

Fundamental Principle of Counting :

If an operation is performed in n different ways and corresponding to it another operation is performed in m different ways, then the total number of ways of performing both the operations is mn . This principle is also true for more than two operations.

 n Factorial

- $n! = \underline{n} = 1 \times 2 \times 3 \times \dots \times (n-1) \times n, \quad n \in \mathbb{N}$
 $= n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$
- $n! = n(n-1)!$
 $= n(n-1)(n-2)!$
- $0! = 1! = 1$

Permutation :

The number of linear permutations of n different objects taken r at a time is

$${}_nP_r = {}^nP_r = P(n, r) = n \times (n-1) \times (n-2) \times \dots \times (n-r+1) = \frac{n!}{(n-r)!} \quad (1 \leq r \leq n)$$

- The number of permutations of n different objects taken all at a time is ${}_nP_n = n!$

Permutation with Repetition :

Number of permutations of n distinct objects taken r at a time with repetitions allowed is $n \times n \times n \times \dots$ r times $= n^r$

Permutation with alike objects :

If p_1 objects are alike, p_2 objects are alike different from earlier ones, ..., p_k objects are alike different from earlier ones and $n = p_1 + p_2 + \dots + p_k$, then the number of arrangements of n things is $\frac{n!}{p_1! p_2! p_3! \dots p_k!}$.

Circular Permutation :

The number of ways of arranging n different objects on a circle is called the number of circular permutations of n objects.

If clockwise and anticlockwise arrangements are considered different, then n distinct objects can be arranged on a circle in $(n-1)!$ ways and if these arrangements are not different, then the number of ways of the arrangement is $\frac{1}{2}(n-1)!$.

Combination :

The number of ways of selecting r things out of n different things is called combination.

Total number of combinations

$$= \binom{n}{r} = {}_nC_r = C(n, r) = \frac{n!}{(n-r)!r!} = \frac{{}_nP_r}{r!} = \frac{n(n-1)(n-2)\dots(n-r+1)}{r!} \quad 0 \leq r \leq n$$

- $\binom{n}{r} = \binom{n}{n-r}$
- $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$
- $\binom{n}{r} = \binom{n}{s} \Rightarrow r = s \text{ or } r + s = n$
- If a set A has n elements, then the number of its subsets with r elements is $\binom{n}{r}$.
- **Maximum Values of $\binom{n}{r}$:**
 - (1) If n is even, then the maximum value of $\binom{n}{r} = \binom{n}{\frac{n}{2}}$
 - (2) If n is odd, then the maximum value of $\binom{n}{r} = \binom{n}{\frac{n-1}{2}} = \binom{n}{\frac{n+1}{2}}$
- From $r = 0$ to $r = \left\lfloor \frac{n}{2} \right\rfloor$; $\binom{n}{r}$ increases as r increases.
- From $r = \left\lfloor \frac{n+1}{2} \right\rfloor$ to $r = n$; $\binom{n}{r}$ decreases as r increases;
where $[x]$ = the greatest integer not exceeding x .
- $\binom{n}{r} = k$, $k \in \mathbb{N} \cup \{0\}$ has at most two solutions.
- The largest index of prime p occurring in $n!$ is $\left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \dots$.
- If $n(A) = k$, then the number of all subsets of the set A = $\binom{k}{0} + \binom{k}{1} + \binom{k}{2} + \dots + \binom{k}{k} = 2^k$
- If $n(A) = k$, then the number of subsets of a set A with m elements = $\binom{k}{m}$
- $\binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n} = 2^n - 1$
- $\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \dots = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots = 2^{n-1}$

- The number of diagonals of a convex polygon with n sides $= \binom{n}{2} - n$ $(n \geq 3)$

- n points are given in a plane, no three of which are collinear.

The total number of triangles obtained by taking these points as vertices of a triangle is $\binom{n}{3}$.

- If n points are given in a plane, then the number of line-segments joining them is $\binom{n}{2}$.

- The number of ways of arranging n different things, taken r at a time;

(i) when m particular things are to be always included in each arrangement, is

$${}_nC_m \times {}_{(n-m)}C_{(r-m)} \times r! \quad (m < r \leq n)$$

(ii) when m particular things are never taken in each arrangement is,

$${}_{(n-m)}P_r = {}_{(n-m)}C_r \times r! \quad (m < r \leq n)$$

- n distinct objects can be arranged in n places in ${}_nP_n = n!$ ways.

