

Random Experiment :

An experiment all of whose possible results are known but which will occur is known only at the end of the experiment is called a random experiment.

Sample Space :

The set of all possible outcomes of a random experiment is called a sample space.

Event :

A subset of a sample space is called an event.

Elementary Events :

Let U be a finite sample space. $U = \{x_1, x_2, \dots, x_n\}$. The singleton $\{x_i\}$, $i = 1, 2, \dots, n$ of U are called elementary events.

Impossible Events :

The special subset $\emptyset$ of the sample space U is called the impossible event

Certain Event :

The special subset U of the sample space U is called the certain event.

Complementary Event :

Let A be an event. The set consisting of all the elements of the sample space U other than the elements of A is called the complementary event of A .

$$A' = \{x \mid x \in U, x \notin A\}$$

Union of Events :

Let A and B be events. The set consisting of all the elements of the sample space U which are in A or in B is called the union of events A and B . It is denoted by $A \cup B$

$$A \cup B = \{x \mid x \in U, x \in A \text{ or } x \in B\}$$

Intersection of Events :

Let A and B be events. The set consisting of all the elements of the sample space U which are in A as well as in B is called the intersection of events A and B .

$$A \cap B = \{x \mid x \in U, x \in A \text{ and } x \in B\}$$

Mutually Exclusive Events :

If for events $A, B \in P(U)$, $A \cap B = \emptyset$, then A and B are called mutually exclusive events.

Exhaustive Events

If for events $A, B \in P(U)$, $A \cup B = U$, then A and B are called exhaustive events.

Mutually Exclusive and Exhaustive Events :

If for events $A, B \in P(U)$, $A \cap B = \emptyset$ and $A \cup B = U$, then A and B are called mutually exclusive and exhaustive events.

Set Function :

Let S be the power set of the sample space U and R be the set of real numbers. A function $T : S \rightarrow R$ is called a real valued set function.

Additive Set Function :

Let $T : S \rightarrow R$ be a set function. If $T(A_1 \cup A_2) = T(A_1) + T(A_2)$, whenever $\forall A_1, A_2 \in S$ and $A_1 \cap A_2 = \emptyset$, then T is called an additive set function on S .

Axiomatic Definition of Probability :

Let U be a finite sample space and S be its power set. Suppose that a set function $P : S \rightarrow R$ satisfies following axioms.

Axiom 1 : $\forall A \in S, P(A) \geq 0$

Axiom 2 : $P(U) = 1$

Axiom 3 : $\forall A_1, A_2 \in S, A_1 \cap A_2 = \emptyset \Rightarrow P(A_1 \cup A_2) = P(A_1) + P(A_2)$

Then P is called a probability function and $P(A)$ is the probability of A .

Classical Definition of Probability :

If a finite sample space, associated with a random experiment, has r equally likely outcomes favourable for the occurrence of an event A , and

if $n(U) = n, n(A) = r, 0 \leq r \leq n$, then $P(A) = \frac{r}{n}$

Difference Event :

Let $A, B \in P(U)$. The set consisting of all the elements of U which are in A but not in B is called the difference event of A and B . It is denoted by $A - B$.

- $P(A') = 1 - P(A)$
- $P(\emptyset) = 0$
- $P(U) = 1$
- If $A \subset B$, then $P(B - A) = P(B) - P(A)$
- $P(A - B) = P(A \cap B') = P(A) - P(A \cap B)$
(The probability that only event A out of A and B occurs)
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- If $A \subset B$, then $P(A) \leq P(B)$
- $0 \leq P(A) \leq 1$
- Only one of the two events A and B occurs,
 $P(A \Delta B) = P(A) + P(B) - 2P(A \cap B)$
- At least two out of three events A, B, C occur,
 $P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$
- Exactly two events of the events A, B and C occur,
 $P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$
- Exactly one event of the events A, B and C occurs,
 $P(A) + P(B) + P(C) - 2P(A \cap C) - 2P(B \cap C) - 2P(C \cap A) + 3P(A \cap B \cap C)$

Conditional Probability :

If A and B are any two events of $S = P(U)$, $P(B) \neq 0$, the condition probability of event A given event B to occur is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Rule of Probability :

$$P(A \cap B) = P(A) \cdot P(B | A) \quad (\text{If } P(A) \neq 0)$$

$$P(A \cap B) = P(B) \cdot P(A | B) \quad (\text{If } P(B) \neq 0)$$

For events A, B and C,

$$P(A \cap B \cap C) = P(A) \cdot P(B | A) \cdot P(C | (A \cap B))$$

Total Probability :

Let B_1 and B_2 be mutually exclusive and exhaustive events. Let $P(B_1) \neq 0$ and $P(B_2) \neq 0$. Then for any event A

$$P(A) = P(B_1) \cdot P(A | B_1) + P(B_2) \cdot P(A | B_2)$$

Bay's Rule :

If B_1, B_2 and B_3 are mutually exclusive and exhaustive events and if for event A, $P(A) \neq 0$, then

$$P(B_i | A) = \frac{P(A | B_i) P(B_i)}{P(A | B_1) P(B_1) + P(A | B_2) P(B_2) + P(A | B_3) P(B_3)}, \quad i = 1, 2, 3$$

Independent Events :

If probability of occurrence of event A does not affect the probability of occurrence of B,

$P(A | B) = P(A)$ and $P(B | A) = P(B)$, then A, B are called independent events.

- If $P(A \cap B) = P(A) \cdot P(B)$, then A and B are said to be independent events.

Random Variable :

Let U be the sample space of a random experiment. A real valued function $X : U \rightarrow R$ is said to be a random variable function.

Probability Distribution of a Random Variable :

Let $X : U \rightarrow R$ be a random variable with the range of random variable $X = \{x_1, x_2, \dots, x_n\}$.

Suppose X assumes a value x_i with probability $p(x_i) = P(X = x_i)$.

If (i) $p(x_i) \geq 0, i = 1, 2, \dots, n$

$$(ii) \sum_{i=1}^n p(x_i) = 1 \text{ for real values } x_1, x_2, \dots, x_n,$$

$\{p(x_1), p(x_2), \dots, p(x_n)\}$ is called the probability distribution of the random variable X.

It can be written as :

X = x	x_1	x_2	x_3	...	x_{n-1}	x_n
p(x)	$p(x_1)$	$p(x_2)$	$p(x_3)$	...	$p(x_{n-1})$	$p(x_n)$

(i)

where $p(x_i) \geq 0$ and $\sum p(x_i) = 1$

Mean :

The probability distribution of a random variable X is given as per (i) The mathematical expectation of a random variable X is denoted by E(X) or mean of x_i is μ .

$$E(X) = \sum_{i=1}^n x_i p(x_i)$$

It is called the mean of X and denoted by E(X) or μ .

Variance :

$$V(X) = \sigma_X^2 = E(X^2) - [E(X)]^2$$

$$= \sum_{i=1}^n x_i^2 p(x_i) - \left[\sum_{i=1}^n x_i p(x_i) \right]^2$$

Standard Deviation :

The positive square root of variance of a random variable X is called its standard deviation. It is denoted by s and it is σ_X or $\sqrt{V(X)}$.

Bernoulli Trials :

Suppose a random experiment has two possible outcomes namely success (S) and failure (F). If constant p ($0 < p < 1$) is the probability of getting success at each of the n trials of the experiment, then the trials are known as Bernoulli trials.

Suppose X denotes number of successes in an experiment of n Bernoulli trials of a random experiment in which p is the probability of getting success. The probability distribution of X is defined as

$$p(x) = P(X = x) = \binom{n}{x} p^x \cdot q^{n-x}, \quad x = 0, 1, 2, \dots, n.$$

where $0 < p < 1$ and $q = 1 - p$

Here probability distribution of random variable X is called binomial random variable.

- Mean and variance of binomial distribution of a random variable with parameters n and p are np and npq respectively.

: Multiple Choice Questions :

- (1) There are 125 students in a class. Out of them 90 students like Mathematics, 75 students like Physics, 60 students like Chemistry, 30 students like only Mathematics and Physics. 40 students like Physics and Chemistry. 10 students like only Mathematics and Chemistry. All the students like at least one subject. The probability that a randomly selected student likes all the three subjects is
- (A) $\frac{10}{125}$ (B) $\frac{2}{5}$ (C) $\frac{4}{25}$ (D) $\frac{1}{5}$

Sol. :

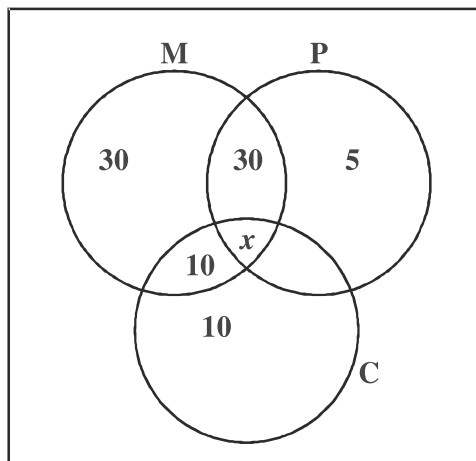


Figure 22.1

M = the set of students who like Mathematics.

P = the set of students who like Physics.

C = the set of students who like Chemistry.

Suppose $n(M \cap P \cap C) = x$

$$P(M \cup P \cup C) = P(M) + P(P) + P(C) - P(M \cap P) - P(M \cap C) - P(P \cap C) + P(M \cap P \cap C)$$

$$\therefore 1 = \frac{90}{125} + \frac{75}{125} + \frac{60}{125} - \frac{(30+x)}{125} - \frac{(10+x)}{125} - \frac{40}{125} + \frac{x}{125}$$

$$\therefore 1 = \frac{145-x}{125}$$

$$\therefore x = 20$$

$$\therefore P(M \cap P \cap C) = \frac{r}{n} = \frac{20}{125} = \frac{4}{25}$$

Ans. : (C)

(2) If $P(A \cup B) = \frac{1}{2}$ and $P(A') = \frac{3}{4}$, then $P(A' \cap B) = \dots\dots$

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) 0

Sol. : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore \frac{1}{2} = \left(1 - \frac{3}{4}\right) + P(B) - P(A \cap B)$$

$$\therefore \frac{1}{2} = \frac{1}{4} + P(A' \cap B)$$

$$\therefore P(A' \cap B) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

Ans. : (A)

(3) 8 boys and 2 girls are seated in a row. The probability that two girls do not sit adjacent to each other is $\dots\dots$

(A) $\frac{1}{5}$

(B) $\frac{4}{5}$

(C) $-\frac{1}{4}$

(D) $\frac{5}{4}$

Sol. : Total number of arrangements of ten persons is $= 10!$

Let A be the event that two girls sit adjacent to each other $n(A) = 2(9!)$

$$\therefore P(A) = \frac{2 \cdot 9!}{10!} = \frac{2 \cdot 9!}{10 \cdot 9!} = \frac{1}{5}$$

$$\therefore \text{The probability of the event that two girls do not sit adjacent to each other} = 1 - \frac{1}{5} = \frac{4}{5}$$

Ans. : (B)

(4) The probability of getting 53 Sundays in a randomly selected leap year is $\dots\dots$

(A) $\frac{1}{365}$

(B) $\frac{53}{366}$

(C) $\frac{2}{7}$

(D) $\frac{1}{7}$

Sol. : There are 366 days in leap year. So there are 52 weeks and 2 consecutive days. So there are 52 Sundays and possibility of 2 more days out of 7 days in the following order.

(1) Sunday, Monday

(2) Monday, Tuesday

(3) Tuesday, Wednesday

(4) Wednesday, Thursday

(5) Thursday, Friday

(6) Friday, Saturday

(7) Saturday, Sunday

So, out of 7 possibilities there are only two possibilities in which we get Sunday in (1) and (7).

$$\text{So probability of a leap year with 53 Sundays} = \frac{2}{7}.$$

Ans. : (C)

- (5) A balanced die is rolled thrice in a random experiment. The probability of not getting the same digit in all the trials is

(A) $\frac{6}{216}$ (B) $\frac{35}{16}$ (C) $\frac{1}{6}$ (D) $\frac{35}{36}$

Sol. : Total number of possible results $n = 6 \times 6 \times 6 = 216$

There are 6 possible ways in which we get the same digit in all the trials, (1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6).

$\therefore$ So the number of possible ways of not getting same digit in all the trials $= 216 - 6 = 210$.
So, $r = 210$

$\therefore$ Probability $= \frac{r}{n} = \frac{210}{216} = \frac{35}{36}$ **Ans. : (D)**

- (6) There are 10 boys and 8 girls in a class. Three students are selected at random. The probability of the event that 2 boys and 1 girl are selected is

(A) $\frac{15}{34}$ (B) $\frac{5}{34}$ (C) $\frac{5}{34}$ (D) $\frac{1}{6}$

Sol. : Here, $n = \binom{18}{3} = \frac{18 \cdot 17 \cdot 16}{6} = 816$

Event A = Two boys and one girl are selected

$$P(A) = \frac{\binom{10}{2} \binom{8}{1}}{\binom{18}{3}} = \frac{45 \times 8}{816} = \frac{15}{34}$$
 Ans. : (A)

- (7) The probability that a certain team will win a competition is 0.7, probability for losing is 0.2 and 0.1 is the probability of no result. If the team plays three times, the probability that they win at least two times is (The team does not lose the match)

(A) 0.9 (B) 0.49 (C) 0.7 (D) 0.8

Sol. : Let W denote win and T for a draw.

All events are mutually exclusive.

$P(WWT) + P(WTW) + P(TWW) + P(WWW)$

$= P(W) \cdot P(W) \cdot P(T) + P(W) \cdot P(T) \cdot P(W) + P(T) \cdot P(W) \cdot P(W) + P(W)P(W)P(W)$

$= \left(\frac{7}{10}\right)\left(\frac{7}{10}\right)\left(\frac{1}{10}\right) + \left(\frac{7}{10}\right)\left(\frac{1}{10}\right)\left(\frac{7}{10}\right) + \left(\frac{1}{10}\right)\left(\frac{7}{10}\right)\left(\frac{7}{10}\right) + \left(\frac{7}{10}\right)\left(\frac{7}{10}\right)\left(\frac{7}{10}\right)$

$= \frac{49}{1000} + \frac{49}{1000} + \frac{49}{1000} + \frac{343}{1000} = \frac{490}{1000} = 0.49$ **Ans. : (B)**

- (8) The probability of getting only 52 Sundays in a year that is not a leap year is

(A) $\frac{1}{7}$ (B) $\frac{6}{7}$ (C) $\frac{52}{365}$ (D) $\frac{312}{365}$

Sol. : In a year not a leap year, there are 365 days = 52 weeks + 1 day

The probability of 1 day to be Sunday is $\frac{1}{7}$.

$\therefore$ Probability that it's not Sunday is $1 - \frac{1}{7} = \frac{6}{7}$ **Ans. : (B)**

- (9) Events A, B, C are mutually exclusive events such that.

