$$\begin{aligned}\therefore \text{The polar form of } z &= r (\cos \theta + i \sin \theta) \\ &= \sqrt{2} (\cos 135^\circ + i \sin 135^\circ) \\ &= \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)\end{aligned}$$

$$\text{The exponential form of } z = re^{i\theta} = \sqrt{2} e^{\frac{3\pi}{4}i}$$

Question 5.

Express the following numbers in the form $x + iy$:

(i) $\sqrt{3} \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$

Solution:

$$\begin{aligned}\sqrt{3} \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right) &= \sqrt{3} \left(\frac{\sqrt{3}}{2} + i \left(\frac{1}{2} \right) \right) \\ &= \frac{3}{2} + \frac{\sqrt{3}}{2} i\end{aligned}$$

(ii) $\sqrt{2} \cdot \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$

Solution:

$$\begin{aligned}&\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) \\ &= \sqrt{2} \left[\cos \left(2\pi - \frac{\pi}{4} \right) + i \sin \left(2\pi - \frac{\pi}{4} \right) \right] \\ &= \sqrt{2} \left[\cos \frac{\pi}{4} - i \sin \frac{\pi}{4} \right] \\ &= \sqrt{2} \left[\frac{1}{\sqrt{2}} + i \left(-\frac{1}{\sqrt{2}} \right) \right] \\ &= 1 - i\end{aligned}$$

(iii) $7 \left(\cos \left(-\frac{5\pi}{6} \right) + i \sin \left(-\frac{5\pi}{6} \right) \right)$

Solution:

$$\begin{aligned} & 7 \left[\cos \left(-\frac{5\pi}{6} \right) + i \sin \left(-\frac{5\pi}{6} \right) \right] \\ &= 7 \left[\cos \frac{5\pi}{6} - i \sin \frac{5\pi}{6} \right] \\ &= 7 \left[\cos \left(\pi - \frac{\pi}{6} \right) - i \sin \left(\pi - \frac{\pi}{6} \right) \right] \\ &= 7 \left[-\cos \frac{\pi}{6} - i \sin \frac{\pi}{6} \right] \\ &= 7 \left[-\frac{\sqrt{3}}{2} - i \left(\frac{1}{2} \right) \right] \\ &= -\frac{7\sqrt{3}}{2} - \frac{7}{2}i \end{aligned}$$

(iv) $e^{\frac{\pi}{3}i}$

Solution:

$$\begin{aligned} z &= re^{i\theta} = e^{\frac{\pi}{3}i} \\ \therefore r &= 1, \theta = \frac{\pi}{3} \\ \text{Polar form of } z &= r(\cos \theta + i \sin \theta) \\ &= 1 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \end{aligned}$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

(v) $e^{\frac{-4\pi}{3}i}$

Solution:

$$z = re^{i\theta} = e^{\frac{-4\pi}{3}i}$$

$$\therefore r = 1, \theta = \frac{-4\pi}{3}$$

Polar form of $z = r(\cos \theta + i \sin \theta)$

$$= 1 \left[\cos \left(-\frac{4\pi}{3} \right) + i \sin \left(-\frac{4\pi}{3} \right) \right]$$

$$= \cos \frac{4\pi}{3} - i \sin \frac{4\pi}{3}$$

$$= \cos \left(\pi + \frac{\pi}{3} \right) - i \sin \left(\pi + \frac{\pi}{3} \right)$$

$$= -\cos \frac{\pi}{3} - i \left(-\sin \frac{\pi}{3} \right)$$

$$= \frac{-1}{2} - i \left(-\frac{\sqrt{3}}{2} \right)$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

(vi) $e^{\frac{5\pi}{6}i}$

Solution:

$$z = re^{i\theta} = e^{\frac{5\pi}{6}i}$$

$$\therefore r = 1, \theta = \frac{5\pi}{6}$$

Polar form of $z = r(\cos \theta + i \sin \theta)$

$$= 1 \left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} \right)$$

$$= \cos \left(\pi - \frac{\pi}{6} \right) + i \sin \left(\pi - \frac{\pi}{6} \right)$$

$$= -\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$$

$$= -\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}$$

$$= -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$

Question 6.

Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$.

Solution:

$$\text{Let } z = \frac{1+2i}{1-3i}$$

$$\begin{aligned} &= \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} \\ &= \frac{1+3i+2i+6i^2}{1-9i^2} \\ &= \frac{1+5i-6}{1+9} \quad \dots [\because i^2 = -1] \\ \therefore z &= \frac{-5+5i}{10} \\ &= -\frac{1}{2} + \frac{1}{2}i \end{aligned}$$

This is of the form $a + bi$, where $a = -\frac{1}{2}$, $b = \frac{1}{2}$

$\therefore$ modulus = r

$$\begin{aligned} &= \sqrt{a^2 + b^2} \\ &= \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \\ &= \sqrt{\frac{1}{4} + \frac{1}{4}} \end{aligned}$$

$$= \frac{1}{\sqrt{2}}$$

If θ is the argument, then

$$\cos \theta = \frac{a}{r}$$

$$= \frac{\left(-\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$$

$$= -\frac{1}{\sqrt{2}}$$

$$\text{and } \sin \theta = \frac{b}{r}$$

$$= \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \frac{3\pi}{4} \dots \left[\begin{array}{l} \because \cos \frac{3\pi}{4} = \cos\left(\pi - \frac{\pi}{4}\right) = -\cos \frac{\pi}{4} = \\ -\frac{1}{\sqrt{2}} \text{ and } \sin \frac{3\pi}{4} = \sin\left(\pi - \frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \end{array} \right]$$

Hence, modulus = $\frac{1}{\sqrt{2}}$ and argurement = $\frac{3\pi}{4}$

Question 7.