- The number of selections of r things out of n distinct things; when m particular things are always included;

$$= \binom{m}{m} \binom{n-m}{r-m} = \binom{n-m}{r-m} \quad (m < r \leq n)$$

- The number of selections of atleast one thing out of n identical things $= n$

- The number of selections of atmost n things from n identical things $= n + 1$

- If m alike objects are of one kind, n alike objects of second kind, p alike objects are of third kind, then the total number of ways of selections of at least one thing $= (m + 1)(n + 1)(p + 1) - 1$.

- If we have a group of one kind of m alike objects, another group of second kind of n alike objects, third group of third kind of p alike objects, then total number of ways of selections of atleast one thing from each group is $m n p$.

- Number of selections of atleast one thing out of n distinct things and m identical things is $2^n (m + 1) - 1$.

- Number of arrangements of n distinct things in n places such that all these things lie at wrong places is ${}_nP_n - {}_nP_{n-1} + {}_nP_{n-2} - {}_nP_{n-3} + \dots + (-1)^n {}_nP_0$.

- The sum of all the numbers formed by using m digits (without repetition) from the digits $a_1, a_2, a_3, \dots, a_n \in \{1, 2, 3, \dots, 9\} = (n-1)P_{(m-1)} \times (a_1 + a_2 + a_3 + \dots + a_n)$ (111 ... 1 m times)

- The sum of all the numbers made by using m digits (with repetition) from the digits $a_1, a_2, a_3, \dots, a_n \in \{1, 2, 3, \dots, 9\} = n^{m-1} \times (a_1 + a_2 + a_3 + \dots + a_n)$ (111 ... 1 m times)

- If the number of identical things is p and the number of distinct things is $n - p$, then total number of ways of selections of r things out of n things,

$$= \begin{cases} \text{if } r \leq p, \binom{n-p}{r} + \binom{n-p}{r-1} + \binom{n-p}{r-2} + \dots + \binom{n-p}{0} \\ \text{if } r > p, \binom{n-p}{r} + \binom{n-p}{r-1} + \binom{n-p}{r-2} + \dots + \binom{n-p}{r-p} \end{cases}$$

- n distinct things can be distributed in l groups of p things and m groups of q things in

$$\frac{n!}{(p!)^l (q!)^m l! m!} \text{ ways, where } lp + mq = n.$$

- If n distinct things are distributed in l groups of p things and m groups of q things, then the total

$$\text{number of ways of the arrangement of } (l + m) \text{ groups} = \frac{n!(l+m)!}{(p!)^l (q!)^m l! m!}; \text{ where } lp + mq = n$$

: Multiple Choice Questions :

$$(1) \binom{43}{4} + \sum_{r=1}^5 \binom{48-r}{3} = \dots\dots$$

$$(A) \binom{48}{3}$$

$$(B) \binom{48}{4}$$

$$(C) \binom{47}{4}$$

$$(D) \binom{48}{5}$$

$$\text{Sol. : } \binom{43}{4} + \sum_{r=1}^5 \binom{48-r}{3}$$

$$= \binom{43}{4} + \binom{47}{3} + \binom{46}{3} + \binom{45}{3} + \binom{44}{3} + \binom{43}{3}$$

$$= \binom{43}{4} + \binom{43}{3} + \binom{44}{3} + \binom{45}{3} + \binom{46}{3} + \binom{47}{3}$$

$$= \binom{44}{4} + \binom{44}{3} + \binom{45}{3} + \binom{46}{3} + \binom{47}{3}$$

$$= \binom{45}{4} + \binom{45}{3} + \binom{46}{3} + \binom{47}{3}$$

$$= \binom{46}{4} + \binom{46}{3} + \binom{47}{3} = \binom{47}{4} + \binom{47}{3} = \binom{48}{4}$$

Ans. : (B)

$$(2) \text{ If } m = \sum_{r=0}^{100} \frac{1}{\binom{100}{r}}, \text{ then } \sum_{r=0}^{100} \frac{r}{\binom{100}{r}} = \dots\dots$$

$$(A) 50m$$

$$(B) 100m$$

$$(C) 99m$$

$$(D) 200m$$

$$\text{Sol. : Let } I = \sum_{r=0}^{100} \frac{r}{\binom{100}{r}}$$

$$= \frac{0}{\binom{100}{0}} + \frac{1}{\binom{100}{1}} + \frac{2}{\binom{100}{2}} + \frac{3}{\binom{100}{3}} + \dots + \frac{100}{\binom{100}{100}} \quad (1)$$

$$= \frac{100}{\binom{100}{100}} + \frac{99}{\binom{100}{99}} + \frac{98}{\binom{100}{98}} + \frac{97}{\binom{100}{97}} + \dots + \frac{0}{\binom{100}{0}}$$

$$\therefore I = \frac{100}{\binom{100}{0}} + \frac{99}{\binom{100}{1}} + \frac{98}{\binom{100}{2}} + \frac{97}{\binom{100}{3}} + \dots + \frac{0}{\binom{100}{100}} \quad \text{as } \binom{n}{r} = \binom{n}{n-r} \dots (2)$$