$P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. The set of possible values for x is
[AIEEE : 2003]

- (A) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (B) $\left[\frac{1}{3}, \frac{13}{3}\right]$ (C) $[0, 1]$ (D) $\left[\frac{1}{3}, \frac{1}{2}\right]$

Sol. : $0 \leq P(A \cup B \cup C) \leq 1$

$$\therefore 0 \leq P(A) + P(B) + P(C) \leq 1$$

$$\therefore 0 \leq \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \leq 1$$

$$\therefore 0 \leq \frac{12x+4}{12} + \frac{3-3x}{12} + \frac{6-12x}{12} \leq 1$$

$$\therefore 0 \leq 13 - 3x \leq 12$$

$$\therefore 0 \leq 13 - 3x$$

$$\therefore 3x \leq 13$$

$$\therefore x \leq \frac{13}{3}$$

$$\therefore x \in \left[\frac{1}{3}, \frac{13}{3}\right] \quad \dots(1)$$

$$\text{Also, } 0 \leq \frac{3x+1}{3} \leq 1, 0 \leq \frac{1-x}{4} \leq 1 \text{ and}$$

$$0 \leq \frac{1-2x}{2} \leq 1.$$

$$\therefore -\frac{1}{3} \leq x \leq \frac{2}{3}, -3 \leq x \leq 1 \text{ and}$$

$$-\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$\therefore x \in \left[-\frac{1}{3}, \frac{1}{2}\right] \quad \dots(2)$$

$$\text{From (1) and (2), } x \in \left[\frac{1}{3}, \frac{1}{2}\right]$$

Ans. : (D)

- (10) A person writes four letters and writes addresses on four envelopes. The letters are randomly inserted in the envelopes. The probability of placing at least one letter not in correct envelope is

- (A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) $\frac{1}{24}$ (D) $\frac{23}{24}$

Sol. : Total number of arrangements of letters is $= {}_4P_4 = 4! = 24$

There is only one possibility that all letters being placed in correct envelopes.

$$\therefore \text{ So, the probability that at least one letters is not inserted in correct envelope } = 1 - \frac{1}{24} = \frac{23}{24}$$

Ans. : (D)

- (11) Two balanced dice are rolled simultaneously. The probability of the event that sum of the integers on the two dice is 10 or more is

- (A) $\frac{10}{36}$ (B) $\frac{1}{9}$ (C) $\frac{32}{36}$ (D) $\frac{1}{6}$

Sol. : Here, $n = 36$

$$A = \{(5, 5), (5, 6), (6, 5), (6, 6), (6, 4), (4, 6)\}$$

$$\therefore r = 6$$

$$\therefore P(A) = \frac{r}{n} = \frac{6}{36} = \frac{1}{6}$$

Ans. : (D)

- (12) Two cards are drawn at random from a well shuffled pack of 52 cards. The probability of the event that one card is a king and the other is a queen is

- (A) $\frac{12}{52 \times 51}$ (B) $\frac{8}{663}$ (C) $\frac{1}{26}$ (D) $\frac{8}{52 \times 51}$

Sol. : Here, $n = \binom{52}{2}$, $r = \binom{4}{1} \binom{4}{1}$

$$\therefore P(1 \text{ king and } 1 \text{ queen}) = \frac{4 \times 4}{\binom{52}{2}} = \frac{16 \times 2}{52 \times 51} = \frac{8}{663}$$

Ans. : (B)

- (13) A, B and C are mutually exclusive and exhaustive events. If $3P(A) = P(B) = 2P(C)$, then $P(B) = \dots\dots$

(A) $\frac{6}{11}$ (B) $\frac{2}{11}$ (C) $\frac{3}{11}$ (D) $\frac{1}{3}$

Sol. : Here, $P(A) + P(B) + P(C) = 1$ (1)

$3P(A) = P(B)$ and $P(B) = 2P(C)$

$P(A) = \frac{1}{3}P(B)$ $P(C) = \frac{1}{2}P(B)$

From (1), $\frac{1}{3}P(B) + P(B) + \frac{1}{2}P(B) = 1$

$\therefore \frac{2P(B) + 6P(B) + 3P(B)}{6} = 1$

$\therefore \frac{11P(B)}{6} = 1$

$\therefore P(B) = \frac{6}{11}$ **Ans. : (A)**

- (14) If for events A and B, $P(A) > 0$ and $P(B) \neq 1$, then $P(A' | B') = \dots\dots$

(A) $1 - P(A | B')$ (B) $1 - P(A | B)$ (C) $\frac{1 - P(A \cup B)}{P(B')}$ (D) $\frac{P(B')}{P(A)}$

Sol. : $P(A' | B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{P(B')}$ **Ans. : (C)**

- (15) 5 men, 2 women and 3 children took part in a music competition. If three persons get the best award for best music, then the probability that out of the three, two are children is $\dots\dots$.

(A) $\frac{7}{40}$ (B) $\frac{2}{3}$ (C) $\frac{2}{10}$ (D) None of these

Sol. : Here, $n = \binom{10}{3} = \frac{10 \cdot 9 \cdot 8}{6} = 120$, $r = \binom{3}{2} \binom{7}{1}$

The probability of the required event = $\frac{\binom{3}{2} \binom{7}{1}}{\binom{10}{3}} = \frac{21}{120} = \frac{7}{40}$ **Ans. : (A)**

- (16) A balanced die is thrown. Let A be the event that the number obtained is greater than 3. Let B be event that the number obtained is less than 5. Then $P(A \cup B)$ is = $\dots\dots$ [AIEEE : 2008]

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) 0 (D) 1

Sol. : $A = \{4, 5, 6\}$, $B = \{1, 2, 3, 4\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\} = U$ (We have 1, 2, 3, 4, 5, 6 only on a die.)

$\therefore P(A \cup B) = P(U) = 1$ **Ans. : (D)**

- (17) If $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$, $P(A') = \frac{2}{3}$ for events A and B, then $P(A' \cap B) = \dots\dots$ [AIEEE : 2002]

(A) $\frac{5}{12}$ (B) $\frac{3}{8}$ (C) $\frac{4}{5}$ (D) $\frac{5}{4}$

Sol. : Here, $P(A') = \frac{2}{3}$. So, $P(A) = 1 - P(A') = 1 - \frac{2}{3} = \frac{1}{3}$

$$\text{Now, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore \frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4}. \text{ So, } P(B) = \frac{2}{3}$$

$$P(A' \cap B) = P(B) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

Ans. : (A)

Second method :

$$\frac{3}{4} = \frac{1}{3} + P(B \cap A')$$

$$\therefore P(A' \cap B) = \frac{3}{4} - \frac{1}{3} = \frac{5}{12}$$

- (18) Five horses take part in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is [AIEEE : 2003]

(A) $\frac{3}{5}$ (B) $\frac{1}{5}$ (C) $\frac{2}{5}$ (D) $\frac{4}{5}$

Sol. : One of the horses will win the race.

$$\text{So, total number of ways for winning horse } n = \binom{5}{1} = 5.$$

$$\text{Possibility of winning the race by one of the selected horses} = \binom{2}{1}$$

$$\therefore \text{Probability that the selected horse will win the race} = \frac{r}{n} = \frac{2}{5} \quad \text{Ans. : (C)}$$

- (19) A problem in mathematics is given to three students A, B, C and the probabilities of the problem being solved by them are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that the problem is solved is

[AIEEE : 2002]

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

Sol. : 'That the problem is not solved' is the event $A' \cap B' \cap C'$.

$$P(A' \cap B' \cap C') = P(A') \cdot P(B') \cdot P(C')$$

$$= \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{4}\right) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{4}$$

$$\therefore \text{Probability that the problem is solved} = 1 - \frac{1}{4} = \frac{3}{4} \quad \text{Ans. : (A)}$$

- (20) The probability that a person A speaks truth is $\frac{4}{5}$ and that probability that a person B speaks truth is $\frac{3}{4}$. The probability that they contradict each other is in an event

(A) $\frac{7}{20}$ (B) $\frac{1}{5}$ (C) $\frac{3}{20}$ (D) $\frac{4}{5}$

Sol. : Probability of the event that they contradict each other

$$= P(A \cap B') + P(A' \cap B)$$

$$= P(A) \cdot P(B') + P(A')P(B)$$

$$= P(A)[1 - P(B)] + [1 - P(A)] \cdot P(B)$$

$$= \frac{4}{5} \left(1 - \frac{3}{4}\right) + \left(1 - \frac{4}{5}\right) \cdot \frac{3}{4} = \frac{4}{20} + \frac{3}{20} = \frac{7}{20}$$

Ans. : (A)

(21) If $P(A) = 0.35$, $P(B) = 0.45$, $P(A \cup B) = 0.65$, then $P(A | B) = \dots$

- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 0.1 (D) 1

Sol. : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore 0.65 = 0.35 + 0.45 - P(A \cap B)$$

$$\therefore P(A \cap B) = 0.15$$

$$\text{Now, } P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.45} = \frac{1}{3}$$

Ans. : (A)

(22) $P((A \cup A') | B) = \dots$

- (A) $P(A | B) + P(A' | B)$ (B) $P(B)$ (C) 0 (D) 1

Sol. : $A \cup A' = U$

$$\therefore P((A \cup A') | B) = P(U | B) = 1$$

Ans. : (D)

(23) If $P(A) = \frac{1}{8}$, $P(A | B) = \frac{1}{4}$, $P(B | A) = \frac{1}{5}$ for events A and B, then $P(B) = \dots$

- (A) $\frac{1}{10}$ (B) $\frac{1}{20}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Sol. : $P(A | B) = \frac{P(A \cap B)}{P(B)}$, $P(B | A) = \frac{P(A \cap B)}{P(A)}$

$$\therefore P(A | B) \cdot P(B) = P(B | A) \cdot P(A)$$

$$\therefore \frac{1}{4} \cdot P(B) = \frac{1}{5} \cdot \frac{1}{8}$$

$$P(B) = \frac{1}{10}$$

Ans. : (A)

(24) An officer from the weight and measure department has checked 100 weigh machines. In his observation he found that 10 weigh machines have A type defect and 5 machines have B type defect. Two weigh machines have both type of defects. If it is given that a randomly selected weigh machine has A type defect, then the probability of it having B type defect is

- (A) 0.1 (B) 0.2 (C) 0.3 (D) $\frac{1}{20}$

Sol. : $P(A) = \frac{10}{100} = 0.1$, $P(B) = \frac{5}{100} = 0.05$, $P(A \cap B) = \frac{2}{100} = 0.02$

$$P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{0.02}{0.1} = 0.2$$

Ans. : (B)

(25) In a survey of a route of a private bus of a travel agency, it was found that 91 % buses arrive in time and 95 % buses depart in time. 85 % buses arrive and depart in time. The probability that given a bus departs in time but does not arrive in time is

- (A) $\frac{81}{100}$ (B) $\frac{85}{100}$ (C) $\frac{2}{19}$ (D) 1

Sol. : A : The event that a bus arrives in time, B : the event that the bus departs in time. Required event $A' | B$.

$$P(A) = \frac{91}{100}, P(B) = \frac{95}{100}, P(A \cap B) = \frac{85}{100}$$

$$P(A' | B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{\frac{95}{100} - \frac{85}{100}}{\frac{95}{100}} = \frac{10}{95} = \frac{2}{19}$$

Ans. : (C)

- (26) Three cards are drawn in succession without replacement, from a well shuffled pack of 52 cards. A_1 is the event that first card is a red colour ace, A_2 is the event that the second card is a jack or a king and A_3 is the event that the third card is greater than 3 but less than 7. The value of $P(A_1 \cap A_2 \cap A_3) = \dots$

(A) 1 (B) $\frac{96}{2250}$ (C) $\frac{8}{5525}$ (D) $\frac{96}{521}$

Sol. : $P(A_1) = \frac{2}{52}$ $P(A_2|A_1) = \frac{8}{51}$ $P(A_3|A_1 \cap A_2) = \frac{12}{50}$

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) = \frac{2}{52} \cdot \frac{8}{51} \cdot \frac{12}{50} = \frac{8}{5525} \quad \text{Ans. : (C)}$$

- (27) A purse contains 6 silver coins and 3 gold coins. Another purse contains 4 silver and 5 gold coins. A purse is selected at random and a coin is drawn from it. The probability that the selected coin is a silver coin is

(A) $\frac{3}{7}$ (B) $\frac{1}{3}$ (C) $\frac{4}{18}$ (D) $\frac{5}{9}$

Sol. : Event B_1 : First purse is selected. Then $P(B_1) = \frac{1}{2}$

Event B_2 : Second purse is selected. Then $P(B_2) = \frac{1}{2}$

A : The selected coin is silver coin

$$\therefore P(A|B_1) = \frac{6}{9} \quad P(A|B_2) = \frac{4}{9}$$

$$\therefore P(A) = P(B_1) \cdot P(A|B_1) + P(B_2) \cdot P(A|B_2) = \frac{1}{2} \cdot \frac{6}{9} + \frac{1}{2} \cdot \frac{4}{9} = \frac{6}{18} + \frac{4}{18} = \frac{10}{18} = \frac{5}{9} \quad \text{Ans. : (D)}$$

- (28) $P(A') = 0.15$, $P(A \cup B) = 0.95$. A and B are independent. $P(B|A) = \dots$

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 0.95 (D) 0.15

Sol. : $P(B|A) = P(B)$ (A, B are independent)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\therefore 0.95 = (1 - 0.15) + P(B) - (1 - 0.15) \cdot P(B)$$

$$\therefore 0.95 = 0.85 + 0.15 P(B)$$

$$\therefore P(B) = \frac{2}{3} \quad \text{Ans. : (B)}$$

- (29) If for a random variable X, the mean $E(X) = 5$ and the standard deviation $\sigma_X = 3$, then $E(2X + 3)^2 = \dots$

(A) 502 (B) 205 (C) 360 (D) 302

Sol. : $\sigma_X^2 = E(X^2) - [E(X)]^2$

$$\therefore 9 = E(X^2) - 5^2$$

$$\therefore E(X^2) = 34$$

$$\text{Now, } E((2X + 3)^2) = E(4X^2 + 12X + 9)$$

$$= 4E(X^2) + 12E(X) + 9$$

$$= 4(34) + 12(5) + 9 = 136 + 60 + 9 = 205$$

Ans. : (B)

- (30) Probability distribution of a random variable X is given as

$X = x$	-2	-1	0	1	2
$p(x)$	0.2	0.1	k	k	0.1

then $2k^2 = \dots$

- (A) 0.36 (B) 0.18 (C) 0.072 (D) 7.2

Sol. : $\sum p(x) = 1$

$$\therefore 0.2 + 0.1 + k + k + 0.1 = 1$$

$$\therefore 2k = 1 - 0.4 = 0.6. \text{ So, } k = 0.3$$

$$\therefore 2k^2 = 2(0.3)^2 = 0.18$$

Ans. : (B)

- (31) Probability distribution of a random variable X is given. Then $V(X) = \dots$

$X = x$	-1	0	1	2
$p(x)$	k	0.1	0.2	0.2

- (A) 1.49 (B) 1.9 (C) 0.1 (D) 0.61

Sol. : $\sum p(x) = 1$

$$\therefore k + 0.1 + 0.2 + 0.2 = 1$$

$$\therefore k = 0.5$$

$$E(X) = \sum x_i p(x_i)$$

$$= (-1) \cdot (0.5) + 0 \cdot (0.1) + 1 \cdot (0.2) + 2 \cdot (0.2) = -0.5 + 0.2 + 0.4 = 0.1$$

$$E(X^2) = \sum x_i^2 p(x_i)$$

$$= (-1)^2 \cdot 0.5 + 0^2 (0.1) + 1^2(0.2) + 2^2 (0.2) = 0.5 + 0 + 0.2 + 0.8 = 1.5$$

$$V(X) = E(X^2) - [E(X)]^2 = 1.5 - (0.1)^2 = 1.5 - 0.01 = 1.49$$

Ans. : (A)

- (32) If the mean and the standard deviation of a random variable X are $E(X) = 4$ and $\sigma_X = 3$ respectively, then $V(2 - 3X) = \dots$

- (A) -23 (B) 12 (C) 81 (D) 144

Sol. : $V(2 - 3X) = |-3|^2 V(X) = 9V(X) = 9\sigma_X^2 = 9 \cdot (9) = 81$

Ans. : (C)

- (33) If the parameters of a binomial distribution are $n = 5$ and $p = 0.30$, then the mean is and the variance is

- (A) 1.5, 1.05 (B) 1.05, 1.5 (C) 1.5, 1.5 (D) 1.5, 1.15

Sol. : $q = 1 - p$

$$\therefore q = 1 - 0.3$$

$$\therefore q = 0.7$$

$$\text{Mean} = np = 5 \times 0.30 = 1.5$$

$$\text{Variance} = npq = 5 \times 0.3 \times 0.7$$

$$= 5 \times \frac{3}{10} \times \frac{7}{10} = 1.05$$

Ans. : (A)

- (34) From the set of 2×2 determinants whose elements are -1, 1, 0, a determinant is selected at random. The probability of the event that selected determinant's value is not zero is

- (A) $\frac{33}{81}$ (B) $\frac{16}{27}$ (C) $\frac{4}{81}$ (D) $\frac{77}{81}$

Sol. : Let $D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ be the required determinant.

Elements a_{11} , a_{12} , a_{21} and a_{22} are selected from amongst -1, 1, 0.

So, the total number of determinants $= 3 \times 3 \times 3 \times 3 = 81$

Let A be the event that $D = 0$

If $a_{11} a_{22} = a_{12} a_{21} = 0$ or $a_{11} a_{22} = a_{12} a_{21} = 1$ or $a_{11} a_{22} = a_{12} a_{21} = -1$, then $D = 0$.

Case I :

$$a_{11} a_{22} = 0.$$

$$\{(1, 0), (0, 1), (-1, 0), (0, -1), (0, 0)\}$$

$$\text{Similarly } a_{12} a_{21} = 0.$$

So, there are total $5 \times 5 = 25$ ways, to make $D = 0$ in this way.

Case II :

Similarly $a_{11} a_{22} = a_{12} a_{21} = 1$ is possible for $\{(1, 1), (-1, -1)\}$

The total number of determinants $2 \times 2 = 4$

Case III :

$a_{11} a_{22} = a_{12} a_{21} = -1$ is possible for $\{(1, -1), (-1, 1)\}$

The total number of determinants $2 \times 2 = 4$

So, the total number of determinants for A = $25 + 4 + 4 = 33$

$$P(A) = \frac{33}{81} = \frac{11}{27}. \quad \text{So, } P(A') = 1 - \frac{11}{27} = \frac{16}{27}. \quad \text{Ans. : (B)}$$

- (35) A die is rolled in a random experiment till 1 appears on it. The probability that 1 appears at even number of trials is

$$(A) \frac{1}{6} \quad (B) \frac{5}{6} \quad (C) \frac{5}{11} \quad (D) \frac{6}{11}$$

Sol. : $\{1, 2, 3, 4, 5, 6\}$ are integers on die.