Convert the complex number $z = \frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$ in the polar form.

Solution:

$$\begin{aligned} z &= \frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}} \\ &= \frac{i-1}{\frac{1}{2} + \frac{\sqrt{3}}{2}i} = \frac{2(i-1)}{1+\sqrt{3}i} \\ &= \frac{2(i-1)(1-\sqrt{3}i)}{(1+\sqrt{3}i)(1-\sqrt{3}i)} = \frac{2(i-\sqrt{3}i^2-1+\sqrt{3}i)}{1-3i^2} \\ &= \frac{2[i-\sqrt{3}(-1)-1+\sqrt{3}i]}{1-3(-1)} \quad \dots [\because i^2 = -1] \\ &= \frac{2}{4} (i+\sqrt{3}-1+\sqrt{3}i) \\ \therefore z &= \frac{\sqrt{3}-1}{2} + \frac{(\sqrt{3}+1)}{2}i \\ \therefore a &= \frac{\sqrt{3}-1}{2}, b = \frac{\sqrt{3}+1}{2} \end{aligned}$$

$$\begin{aligned}\therefore |z| = r = \sqrt{a^2 + b^2} &= \sqrt{\left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}+1}{2}\right)^2} \\ &= \sqrt{\frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{4}} \\ &= \sqrt{\frac{3-2\sqrt{3}+1+3+2\sqrt{3}+1}{4}} \\ &= \sqrt{\frac{8}{4}} = \sqrt{2}\end{aligned}$$

Here, $\left(\frac{\sqrt{3}-1}{2}, \frac{\sqrt{3}+1}{2}\right)$ lies in 1st quadrant.

$$\begin{aligned}\therefore \text{amp}(z) = \theta &= \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{\frac{\sqrt{3}+1}{2}}{\frac{\sqrt{3}-1}{2}}\right) \\ &= \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\end{aligned}$$

$$\begin{aligned}\therefore \text{The polar form of } z &= r(\cos\theta + i.\sin\theta) \\ &= \sqrt{2}(\cos\theta + i.\sin\theta), \text{ where } \theta = \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\end{aligned}$$

Question 8.

For $z = 2 + 3i$, verify the following:

(i) $\overline{\bar{z}} = z$

Solution:

$$z = 2 + 3i$$

$$\therefore \bar{z} = 2 - 3i$$

$$\therefore \overline{\bar{z}} = 2 + 3i = z$$

(ii) $\overline{z\bar{z}} = |z|^2$

Solution:

$$z\bar{z} = (2 + 3i)(2 - 3i)$$

$$= 4 - 9i^2$$

$$= 4 - 9(-1) \dots [\because i^2 = -1]$$

$$= 13$$

$$|z|^2 = \left(\sqrt{2^2 + 3^2}\right)^2$$

$$= 2^2 + 3^2$$

$$= 4 + 9$$

$$= 13$$

$$\therefore \overline{z\bar{z}} = |z|^2$$

(iii) $(z + \bar{z})$ is real

Solution:

$$(z + \bar{z}) = (2 + 3i) + (2 - 3i)$$

$$= 2 + 3i + 2 - 3i$$

$= 4$, which is a real number.

$\therefore z + \bar{z}$ is real.

(iv) $z - \bar{z} = 6i$

Solution:

$$z - \bar{z} = (2 + 3i) - (2 - 3i)$$

$$= 2 + 3i - 2 + 3i$$

$$= 6i$$

Question 9.

$z_1 = 1 + i$, $z_2 = 2 - 3i$, verify the following:

(i) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

Solution:

$$z_1 = 1 + i, z_2 = 2 - 3i$$

$$\therefore \overline{z_1} = 1 - i, \overline{z_2} = 2 + 3i$$

$$z_1 + z_2 = (1 + i) + (2 - 3i) = 3 - 2i$$

$$\therefore \overline{z_1 + z_2} = 3 + 2i$$

$$\begin{aligned} \overline{z_1} + \overline{z_2} &= (1 - i) + (2 + 3i) \\ &= 3 + 2i \end{aligned}$$

$$\therefore \overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

$$(ii) \overline{Z_1 - Z_2} = \overline{Z_1} - \overline{Z_2}$$

Solution:

$$\begin{aligned} z_1 - z_2 &= (1 + i) - (2 - 3i) \\ &= 1 + i - 2 + 3i \\ &= -1 + 4i \\ \therefore \overline{z_1 - z_2} &= -1 - 4i \\ \overline{z_1} - \overline{z_2} &= (1 - i) - (2 + 3i) \\ &= 1 - i - 2 - 3i \\ &= -1 - 4i \\ \therefore \overline{z_1 - z_2} &= \overline{z_1} - \overline{z_2} \end{aligned}$$

$$(iii) \overline{Z_1 \cdot Z_2} = \overline{Z_1} \cdot \overline{Z_2}$$

Solution:

$$\begin{aligned} (z_1 \cdot z_2) &= (1 + i)(2 - 3i) \\ &= 2 - 3i + 2i - 3i^2 \\ &= 2 - i - 3(-1) \quad \dots[\because i^2 = -1] \\ &= 5 - i \\ \therefore \overline{z_1 \cdot z_2} &= 5 + i \\ \overline{z_1} \cdot \overline{z_2} &= (1 - i)(2 + 3i) \\ &= 2 + 3i - 2i - 3i^2 \\ &= 2 + i - 3(-1) \quad \dots[\because i^2 = -1] \\ &= 5 + i \\ \therefore \overline{z_1 z_2} &= \overline{z_1} \cdot \overline{z_2} \end{aligned}$$

$$(iv) \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

Solution:

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{1+i}{2-3i} = \frac{(1+i)(2+3i)}{(2-3i)(2+3i)} \\ &= \frac{2+3i+2i+3i^2}{4-9i^2} = \frac{2+5i+3(-1)}{4-9(-1)} \end{aligned}$$

...[$\because i^2 = -1$]

$$= \frac{-1+5i}{13} = \frac{-1}{13} + \frac{5}{13}i$$

$$\therefore \overline{\left(\frac{z_1}{z_2}\right)} = \frac{-1}{13} - \frac{5}{13}i$$

$$\frac{\bar{z}_1}{\bar{z}_2} = \frac{1-i}{2+3i}$$

$$= \frac{(1-i)(2-3i)}{(2+3i)(2-3i)}$$

$$= \frac{2-3i-2i+3i^2}{4-9i^2}$$

$$= \frac{2-5i+3(-1)}{4-9(-1)} \quad \dots[\because i^2 = -1]$$

$$= \frac{-1-5i}{13} = \frac{-1}{13} - \frac{5}{13}i$$

$$\therefore \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

Ex 1.4

Question 1.

Find the value of

(i) ω^{18}

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(ii) ω^{21}

(iii) ω^{-30}

(iv) ω^{-105}

Solution:

$$\begin{aligned}\omega^3 &= 1 \\ \text{i. } \omega^{18} &= (\omega^3)^6 = (1)^6 = 1 \\ \text{ii. } \omega^{21} &= (\omega^3)^7 = (1)^7 = 1 \\ \text{iii. } \omega^{-30} &= (\omega^3)^{-10} = (1)^{-10} = \frac{1}{(1)^{10}} = 1 \\ \text{iv. } \omega^{-105} &= (\omega^3)^{-35} = (1)^{-35} = \frac{1}{(1)^{35}} = 1\end{aligned}$$

Question 2.

If ω is the complex cube root of unity, show that

(i) $(2 - \omega)(2 - \omega^2) = 7$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\text{L.H.S.} = (2 - \omega)(2 - \omega^2)$$

$$= 4 - 2\omega^2 - 2\omega + \omega^3$$

$$= 4 - 2(\omega^2 + \omega) + 1$$

$$= 4 - 2(-1) + 1$$

$$= 4 + 2 + 1$$

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$$= 7$$

$$= \text{R.H.S.}$$

$$(ii) (1 + \omega - \omega^2)^6 = 64$$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\begin{aligned} \text{L.H.S.} &= (1 + \omega - \omega^2)^6 \\ &= [(1 + \omega) - \omega^2]^6 \\ &= (-\omega^2 - \omega^2)^6 \\ &= (-2\omega^2)^6 \\ &= 64 \cdot \omega^{12} \\ &= 64(\omega^3)^4 \\ &= 64(1)^4 \\ &= 64 = \text{R.H.S.} \end{aligned}$$

$$(iii) (1 + \omega)^3 - (1 + \omega^2)^3 = 0$$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\begin{aligned}\text{L.H.S.} &= (1 + \omega)^3 - (1 + \omega^2)^3 \\ &= (-\omega^2)^3 - (-\omega)^3 \\ &= -\omega^6 + \omega^3 \\ &= -(\omega^3)^2 + \omega^3 \\ &= -(1)^2 + 1 \\ &= -1 + 1 = 0 = \text{R.H.S.}\end{aligned}$$

$$\text{(iv) } (2 + \omega + \omega^2)^3 - (1 - 3\omega + \omega^2)^3 = 65$$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\begin{aligned}\text{L.H.S.} &= (2 + \omega + \omega^2)^3 - (1 - 3\omega + \omega^2)^3 \\ &= [2 + (\omega + \omega^2)]^3 - [-3\omega + (1 + \omega^2)]^3 \\ &= (2 - 1)^3 - (-3\omega - \omega)^3 \\ &= 1^3 - (-4\omega)^3 \\ &= 1 + 64\omega^3 \\ &= 1 + 64(1) = 65 \\ &= \text{R.H.S.}\end{aligned}$$

$$\text{(v) } (3 + 3\omega + 5\omega^2)^6 - (2 + 6\omega + 2\omega^2)^3 = 0$$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\begin{aligned}
 \text{L.H.S.} &= (3 + 3\omega + 5\omega^2)^6 - (2 + 6\omega + 2\omega^2)^3 \\
 &= [3(1 + \omega) + 5\omega^2]^6 - [2(1 + \omega^2) + 6\omega]^3 \\
 &= [3(-\omega^2) + 5\omega^2]^6 - [2(-\omega) + 6\omega]^3 \\
 &= (2\omega^2)^6 - (4\omega)^3 \\
 &= 64\omega^{12} - 64\omega^3 \\
 &= 64(\omega^3)^4 - 64\omega^3 \\
 &= 64(1) - 64(1) \\
 &= 0 = \text{R.H.S.}
 \end{aligned}$$