(1) + (2) $\Rightarrow$

$$2I = \frac{100}{\binom{100}{0}} + \frac{100}{\binom{100}{1}} + \frac{100}{\binom{100}{2}} + \frac{100}{\binom{100}{3}} + \dots + \frac{100}{\binom{100}{100}}$$

$$= 100 \left[\frac{1}{\binom{100}{0}} + \frac{1}{\binom{100}{1}} + \frac{1}{\binom{100}{2}} + \frac{1}{\binom{100}{3}} + \dots + \frac{1}{\binom{100}{100}} \right]$$

$$= 100 \sum_{r=0}^{100} \frac{1}{\binom{100}{r}}$$

$$= 100 m$$

$$\therefore I = 50 m$$

Ans. : (A)

- (3) 30 persons can be selected out of 50 persons in ways, if 4 particular persons are included.

(A) $\binom{46}{26}$

(B) $\binom{46}{30}$

(C) ${}_{46}P_{26}$

(D) $4 \binom{46}{26}$

Sol. : The number of ways of selection of 4 persons out of 4 particular persons = $\binom{4}{4} = 1$

The number of ways of selection of remaining 26 persons out of 46 persons = $\binom{46}{26}$

$$\text{The total number of ways} = 1 \times \binom{46}{26} = \binom{46}{26}$$

Ans. : (A)

- (4) By using the alphabets of the word MONDAY, words of 4 letters can be formed in which Y does not occur.

(A) 119

(B) 24

(C) 5

(D) 120

Sol. : Here, $n = 6$ $m = 1$ $r = 4$

(Y is not included at any place. So, 5 alphabets are to be arranged in 4 places.)

$$\text{Total number of words} = {}_{(n-m)}P_r = {}_5P_4 = 120$$

Ans. : (D)

- (5) 30 persons out of 100 persons can be arranged in a queue in ways, if 20 particular persons are to be included.

(A) $\binom{80}{10} 30!$

(B) $\binom{100}{20} \binom{80}{10} 30!$

(C) $20 \times {}_{80}P_{10}$

(D) ${}_{80}P_{10} \times 30!$

Sol. : The total number of ways of the arrangement = $\binom{20}{20} \binom{80}{10} \cdot 30! = \binom{80}{10} \cdot 30!$ **Ans. : (A)**

- (6) 80 persons out of n persons can be arranged on a circular table in ways in which 5 particular persons are not included.

(A) $\binom{n-5}{80} 80!$ (B) ${}_{(n-5)}P_{80}$ (C) $\frac{1}{2} \binom{n-5}{80} 79!$ (D) $\binom{n-5}{n-85} 79!$

Sol. : 80 persons are selected out of $n - 5$ persons and they can be arranged on a circular table in

$$\binom{n-5}{80} (80 - 1)! = \binom{n-5}{n-85} 79! \quad \text{Ans. : (D)}$$

- (7) m , n and p things from $m + n + p$ things can be arranged in three boxes in ways.

(A) $\frac{(m+n+p)!}{m!n!p!} \times 3$ (B) $\frac{(m+n+p)!}{m!n!p!} \times \frac{1}{6}$ (C) $\frac{(m+n+p)!}{m!n!p!}$ (D) $\frac{(m+n+p)!}{m!n!p!} \times 3!$

Sol. : The total number of ways = $\binom{m+n+p}{m} \times \binom{n+p}{n} \times \binom{p}{p} \times 3!$

$$= \frac{(m+n+p)!}{(n+p)!m!} \times \frac{(n+p)!}{p!n!} \times \frac{p!}{0!p!} \times 3! = \frac{(m+n+p)!}{m!n!p!} \times 3! \quad \text{Ans. : (D)}$$

- (8) $m + n + p$ things can be distributed in three parts of m things, n things and p things in ways.