The probability of getting 1 on the die = $\frac{1}{6}$.

$\therefore$ The probability of not getting 1 is = $1 - \frac{1}{6} = \frac{5}{6}$

The probability that 1 appears at even number of trials = $\frac{5}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} + \dots$

$$= \frac{1}{6} \left[\frac{5}{6} + \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^5 + \dots \infty \right]$$

$$= \frac{1}{6} \left(\frac{\frac{5}{6}}{1 - \frac{25}{36}} \right) = \frac{5}{11} \quad \text{Ans. : (C)}$$

- (36) Three out of six vertices of a regular hexagon are selected in a random experiment. They form a triangle. The probability that the selected triangle is an equilateral triangle is

$$(A) \frac{1}{2} \quad (B) \frac{5}{6} \quad (C) \frac{1}{10} \quad (D) \frac{1}{20}$$

Sol. : $n = \binom{6}{3} = 20$

There are only two possibilities to get equilateral triangle.

So, $r = 2$

$$P(A) = \frac{r}{n} = \frac{1}{10} \quad \text{Ans. : (C)}$$

- (37) In a random experiment, a die is tossed twice. The probability of the event of getting 1, 2 or 3 at first trial and 2, 4, 5 or 6 at second trial is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{1}{4}$

Sol. : A : 1, 2 or 3 appear on the die, $P(A) = \frac{3}{6}$

B : 2, 4, 5 or 6 appear on the die, $P(B) = \frac{4}{6}$

$$P(A \cap B) = P(A) \cdot P(B) = \frac{3}{6} \cdot \frac{4}{6} = \frac{12}{36} = \frac{1}{3}$$

Ans. : (A)

- (38) Raj speaks truth 90 % times in the presence of his parents at home, speaks truth 60 % times in the presence of his wife. The probability of the event that he contradicts in presence of parents and wife is %.

(A) 32 (B) 100 (C) 40 (D) 42

Sol. : Event A : Raj speaks truth to parents. So $P(A) = \frac{90}{100}$. So, $P(A') = \frac{10}{100}$

Event B : Raj speaks truth to wife. So $P(B) = \frac{60}{100}$. So, $P(B') = \frac{40}{100}$

Probability of the event that Raj contradicts in presence of both,

$$= P(A \cap B') + P(A' \cap B)$$

$$= P(A) \cdot P(B') + P(A') \cdot P(B)$$

$$= \frac{90}{100} \cdot \frac{40}{100} + \frac{10}{100} \cdot \frac{60}{100} = \frac{36}{100} + \frac{6}{100} = \frac{42}{100} = 0.42$$

Ans. : (D)

- (39) There are 100 questions in a question bank. Kautilya answers 90 % questions correctly and Saptesh answers 70 % questions correctly from the same question bank. In a random experiment, one question is selected and given them to solve. Then the probability of the event that at least one of them answered correctly is

(A) 0.03 (B) 0.97 (C) 0.7 (D) 0.07

Sol. : Event A : Kautilya answers correctly.

$$P(A) = \frac{90}{100}. \quad \text{So, } P(A') = \frac{10}{100}$$

Event B : Saptesh answers correctly.

$$P(B) = \frac{70}{100}. \quad \text{So, } P(B') = \frac{30}{100}$$

The probability of the event that both have given incorrect answers of selected questions is

$$P(A' \cap B') = P(A') \cdot P(B') = \frac{10}{100} \cdot \frac{30}{100} = \frac{3}{100} = 0.03$$

$\therefore$ The probability that at least one of them answered correctly is

$$1 - P(A' \cap B') = 1 - 0.03 = 0.97$$

Ans. : (B)

- (40) The probability of the event that Haresh successfully shoots a target is $\frac{1}{4}$. If Haresh shoots 25 times, then the variance is

(A) $\frac{5}{8}$ (B) $\frac{75}{16}$ (C) $\frac{75}{4}$ (D) 25

Sol. : Probability that Haresh successfully shoots a target, $p = \frac{1}{4}$. So, $q = 1 - p = \frac{3}{4}$

$$n = 25. \text{ So, variance} = npq = 25 \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{75}{16}$$

Ans. : (B)

- (41) A car manufacturing company manufactures two models of cars namely A and B. 6 cars out of 100 cars are defective in model A and 2 cars out of 100 cars are defective in model B. The probability of the event that a randomly selected car is a non defective car is
- (A) 0.96 (B) 0.94 (C) 0.98 (D) 0.02

Sol. : Suppose C is the event that a randomly selected car is non defective.

If the selected car is of model A, then $P(C|A) = \frac{94}{100}$

If the selected car is of model B, then $P(C|B) = \frac{98}{100}$

The probability of selecting models A or B = $\frac{1}{2}$

$$P(C) = P(A) \cdot P(C|A) + P(B) \cdot P(C|B)$$

$$= \frac{1}{2} \cdot \frac{94}{100} + \frac{1}{2} \cdot \frac{98}{100} = \frac{47}{100} + \frac{49}{100} = \frac{96}{100} = 0.96 \quad \text{Ans. : (A)}$$

- (42) The probability that the response of husband in a practical problem is positive is $\frac{3}{5}$ and probability that the response of wife is positive is $\frac{2}{5}$. The probability that they have different response in some problem is

- (A) $\frac{13}{25}$ (B) $\frac{12}{25}$ (C) $\frac{6}{25}$ (D) $\frac{19}{25}$

Sol. : Event A : Husband has a positive response,

$$P(A) = \frac{3}{5}. \text{ So, } P(A') = \frac{2}{5}$$

Event B : Wife has a positive response,

$$\therefore P(B) = \frac{2}{5}. \text{ So, } P(B') = \frac{3}{5}$$

$\therefore$ Probability of the required event

$$= P(A \cap B') + P(A' \cap B)$$

$$= P(A) \cdot P(B') + P(A') \cdot P(B) = \frac{3}{5} \cdot \frac{3}{5} + \frac{2}{5} \cdot \frac{2}{5} = \frac{13}{25} \quad \text{Ans. : (A)}$$

- (43) A die is rolled 10 times. The probability of the event that 5 appears exactly two times is

- (A) $\frac{5^9}{4 \cdot 6^8}$ (B) $\frac{3 \left(\frac{1}{6}\right)^8}{2}$ (C) $\frac{3}{2} \cdot \frac{55}{65}$ (D) $\frac{3}{2} \cdot \frac{56}{65}$

Sol. : $n = 10$

A : Five appears on die. So, $p = P(A) = \frac{1}{6}$, $q = \frac{5}{6}$

$$\therefore P(X = 2) = \binom{10}{2} \cdot \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^8 = \frac{10 \times 9}{2} \times \frac{1}{6^2} \times \frac{5^8}{6^8} = \frac{5^9}{4 \cdot 6^8} \quad \text{Ans. : (A)}$$

- (44) 10 % students failed in a school. 4 students are selected at random with replacement, then the probability that they are successful students is

- (A) 0.081 (B) 0.6561 (C) 0.3561 (D) 0.046

Sol. : Event A : The selected student has failed. So, $P(A) = \frac{10}{100} = \frac{1}{10} = 0.1$

Event A' = The selected student is successful. So, $P(A') = 0.9$

Here, $n = 4$

$$P(X = 4) = {}^4C_4 (0.9)^4 \cdot (0.1)^0 = 0.6561$$

Ans. : (B)

(45) The mean and the variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. Then $P(X \geq 1) = \dots$

- (A) 1 (B) 0 (C) $\frac{1}{729}$ (D) $\frac{728}{729}$

Sol. : The mean = $np = 4$, The variance = $npq = \frac{4}{3}$

$$\therefore \frac{npq}{np} = \frac{\frac{4}{3}}{4} = \frac{1}{3}. \text{ So, } q = \frac{1}{3},$$

So, $p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$. So $n = 6$ (because $np = 4$)

$$P(X \geq 1) = 1 - P(X = 0)$$

$$= 1 - {}^6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6$$

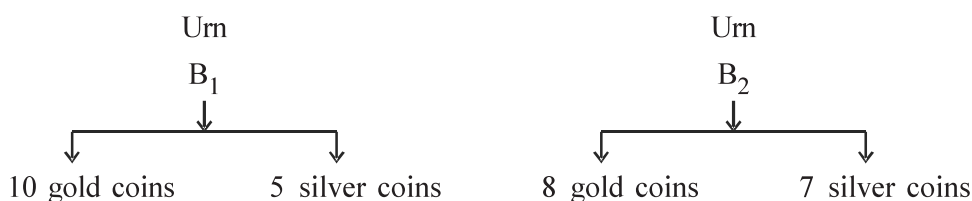
$$= 1 - 1 \cdot 1 \cdot \frac{1}{3^6} = 1 - \frac{1}{3^6} = 1 - \frac{1}{729} = \frac{728}{729}$$

Ans. : (D)

(46) There are 10 gold coins and 5 silver coins in an urn and there are 8 gold coins and 7 silver coins in another urn. An urn is selected at random and one coin is selected from it. The probability that the selected coin is a gold coin is

- (A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{80}{225}$ (D) $\frac{12}{30}$

Sol. :



Event A : Selected coin is gold coin.

Events B_1 and B_2 represent selection of first and second urn respectively.

$$P(A) = P(B_1) \cdot P(A|B_1) + P(B_2) \cdot P(A|B_2) = \frac{1}{2} \cdot \frac{10}{15} + \frac{1}{2} \cdot \frac{8}{15} = \frac{18}{30} = \frac{3}{5}$$

Ans. : (B)

(47) There are 20 apples in a box. 4 of them are rotten. The mathematical expectation of two apples selected at random being rotten is

- (A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) 1

Sol. : The total number of apple is 20. 16 of them are fresh apples and 4 are rotten apples.

X = A randomly selected apple is a rotten apple.

$$P(X = 0) = \frac{{}^{16}C_2 {}^4C_0}{{}^{20}C_2} = \frac{16 \times 15}{20 \times 19} = \frac{12}{19}$$

$$P(X = 1) = \frac{\binom{16}{1} \binom{4}{1}}{\binom{20}{2}} = \frac{16 \times 4 \times 2}{20 \times 19} = \frac{32}{95}$$

$$P(X = 2) = \frac{\binom{16}{0} \binom{4}{2}}{\binom{20}{2}} = \frac{4 \times 3}{20 \times 19} = \frac{3}{95}$$

x_i	0	1	2
$p(x_i)$	$\frac{12}{19}$	$\frac{32}{95}$	$\frac{3}{95}$

$$\text{Required mathematical expectation} = \sum_{i=1}^n x_i p(x_i) = 0 \cdot \left(\frac{12}{19}\right) + 1 \cdot \left(\frac{32}{95}\right) + 2 \cdot \left(\frac{3}{95}\right) = \frac{32+6}{95} = \frac{38}{95} = \frac{2}{5}$$

Ans. : (B)

- (48) Two dice are rolled five times. If identical digits appearing on dice is considered as a success, then probability of getting exactly two successes is

(A) $\frac{625}{3888}$ (B) $\frac{125}{1296}$ (C) $\frac{25}{216}$ (D) $\frac{1}{10}$

Sol. : Here, dice are rolled five times. So, $n = 5$

Two dice are rolled and event A is getting same digits. Then,

$$\therefore A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

$$\therefore \text{Probability of success } p = \frac{6}{36} = \frac{1}{6}. \text{ Probability of failure } q = \frac{5}{6}$$

$$P(X = 2) = \binom{n}{x} p^x \cdot q^{n-x}$$

$$= \binom{5}{2} \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^3 = 10 \cdot \frac{1}{6} \times \frac{1}{6} \cdot \frac{5 \times 5 \times 5}{6 \times 6 \times 6} = \frac{1250}{7776} = \frac{625}{3888}$$

Ans. : (A)

- (49) A player playing a game of tossing a balanced die receives ₹ 10 from his opponent, if he throws the integer 1 or 2. How much should he pay to his opponent, if he throws 3, 4, 5 or 6, so that the game becomes fair ?

(A) 2 (B) 3 (C) 5 (D) 10

Sol. : Suppose a is the amount which the player has to pay to his opponent.

Let X = The amount received by the player.

Here, $E(X) = 0$

$$X(x) = \begin{cases} 10 & x = 1, 2 \\ a & x = 3, 4, 5, 6 \end{cases}$$

x_i	10	a
$p(x_i)$	$\frac{2}{6}$	$\frac{4}{6}$

$$E(X) = \sum x_i p(x_i) = 10 \left(\frac{2}{6}\right) + a \left(\frac{4}{6}\right) = \frac{20}{6} + \frac{4a}{6}$$

$$\therefore 20 + 4a = 0$$

$$\therefore a = -5$$

$\therefore$ The player has to pay ₹ 5 to his opponent.

Ans. : (C)

- (50) There are cards marked with the integers 1 to 25 in a box. Two cards are selected one by one without replacement. The probability of getting even integers marked on both the cards is

(A) $\frac{12}{25}$ (B) $\frac{11}{50}$ (C) $\frac{2}{5}$ (D) $\frac{1}{2}$

Sol. : The set of even integers between 1 to 25 = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24}

A : Selecting first card with an even integer marked on it. So, $P(A) = \frac{12}{25}$

B : Selecting second card with an even integer marked on it. So, $P(B|A) = \frac{11}{24}$

$\therefore$ Required probability $P(A \cap B) = P(A) \cdot P(B|A) = \frac{12}{25} \cdot \frac{11}{24} = \frac{11}{50}$ **Ans. : (B)**

- (51) E and F are independent events. $P(E) = 0.35$ and $P(E \cup F) = 0.60$. The probability that event F does not occur is

(A) $\frac{5}{13}$ (B) $\frac{8}{13}$ (C) $\frac{1}{13}$ (D) $\frac{7}{13}$

Sol. : For independent events E and F,

$$P(E \cap F) = P(E) \cdot P(F) = 0.35 P(F)$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$\therefore 0.60 = 0.35 + P(F) - 0.35 P(F)$$

$$\therefore 0.25 = (1 - 0.35) P(F)$$

$$\therefore P(F) = \frac{0.25}{0.65} = \frac{5}{13}$$

$$\therefore \text{Probability that event F does not occur} = P(F') = 1 - P(F) = 1 - \frac{5}{13} = \frac{8}{13} \quad \text{Ans. : (B)}$$

- (52) A box contains 3 white, 3 red and 4 black balls. Two balls are selected at random without replacement. The probability of selecting at least one black ball is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{2}{15}$ (D) $\frac{13}{15}$

Sol. : 3 white + 3 red + 4 black = 10 balls are there in the box.

$\therefore$ Event B_1 : First ball selected is not a black ball

Event B_2 : Second ball selected is not a black ball

$$P(B_1) = \frac{\binom{6}{1}}{\binom{10}{1}} = \frac{6}{10}, \quad P(B_2|B_1) = \frac{\binom{5}{1}}{\binom{9}{1}} = \frac{5}{9}$$

$\therefore$ Probability that none of the balls selected is a black ball.

$$P(B_1 \cap B_2) = P(B_1) \cdot P(B_2|B_1) = \frac{6}{10} \cdot \frac{5}{9} = \frac{1}{3}$$

$$\therefore \text{Probability that at least one ball of black colour is selected} = 1 - P(B_1 \cap B_2) = 1 - \frac{1}{3} = \frac{2}{3}$$

Ans. : (B)

Second method :

Required probability = One or two black colour balls are selected.

$$= \frac{\binom{4}{1}\binom{6}{1}}{\binom{10}{2}} + \frac{\binom{4}{2}}{\binom{10}{2}}$$

$$= \frac{24}{45} + \frac{6}{45} = \frac{30}{45} = \frac{2}{3}$$

Ans. : (B)

- (53) The probability of getting integers 1, 2, 3, 4, 5, 6 on the top face of unbalanced die are given as

Digits	1	2	3	4	5	6
Probability	0.15	0.05	0.25	0.15	0.25	0.15

‘If the digit on top face of a die is 3 or 4’ is given, then probability of the event that digit 3 occurs on the die is

- (A) $\frac{5}{8}$ (B) $\frac{3}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Sol. : Event A = The digit occurring on the die is 3, B = The digit occurring on the die is 4.

$$P(A) = 0.25, \quad P(B) = 0.15$$

$$\therefore P(A \cup B) = P(A) + P(B) = 0.25 + 0.15 = 0.40$$

$$P(A|A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$$

$$= \frac{P(A)}{P(A \cup B)} = \frac{0.25}{0.40} = \frac{5}{8}$$

Ans. : (A)

- (54) A box contains five white and five black balls. Two balls are selected in two attempts at random (1) without replacement (2) with replacement. The probability that in the selection at the first attempt, both the balls are white and in the selection at the second attempt both the balls are black is and

- (A) $\frac{5}{63}, \frac{4}{81}$ (B) $\frac{4}{81}, \frac{5}{63}$ (C) $\frac{4}{63}, \frac{5}{81}$ (D) $\frac{5}{81}, \frac{4}{63}$

Sol. : Without replacement :

Event A : Both the balls selected are white

Event B : Both the balls selected are black

Consider experiment without replacement.

$$\text{For first event } P(A) = \frac{\binom{5}{2}}{\binom{10}{2}} = \frac{5 \times 4}{10 \times 9} = \frac{2}{9}, \text{ for the second event } P(B|A) = \frac{\binom{5}{2}}{\binom{8}{2}} = \frac{5 \times 4}{8 \times 7} = \frac{5}{14}$$

$\therefore$ Probability of the required event,

$$P(A \cap B) = \frac{2}{9} \cdot \frac{5}{14} = \frac{5}{63}$$

(At this stage, we can say that A is the correct answer.)