(vi) $a + b\omega + c\omega^2 + a\omega + b\omega^2 = \omega^2$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\begin{aligned}
 \text{L.H.S.} &= \frac{a + b\omega + c\omega^2}{c + a\omega + b\omega^2} = \frac{a\omega^3 + b\omega^4 + c\omega^2}{c + a\omega + b\omega^2} \\
 &\quad \dots [\because \omega^3 = 1, \therefore \omega^4 = \omega] \\
 &= \frac{\omega^2(c + a\omega + b\omega^2)}{c + a\omega + b\omega^2} = \omega^2 = \text{R.H.S.}
 \end{aligned}$$

(vii) $(a + b) + (a\omega + b\omega^2) + (a\omega^2 + b\omega) = 0$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\begin{aligned}\text{L.H.S.} &= (a + b) + (a\omega + b\omega^2) + (a\omega^2 + b\omega) \\ &= (a + a\omega + a\omega^2) + (b + b\omega + b\omega^2) \\ &= a(1 + \omega + \omega^2) + b(1 + \omega + \omega^2) \\ &= a(0) + b(0) \\ &= 0 = \text{R.H.S.}\end{aligned}$$

$$\text{(viii) } (a - b)(a - b\omega)(a - b\omega^2) = a^3 - b^3$$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\begin{aligned}\text{L.H.S.} &= (a - b)(a - b\omega)(a - b\omega^2) \\ &= (a - b)(a^2 - ab\omega^2 - ab\omega + b^2\omega^3) \\ &= (a - b)[a^2 - ab(\omega^2 + \omega) + b^2(1)] \\ &= (a - b)[a^2 - ab(-1) + b^2] \\ &= (a - b)(a^2 + ab + b^2) \\ &= a^3 - b^3 = \text{R.H.S.}\end{aligned}$$

$$\text{(ix) } (a + b)^2 + (a\omega + b\omega^2)^2 + (a\omega^2 + b\omega)^2 = 6ab$$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\begin{aligned}
 \text{L.H.S.} &= (a+b)^2 + (a\omega + b\omega^2)^2 + (a\omega^2 + b\omega)^2 \\
 &= (a+b)^2 + \omega^2(a+b\omega)^2 + \omega^2(a\omega + b)^2 \\
 &= (a+b)^2 + \omega^2[(a+b\omega)^2 + (a\omega + b)^2] \\
 &= a^2 + b^2 + 2ab + \omega^2(a^2 + 2ab\omega + b^2\omega^2 \\
 &\quad + a^2\omega^2 + 2ab\omega + b^2) \\
 &= a^2 + b^2 + 2ab + a^2\omega^2 + 2ab\omega^3 + b^2\omega^4 \\
 &\quad + a^2\omega^4 + 2ab\omega^3 + b^2\omega^2 \\
 &= a^2 + b^2 + 2ab + a^2\omega^2 + 2ab(1) + b^2(\omega) \\
 &\quad + a^2(\omega) + 2ab(1) + b^2\omega^2 \\
 &= a^2(1 + \omega + \omega^2) + b^2(1 + \omega + \omega^2) + 2ab \\
 &\quad + 2ab + 2ab \\
 &= a^2(0) + b^2(0) + 6ab \\
 &= 6ab = \text{R.H.S.}
 \end{aligned}$$

Question 3.

If ω is the complex cube root of unity, find the value of

(i) $\omega + 1\omega$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\omega + 1\omega = \omega + \omega = 2\omega = -\omega^2 = -1$$

(ii) $\omega^2 + \omega^3 + \omega^4$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

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$$\begin{aligned}\omega^2 + \omega^3 + \omega^4 \\&= \omega^2(1 + \omega + \omega^2) \\&= \omega^2(0) \\&= 0\end{aligned}$$

(iii) $(1 + \omega^2)^3$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\begin{aligned}(1 + \omega^2)^3 \\&= (-\omega)^3 \\&= -\omega^3 \\&= -1\end{aligned}$$

(iv) $(1 - \omega - \omega^2)^3 + (1 - \omega + \omega^2)^3$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$\begin{aligned}(1 - \omega - \omega^2)^3 + (1 - \omega + \omega^2)^3 \\&= [1 - (\omega + \omega^2)]^3 + [(1 + \omega^2) - \omega]^3 \\&= [1 - (-1)]^3 + (-\omega - \omega)^3 \\&= 2^3 + (-2\omega)^3\end{aligned}$$

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$$= 8 - 8\omega^3$$

$$= 8 - 8(1)$$

$$= 0$$

$$(v) (1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$$

Solution:

ω is the complex cube root of unity.

$$\omega^3 = 1 \text{ and } 1 + \omega + \omega^2 = 0$$

$$\text{Also, } 1 + \omega^2 = -\omega, 1 + \omega = -\omega^2 \text{ and } \omega + \omega^2 = -1$$

$$(1 + \omega)(1 + \omega^2)(1 + \omega^4)(1 + \omega^8)$$

$$= (1 + \omega)(1 + \omega^2)(1 + \omega)(1 + \omega^2) \dots [\because \omega^3 = 1, \omega^4 = \omega]$$

$$= (-\omega^2)(-\omega)(-\omega^2)(-\omega)$$

$$= \omega^6$$

$$= (\omega^3)^2$$

$$= (1)^2$$

$$= 1$$

Question 4.