(A) $\frac{(m+n+p)!}{m!n!p!} \times 3$ (B) $\frac{(m+n+p)!}{m!n!p!} \times \frac{1}{6}$ (C) $\frac{(m+n+p)!}{m!n!p!}$ (D) $\frac{(m+n+p)!}{m!n!p!} \times 3!$

Sol. : The total number of ways = $\binom{m+n+p}{m} \times \binom{n+p}{n} \times \binom{p}{p}$

$$= \frac{(m+n+p)!}{(n+p)!m!} \times \frac{(n+p)!}{p!n!} \times 1 = \frac{(m+n+p)!}{m!n!p!} \quad \text{Ans. : (C)}$$

- (9) mn distinct things can be distributed in n equal parts in ways.

(A) $\frac{(mn)!}{(m!)^n} \times n!$ (B) $\frac{(mn)!}{(m!)^n} \times \frac{1}{n}$ (C) $\frac{(mn)!}{(m!)^n} \times \frac{1}{n!}$ (D) $\frac{(mn)!}{(m!)^n}$

Sol. : There are $\frac{mn}{n} = m$ things in each part.

$$\text{The total number of ways of distribution} = \frac{\binom{mn}{m} \binom{mn-m}{m} \cdots \binom{m}{m}}{n!}$$

$$= \frac{\frac{(mn)!}{m!(mn-m)!} \times \frac{(mn-m)!}{m!(mn-2m)!} \cdots \frac{m!}{m!0!}}{n!} = \frac{(mn)!}{(m!)^n} \times \frac{1}{n!}$$

Ans. : (C)

(10) mn distinct things can be distributed equally among n persons in ways .

- (A) $\frac{(mn)!}{(m!)^n} \times n!$ (B) $\frac{(mn)!}{(m!)^n}$ (C) $\frac{(mn)!}{n! (m!)^n}$ (D) $\frac{(mn)!}{(m!)^n} \times n!$

Sol. : Here, each person gets $\frac{mn}{n} = m$ things.

$\therefore$ The total number of ways of distribution

$$= \binom{mn}{m} \binom{mn-m}{m} \dots \binom{m}{m} \frac{1}{n!} \times n!$$

$$= \frac{(mn)!}{m!(mn-m)!} \times \frac{(mn-m)!}{m!(mn-2m)!} \times \dots \times \frac{m!}{0! m!} = \frac{(mn)!}{(m!)^n} \quad \text{Ans. : (B)}$$

(11) nine digit numbers can be formed by interchanging the digits of the number 221177666 such that odd digits lie at even positions.

- (A) 16 (B) 54 (C) 60 (D) 90

Sol. : The number of ways of arrangements of odd digits 1, 1, 7, 7 in 4 even positions = $\frac{4!}{2!2!} = 6$

The number of ways of the arrangements of even digits 2, 2, 6, 6, 6 in remaining 5 positions

$$= \frac{5!}{2! \times 3!} = \frac{120}{12} = 10$$

The total number of ways = $6 \times 10 = 60$

Ans. : (C)

(12) Ten's digit of $6! + 7! + 8! + \dots + 100!$ is

- (A) 4 (B) 6 (C) 2 (D) 8

Sol. : Ten's digits of $10! + 11! + 12! + \dots + 100!$ is 0.

$$6! = 720, 7! = 5040, 8! = 40320, 9! = 362880$$

$$\therefore 6! + 7! + 8! + 9! = 408960$$

$$\therefore \text{Ten's digit of the number} = 6$$

Ans. : (B)

(13) By using all the alphabets of the word MONDAY; all possible words are to be made. Arranging them in dictionary order, the position of the word MONDAY in the dictionary order is (Without repetition.)

- (A) 326 (B) 303 (C) 327 (D) 300

Sol. :

2	3	2	1	0	0
M	O	N	D	A	Y
5!	4!	3!	2!	1!	0!

$$\text{Position of this word} = 2 \times 5! + 3 \times 4! + 2 \times 3! + 1 \times 2! + 0 \times 1! + 0 \times 0! + 1$$

$$= 240 + 72 + 12 + 2 + 1 = 327$$

Ans. : (C)

Explanation : Write MONDAY .

There are two alphabets A and D, which precede M lying to the right of M. So, write 2 above M.

There are three alphabets A, D and N, which precede O lying to the right of O. So, write 3 above O.

There are two alphabets A and D, which precede N lying to the right of N. So, write 2 above N.

Similarly, write 1, 0, 0 above D, A and Y respectively. In the III row; start to write 0! 1! 2! ... 5! from the right sides.

Do the sum of the products of the corresponding elements of first and third row and add 1 to this result. So; we get the position of the word in a dictionary order.