With replacement :

A and B are independent events.

$$P(A \cap B) = P(A) \cdot P(B)$$

$$= \frac{\binom{5}{2}}{\binom{10}{2}} \cdot \frac{\binom{5}{2}}{\binom{10}{2}} = \frac{2}{9} \cdot \frac{2}{9} = \frac{4}{81}$$

Ans. : (A)

- (55) There are two types of treatment, ayurvedic (A) and homeopathic (H) available for certain disease. The probability of the recovery from the disease by ayurvedic treatment is $\frac{1}{2}$ and that from by homeopathic treatment is $\frac{5}{8}$. (i) What is the probability that the patient is cured from the disease ? (ii) Given that if the patient is cured, what is the probability that he opted for ayurvedic treatment ?

(A) $\frac{9}{16}, \frac{4}{9}$

(B) $\frac{2}{3}, \frac{1}{3}$

(C) $\frac{3}{4}, \frac{2}{3}$

(D) $1, \frac{1}{2}$

Sol. : A : The patient has opted for ayurvedic treatment. $P(A) = \frac{1}{2}$

B : The patient has opted for homeopathic treatment. $P(B) = \frac{1}{2}$

C : The patient is cured. $P(C|A) = \frac{1}{2}$, $P(C|B) = \frac{5}{8}$

$$P(C) = P(A) \cdot P(C|A) + P(B)P(C|B) = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{5}{8} = \frac{9}{16}$$

(So it is clear that answer is A.)

Now, given that the patient is cured. Probability that patient has chosen ayurvedic treatment is required to be known.

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{P(A)P(C|A)}{P(C)} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{9}{16}} = \frac{16}{9 \times 4} = \frac{4}{9}$$

Ans. : (A)

- (56) If $P(A \cup B) = P(A)$, then $P(A|B) = \dots\dots$

(A) $\frac{P(A)}{P(B)}$

(B) 1

(C) $P(A \cap B)$

(D) $P(A)$

Sol. : $P(A) + P(B) - P(A \cap B) = P(A)$

$$\therefore P(A \cap B) = P(B)$$

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

Ans. : (B)

- (57) If A and B are independent events, $P(A \cup B) = 0.5$ and $P(A) = 0.2$, then $P(B) = \dots\dots$

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{3}{8}$

(D) $\frac{2}{5}$

Sol. : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$$

$$\therefore 0.5 = 0.2 + P(B) - 0.2P(B)$$

$$\therefore 0.3 = 0.8 P(B)$$

$$\therefore P(B) = \frac{3}{8}$$

Ans. : (C)

(58) Eight coins are tossed once. The probability of getting heads at least six times is

- (A) $\frac{65}{256}$ (B) $\frac{63}{256}$ (C) $\frac{37}{128}$ (D) $\frac{37}{256}$

Sol. : Probability of getting head $p = \frac{1}{2}$. So, $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

Here, eight coins are tossed. So, $n = 8$

$$p(x) = \binom{8}{x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{8-x}$$

$$= \binom{8}{x} \left(\frac{1}{2}\right)^8$$

$$P(X \geq 6) = p(6) + p(7) + p(8) = \left[\binom{8}{6} + \binom{8}{7} + \binom{8}{8} \right] \left(\frac{1}{2}\right)^8 = \frac{(28+8+1)}{256} = \frac{37}{256} \quad \text{Ans. : (D)}$$

(59) Distribution of a random variable is as follows :

X = x	0	1	2	3
p(x)	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

Variance of $g(X) = 2X + 3$ is

- (A) 6 (B) 36 (C) 4 (D) 8

Sol. : $E(X) = \sum x_i p(x_i)$

$$= 0 \cdot \frac{1}{4} + \frac{1}{8} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{8} = \frac{3}{2}$$

$$E(X^2) = \sum x_i^2 p(x_i)$$

$$= 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{8} + 4 \cdot \frac{1}{2} + 9 \cdot \frac{1}{8} = \frac{13}{4}$$

$$V(X) = E(X^2) - (E(X))^2 = \frac{13}{4} - \left(\frac{3}{2}\right)^2 = 1$$

$\therefore$ Variance of $g(X) = 2X + 3$ is $2^2 V(X) = 4V(X) = 4(1) = 4$ **Ans. : (C)**

(60) $p(x) = \binom{6}{x} p^x \cdot q^{6-x}$, $x = 0, 1, 2, \dots, 6$ is a binomial distribution. If $3p(2) = 2p(3)$, then $p = \dots$

- (A) $\frac{1}{17}$ (B) $\frac{8}{9}$ (C) $\frac{9}{17}$ (D) $\frac{3}{4}$

Sol. : Here, $3p(2) = 2p(3)$

$$\therefore 3 \binom{6}{2} \cdot p^2 \cdot q^4 = 2 \binom{6}{3} p^3 \cdot q^3$$

$$\therefore 3 \cdot 15 p^2 q^4 = 2 \cdot 20 p^3 q^3$$

$$\therefore 9q = 8p$$

$$\therefore 9(1 - p) = 8p$$

$$\therefore p = \frac{9}{17}$$

Ans. : (C)

- (61) The probability distribution of a random variable X is given as

$X = x$	-2	-1	0	1	2
$p(x)$	0.2	0.1	0.3	0.3	0.1

then $E(X) = \dots$ and $E(3X + 2) = \dots$ $V(3X + 2) = \dots$

- (A) 0, 2, 14.4 (B) 0, 0, 4 (C) 0, 2, 4.8 (D) 2, 2, 14.4

Sol. : $E(X) = \sum x_i p(x_i)$
 $= -2 \cdot (0.2) + (-1) \cdot (0.1) + 0 \cdot (0.3) + 1 \cdot (0.3) + 2 \cdot (0.1)$
 $= -0.4 - 0.1 + 0 + 0.3 + 0.2 = 0$

$E(3X + 2) = 3E(X) + 2 = 3(0) + 2 = 2$

$E(X^2) = \sum x_i^2 p(x_i)$
 $= 4(0.2) + 1(0.1) + 0(0.3) + 1 \cdot (0.3) + 4 \cdot (0.1)$
 $= 0.8 + 0.1 + 0 + 0.3 + 0.4 = 1.6$

$V(X) = E(X^2) - [E(X)]^2 = 1.6 - 0 = 1.6$

$\therefore V(3X + 2) = 3^2 V(X) = 9(1.6) = 14.4$

Ans. : (A)

- (62) A balanced coin is tossed n times. The probability of getting head (H) at least once is

- (A) $1 - \left(\frac{1}{2}\right)^n$ (B) $1 - \left(\frac{1}{6}\right)^n$ (C) $1 - \left(\frac{5}{6}\right)^n$ (D) $\frac{6^n - 5^n}{7}$

Sol. : A coin is tossed n times. The number of elements of sample space $= 2^n$

Probability of the event A that head does not occur in any attempt is

$P(A) = \frac{1}{2^n}$ ($A = \{TTT \dots T\}$)

Probability that head occurs at least once $= 1 - \frac{1}{2^n}$

Ans. : (A)

- (63) Complex number ω is the cube root of 1, $\omega \neq 1$. A balanced die is tossed thrice, r_1, r_2 and r_3 , are the digits on the die occurring respectively. The probability of the event associated with $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

- (A) $\frac{1}{36}$ (B) $\frac{1}{216}$ (C) $\frac{1}{6}$ (D) $\frac{5}{6}$

Sol. : A balanced die is tossed thrice,

$n = 6 \times 6 \times 6 = 216$

Event $A = \{(r_1, r_2, r_3) \mid r_1, r_2, r_3 = 1, 2, 3, 4, 5, 6, \omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0, \omega \neq 1\}$

(1, 2, 3) (2, 3, 4), (3, 4, 5), (4, 5, 6) (1, 3, 5), (2, 4, 6) are r_1, r_2, r_3 in some order.

$r = 6 \cdot 3! = 6 \cdot 6 = 36$

$\therefore P(A) = \frac{36}{216} = \frac{1}{6}$

Ans. : (C)

- (64) The number of elements in sets A and B are 4 and 2 respectively. A subset of $A \times B$ is selected at random. The probability that the selected subset contains at least three elements is

- (A) $\frac{219}{256}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{37}{256}$

Sol. : The number of elements in set A = 4 and the number of elements in set B = 2.

$\therefore$ The number of elements in $A \times B = 4 \times 2 = 8$

$\therefore$ The number of subsets of $A \times B = 2^8 = 256$

A subset which does not contain any element is $\emptyset$.

The number of subsets which contain only one element = $\binom{8}{1} = 8$

The number of subsets which contain exactly 2 elements = $\binom{8}{2} = \frac{8 \times 7}{2} = 28$

$\therefore$ The number of subsets which contain at least three elements = $256 - 1 - 8 - 28$
 $= 256 - 37 = 219$

$\therefore$ The probability of the required event = $\frac{219}{256}$ **Ans. : (A)**

- (65) A problem in mathematics is given to three students who can solve it with probabilities $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$ respectively. The probability that the problem is solved is

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

Sol. : $P(A) = \frac{1}{2}$ $P(B) = \frac{1}{3}$ $P(C) = \frac{1}{4}$

Event A : First student cannot solve the problem. So, $P(A') = 1 - P(A) = \frac{3}{4}$

Event B : Second student cannot solve the problem. So, $P(B') = 1 - P(B) = \frac{4}{5}$

Event C : Third student cannot solve the problem. So, $P(C') = 1 - P(C) = \frac{5}{6}$

Probability that the problem is solved,

$$\begin{aligned} P(A \cup B \cup C) &= 1 - P((A \cup B \cup C)') \\ &= 1 - P(A' \cap B' \cap C') \\ &= 1 - P(A') \cdot P(B') \cdot P(C') = 1 - \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6} = 1 - \frac{1}{2} = \frac{1}{2} \end{aligned} \quad \text{Ans. : (B)}$$

- (66) A balanced die is tossed five times in a random experiment. A success is getting 'an odd digit' on the top face of the die. The variance of the number of successes is

(A) $\frac{8}{3}$ (B) $\frac{3}{8}$ (C) $\frac{4}{5}$ (D) $\frac{5}{4}$

Sol. : $n = 5$

$\{1, 3, 5\}$ is the event of getting an odd digit on the top face of the die.

$\therefore$ Probability of success $p = \frac{3}{6} = \frac{1}{2}$. Probability of failure $q = 1 - \frac{1}{2} = \frac{1}{2}$

Variance = $npq = 5 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{4}$ **Ans. : (D)**

- (67) If the mean and the variance of a binomial distribution of random variable X are 4 and 2 respectively, then $P(X = 1) = \dots$

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{32}$

Sol. : The mean = $np = 4$. The variance = $npq = 2$

$$\therefore \frac{npq}{np} = \frac{2}{4} \Rightarrow q = \frac{1}{2}. \text{ So, } p = \frac{1}{2}$$

$$np = 4 \Rightarrow n \cdot \frac{1}{2} = 4 \Rightarrow n = 8$$

$$\begin{aligned} \text{The probability distribution of variable X} = p(x) &= \binom{n}{x} p^x \cdot q^{n-x} = \binom{8}{x} \left(\frac{1}{2}\right)^x \cdot \left(\frac{1}{2}\right)^{8-x} \\ &= \binom{8}{x} \left(\frac{1}{2}\right)^8 = \binom{8}{x} \frac{1}{256} \end{aligned}$$

$$\text{Now, } p(1) = \binom{8}{1} \frac{1}{256} = \frac{1}{32}$$

Ans. : (D)

- (68) The probability that husband does not speak truth is $\frac{1}{5}$ and wife does not speak truth is $\frac{1}{4}$.
The probability that they contradict each other is

(A) $\frac{4}{5}$ (B) $\frac{1}{5}$ (C) $\frac{7}{20}$ (D) $\frac{3}{20}$

Sol. : Here, $P(A) = \frac{1}{5}$, $P(A') = \frac{4}{5}$. $P(B) = \frac{1}{4}$, $P(B') = \frac{3}{4}$

$$\begin{aligned}\text{The probability of required event} &= P(A \cap B') + P(A' \cap B) = P(A) \cdot P(B') + P(A') \cdot P(B) \\ &= \frac{1}{5} \cdot \frac{3}{4} + \frac{4}{5} \cdot \frac{1}{4} = \frac{7}{20}\end{aligned}$$

Ans. : (C)

- (69) The probability distribution of random variable X is given as

$X = x$	1	2	3	4	5	6	7	8
$p(x)$	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

If the event $E = \{X \text{ is prime}\}$ and $F = \{X < 4\}$, then the probability of the event $E \cup F$ is

(A) 0.50 (B) 0.35 (C) 0.77 (D) 0.89

Sol. : $E = \{2, 3, 5, 7\}$ and $F = \{1, 2, 3\}$

$$E \cup F = \{1, 2, 3, 5, 7\}$$

$$\begin{aligned}P(E \cup F) &= p(1) + p(2) + p(3) + p(5) + p(7) \\ &= 0.15 + 0.23 + 0.12 + 0.20 + 0.07 = 0.77\end{aligned}$$

Ans. : (C)

- (70) If the mean and the variance of a binomial distribution of a random variable are 4 and 2 respectively, then $p(2) = \dots$

(A) $\frac{8}{64}$ (B) $\frac{7}{64}$ (C) $\frac{57}{64}$ (D) $\frac{37}{256}$

Sol. : Here, $np = 4$, $npq = 2$. So, $n = 8$, $p = \frac{1}{2}$, $q = \frac{1}{2}$

$$p(x) = \binom{n}{x} p^x \cdot q^{n-x}$$

$$p(2) = \binom{8}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^6 = \binom{8}{2} \left(\frac{1}{2}\right)^8 = \frac{8 \times 7}{2} \times \frac{1}{2^8} = \frac{7}{2^6} = \frac{7}{64}$$

Ans. : (B)

- (71) If for events A and B, $P((A \cup B)') = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$, $P(A') = \frac{1}{4}$, then the events A and B are (JEE 2014)

(A) equally likely and independent (B) equally likely but not independent
(C) independent but not equally likely (D) mutually exclusive and independent

Sol. : $P((A \cup B)') = \frac{1}{6}$. So, $P(A \cup B) = 1 - \frac{1}{6} = \frac{5}{6}$

$$\therefore P(A') = \frac{1}{4}. \text{ So, } P(A) = 1 - P(A') = \frac{3}{4}. \text{ Also, } P(A \cap B) = \frac{1}{4}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$$

$$\therefore P(B) = \frac{1}{3}$$

$$P(A) = \frac{3}{4} \text{ and } P(B) = \frac{1}{3}.$$

∴ A and B are not equally likely.

$$P(A \cap B) = \frac{1}{4}. \text{ Also, } P(A) \cdot P(B) = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$$

$$\therefore P(A \cap B) = P(A) \cdot P(B)$$

∴ A and B are independent events.

Ans. : (C)

(72) If for events A and B, $P(A) = \frac{1}{4}$, $P(A|B) = \frac{1}{2}$, $P(B|A) = \frac{2}{3}$, then $P(B) = \dots$

- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$

$$\text{Sol. : } P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$\therefore P(B) \cdot P(A|B) = P(A) \cdot P(B|A). \text{ So, } P(B) \left(\frac{1}{2}\right) = \frac{1}{4} \cdot \frac{2}{3}$$

$$\therefore P(B) = \frac{1}{3}$$

Ans. : (B)

(73) One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, given that the product of these digits is zero, is [JEE : 2009]

- (A) $\frac{1}{50}$ (B) $\frac{1}{40}$ (C) $\frac{1}{14}$ (D) $\frac{1}{49}$

Sol. : $A = \{00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40\}$
 $B = \{08, 17, 26, 35, 44\}$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1}{14}$$

Ans. : (C)

(74) Two numbers are selected at random from $U = \{1, 2, 3, 4, 5, 6\}$ in succession without replacement. The probability that the minimum of these two numbers is less than 4 is

- (A) $\frac{1}{15}$ (B) $\frac{14}{15}$ (C) $\frac{1}{5}$ (D) $\frac{4}{5}$

Sol. : Set $U = \{1, 2, 3, 4, 5, 6\}$

The total number of ways that the two numbers are selected in succession without replacement is

$$n(U) = 6 \times 5 = 30$$

A : The minimum of these two selected numbers is less than 4.