If α and β are the complex cube roots of unity, show that

$$(i) \alpha^2 + \beta^2 + \alpha\beta = 0$$

$$(ii) \alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} = 0$$

Solution:

α and β are the complex cube roots of unity.

$$\therefore \alpha = \frac{-1+i\sqrt{3}}{2} \text{ and } \beta = \frac{-1-i\sqrt{3}}{2}$$

$$\begin{aligned}\therefore \alpha\beta &= \left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{-1-i\sqrt{3}}{2}\right) \\ &= \frac{(-1)^2 - (i\sqrt{3})^2}{4} = \frac{1 - (-1)(3)}{4} \\ &= \frac{1+3}{4}\end{aligned}$$

$$\therefore \alpha\beta = 1$$

$$\begin{aligned}\text{Also, } \alpha + \beta &= \frac{-1+i\sqrt{3}}{2} + \frac{-1-i\sqrt{3}}{2} \\ &= \frac{-1+i\sqrt{3}-1-i\sqrt{3}}{2} = \frac{-2}{2}\end{aligned}$$

$$\therefore \alpha + \beta = -1$$

$$\begin{aligned}\text{i. } \alpha^2 + \beta^2 + \alpha\beta &= \alpha^2 + 2\alpha\beta + \beta^2 + \alpha\beta - 2\alpha\beta \\ &\quad \dots [\text{Adding and subtracting } 2\alpha\beta] \\ &= (\alpha^2 + 2\alpha\beta + \beta^2) - \alpha\beta \\ &= (\alpha + \beta)^2 - \alpha\beta \\ &= (-1)^2 - 1 = 1 - 1 \\ &= 0\end{aligned}$$

$$\begin{aligned}\text{ii. } & \alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} \\ &= \alpha^4 + \beta^4 + 2\alpha^2\beta^2 - 2\alpha^2\beta^2 + \frac{1}{\alpha\beta} \\ & \quad \dots [\text{Adding and subtracting } 2\alpha^2\beta^2] \\ &= (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 + \frac{1}{\alpha\beta} \\ &= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2 + \frac{1}{\alpha\beta} \\ &= [(-1)^2 - 2(1)]^2 - 2(1)^2 + \frac{1}{1} \\ &= (1 - 2)^2 - 2 + 1 \\ &= (-1)^2 - 1 \\ &= 1 - 1 \\ &= 0\end{aligned}$$

Question 5.

If $x = a + b$, $y = \alpha a + \beta b$ and $z = a\beta + b\alpha$, where α and β are complex cube roots of unity, show that $xyz = a^3 + b^3$.

Solution:

$$x = a + b, y = \alpha a + \beta b, z = a\beta + b\alpha$$

α and β are the complex cube roots of unity.

$$\therefore \alpha = -1 + i\sqrt{3}/2 \text{ and } \beta = -1 - i\sqrt{3}/2$$

$$\begin{aligned}\therefore \alpha\beta &= \left(\frac{-1+i\sqrt{3}}{2}\right)\left(\frac{-1-i\sqrt{3}}{2}\right) \\ &= \frac{(-1)^2 - (i\sqrt{3})^2}{4} \\ &= \frac{1 - (-1)(3)}{4} \quad \dots [\because i^2 = -1] \\ &= \frac{1+3}{4}\end{aligned}$$

$$\therefore \alpha\beta = 1$$

$$\begin{aligned}\text{Also, } \alpha + \beta &= \frac{-1+i\sqrt{3}}{2} + \frac{-1-i\sqrt{3}}{2} \\ &= \frac{-1+i\sqrt{3}-1-i\sqrt{3}}{2} = \frac{-2}{2}\end{aligned}$$

$$\therefore \alpha + \beta = -1$$

$$\begin{aligned}\therefore xyz &= (a+b)(\alpha a + \beta b)(a\beta + b\alpha) \\ &= (a+b)(\alpha\beta a^2 + \alpha^2 ab + \beta^2 ab + \alpha\beta b^2) \\ &= (a+b)[1.(a^2) + (\alpha^2 + \beta^2)ab + 1.(b^2)] \\ &= (a+b)\{a^2 + [(\alpha + \beta)^2 - 2\alpha\beta]ab + b^2\} \\ &= (a+b)\{a^2 + [(-1)^2 - 2(1)]ab + b^2\} \\ &= (a+b)[a^2 + (1-2)ab + b^2] \\ &= (a+b)(a^2 - ab + b^2) = a^3 + b^3\end{aligned}$$

Question 6.

Find the equation in cartesian coordinates of the locus of z if

(i) $|z| = 10$

Solution:

Let $z = x + iy$

$|z| = 10$

$|x + iy| = 10$

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$$\sqrt{x^2 + y^2} = 10$$

$$\therefore x^2 + y^2 = 100$$

$$(ii) |z - 3| = 2$$

Solution:

$$\text{Let } z = x + iy$$

$$|z - 3| = 2$$

$$|x + iy - 3| = 2$$

$$|(x - 3) + iy| = 2$$

$$\sqrt{(x-3)^2 + y^2} = 2$$

$$\therefore (x - 3)^2 + y^2 = 4$$

$$(iii) |z - 5 + 6i| = 5$$

Solution:

$$\text{Let } z = x + iy$$

$$|z - 5 + 6i| = 5$$

$$|x + iy - 5 + 6i| = 5$$

$$|(x - 5) + i(y + 6)| = 5$$

$$\sqrt{(x-5)^2 + (y+6)^2} = 5$$

$$\therefore (x - 5)^2 + (y + 6)^2 = 25$$

$$(iv) |z + 8| = |z - 4|$$

Solution:

$$\text{Let } z = x + iy$$

$$|z + 8| = |z - 4|$$

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$$|x + iy + 8| = |x + iy - 4|$$

$$|(x + 8) + iy| = |(x - 4) + iy|$$

$$\sqrt{(x+8)^2+y^2} = \sqrt{(x-4)^2+y^2}$$

$$(x + 8)^2 + y^2 = (x - 4)^2 + y^2$$

$$x^2 + 16x + 64 + y^2 = x^2 - 8x + 16 + y^2$$

$$16x + 64 = -8x + 16$$

$$24x + 48 = 0$$

$$\therefore x + 2 = 0$$

$$(v) |z - 2 - 2i| = |z + 2 + 2i|$$

Solution:

Let $z = x + iy$

$$|z - 2 - 2i| = |z + 2 + 2i|$$

$$|x + iy - 2 - 2i| = |x + iy + 2 + 2i|$$

$$|(x - 2) + i(y - 2)| = |(x + 2) + i(y + 2)|$$

$$\sqrt{(x-2)^2+(y-2)^2} = \sqrt{(x+2)^2+(y+2)^2}$$

$$(x - 2)^2 + (y - 2)^2 = (x + 2)^2 + (y + 2)^2$$

$$x^2 - 4x + 4 + y^2 - 4y + 4 = x^2 + 4x + 4 + y^2 + 4y + 4$$

$$-4x - 4y = 4x + 4y$$

$$8x + 8y = 0$$

$$x + y = 0$$

$$y = -x$$

$$(vi) |z+3i||z-6i|=1$$

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Solution:

Let $z = x + iy$

$$\left| \frac{z + 3i}{z - 6i} \right| = 1$$

$$\left| \frac{x + iy + 3i}{x + iy - 6i} \right| = 1$$

$$\left| \frac{x + (y + 3)i}{x + (y - 6)i} \right| = 1$$

$$\frac{\sqrt{x^2 + (y + 3)^2}}{\sqrt{x^2 + (y - 6)^2}} = 1$$

$$\frac{x^2 + (y + 3)^2}{x^2 + (y - 6)^2} = 1$$

$$x^2 + (y + 3)^2 = x^2 + (y - 6)^2$$

$$y^2 + 6y + 9 = y^2 - 12y + 36$$

$$18y - 27 = 0$$

$$2y - 3 = 0$$

Question 7.

Use De Moivre's theorem and simplify the following:

(i) $(\cos 2\theta + i \sin 2\theta)^7 (\cos 4\theta + i \sin 4\theta)^3$

Solution:

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$$\begin{aligned}
 & \frac{(\cos 2\theta + i \sin 2\theta)^7}{(\cos 4\theta + i \sin 4\theta)^3} \\
 &= \frac{(\cos \theta + i \sin \theta)^{2 \times 7}}{(\cos \theta + i \sin \theta)^{4 \times 3}} \\
 & \dots [\because \cos n\theta + i \sin n\theta = (\cos \theta + i \sin \theta)^n] \\
 &= (\cos \theta + i \sin \theta)^{14} (\cos \theta + i \sin \theta)^{-12} \\
 &= (\cos \theta + i \sin \theta)^{14-12} \\
 &= (\cos \theta + i \sin \theta)^2 \\
 &= \cos 2\theta + i \sin 2\theta \\
 & \dots [\because (\cos \theta + i \sin \theta)^n = (\cos n\theta + i \sin n\theta)]
 \end{aligned}$$

Alternate Method:

$$\begin{aligned}
 & \frac{(\cos 2\theta + i \sin 2\theta)^7}{(\cos 4\theta + i \sin 4\theta)^3} \\
 &= \frac{\cos 7(2\theta) + i \sin 7(2\theta)}{\cos 3(4\theta) + i \sin 3(4\theta)} \\
 & \dots [\because (\cos \theta + i \sin \theta)^n = (\cos n\theta + i \sin n\theta)] \\
 &= \frac{\cos 14\theta + i \sin 14\theta}{\cos 12\theta + i \sin 12\theta} \\
 &= \frac{(\cos 14\theta + i \sin 14\theta)(\cos 12\theta - i \sin 12\theta)}{(\cos 12\theta + i \sin 12\theta)(\cos 12\theta - i \sin 12\theta)} \\
 &= \frac{\cos 14\theta \cos 12\theta - i \cos 14\theta \sin 12\theta + i \sin 14\theta \cos 12\theta - i^2 \sin 14\theta \sin 12\theta}{\cos^2 12\theta - i^2 \sin^2 12\theta} \\
 &= \frac{(\cos 14\theta \cos 12\theta + \sin 14\theta \sin 12\theta) + i(\sin 14\theta \cos 12\theta - \cos 14\theta \sin 12\theta)}{\cos^2 12\theta + \sin^2 12\theta} \\
 &= \frac{\cos(14\theta - 12\theta) + i \sin(14\theta - 12\theta)}{(1)^2} \\
 &= \cos 2\theta + i \sin 2\theta
 \end{aligned}$$