We can use this method only if all the alphabets of a word are distinct.

Second method : Arrange all the alphabets of the word MONDAY in dictionary order; i.e. A, D, M, N, O, Y.

First, the words starting with A are = $5! = 120$ in number. Then the words starting with D = $5! = 120$ in number.

Now, M is at the first position. The number of words with MA = $4! = 24$.

The number of words with MD = $4! = 24$. The number of words with MN = $4! = 24$.

Now, O is at second position.

The number of words with MOA = $3! = 6$. The number of words with MOD = $3! = 6$.

Now, N is at third position.

The number of words with MONA = $2! = 2$.

Now, D is fourth position. Then in dictionary order we get the word MONDAY. For the position of MONDAY, before the word MONDAY; we have $120 + 120 + 24 + 24 + 24 + 6 + 6 + 2$ words.

The position of the word MONDAY (in dictionary order) is 327.

(14) Using four alphabets out of the alphabets of the word MATHEMATICS words can be formed.

(A) 1680

(B) 18

(C) 756

(D) 2454

Sol. : There are 11 alphabets in the given word MATHEMATICS. i.e. M, M, T, T, A, A, H, E, I, C, S.

(1) The number of ways of forming four alphabet words from eight distinct alphabets MTAHEICS

$$= {}_8P_4 = 8 \times 7 \times 6 \times 5 = 1680$$

(2) The number of words of four alphabets formed by using two same and two distinct alphabets

$$= \binom{3}{1} \binom{7}{2} \times \frac{4!}{2!} \quad (\text{Taking one pair and two distinct alphabets from 3})$$
$$= 3 \times 21 \times 12 = 756$$

(3) The number of words formed by two pairs of two same alphabets

$$= \binom{3}{2} \frac{4!}{2! 2!} = 3 \times 6 = 18$$

$\therefore$ The total number of words of four alphabet words = $1680 + 756 + 18 = 2454$ **Ans. : (D)**

(15) By using the digits 0, 1, 2, 3, 4, 5 only once five digit numbers divisible by 3 can be formed.

(A) 96

(B) 600

(C) 120

(D) 216

Sol. : If the sum of all the digits of a number is divisible by 3, then the number is divisible by three.

Here, the sum of all the given digits is 15. Forming a five digit number using the digits 1, 2, 3, 4, 5 such that sum of digits is 15 and using the digits 0, 1, 2, 4, 5 such that sum of digits is 12 is only possible. We cannot find five digits whose sum is less than 12 or greater than 15, such that the number is divisible by three.

$\therefore$ We can form $5! = 120$ numbers of five digits with the help of the digits 1, 2, 3, 4, 5 and they are divisible by 3.

We can form five digit numbers using 0, 1, 2, 4, 5 which are divisible by 3 and their numbers is $4(4!) = 96$.

The total number of five digit numbers which are divisible by three = $120 + 96 = 216$ **Ans. : (D)**

(16) 2, 3 and 5 points lie on $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ of ΔABC respectively. triangles can be formed by taking these points as vertices.

(A) 109

(B) 120

(C) 107

(D) 99

Sol. : Here, $2 + 3 + 5 = 10$ points are given.

$\therefore$ The number of triangles = $\binom{10}{3} - \binom{3}{3} - \binom{5}{3} = 120 - 1 - 10 = 109$ **Ans. : (A)**

(17) 2, 3 and 5 points lie on $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ of ΔABC respectively. lines can be formed by using these points.

(A) 32

(B) 45

(C) 34

(D) 33

Sol. : A unique line passes through two distinct points. 10 points are given.

$\therefore$ The number of lines = $\binom{10}{2} - \binom{3}{2} - \binom{5}{2} + 2$
 $= \frac{10 \times 9}{2} - 3 - 10 + 2 = 34$ **Ans. : (C)**

(18) If $\binom{189}{35} + \binom{189}{r} = \binom{190}{r}$, then $r = \dots$

(A) 36 or 154

(B) 35 or 155

(C) 36 or 155

(D) 36 or 153

Sol. : Here, $\binom{189}{35} + \binom{189}{r} = \binom{190}{r} = \binom{189}{r-1} + \binom{189}{r}$

$$\therefore \binom{189}{35} = \binom{189}{r-1}$$

$$\therefore 35 = r - 1 \quad \text{or} \quad 35 + r - 1 = 189$$

$$\therefore r = 36 \quad \text{or} \quad r = 155$$

Ans. : (C)