∴ A' : Both selected numbers are from $\{4, 5, 6\}$.

$$\therefore n(A') = 3 \times 2 = 6$$

$$\therefore n(A) = 30 - 6 = 24$$

$$\therefore P(A) = \frac{n(A)}{n(U)} = \frac{24}{30} = \frac{4}{5}$$

Ans. : (D)

(75) India plays two ODI matches each with England and Australia. For each win 2 points, for losing the match 0 point and for a tie in the match 1 point are allotted. The probability for India to get the points 0, 1, 2 are 0.45, 0.05 and 0.50 respectively. The probability that India gets at least 7 points in the matches is

- (A) 0.625 (B) 0.0250 (C) 0.0875 (D) None of these

Sol. : Here, 4 matches are played. India will get at least 0 and atmost 8 points.

Here 'India gets at least 7 points' means 'India gets 7 or 8 points'.

The number of the possible ways of getting 7 points in four matches is

1 + 2 + 2 + 2 or 2 + 1 + 2 + 2 or 2 + 2 + 1 + 2 or 2 + 2 + 2 + 1 in four matches respectively.

$$p(7) = 4 (0.050 \times 0.5 \times 0.5 \times 0.5) = 0.025$$

8 points are possible in only one way : 2 + 2 + 2 + 2 in each match.

$$p(8) = 0.5 \times 0.5 \times 0.5 \times 0.5 = 0.0625$$

∴ So, probability that India gets atleast 7 points in four matches is

$$p(7) + p(8) = 0.025 + 0.0625 = 0.0875$$

Ans. : (C)

- (76) Three numbers are chosen at random without replacement from {1, 2, 3, 4, 5, 6, 7, 8}. The probability that their minimum is 3, given that their maximum is 5 is

(A) $\frac{3}{8}$ (B) $\frac{1}{10}$ (C) $-\frac{1}{10}$ (D) $\frac{2}{10}$

Sol. : S = {1, 2, 3, ..., 8}

The maximum is 5. So we can select the number from A = {1, 2, 3, 4, 5}.

The minimum is 3. So, we can select the number from B = {3, 4, 5, 6, 7, 8}.

$$P(B | A) = \frac{\binom{3}{3}}{\binom{5}{3}} = \frac{4}{20} = \frac{1}{10}$$

Ans. : (B)

- (77) A pair of fair dice is thrown independently thrice. The probability of getting a score of exactly 9 twice is

(A) $\frac{8}{729}$ (B) $\frac{8}{243}$ (C) $\frac{1}{729}$ (D) $\frac{8}{9}$

Sol. : The event that sum of digits is 9 considered as success is

$$A = \{(3, 6), (4, 5), (5, 4), (6, 3)\}$$

$$\therefore \text{Probability of success } p = \frac{4}{36} = \frac{1}{9}. \text{ So } q = 1 - \frac{1}{9} = \frac{8}{9}$$

Here dice are tossed thrice.

$$\text{So, } n = 3$$

$$\therefore p(x) = \binom{n}{x} p^x q^{n-x}$$

$$\therefore p(2) = \binom{3}{2} \left(\frac{1}{9}\right)^2 \left(\frac{8}{9}\right)^1 = \binom{3}{2} \left(\frac{1}{9}\right)^2 \left(\frac{8}{9}\right)^1 = 3 \cdot \frac{1}{9 \times 9} \times \frac{8}{9} = \frac{8}{243}$$

Ans. : (B)

- (78) The probability that the two bomber planes I and II hit the target is 0.3 and 0.2 respectively. If plane I does not hit the target, then plane II will throw the bomb. The target is destroyed. The probability that plane II has hit the target is

(A) $\frac{8}{21}$ (B) $\frac{5}{21}$ (C) $\frac{7}{22}$ (D) $\frac{5}{22}$

Sol. : Event A : Plane I hits the target. Event B : Plane II hits the target.

$$P(A) = 0.3, P(B) = 0.2, P(A') = 0.7, P(B') = 0.8$$

Required probability

$$\begin{aligned} P(B) &= P(A') P(B) + P(A') \cdot P(B') \cdot P(A') \cdot P(B) + P(A') \cdot P(B') \cdot P(A') \cdot P(B') \cdot P(A') \cdot P(B) + \dots \infty \\ &= (0.7) \cdot (0.2) + (0.7) \cdot (0.8) \cdot (0.7) \cdot (0.2) + (0.7) (0.8) (0.7) (0.8) (0.7) (0.2) + \dots \infty \\ &= (0.7) (0.2) [1 + (0.7) (0.8) + (0.7)^2 (0.8)^2 + \dots] \\ &= (0.7) (0.2) \left[\frac{1}{1 - (0.7)(0.8)} \right] \end{aligned}$$

(infinite series)

$$= \frac{14}{100} \left(\frac{100}{100 - 56} \right) = \frac{14}{44} = \frac{7}{22}$$

Ans. : (C)

- (79) An urn contains 9 balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls are of different colours is

(A) $\frac{2}{7}$ (B) $\frac{1}{21}$ (C) $\frac{2}{23}$ (D) $\frac{1}{3}$

Sol. : $n(A) = \binom{3}{1}\binom{4}{1}\binom{2}{1} = 24.$

$$n(U) = \binom{9}{3} = \frac{9 \cdot 8 \cdot 7}{6} = 84$$

$$P(A) = \frac{24}{84} = \frac{2}{7}$$

Ans. : (A)

- (80) There are n urns. Each contains $n + 1$ balls. i th urn contains i white balls and $(n + 1 - i)$ red balls. The event u_i indicates i th urn is selected at the random where $i = 1, 2, \dots, n$ and one ball is selected from the selected urn. Event w is that selected ball is a white ball. Then

- (1) If $P(u_i) \propto i$ where $i = 1, 2, \dots, n$, then $\lim_{n \rightarrow \infty} P(w) = \dots$

(A) 1 (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{1}{4}$

Sol. : $P(u_i) \propto i$

$$\therefore P(u_i) = ki$$

$$\sum P(u_i) = 1. \text{ So, } \sum ki = 1$$

$$k \sum i = 1. \text{ So, } \frac{kn(n+1)}{2} = 1. \text{ So } k = \frac{2}{n(n+1)}$$

$$\text{Now, } P(w) = \sum P(u_i)P(w|u_i)$$

$$= \sum ki \cdot \frac{i}{n+1} = \sum \frac{2i}{n(n+1)} \cdot \frac{i}{n+1} = \sum \frac{2i^2}{n(n+1)^2}$$

$$\therefore \lim_{n \rightarrow \infty} P(w) = \lim_{n \rightarrow \infty} \sum \frac{2i^2}{n(n+1)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{2}{n(n+1)^2} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n \rightarrow \infty} \frac{(2n+1)}{3n+3} = \lim_{n \rightarrow \infty} \frac{2+\frac{1}{n}}{3+\frac{3}{n}} = \frac{2+0}{3+0} = \frac{2}{3}$$

Ans. : (B)

- (2) If $P(u_i) = c$, where c is constant and $i = 1, 2, 3, \dots, n$, then $P(u_n|w) = \dots$

(A) $\frac{2}{n+1}$ (B) $\frac{1}{n+1}$ (C) $\frac{n}{n+1}$ (D) $\frac{1}{2}$

Sol. : $P(u_n|w) = \frac{P(w|u_n)P(u_n)}{P(w)}$

$$= \frac{c \binom{n}{n+1}}{c \left(\sum \frac{i}{n+1} \right)} = \frac{cn}{n+1} \times \frac{2(n+1)}{cn(n+1)} = \frac{2}{n+1} \quad (P(w) = \frac{c}{n+1} + \frac{2c}{n+1} + \frac{3c}{n+1} + \dots + \frac{nc}{n+1})$$

Ans. : (A)

- (81) There are three boxes I_1, I_2 and I_3 . In box (I_1) there are three cards marked with 1, 2 and 3. In box (I_2) there are five cards marked with 1, 2, 3, 4 and 5. In box (I_3) there are seven cards marked with 1, 2, 3, 4, 5, 6 and 7. A card is selected from each box. If x_i is the event that card selected from i th box is marked x_i ($i = 1, 2, 3$) then

(1) The probability that $x_1 + x_2 + x_3$ is odd.

- (A) $\frac{29}{105}$ (B) $\frac{53}{105}$ (C) $\frac{57}{105}$ (D) $\frac{1}{2}$

(2) The probability that x_1, x_2, x_3 are in A.P.

- (A) $\frac{9}{105}$ (B) $\frac{10}{105}$ (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

Sol. : (1) (i) There are two possibilities for $x_1 + x_2 + x_3$ to be odd.

(i) x_1, x_2 and x_3 are all odd.

(ii) Any two of them are even and one is odd.

(i) If x_1, x_2, x_3 are odd, then the total number of possibilities = $2 \times 3 \times 4 = 24$

(ii) Two even and one odd (Even from box I_1 , even from box I_2 , odd from box I_3 in this way)
 $1 \cdot 2 \cdot 4 + 1 \cdot 3 \cdot 3 + 2 \cdot 2 \cdot 3 = 8 + 9 + 12 = 29$ ways to select cards.

The total number of possibilities that $x_1 + x_2 + x_3$ is odd = $r = 24 + 29 = 53$

One card is selected from each box. So total number of possibilities

$$n = 3 \times 5 \times 7 = 105$$

Ans. : (B)

$\therefore$ The probability of the required event = $\frac{r}{n} = \frac{53}{105}$

(2) The probability that x_1, x_2, x_3 are in A.P.

B = $\{(1, 2, 3), (1, 3, 5), (1, 4, 7), (2, 3, 4), (2, 4, 6), (3, 4, 5), (3, 5, 7), (3, 2, 1),$
 $(3, 3, 3), (1, 1, 1) (2, 2, 2)\}$

$$P(B) = \frac{r}{n} = \frac{11}{105}$$

Ans. : (C)

- (82) A pair of dice is rolled till the sum of digits obtained on the dice is either 4 or 6. The probability that 6 appears first is

- (A) $\frac{3}{8}$ (B) $\frac{5}{8}$ (C) $\frac{7}{8}$ (D) $\frac{1}{8}$

Sol. : Let E_n be the event that the sum of the digits on the top faces of the dice is six at the n th trial.

Thus for trials 1, 2, 3, ..., $n - 1$ the sum of the digits is neither 4 nor 6.

The sum 4 occurs for the event $\{(1, 3), (2, 2), (3, 1)\}$.

The sum 6 occurs for the event $\{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$.

Also $n(U) = 36$.

The probability of getting sum 4 or 6 on the top faces of the dice is = $\frac{8}{36} = \frac{2}{9}$.

$$\therefore P(E_n) = \left(1 - \frac{2}{9}\right)^{n-1} \frac{2}{9} = \left(\frac{7}{9}\right)^{n-1} \times \frac{2}{9}$$

$\therefore$ Probability of getting 6 first.

$$\begin{aligned} P(E_1 \cup E_2 \cup \dots) &= P(E_1) + P(E_2) + \dots \infty \\ &= \frac{2}{9} + \frac{7}{9} \times \frac{2}{9} + \left(\frac{7}{9}\right)^2 \frac{2}{9} + \dots \infty \end{aligned}$$

$$= \frac{2}{9} \left(\frac{1}{1 - \frac{7}{9}} \right) = \frac{2}{9} \times \frac{9}{2} = 1 \quad \text{(Infinite G.P., } a + ar + ar^2 + \dots = \frac{a}{1-r}, |r| < 1) \quad \text{Ans. : (B)}$$

(83) A traveller puts his luggage in the locker of a railway station. The locker can be opened by a three digit code. One unknown person tries to open the locker,

(1) Probability that the locker is opened at the first trial is

(2) Probability that the locker is opened at the $(k + 1)$ th trial is ($k < 1000$)

(A) (1) 0.001 (2) $\frac{k}{100}$ (B) (1) 0.01 (2) $\frac{k}{1000}$ (C) (1) 0.001 (2) $\frac{k}{1000}$ (D) (1) 1 (2) $\frac{k}{10000}$

Sol. : (1) Suppose event A : locker can be opened at first attempt. There are 0 to 9 digits in the three digit code.

$$\therefore P(A) = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = 0.001$$

(2) Suppose event B : locker can be opened at the $(k + 1)$ th trial

B' : He cannot open the locker in first k trials.

$$\therefore P(B) = 1 - P(B')$$

$$= 1 - \frac{999}{1000} \cdot \frac{998}{999} \cdots \frac{1000 - (k-1)}{1000 - (k-2)} \cdot \frac{1000 - k}{1000 - (k-1)}$$

$$= 1 - \frac{1000 - k}{1000} = \frac{k}{1000}$$

Ans. : (C)

(84) A communication system has n parts. The probability that each part works independently is k . The whole system is functional only when more than half of its parts are in working order. For which k does a communication system with 5 parts works better than a communication system with 3 parts ?

(A) $< \frac{1}{2}$

(B) $> \frac{1}{2}$

(C) $> \frac{4}{5}$

(D) $> \frac{3}{4}$

Sol. : Here, communication system is a binomial distribution with parameters n, p . Probability that the system with 5 parts works is ${}_5C_3 k^3 (1 - k)^2 + {}_5C_4 k^4 (1 - k) + k^5$.

and probability that system with 3 parts works is ${}_3C_2 k^2 (1 - k) + k^3$

If communication system with five parts works better, than the system with 3 parts is

$$10k^3(1 - k)^2 + 5k^4(1 - k) + k^5 > 3k^2(1 - k) + k^3$$

$$\therefore 10k(1 - k)^2 + 5k^2(1 - k) + k^3 > 3(1 - k) + k$$

$$\therefore 10k(1 - k)^2 + 5k^2(1 - k) - 3(1 - k) - k(1 - k)(1 + k) > 0$$

$$\therefore (1 - k)[10k - 10k^2 + 5k^2 - 3 - k - k^2] > 0$$

$$\therefore (1 - k)(-6k^2 + 9k - 3) > 0$$

$$\therefore -3(1 - k)(2k^2 - 3k + 1) > 0$$

$$\therefore 3(k - 1)^2(2k - 1) > 0$$

$$\therefore k > \frac{1}{2}$$

Ans. : (B)

: Assertion and Reason Type Questions :

Explanation : Each of these questions contains statement 1 (Assertion) and statement 2 (Reason). Each question has four choices, option (A), (B), (C) and (D), which are as follows. You have to select the correct choice.

(A) Statement 1 is true, statement 2 is true. Statement 2 is the correct explanation for statement 1.

(B) Statement 1 is true, statement 2 is true. Statement 2 is not a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

(85) **Statement 1** : Suppose $A_i (i = 1, 2, \dots, n)$ are n independent events where

$$P(A'_i) = \frac{i}{i+1}, 1 \leq i \leq n. \text{ Probability that none of the events occur is } \frac{n}{n+1}.$$

Statement 2 : If $A_1, A_2, \dots, A_n$ are independent events, then

$$P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1) \cdot P(A_2) \dots P(A_n)$$

Sol. : $P(\text{None of the event occurs})$

$$= P(A'_1 \cap A'_2 \cap \dots \cap A'_n)$$

$$= P(A'_1) P(A'_2) \dots P(A'_n)$$

(If $A_i (i = 1, 2, \dots, n)$ are independent events, then A'_i are also independent events.)

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \dots \times \frac{n-1}{n} \times \frac{n}{n+1} = \frac{1}{n+1}$$

$\therefore$ So, statement 1 is false and statement 2 is true.

Ans. : (D)

(86) At a bus-stop there are 10 passengers who can occupy a seat in any of three buses. (There are no passengers sitting in the bus earlier.) The probability that there is at least one passenger in every bus is

(A) $\frac{{}^{10}P_3 \cdot 3^7}{3^{10}}$

(B) $1 - \frac{{}^{10}C_3 \cdot 3^7}{3^{10}}$

(C) $1 - \frac{2^{10}}{3^{10}}$

(D) $\frac{3^9 - 2^{10} + 1}{3^9}$

Sol. : 10 passengers can occupy a seat in 3 buses in 3^{10} ways.

There is at least one passenger in every bus.

If all persons sit in a bus, they can choose the bus in $\binom{3}{1}$ ways and all will take a seat in it.