(ii) $\cos 5\theta + i \sin 5\theta (\cos 3\theta - i \sin 3\theta)^2$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-1-complex-numbers/>

$$\begin{aligned}
 & \frac{\cos 5\theta + i \sin 5\theta}{(\cos 3\theta - i \sin 3\theta)^2} \\
 &= \frac{\cos 5\theta + i \sin 5\theta}{[\cos(-3\theta) + i \sin(-3\theta)]^2} \\
 & \quad \dots [\because \sin(-\theta) = -\sin \theta, \cos(-\theta) = \cos \theta] \\
 &= \frac{(\cos \theta + i \sin \theta)^5}{(\cos \theta + i \sin \theta)^{-3 \times 2}} \\
 & \quad \dots [\because \cos n \theta + i \sin n \theta = (\cos \theta + i \sin \theta)^n] \\
 &= \frac{(\cos \theta + i \sin \theta)^5}{(\cos \theta + i \sin \theta)^{-6}} \\
 &= (\cos \theta + i \sin \theta)^5 (\cos \theta + i \sin \theta)^6 \\
 &= (\cos \theta + i \sin \theta)^{5+6} \\
 &= (\cos \theta + i \sin \theta)^{11} \\
 &= \cos 11\theta + i \sin 11\theta \\
 & \quad \dots [\because (\cos \theta + i \sin \theta)^n = (\cos n \theta + i \sin n \theta)]
 \end{aligned}$$

(iii) $(\cos 7\pi/13 + i \sin 7\pi/13)^4 (\cos 4\pi/13 - i \sin 4\pi/13)^6$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-1-complex-numbers/>

$$\begin{aligned}
 & \frac{\cos 5\theta + i \sin 5\theta}{(\cos 3\theta - i \sin 3\theta)^2} \\
 &= \frac{\cos 5\theta + i \sin 5\theta}{[\cos(-3\theta) + i \sin(-3\theta)]^2} \\
 & \quad \dots [\because \sin(-\theta) = -\sin \theta, \cos(-\theta) = \cos \theta] \\
 &= \frac{(\cos \theta + i \sin \theta)^5}{(\cos \theta + i \sin \theta)^{-3 \times 2}} \\
 & \quad \dots [\because \cos n\theta + i \sin n\theta = (\cos \theta + i \sin \theta)^n] \\
 &= \frac{(\cos \theta + i \sin \theta)^5}{(\cos \theta + i \sin \theta)^{-6}} \\
 &= (\cos \theta + i \sin \theta)^5 (\cos \theta + i \sin \theta)^6 \\
 &= (\cos \theta + i \sin \theta)^{5+6} \\
 &= (\cos \theta + i \sin \theta)^{11} \\
 &= \cos 11\theta + i \sin 11\theta \\
 & \quad \dots [\because (\cos \theta + i \sin \theta)^n = (\cos n\theta + i \sin n\theta)]
 \end{aligned}$$

Question 8.

Express the following in the form $a + ib$, $a, b \in \mathbb{R}$, using De Moivre's theorem.

(i) $(1 - i)^5$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-1-complex-numbers/>

Let $z = 1 - i$

$\therefore a = 1, b = -1$, i.e. $a > 0, b < 0$

$$\therefore |z| = \sqrt{a^2 + b^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

Here, $(1, -1)$ lies in 4th quadrant.

$$\therefore \text{amp}(z) = 2\pi - \tan^{-1} \left| \frac{b}{a} \right|$$

$$= 2\pi - \tan^{-1} \left| \frac{-1}{1} \right|$$

$$= 2\pi - \tan^{-1} |-1|$$

$$= 2\pi - \tan^{-1} (1)$$

$$= 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$$z^5 = (1 - i)^5 = \left[\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) \right]^5$$

$$= 4\sqrt{2} \left[\cos \left(2\pi - \frac{\pi}{4} \right) + i \sin \left(2\pi - \frac{\pi}{4} \right) \right]^5$$

$$= 4\sqrt{2} \left[\cos \frac{\pi}{4} - i \sin \frac{\pi}{4} \right]^5$$

$$= 4\sqrt{2} \left[\cos \frac{5\pi}{4} - i \sin \frac{5\pi}{4} \right]$$

$$= 4\sqrt{2} \left[\cos \left(\pi + \frac{\pi}{4} \right) - i \sin \left(\pi + \frac{\pi}{4} \right) \right]$$

$$= 4\sqrt{2} \left(\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i \right)$$

$$= -4 + 4i$$

(ii) $(1 + i)^6$

Solution:

Let $z = 1 + i$

$\therefore a = 1, b = 1$, i.e. $a > 0, b > 0$

$\therefore |z| = \sqrt{a^2 + b^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

Here, $(1, 1)$ lies in 1st quadrant.