Two buses can be selected in $\binom{3}{2}$ ways. All persons can occupy a seat in these two buses in 2^{10} ways.

But there is at least one passenger in each of the two buses. So, all persons can sit in two buses in $\binom{3}{2} (2^{10} - 2)$ ways as required.

$$\therefore \text{ The number of ways as per given condition } = 3^{10} - \binom{3}{1} - \binom{3}{2} (2^{10} - 2)$$

$$\therefore \text{ Required probability } = \frac{3^{10} - 3 - 3(2^{10} - 2)}{3^{10}}$$

$$= \frac{3^9 - 2^{10} + 1}{3^9}$$

Ans. : (D)

(87) Two randomly selected numbers are multiplied. The probability that the digit at their unit place is even is p and the probability that the odd digit occurs at unit place is q , then $\frac{10p}{q} = \dots$

(A) 10

(B) 20

(C) 30

(D) $\frac{1}{30}$

Sol. : We know that the product of two odd numbers is odd. In all other possibilities, we get an even number.

$$\therefore p = \frac{3}{4} \text{ and } q = \frac{1}{4}$$

$$\therefore \frac{10p}{q} = 30$$

Ans. : (C)

- (88) One letter is selected from the words MATHEMATICS and STATISTICS. If the probability that a common letter is selected is $\frac{a}{b}$, then $a + b + 3 = \dots$ (a, b are coprime.)
- (A) 50 (B) 62 (C) 60 (D) 65

Sol. : There are 11 letters in MATHEMATICS.

$\therefore$ One letter can be selected at random with probability = $\frac{1}{11}$.

There are 10 letters in the word STATISTICS.

$\therefore$ Probability of selecting a letter from them is $\frac{1}{10}$. In both the words A, T, I, C, S are common letters.

$\therefore$ Probability of selecting common letters

$$= \frac{2}{11} \times \frac{1}{10} + \frac{2}{11} \times \frac{3}{10} + \frac{1}{11} \times \frac{2}{10} + \frac{1}{11} \times \frac{1}{10} + \frac{1}{11} \times \frac{3}{10} = \frac{7}{55} = \frac{a}{b}. \text{ So, } a = 7, b = 55$$

$$\therefore a + b + 3 = 7 + 55 + 3 = 65$$

Ans. : (D)

- (89) In a multiple choice questions test, one option out of four is correct. Such type of questions are given as a homework to students. Students give the answers by guess work or by helping each other or by their own knowledge. The probability of giving answers by guess work is $\frac{1}{3}$. The probability of getting answers by helping each other is $\frac{1}{6}$. Probability of getting correct answer when he had taken help from friends is $\frac{1}{8}$. If a student has given correct answer, then the probability that he knows the correct answer is $\dots$.

- (A) $\frac{29}{35}$ (B) $\frac{24}{29}$ (C) $\frac{1}{7}$ (D) $\frac{1}{9}$

Sol. : Suppose Event A : The answer given by the student is by guess work.

Event B : The answer obtained is by each other's help.

Event C : The student knows the answer.

Event D : The student gives the correct answer.

$$P(A) = \frac{1}{3}, P(B) = \frac{1}{6}, P(C) = \frac{1}{2}$$

$$P(D|B) = \frac{1}{8}, P(D|A) = \frac{1}{4} \text{ (By assuming one of four to be correct), } P(D|C) = 1$$

(gives correct answers)

$$\text{Required probability } P(C|D) = \frac{P(C) P(D|C)}{P(A) P(D|A) + P(B) P(D|B) + P(C) P(D|C)}$$

$$= \frac{\frac{1}{2}}{\frac{1}{3} \cdot \frac{1}{4} + \frac{1}{6} \cdot \frac{1}{8} + \frac{1}{2} \cdot 1} = \frac{24}{29}$$

Ans. : (B)

- (90) Find the probability that the birthdays of 6 persons fall in two particular calendar months is
- (A) $\frac{341}{12^5}$ (B) $\frac{66}{12^5}$ (C) $\frac{352}{12^5}$ (D) $\frac{11}{12^5}$

Sol. : The number of ways of birthday coming in any month = 12^6

The number of ways of selection of two particular months = ${}_{12}C_2$

The number of ways of birthdays of 6 persons coming in these two months = 2^6 .

Out of these 2^6 ways, there are two ways, when all the 6 persons will have same month of birth.

$\therefore$ The number of ways birthday comes in two different months $= 2^6 - 2$.

$$\therefore \text{ Required probability} = \frac{{}^{12}C_2(2^6-2)}{12^6} = \frac{341}{12^5} \quad \text{Ans. : (A)}$$

- (91) An urn contains 10 white and 3 black balls. Balls are selected at random one by one till we exhaust all the black balls without replacement. The probability that this process is complete at the seventh trial is

(A) $\frac{5}{286}$ (B) $\frac{3}{286}$ (C) $\frac{7}{286}$ (D) $\frac{15}{286}$

Sol. : Required probability

= Probability of getting two black and four white balls at the first six attempts $\times$ probability of getting a black ball at seventh trial.

$$= \frac{\binom{3}{2}\binom{10}{4}}{\binom{13}{6}} \times \frac{1}{7} = \frac{3 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 720}{24 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8} \times \frac{1}{7} = \frac{15}{286} \quad \text{Ans. : (D)}$$

- (92) A box contains 3 white and 3 black balls. Three balls are selected at random and then three red balls are inserted in the box. Again three balls are selected. The probability that all the different coloured balls are selected is

(A) $> 20\%$ (B) $> 25\%$ (C) $> 30\%$ (D) $> 33\%$

Sol. : If three white balls or three black balls are selected at the first attempt, then it is not possible to get all different colour balls at the second attempt.

So two white and one black or one white and two black balls can be selected at the first attempt.

A : Two white balls, one black ball selected

B : Two black balls, one white ball selected

C : Three different colour balls selected.

$$\begin{aligned} P(A) = P(B) &= \frac{\binom{3}{2}\binom{3}{1}}{\binom{6}{3}} \\ P(C|A) = P(C|B) &= \frac{\binom{1}{1} \times \binom{2}{1} \times \binom{3}{1}}{\binom{6}{3}} \\ \therefore P(C) &= P(A)P(C|A) + P(B)P(C|B) \end{aligned} \quad \left| \quad \begin{aligned} &= 2 \left[\frac{\binom{3}{2}\binom{3}{1}}{\binom{6}{3}} \times \frac{1 \times 2 \times 3}{\binom{6}{3}} \right] \\ &= \frac{2 \times 3 \times 3 \times 6}{20 \times 20} \\ &= \frac{27}{100} = 27\% \end{aligned} \quad \text{Ans. : (A), (B)}$$

- (93) A person parks his car where already $(n-1)$ cars are parked. He does not park at either end. When he returns, he finds that m places out of n places are still occupied. The probability that cars parked in his neighbouring places have left is

(A) $\frac{n-m}{(n-1)!}$ (B) $\frac{n-m!}{(n-1)!}$ (C) $\frac{(n-m)(n-m-1)}{(n-1)(n-2)}$ (D) $\frac{(n-m)(n-m+1)}{(n-1)(n-2)}$

Sol. : Suppose in all m cars are parked. So, excluding one car (owner's car) $m-1$ cars are parked in $(n-1)$ places.

$$\therefore n(U) = \binom{n-1}{m-1}.$$

If two neighbouring places are empty $(m - 1)$ cars are left from $(n - 3)$ places.

$$\therefore n(A) = n - 3$$

$$\therefore P(A) = \frac{n(A)}{n(U)} = \frac{\binom{n-3}{m-1}}{\binom{n-1}{m-1}}$$

$$= \frac{(n-3)!}{(m-1)!(n-m-2)!} \cdot \frac{(m-1)!(n-m)!}{(n-1)!}$$

$$= \frac{(n-m)(n-m-1)}{(n-1)(n-2)}$$

Ans. : (C)

- (94) Martin, Iqbal and Deepak roll a die one by one. Martin starts the game, then Iqbal continues and then Deepak continues the experiment. The probability that Deepak gets digit 6 on the top face of the die first is

(A) $\frac{1}{3}$

(B) $\frac{2}{9}$

(C) $\frac{25}{91}$

(D) $\frac{5}{18}$

Sol. : Probability of getting 6 on the dice at the k th trial first = $\left(\frac{5}{6}\right)^{k-1} \cdot \left(\frac{1}{6}\right)$

Deepak can get success at the third trial or at the sixth trial or at the ninth trial in the successive attempts.

$$\therefore \text{Required probability} = \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \dots + \left(\frac{5}{6}\right)^{3n-1} \cdot \frac{1}{6} + \dots \infty$$

In this infinite G.P., first term $a = \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}$; common ratio $r = \left(\frac{5}{6}\right)^3$

$$\therefore \text{Required probability} = \frac{a}{1-r} = \frac{\frac{5^2}{6^3}}{1 - \frac{5^3}{6^3}} = \frac{25}{91}$$

Ans. : (C)

- (95) If the probability of at least one success in a binomial distribution $B\left(n, p = \frac{1}{4}\right)$ is greater than or equal to $\frac{9}{10}$, then n is greater than [AIEEE : 2009]

(A) $\frac{1}{\log_{10} 4 + \log_{10} 3}$

(B) $\frac{9}{\log_{10} 4 - \log_{10} 3}$

(C) $\frac{4}{\log_{10} 4 - \log_{10} 3}$

(D) $\frac{1}{\log_{10} 4 - \log_{10} 3}$

Sol. : Here, $q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$, $p = \frac{1}{4}$

$$\text{Now, } p(1) + p(2) + \dots + p(n) \geq \frac{9}{10}$$

$$\therefore 1 - p(0) \geq \frac{9}{10}$$

$$\therefore 1 - \frac{9}{10} \geq p(0)$$

$$\therefore p(0) \leq \frac{1}{10}$$

$$\therefore \binom{n}{0} \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^n \leq \frac{1}{10}$$

$$\therefore \left(\frac{3}{4}\right)^n \leq \frac{1}{10}$$

$$\therefore \left(\frac{4}{3}\right)^n \geq 10$$

$$\therefore \log_{10} \left(\frac{4}{3}\right)^n \geq \log_{10} 10$$

$$\therefore n[\log_{10} 4 - \log_{10} 3] \geq 1$$

$$\therefore n \geq \frac{1}{\log_{10} 4 - \log_{10} 3}$$

Ans. : (D)

- (96) The probability that there are 53 Sundays in a randomly selected year of 22nd century is
 (A) $\frac{3}{28}$ (B) $\frac{2}{28}$ (C) $\frac{7}{28}$ (D) $\frac{5}{28}$

Sol. : Suppose event B_1 : A leap year is selected from the 22nd century

B_2 : The selected year is not a leap year of 22nd century

A : 53 Sundays occur in the leap year

There are 25 leap years in 100 years.

$$P(B_1) = \frac{25}{100} = \frac{1}{4}, P(B_2) = \frac{75}{100} = \frac{3}{4},$$

There are 52 weeks and 2 days in leap years.

$\therefore U = \{(\text{Sunday, Monday}), (\text{Monday, Tuesday}), (\text{Tuesday, Wednesday}),$
 $(\text{Wednesday, Thursday}), (\text{Thursday, Friday}), (\text{Friday, Saturday}), (\text{Saturday, Sunday})\}$

$\therefore$ Probability of 53 Sundays in a leap year $P(A | B_1) = \frac{2}{7}$

There are 52 weeks and 1 day in a year which is not a leap year.

$\therefore U = \{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}$

$\therefore$ Probability of 53 Sundays in a year other than leap year $P(A | B_2) = \frac{1}{7}$

$\therefore$ Required probability $P(A) = P(B_1) \cdot P(A | B_1) + P(B_2) \cdot P(A | B_2)$

$$= \left(\frac{1}{4} \times \frac{2}{7}\right) + \left(\frac{3}{4} \times \frac{1}{7}\right) = \frac{5}{28}$$

Ans. : (D)

- (97) A random variable X takes the values 0, 1, 2, 3 and its mean is 1.3.

If $P(X = 3) = 2P(X = 1)$ and $P(X = 2) = 0.3$, then $P(X = 0) = \dots\dots$

- (A) 0.1 (B) 0.2 (C) 0.3 (D) 0.4

Sol. : Suppose, $P(X = 0) = a$ and $P(X = 1) = b$, then $P(X = 3) = 2b$

x_i	$p(x_i)$	$x_i p(x_i)$
0	a	0
1	b	b
2	0.3	0.6
3	$2b$	$6b$
Total	$a + 3b + 0.3$	$7b + 0.6$

$$\text{Mean } E(X) = \sum x_i p(x_i)$$

$$1.3 = 7b + 0.6. \text{ So, } 7b = 0.7. \text{ So, } b = 0.1$$

$$\text{Now, } \sum p(x_i) = 1$$

$$\therefore a + 3b + 0.3 = 1$$

$$\therefore a + 0.3 + 0.3 = 1. \text{ So, } a = 0.4$$

Ans. : (D)

- (98) Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2 respectively. The second plane will bomb only if first misses the target. The probability that the target is hit by the second plane is [AIEEE : 2007]

- (A) 0.14 (B) 0.2 (C) 0.7 (D) 0.06

Sol. : Let Event A : Plane I hits the target.

B : Plane II hits the target

Here, $P(A) = 0.3$ and $P(B | A') = 0.2$. So, $P(A') = 1 - 0.3 = 0.7$

Required probability $P(A' \cap B) = P(A') \cdot P(B | A') = 0.7 \times 0.2 = 0.14$

Ans. : (A)

- (99) If the sum of the mean and the variance of binomial distribution of 18 trials is 10, then binomial distribution is

(A) $\binom{18}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{18-x}$, $x = 0, 1, 2, \dots, 18$ (B) $\binom{18}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{18-x}$, $x = 0, 1, 2, \dots, 18$
 (C) $\binom{18}{x} \left(\frac{1}{2}\right)^{18}$, $x = 0, 1, 2, \dots, 18$ (D) $\binom{18}{x} \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{18-x}$, $x = 0, 1, 2, \dots, 18$

Sol. : Here, $n = 18$, and mean + variance = 10. So, $np + npq = 10$, $p + q = 1$

Now, $18p + 18p(1 - p) = 10$

$\therefore 18p + 18p - 18p^2 - 10 = 0$

$\therefore 18p^2 - 36p + 10 = 0$

$\therefore 9p^2 - 18p + 5 = 0$

$\therefore (3p - 5)(3p - 1) = 0$

$\therefore p = \frac{5}{3}$ (not possible) ($0 < p < 1$) or $p = \frac{1}{3}$. So, $q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

So $p(x) = \binom{n}{x} p^x q^{n-x} = \binom{18}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{18-x}$, $x = 0, 1, 2, \dots, 18$ **Ans. : (A)**

- (100) If 12 identical balls are to be placed in 3 identical boxes, the probability that one of the boxes contains exactly 3 balls is [JEE : 2015]

(A) $220\left(\frac{1}{3}\right)^{12}$ (B) $22\left(\frac{1}{3}\right)^{11}$ (C) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (D) $55\left(\frac{2}{3}\right)^{10}$

Sol. : $r = \binom{12}{3} \cdot 2^9$, $n = 3^{12}$

Required probability = $\frac{r}{n} = \frac{\binom{12}{3} 2^9}{3^{12}} = \frac{55}{3}\left(\frac{2}{3}\right)^{11}$ **Ans. : (C)**

- (101) An examination has 5 multiple choice questions. Each question has three options out of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is

(A) $\frac{11}{3^5}$ (B) $\frac{10}{3^5}$ (C) $\frac{17}{3^5}$ (D) $\frac{13}{3^5}$

Sol. : Total number of ways of answering 5 questions is $n = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$

For the four correct answers, select four questions and one incorrect answer from the question remaining.

The total number of ways = $\left[\binom{5}{4} \times 1\right] \times 2 = 5 \times 2 = 10$

Five correct answers can be given in only one way.

$\therefore$ Total number of elements of event $r = 10 + 1 = 11$

$\therefore$ Required probability = $\frac{r}{n} = \frac{11}{3^5}$ **Ans. : (A)**

Second method :

$$n = 5, p = \frac{1}{3}, q = \frac{2}{3}$$

Required probability = $p(4) + p(5)$

$$\begin{aligned} &= \binom{5}{4} \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1 + \binom{5}{5} \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^0 \\ &= \frac{5 \times 2}{3^5} + \frac{1}{3^5} \\ &= \frac{10+1}{3^5} = \frac{11}{3^5} \end{aligned}$$

Ans. : (A)

- (102) If $P(A) = \frac{3x+1}{3}$ and $P(B) = \frac{1-x}{4}$ for mutually exclusive events A and B, the set of possible value of x in the interval is [JEE : 2013]

(A) $[0, 1]$ (B) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (C) $\left[-\frac{1}{3}, \frac{5}{9}\right]$ (D) $\left[-\frac{7}{9}, \frac{4}{9}\right]$

Sol. : Here, $0 \leq P(A) \leq 1, 0 \leq P(B) \leq 1$

$$\begin{aligned} 0 &\leq \frac{3x+1}{3} \leq 1, 0 \leq \frac{1-x}{4} \leq 1 \\ \therefore 0 &\leq 3x+1 \leq 3, 0 \leq 1-x \leq 4 \\ \therefore -1 &\leq 3x \leq 2, -1 \leq -x \leq 3 \\ \therefore -\frac{1}{3} &\leq x \leq \frac{2}{3}, -3 \leq x \leq 1 \end{aligned} \quad (1)$$

Also, $0 \leq P(A \cup B) \leq 1$

$\therefore 0 \leq P(A) + P(B) \leq 1$ (A, B are mutually exclusive events)

$$\begin{aligned} \therefore 0 &\leq \frac{3x+1}{3} + \frac{1-x}{4} \leq 1 \\ \therefore 0 &\leq \frac{12x+4+3-3x}{12} \leq 1 \\ \therefore 0 &\leq 9x+7 \leq 12. \text{ So, } -\frac{7}{9} \leq x \leq \frac{5}{9} \end{aligned} \quad (2)$$

$$\max \left\{ -\frac{1}{3}, -3, -\frac{7}{9} \right\} \leq x \leq \min \left\{ \frac{2}{3}, 1, \frac{5}{9} \right\}$$

$$-\frac{1}{3} \leq x \leq \frac{5}{9}. \text{ So } x \in \left[-\frac{1}{3}, \frac{5}{9} \right]$$

Ans. : (C)

- (103) For events A and B, $P(A \cup B) = P(A \cap B)$, then which of the following statement is not true. [Online JEE : 2014]

- (A) A and B are equally likely (B) $P(A \cap B') = 0$
(C) $P(A' \cap B) = 0$ (D) $P(A) + P(B) = 1$

Sol. : Here, $P(A \cup B) = P(A \cap B)$

$$\begin{aligned} \therefore P(A) + P(B) - P(A \cap B) &= P(A \cap B) \\ \therefore (P(A) - P(A \cap B)) + P(B) - P(A \cap B) &= 0 \\ \therefore P(A \cap B') + P(A' \cap B) &= 0 \\ \therefore P(A \cap B') = 0 \text{ and } P(A' \cap B) &= 0 \end{aligned} \quad (1)$$

$$\therefore P(A) - P(A \cap B) = 0 \text{ and } P(B) - P(A \cap B) = 0$$

$$\therefore P(A) = P(A \cap B) \text{ and } P(B) = P(A \cap B)$$

$$\therefore P(A) = P(B) \quad (2)$$

$\therefore$ (A), (B) and (C) are correct. (D) is false.

Ans. : (D)

- (104) A set contains 7 elements. A non empty subset A of S and an element x of S are chosen at random. Then the probability that $x \in A$ is [Online JEE : 2014]

(A) $\frac{1}{2}$ (B) $\frac{64}{127}$ (C) $\frac{63}{128}$ (D) $\frac{32}{128}$

Sol. : The number of non empty subsets of S = $2^7 - 1 = 127$. So, $n = 127$

The number of subsets which does not contain x is = $2^6 - 1 = 63$.

So, number of subsets which contains x = $127 - 63 = 64$. So, $r = 64$

$$\therefore \text{Required probability} = \frac{r}{n} = \frac{64}{127}$$

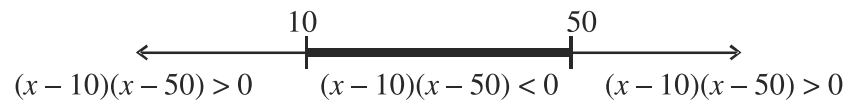
Ans. : (B)

- (105) A number x is chosen at random from the set $\{1, 2, 3, \dots, 100\}$. Define the event

A = the chosen number x satisfies $\frac{(x-10)(x-50)}{(x-30)} \geq 0$. Then P(A) is [Online JEE : 2014]

(A) 0.71 (B) 0.70 (C) 0.51 (D) 0.20

Sol. : $(x-10)(x-50) = 0 \Rightarrow x = 10$ or $x = 50$



Case 1 : If $(x-10)(x-50) \geq 0$ and $(x-30) > 0$, then $\frac{(x-10)(x-50)}{(x-30)} \geq 0$

$\therefore x \in \{1, 2, \dots, 10\} \cup \{50, 51, \dots, 100\}$ and $x > 30$

$\therefore x \in \{50, 51, \dots, 100\}$

Case 2 : If $(x-10)(x-50) \leq 0$ and $(x-30) < 0$, then $\frac{(x-10)(x-50)}{(x-30)} \geq 0$

$\therefore x \in \{10, 11, \dots, 29\}$

So, from cases (1) and (2),

$$A = \{10, 11, \dots, 29\} \cup \{50, 51, \dots, 100\}$$

$\therefore$ The number of elements of A, $r = 20 + 51 = 71$

$$\therefore P(A) = \frac{71}{100} = 0.71$$

Ans. : (A)

- (106) A, B and C are mutually independent events. If $P(C) > 0$ and $P(A \cap B \cap C) = 0$, then $P(A' \cap B' | C) = \dots$

(A) $P(B') - P(C')$ (B) $P(A') + P(B')$ (C) $P(B') - P(B')$ (D) $P(A') - P(B)$

Sol. : $P(A' \cap B' | C) = \frac{P(A' \cap B' \cap C)}{P(C)}$

$$= \frac{P[(A \cup B)' \cap C]}{P(C)}$$

$$= \frac{P(C) - P[(A \cup B) \cap C]}{P(C)}$$

$$= \frac{P(C) - [P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)]}{P(C)}$$

$$\begin{aligned}
&= \frac{P(C) - P(A \cap C) - P(B \cap C) + 0}{P(C)} \\
&= \frac{P(C) - P(A) \cdot P(C) - P(B) P(C)}{P(C)} \\
&= \frac{P(C) [1 - P(A) - P(B)]}{P(C)} \\
&= 1 - P(A) - P(B) \\
&= P(A') - P(B)
\end{aligned}$$

Ans. : (D)

- (107) Consider 5 independent Bernoulli trials each with probability of success p . If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval

(A) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (B) $\left[0, \frac{1}{2}\right]$ (C) $\left(\frac{11}{12}, 1\right]$ (D) $\left(\frac{1}{2}, \frac{3}{4}\right]$

Sol. : Let Event A : Getting at least one failure

$\therefore A' =$ getting success in all trials

$$P(A') = p(5) = \binom{5}{5} p^5 q^0 = p^5$$

Now, $P(A) \geq \frac{31}{32}$

$$\therefore 1 - P(A') \geq \frac{31}{32}$$

$$\therefore P(A') \leq \frac{1}{32}$$

$$\therefore p^5 \leq \left(\frac{1}{2}\right)^5. \text{ So, } p \leq \frac{1}{2}$$

$$\therefore p \in \left[0, \frac{1}{2}\right] \quad (p \geq 0) \quad \text{Ans. : (B)}$$

- (108) Three numbers are chosen at random without replacement from $\{1, 2, 3, \dots, 8\}$. The probability that their smallest is 3 given that their largest is 6 is [AIEEE 2012]

(A) $\frac{3}{8}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$

Sol. : Event A : The largest number from the selected numbers is 6 and the remaining two numbers are from 1 to 5

$$P(A) = \frac{\binom{1}{1} \binom{5}{2}}{\binom{8}{3}} = \frac{10}{8 \times 7} = \frac{5}{28}$$

Event B : The smallest number from the selected numbers is 3 and the remaining numbers are from 4 to 8.

$$\therefore P(B) = \frac{\binom{1}{1} \binom{5}{2}}{\binom{8}{3}} = \frac{5}{28}$$

Event $(A \cap B)$: From selected three numbers the largest is 6, the smallest is 3 and remaining is either 4 or 5.

$$\therefore P(A \cap B) = \frac{\binom{1}{1}\binom{1}{1}\binom{2}{1}}{\binom{8}{3}} = \frac{1}{28}$$

$$\text{Now, required probability } P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{28}}{\frac{5}{28}} = \frac{1}{5}$$

Ans. : (B)

Second method :

The largest number selected is 6. Suppose remaining numbers are a and b ,

$$U = \{(a, b, 6)\}, a = 1, 2, 3, 4, b = 1, 2, 3, 4, 5, a < b$$

$$\therefore n(U) = \binom{5}{2} = 10$$

$$\therefore A = \{\{3, 4, 6\}, \{3, 5, 6\}\}. \text{ So, } n(A) = 2$$

$$\therefore P(A) = \frac{2}{10} = \frac{1}{5}$$

Ans. : (B)

- (109) The probability of a man hitting a target is $\frac{2}{5}$. He fires at a target k times (k , a given positive integer). Then the smallest k , so that the probability of hitting the target at least once is more than $\frac{7}{10}$ is [Online JEE : 2013]

(A) 3 (B) 5 (C) 2 (D) 4

Sol. : Here $p = \frac{2}{5}$, $q = 1 - p = \frac{3}{5}$ and $n = k$

$$\text{So, here } p(1) + p(2) + \dots + p(k) > \frac{7}{10}$$

$$\therefore 1 - p(0) > \frac{7}{10}$$

$$\therefore p(0) < \frac{3}{10}$$

$$\therefore \binom{k}{0} \left(\frac{3}{5}\right)^k < \frac{3}{10}$$

$$\therefore \frac{10}{3} \times \left(\frac{3}{5}\right)^k < 1$$

$$\therefore k = 3, 4, 5, \dots$$

$$\therefore \text{The smallest value of } k \text{ is } 3.$$

Ans. : (A)

- (110) Given two independent events, if the probability that exactly one of them occurs is $\frac{26}{49}$ and the probability that none of them occurs is $\frac{15}{49}$, then the probability of more probable of the two events is [Online JEE : 2013]

(A) $\frac{4}{7}$ (B) $\frac{6}{7}$ (C) $\frac{3}{7}$ (D) $\frac{5}{7}$

Sol. : Suppose $P(A) = x$ and $P(B) = y$, where A and B are independent events.

$$P[(A \cap B') \cup (A' \cap B)] = \frac{26}{49} \text{ and } P(A' \cap B') = \frac{15}{49}$$

$$\therefore P(A \cap B') + P(A' \cap B) = \frac{26}{49} \text{ and } P(A') \cdot P(B') = \frac{15}{49}$$

$$\therefore P(A) \cdot P(B') + P(A') \cdot P(B) = \frac{26}{49} \text{ and } (1 - P(A))(1 - P(B)) = \frac{15}{49}$$

$$\therefore x(1 - y) + (1 - x)y = \frac{26}{49} \text{ and } (1 - x)(1 - y) = \frac{15}{49}$$

$$\therefore x + y - 2xy = \frac{26}{49} \text{ and } 1 - x - y + xy = \frac{15}{49}$$

$$\therefore x + y - xy = 1 - \frac{15}{49} = \frac{34}{49}$$

$$(x + y - xy) - xy = \frac{26}{49}$$

$$\therefore \frac{34}{49} - xy = \frac{26}{49} \cdot \text{So, } xy = \frac{8}{49} \cdot \text{So, } y = \frac{8}{49x}$$

$$\text{Also, } x + y - xy = \frac{34}{49}$$

$$\therefore x + y - \frac{8}{49} = \frac{34}{49}$$

$$\therefore x + \frac{8}{49x} = \frac{34}{49} + \frac{8}{49} = \frac{6}{7}$$

$$\therefore 49x^2 + 8 = 42x$$

$$\therefore 49x^2 - 42x + 8 = 0$$

$$\therefore (7x - 3)^2 = 1$$

$$\therefore 7x - 3 = 1 \text{ or } 7x - 3 = -1$$

$$\therefore x = \frac{4}{7} \text{ or } x = \frac{2}{7}$$

$$\text{Now, } x \cdot y = \frac{8}{49} \cdot \text{So, } y = \frac{2}{7} \text{ or } \frac{4}{7}$$

$$\therefore \text{Required probability} = \frac{4}{7}$$

Ans. : (A)

- (111) If X is a binomial distribution $B(n, p)$ with parameters n and p such that $P(X = 2) = P(X = 3)$, then $E(X)$ = the mean of variable is [Online JEE : 2014]

(A) $2p$

(B) $3 - p$

(C) $\frac{p}{2}$

(D) $\frac{p}{3}$

Sol. : Here $p(2) = p(3)$

$$\therefore \binom{n}{2} p^2 q^{n-2} = \binom{n}{3} p^3 q^{n-3}$$

$$\therefore \frac{n(n-1)q}{2} = \frac{n(n-1)(n-2)p}{6}$$

$$\therefore 3q = (n - 2)p$$

$$\therefore 3q = np - 2p$$

$$\therefore 3(1 - p) = np - 2p$$

$$\therefore 3 - 3p + 2p = np$$

$$\therefore 3 - p = E(X)$$

$$(\text{Mean } E(X) = np)$$

$$\therefore E(X) = 3 - p$$

Ans. : (B)

(112) If A and E are any two events with positive probabilities, $(P(A \cap E) \neq 0)$

Statement 1 : $P(E | A) \geq P(A | E) \cdot P(E)$

Statement 2 : $P(A | E) \geq P(A \cap E)$

[JEE : 2014]

(A) Both the statements are true

(B) Both the statements are false.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false and statement 2 is false.

Sol. : $P(E | A) \geq P(A | E) \cdot P(E)$

$$\Leftrightarrow \frac{P(A \cap E)}{P(A)} \geq \frac{P(A \cap E)}{P(E)} \cdot P(E)$$

$$\Leftrightarrow P(A) \leq 1 \text{ which is true.}$$

So, statement 1 is true.

Also, $P(A | E) \geq P(A \cap E)$

$$\Leftrightarrow \frac{P(A \cap E)}{P(E)} \geq P(A \cap E)$$

$$\Leftrightarrow 1 \geq P(E)$$

$\therefore$ Statement 2 is true.

Ans. : (A)

(113) Two dice A and B are rolled once. If the event E_1 = getting 4 on dice A and the event E_2 = getting 2 on dice B and Event E_3 = sum of digits on the dice is odd, then which of the following statement is false ?

(A) E_1 and E_2 are independent events

(B) E_2 and E_3 are independent events

(C) E_1 and E_3 are independent events

(D) E_1, E_2 and E_3 are independent events

Sol. : $P(E_1) = \frac{1}{6}, P(E_2) = \frac{1}{6}, P(E_3) = \frac{1}{2}$

$$P(E_1 \cap E_2) = \frac{1}{36} = P(E_1) \cdot P(E_2) \quad (\text{only one element } (4, 2))$$

$$P(E_2 \cap E_3) = \frac{3}{36} = \frac{1}{12} = P(E_2) \cdot P(E_3) \quad ((1, 2), (3, 2), (5, 2) \text{ are elements})$$

$$P(E_1 \cap E_3) = \frac{3}{36} = \frac{1}{12} = P(E_1) \cdot P(E_3) \quad ((4, 1), (4, 3), (4, 5) \text{ are elements})$$

$$P(E_1 \cap E_2 \cap E_3) = 0 \neq P(E_1) \cdot P(E_2) \cdot P(E_3)$$

Ans. : (D)

Questions with More Than One Correct Answers :

(114) A and B are two independent events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{5}$, then,

(A) $P(A \cup B) = \frac{3}{5}$

(B) $P(A | B) = \frac{1}{2}$

(C) $P(A | (A \cup B)) = \frac{5}{6}$

(D) $P((A \cap B) | (A' \cup B')) = 0$

Sol. : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= P(A) + P(B) - P(A) \cdot P(B)$$

(A and B are independent events)

$$= \frac{1}{2} + \frac{1}{5} - \left(\frac{1}{2} \times \frac{1}{5}\right) = \frac{3}{5}$$

$$P(A | B) = P(A) = \frac{1}{2}$$

$$P(A | A \cup B) = \frac{P[A \cap (A \cup B)]}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{\frac{1}{2}}{\frac{3}{5}} = \frac{5}{6}$$

$$A' \cup B' = (A \cap B)'$$

$$\therefore P((A \cap B) | (A' \cup B')) = 0$$

Ans. : (A), (B), (C), (D)

(115) If $P(A) = P(A | B) = \frac{1}{4}$ and $P(B | A) = \frac{1}{2}$ for events A and B, then

(A) A and B are independent events

(B) A and B are mutually exclusive

(C) $P(A' | B) = \frac{3}{4}$

(D) $P(B' | A') = \frac{1}{2}$

Sol. : $P(A) = P(A | B)$. So, $P(A) = \frac{P(A \cap B)}{P(B)}$

$\therefore P(A \cap B) = P(A) \cdot P(B)$. So, A and B are independent events.

$$P(A \cap B) = P(A) \cdot P(B | A) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \neq 0$$

So, A and B are not mutually exclusive

$$P(A' | B) = P(A') = 1 - P(A) = 1 - \frac{1}{4} = \frac{3}{4}$$

(A', B are independent)

$$P(B' | A') = P(B') = 1 - P(B) = 1 - \frac{1}{2} = \frac{1}{2}$$

(A', B' are independent)

Ans. : (A), (C), (D)

(116) A student takes tests I, II and III. The student is successful if he passes either in test I and II or test I and III or in all the tests. The probabilities of the students passing in tests I, II, III are p, q and $\frac{1}{2}$ respectively. If the probability that the student is successful is $\frac{1}{2}$, then

(A) $p = \frac{3}{4}, q = \frac{1}{3}$

(B) $p = \frac{2}{3}, q = \frac{1}{2}$

(C) $p = \frac{3}{5}, q = \frac{2}{3}$

(D) there are infinitely many values of p and q

Sol. : Let A, B, C be the events that the student is successful in tests I, II and III respectively. Then,

$\therefore P(A) = p, P(B) = q, P(C) = \frac{1}{2}$. If X is the event that the student is successful, then

$$\therefore P(X) = P(A \cap B \cap C') \cup (A \cap B' \cap C) \cup (A \cap B \cap C)$$

$$\frac{1}{2} = P(A) \cdot P(B) \cdot P(C') + P(A) \cdot P(B') \cdot P(C) + P(A) \cdot P(B) \cdot P(C)$$

$$= pq\left(1 - \frac{1}{2}\right) + p(1 - q)\left(\frac{1}{2}\right) + pq\left(\frac{1}{2}\right)$$

$$= \frac{1}{2}[pq + p - pq + pq]$$

$$\frac{1}{2} = \frac{1}{2}p(1 + q)$$

$$\therefore p(1 + q) = 1$$

This equation is satisfied for all pairs of values in (A), (B) and (C). Also, $p(1 + q) = 1$ satisfied by infinitely many values of p and q .