$\therefore \text{amp}(z) = \tan^{-1}\left(\frac{b}{a}\right)$

$$= \tan^{-1}\left(\frac{1}{1}\right)$$

$$= \frac{\pi}{4}$$

$$z^6 = (1 + i)^6$$

$$= \left[\sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) \right]^6$$

$$= 8 \left[\cos \frac{6\pi}{4} + i \sin \frac{6\pi}{4} \right]$$

$$\dots [\because (\cos \theta + i \sin \theta)^n = (\cos n \theta + i \sin n \theta)]$$

$$= 8 \left[\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right]$$

$$= 8 [0 + i(-1)]$$

$$= -8i$$

(iii) $(1 - \sqrt{3} i)^4$

Solution:

Let $z = 1 - \sqrt{3} i$

$\therefore a = 1, b = -\sqrt{3}$, i.e. $a > 0, b < 0$

$$\therefore |z| = \sqrt{a^2 + b^2} = \sqrt{1^2 + (-\sqrt{3})^2} = 2$$

Here, $(1, -\sqrt{3})$ lies in 4th quadrant.

$$\begin{aligned}\therefore \text{amp}(z) &= 2\pi - \tan^{-1} \left| \frac{b}{a} \right| \\ &= 2\pi - \tan^{-1} \left| \frac{-\sqrt{3}}{1} \right| \\ &= 2\pi - \tan^{-1}(\sqrt{3}) \\ &= 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}\end{aligned}$$

$$\begin{aligned}z^4 &= (1 - \sqrt{3} i)^4 \\ &= \left[2 \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right) \right]^4 \\ &= 16 \left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right)^4 \\ &= 16 \left[\cos \left(2\pi - \frac{\pi}{3} \right) + i \sin \left(2\pi - \frac{\pi}{3} \right) \right]^4 \\ &= 16 \left(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3} \right)^4\end{aligned}$$

$$\begin{aligned} &= 16 \left(\cos \frac{4\pi}{3} - i \sin \frac{4\pi}{3} \right) \\ &\dots [\because (\cos \theta + i \sin \theta)^n = (\cos n \theta + i \sin n \theta)] \\ &= 16 \left[\cos \left(\pi + \frac{\pi}{3} \right) - i \sin \left(\pi + \frac{\pi}{3} \right) \right] \\ &= 16 \left(-\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \\ &= 16 \left[-\frac{1}{2} + \frac{\sqrt{3}}{2} i \right] \\ &= -8 + 8\sqrt{3}i \end{aligned}$$

(iv) $(-2\sqrt{3} - 2i)^5$

Solution:

$$\text{Let } z = -2\sqrt{3} - 2i$$

$$\therefore a = -2\sqrt{3}, b = -2, \text{ i.e. } a < 0, b < 0$$

$$\begin{aligned}\therefore |z| &= \sqrt{a^2 + b^2} = \sqrt{(-2\sqrt{3})^2 + (-2)^2} = \sqrt{12 + 4} \\ &= 4\end{aligned}$$

Here, $(-2\sqrt{3}, -2)$ lies in 3rd quadrant.

$$\begin{aligned}\therefore \text{amp}(z) &= \pi + \tan^{-1}\left|\frac{b}{a}\right| = \pi + \tan^{-1}\left|\frac{-2}{-2\sqrt{3}}\right| \\ &= \pi + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \\ &= \pi + \frac{\pi}{6} = \frac{7\pi}{6}\end{aligned}$$

$$\begin{aligned}z^5 &= (-2\sqrt{3} - 2i)^5 \\ &= \left[4\left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6}\right)\right]^5 \\ &= 1024\left(\cos \frac{35\pi}{6} + i \sin \frac{35\pi}{6}\right) \\ &\dots [\because (\cos \theta + i \sin \theta)^n = (\cos n\theta + i \sin n\theta)] \\ &= 1024\left[\cos\left(6\pi - \frac{\pi}{6}\right) + i \sin\left(6\pi - \frac{\pi}{6}\right)\right] \\ &= 1024\left(\cos \frac{\pi}{6} - i \sin \frac{\pi}{6}\right) \\ &= 1024\left[\frac{\sqrt{3}}{2} - \frac{1}{2}i\right] = 512\sqrt{3} - 512i\end{aligned}$$



Maharashtra Board Solutions

Class 11 Arts & Science Maths

(Part 2)

- Chapter 1- Complex Numbers
- Chapter 2- Sequences and Series
- Chapter 3- Permutations and Combination
- Chapter 4- Methods of Induction and Binomial Theorem
- Chapter 5- Sets and Relations
- Chapter 6- Functions
- Chapter 7- Limits
- Chapter 8- Continuity
- Chapter 9- Differentiation

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About About Maharashtra State Board (MSBSHSE)

The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

The Maharashtra State Board of Secondary & Higher Secondary Education (MSBSHSE), Pune is an independent body of the Maharashtra Government. There are more than 1.4 million students that appear in the examination every year. The Maha State Board conducts the board examination twice a year. This board conducts the examination for SSC and HSC.

The Maharashtra government established the Maharashtra State Bureau of Textbook Production and Curriculum Research, also commonly referred to as Ebalbharati, in 1967 to take up the responsibility of providing quality textbooks to students from all classes studying under the Maharashtra State Board. MSBHSE prepares and updates the curriculum to provide holistic development for students. It is designed to tackle the difficulty in understanding the concepts with simple language with simple illustrations. Every year around 10 lakh students are enrolled in schools that are affiliated with the Maharashtra State Board.

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FAQs

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How many state boards are there in Maharashtra?

The Maharashtra State Board of Secondary & Higher Secondary Education, conducts the HSC and SSC Examinations in the state of Maharashtra through its nine Divisional Boards located at Pune, Mumbai, Aurangabad, Nasik, Kolhapur, Amravati, Latur, Nagpur and Ratnagiri.

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For any clarifications or questions you can write to **info@indcareer.com**

Postal Address

IndCareer.com
52, Shilpa Nagar,
Somalwada
Nagpur - 440015
Maharashtra, India

WhatsApp: +91 9561 204 888

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