Ans. : (A), (B), (C), (D)

(117) If a random variable X follows binomial distribution with mean a and variance b , then

(A) $a > b > 0$

(B) $\frac{a}{b} > 1$

(C) $\frac{a^2}{a-b}$ is an integer

(D) $\frac{a^2}{a+b}$ is an integer

Sol. : Here, $a = np$, $b = npq$

Now, $0 < q < 1$. So, $0 < npq < np$. So, $0 < b < a$

$$\therefore 0 < b < a \Rightarrow a > b > 0 \Rightarrow \frac{a}{b} > 1$$

$$\text{Also, } \frac{a^2}{a-b} = \frac{n^2 p^2}{np - npq} = \frac{np}{1-q} = \frac{np}{p} = n \text{ which is an integer.}$$

It is clear that, (D) is false.

Ans. : (A), (B), (C)

(118) Three numbers are chosen at random without replacement from $\{1, 2, 3, \dots, 15\}$. Let E_1 be the event that smallest of the numbers chosen is 5 and E_2 denotes that the largest of the numbers is 10, then,

(A) $P(E_1) = \frac{9}{91}$

(B) $P(E_2) = \frac{36}{455}$

(C) $P(E_1 \cap E_2) = \frac{4}{455}$

(D) $P(E_1 | E_2) = \frac{1}{9}$

Sol. : $P(E_1) = P(\text{choosing 5 and two numbers from 6 to 15.})$

$$= \frac{{}^1C_1 \times {}^{10}C_2}{{}^{15}C_3} = \frac{45 \times 6}{15 \cdot 14 \cdot 13} = \frac{9}{91}$$

$P(E_2) = P(\text{Choosing 10 and two numbers from 1 to 9})$

$$= \frac{{}^1C_1 \times {}^9C_2}{{}^{15}C_3} = \frac{36 \times 6}{15 \cdot 14 \cdot 13} = \frac{36}{455}$$

$$P(E_1 \cap E_2) = \frac{6 \times 4}{15 \cdot 14 \cdot 13} = \frac{4}{455}$$

$$P(E_1 | E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{4}{36} = \frac{1}{9}$$

Ans. : (A), (B), (C), (D)

: 'Match the Following' Type Questions :

(119) A man takes a step forward with probability 0.4 and backward with probability 0.6. Suppose the man takes 11 steps and p_r denotes the probability that the man is r steps away from his initial position, then values of

(A) p_1

(P) 0

(B) p_3

(Q) ${}^{11}C_5 (0.24)^5$

(C) p_{10}

(R) $(0.4)^{11} + (0.6)^{11}$

(D) p_{11}

(S) ${}^{11}C_4 (0.24)^4 (0.28)$

Sol. : Let X = the number of steps taken in the forward direction, then $n = 11$, $p = 0.4$, $q = 0.6$

$$\begin{aligned}
 p_1 &= P(X = 5) + P(X = 6) \\
 &= {}^{11}C_5 p^5 q^6 + {}^{11}C_6 p^6 q^5 \\
 &= {}^{11}C_5 p^5 q^6 + {}^{11}C_5 p^6 q^5 \\
 &= {}^{11}C_5 p^5 q^5 (q + p) \quad (p + q = 1) \\
 &= {}^{11}C_5 (pq)^5 (1) \\
 &= {}^{11}C_5 (0.24)^5 \quad \text{So, (A) } \rightarrow \text{(Q)}
 \end{aligned}$$

$$\begin{aligned}
 p_3 &= P(X = 4) + P(X = 7) \\
 &= {}^{11}C_4 p^4 q^7 + {}^{11}C_7 p^7 q^4 \\
 &= {}^{11}C_4 (pq)^4 \times [(q^3 + p^3)] \\
 &= {}^{11}C_4 (pq)^4 [(p + q)^3 - 3pq(p + q)] \quad (p + q = 1) \\
 &= {}^{11}C_4 (pq)^4 (1 - 3pq) \\
 &= {}^{11}C_4 (0.24)^4 (0.28) \quad \text{So, (B) } \rightarrow \text{(S)}
 \end{aligned}$$

$$p_{10} = 0 \quad \text{So, (C) } \rightarrow \text{(P)}$$

$$\begin{aligned}
 p_{11} &= P(X = 0) + P(X = 11) \\
 &= \binom{11}{0} p^0 q^{11} + \binom{11}{11} p^{11} q^0 \\
 &= q^{11} + p^{11} \\
 &= (0.6)^{11} + (0.4)^{11} \quad \text{So, (D) } \rightarrow \text{(R)}
 \end{aligned}$$

Note : If X is the number of steps taken in the forward direction and Y is the number of steps taken in the reverse direction, then $X + Y = 11$ and $|X - Y| = r$.

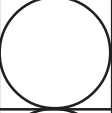
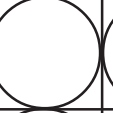
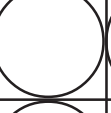
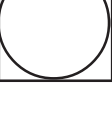
Now, if $r = 1$, then $X = 5$ or $X = 6$

if $r = 3$, then $X = 4$ or $X = 7$

if $r = 11$, then $X = 0$ or $X = 11$

But if we take $r = 0$, then $X = \frac{11}{2}$

which is not possible, so $p_{10} = 0$.

Ans. :	P	Q	R	S
A				
B				
C				
D				

(120) A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required. Then,

- | | |
|-----------------------------------|-----------------------|
| (i) $P(X = 3)$ | (a) $\frac{25}{36}$ |
| (ii) $P(X \geq 3)$ | (b) $\frac{125}{216}$ |
| (iii) $P(X \geq 6 \mid X \geq 3)$ | (c) 6 |
| (iv) $E(X)$ | (d) $\frac{25}{216}$ |

- (A) (i) $\rightarrow$ (d), (ii) $\rightarrow$ (a), (iii) $\rightarrow$ (c), (iv) $\rightarrow$ (b)
 (B) (i) $\rightarrow$ (d), (ii) $\rightarrow$ (a), (iii) $\rightarrow$ (b), (iv) $\rightarrow$ (c)
 (C) (i) $\rightarrow$ (d), (ii) $\rightarrow$ (b), (iii) $\rightarrow$ (a), (iv) $\rightarrow$ (c)
 (D) (i) $\rightarrow$ (a), (ii) $\rightarrow$ (d), (iii) $\rightarrow$ (b), (iv) $\rightarrow$ (c)

Sol. : Here, $p = \frac{1}{6}$, $q = \frac{5}{6}$

$$P(X = r) = pq^{r-1}$$

$$\therefore P(X = 3) = \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2 = \frac{25}{216} \quad \text{(i) } \rightarrow \text{ (d)}$$

$$\therefore P(X \geq 3) = 1 - [P(X = 1) + P(X = 2)] = 1 - \left[\frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right)\right] = 1 - \frac{11}{36} = \frac{25}{36} \quad \text{(ii) } \rightarrow \text{ (a)}$$

$$P(X \geq 6 \mid X \geq 3) = \frac{P[(X \geq 6) \cap (X \geq 3)]}{P(X \geq 3)} = \frac{P(X \geq 6)}{P(X \geq 3)}$$

$$P(X \geq 6) = pq^5 + pq^6 + pq^7 + \dots = \frac{pq^5}{1-q} = \frac{pq^5}{p} = q^5. \text{ Similarly } P(X \geq 3) = q^2$$

$$P(X \geq 6 \mid X \geq 3) = \frac{q^5}{q^2} = q^3 = \frac{125}{216} \quad \text{(iii) } \rightarrow \text{ (b)}$$

$$E(X) = \sum_{r=1}^{\infty} r \cdot P(X = r) = \sum_{r=1}^{\infty} rpq^{r-1} = \frac{p}{(1-q)^2} = \frac{p}{p^2} = 6 \quad \text{(iv) } \rightarrow \text{ (c)}$$

Ans. : (B)

Note : (1) If $\sum_{r=1}^{\infty} rpq^{r-1} = s$, then

$$s = p(1 + 2q + 3q^2 + \dots + \infty)$$

$$\therefore qs = p(q + 2q^2 + 3q^3 + \dots + \infty)$$

$$\therefore (1 - q)s = p(1 + q + q^2 + \dots + \infty) = \frac{p}{1-q}$$

Note : First solving (2), we get (ii) $\rightarrow$ (a). So, answer can be (A) or (B). Now solving (iii), we get (iii) $\rightarrow$ (b). So, answer is (B).

(121) If a random variable X follows binomial distribution with mean 10 and standard deviation 5, then match the following :

(i) $E(X^2)$ (p) 0

(ii) $E[X(X + 1)]$ (q) 135

(iii) $E\left(\frac{X-10}{5}\right)$ (r) 125

(iv) $E\left[\left(\frac{X-10}{5}\right)^2\right]$ (s) 1

(A) (i) $\rightarrow$ (q), (ii) $\rightarrow$ (r), (iii) $\rightarrow$ (p), (iv) $\rightarrow$ (s)

(B) (i) $\rightarrow$ (r), (ii) $\rightarrow$ (p), (iii) $\rightarrow$ (s), (iv) $\rightarrow$ (p)

(C) (i) $\rightarrow$ (r), (ii) $\rightarrow$ (q), (iii) $\rightarrow$ (p), (iv) $\rightarrow$ (s)

(D) (i) $\rightarrow$ (p), (ii) $\rightarrow$ (q), (iii) $\rightarrow$ (r), (iv) $\rightarrow$ (s)

Sol. : Here mean $E(X) = 10$, Standard deviation $\sigma_X = 5$, $V(X) = 25$

$$\therefore \frac{npq}{np} = \frac{5}{10}. \text{ So, } q = \frac{1}{2}. p = 1 - q = \frac{1}{2} \text{ and } np = 10 \Rightarrow n = 20$$

$$V(X) = E(X^2) - (E(X))^2$$

$$25 = E(X^2) - (10)^2$$

$$E(X^2) = 100 + 25 = 125 \quad \text{(i) } \rightarrow (r)$$

$$E[X(X + 1)] = E(X^2 + X) = E(X^2) + E(X) = 125 + 10 = 135 \quad \text{(ii) } \rightarrow (q)$$

$$E\left(\left(\frac{X-10}{5}\right)\right) = E\left(\frac{1}{5}X - 2\right) = \frac{1}{5}E(X) - 2 = \frac{1}{5}(10) - 2 = 0 \quad \text{(iii) } \rightarrow (p)$$

$$E\left(\left(\frac{X-10}{5}\right)^2\right) = E\left(\left(\frac{X^2 - 20X + 100}{25}\right)\right)$$

$$= \frac{1}{25}E(X^2) - \frac{4}{5}E(X) + 4$$

$$= \frac{1}{25}(125) - \frac{4}{5}(10) + 4 = 5 - 8 + 4 = 1 \quad \text{(iv) } \rightarrow (s) \quad \text{Ans. : (C)}$$

Note : (i) $\rightarrow (r)$ and then (ii) $\rightarrow (q)$ gives Ans. (C).

: Assertion and Reason Type Questions :

Explanation : Each of these questions contains statement 1 (Assertion) and statement 2 (Reason). Each question has four choices, option (A), (B), (C) and (D), which are as follows. You have to select the correct choice.

(A) Statement 1 is true, statement 2 is true. Statement 2 is the correct explanation for statement 1.

(B) Statement 1 is true, statement 2 is true. Statement 2 is not a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

(122) Four numbers are chosen at random without replacement from $\{1, 2, 3, \dots, 20\}$.

Statement 1 : The probability that the chosen numbers when arranged in some order will form an A.P. is $\frac{1}{85}$.

Statement 2 : If the four numbers chosen form an A.P. the set of all possible values of common difference is $\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$.

Sol. : $U = \{1, 2, 3, \dots, 20\}$

Let the common difference $d = 6$. We get $20, 20 - 6 = 14, 14 - 6 = 8, 8 - 6 = 2$

So, we get four numbers 20, 14, 8, 2.

$\therefore d = \pm 6$ is possible.

$\therefore$ Statement 2 is false.

Now, 4 numbers with $d = \pm 6$ can be chosen in 2 ways. 2, 8, 14, 20; 1, 7, 13, 19;

4 numbers with $d = \pm 5$ can be chosen in 5 ways.

1, 6, 11, 16; 2, 7, 12, 17; 3, 8, 13, 18; 4, 9, 14, 19 or 5, 10, 15, 20

Proceeding as above,

4 numbers with $d = \pm 4$ we can be chosen in 8 ways.

4 numbers with $d = \pm 3$ we can be chosen in 11 ways.

4 numbers with $d = \pm 2$ we can be chosen in 14 ways.

4 numbers with $d = \pm 1$ we can be chosen in 17 ways.

$$\therefore \text{Required probability} = \frac{2+5+8+11+14+17}{{}^{20}C_4} = \frac{57 \times 24}{20 \times 19 \times 18 \times 17} = \frac{1}{85}$$

$\therefore$ Statement 1 is true.

Ans. : (C)

(123) If $P(A) \neq P(B)$, A, B and C are three mutually independent events, then

Statement 1 : A and $B \cup C$ are independent.

Statement 2 : A and $B \cap C$ are independent.

Sol. : A, B and C are mutually independent events.

$P(A \cap B) = P(A) \cdot P(B)$, $P(B \cap C) = P(B) \cdot P(C)$, $P(A \cap C) = P(A) \cdot P(C)$ and $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$.

$$\begin{aligned} \text{Now, } P[A \cap (B \cup C)] &= P[(A \cap B) \cup (A \cap C)] \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ &= P(A) \cdot P(B) + P(A) \cdot P(C) - P(A) \cdot P(B) \cdot P(C) \\ &= P(A) [P(B) + P(C) - P(B) \cdot P(C)] \\ &= P(A) [P(B) + P(C) - P(B \cap C)] \\ &= P(A) \cdot P(B \cup C) \end{aligned}$$

$$\begin{aligned} \text{and } P[A \cap (B \cap C)] &= P(A \cap B \cap C) \\ &= P(A) \cdot P(B) \cdot P(C) \\ &= P(A) \cdot P(B \cap C) \end{aligned}$$

$\therefore$ Statement 1 and 2 are true, but statement 2 is not a correct explanation of statement 1

Ans. : (B)

(124) **Statement 1 :** The probability of drawing either an ace or a king from a pack of 52 playing cards in single draw is $\frac{1}{13}$.

Statement 2 : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Sol. : Statement 2 is true.

Now, let A denote the event of drawing an ace and B denote the event of drawing a king, then,

$$\therefore P(A) = \frac{4}{52} = \frac{1}{13}, P(B) = \frac{4}{52} = \frac{1}{13}, P(A \cap B) = 0$$

$$\therefore P(A \cup B) = P(A) + P(B) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13} \neq \frac{1}{13}$$

$\therefore$ Statement 1 is false.

Ans. : (D)