- (19) If $n(A) = m$ and $n(B) = 2$, then onto functions from A to B are defined.
 (A) $m^2 - 2$ (B) ${}_m P_2$ (C) $2^m - 2$ (D) $2^m - 1$

Sol. : The total number of functions from A to B = 2^m

and the total numbers of not onto functions from A to B = 2

$\therefore$ The number of onto functions from A to B = The total numbers of functions –

The total number of not onto functions

$$= 2^m - 2$$

Ans. : (C)

- (20) If $A = \{a, b, c, d\}$ and $B = \{1, 2, 3\}$, then onto functions are there from A to B.
 (A) 45 (B) 36 (C) 33 (D) 24

Sol. : The total number of functions from A to B = $3^4 = 81$

The total number of not onto functions having one element in the range is three and those having

$$\text{two elements in the range} = \binom{3}{2} (2^4 - 2) = 3 \times 14 = 42$$

$\therefore$ The total number of not onto functions = $3 + 42 = 45$

$\therefore$ The number of onto functions = $81 - 45 = 36$

Ans. : (B)

Note : If $n(A) = m$ and $n(B) = n$, then the number of onto functions from A to B

$$\begin{aligned} &= \sum_{r=0}^{n-1} (-1)^r \binom{n}{r} (n-r)^m \\ &= (-1)^0 \binom{3}{0} (3)^4 + (-1)^1 \binom{3}{1} (2)^4 + (-1)^2 \binom{3}{2} \\ &= 81 - 48 + 3 = 36 \end{aligned}$$

- (21) Four boys and four girls can be arranged in a row in ways, so that two persons of same gender do not occupy adjacent positions.

- (A) 1152 (B) 576 (C) 2880 (D) 2304

Sol. : B G B G B G B G

or

G B G B G B G B

There are two possibilities. An arrangement starts with a boy or with a girl.

The total number of ways that 4 boys can be arranged in 4 places and 4 girls can be arranged in 4 places = $4! \times 4!$

$\therefore$ The total number of ways = $4! \times 4!$

$\therefore$ The total number of ways of this arrangement = $2 \times 4! \times 4! = 2 \times 24 \times 24 = 1152$ **Ans. :** (A)

- (22) If $A = \{1, 2, 3, \dots, n\}$, then the number of commutative binary operations defined on A is

- (A) $\binom{n^2}{n}$ (B) $\frac{n(n-1)}{2}$ (C) $\frac{n^{n^2}}{2}$ (D) $n \Sigma^n$

Sol. : For any $x, y \in A$, $x * y = y * x$

where $*$ is a function from $A \times A \rightarrow A$.

For every $(x, y) \in A \times A$, $x * y$ can be determined in n ways.

So, $1 * 1$ can be decided in n ways.

$2 * 1 = 1 * 2$ and $2 * 2$ can be decided in $n \cdot n = n^2$ ways.

$3 * 1 = 1 * 3$, $3 * 2 = 2 * 3$ and $3 * 3$ can be decided in $n \cdot n \cdot n = n^3$ ways.

Similarly, $n * 1 = 1 * n$, $n * 2 = 2 * n$, ... the total number of ways to decide the value
 $= n \times n \times \dots \times n$ times $= n^n$

$\therefore$ The number of commutative binary operations is

$$= n \times n^2 \times n^3 \times \dots \times n^n = n^{1+2+3+\dots+n} = n^{\sum n} \quad \text{Ans. : (D)}$$

- (23) If five digit numbers are formed by using the digits 3, 4, 5, 6, 8 only once, then the sum of ten's digit of all such five digit numbers is

(A) 6240 (B) 624 (C) 3120 (D) 156

Sol. : The sum of ten's digit = Sum of unit digit

$$= (n - 1)! \times \text{sum of all the digits}$$

$$= (5 - 1)! \times (3 + 4 + 5 + 6 + 8)$$

$$= 24 \times 26$$

$$= 624$$

The total number of numbers with fixed unit digit = $4!$

The sum of their unit digits = $24 \times 26 = 624$

Ans. : (B)

- (24) The sum of unit digits of all 4 digit numbers that can be formed using the digits 1 to 9 is
 (Without repetition).

(A) 10,800 (B) 12,700 (C) 15,120 (D) 50,400

Sol. : Required sum $= {}_{(n-1)}P_{(m-1)}$ (the sum of all the digits) (1)

$$= {}_{(9-1)}P_{(4-1)} (1 + 2 + \dots + 9) \quad (n = 9, m = 4)$$

$$= {}_8P_3 (45) = 8 \times 7 \times 6 \times 45 = 15,120 \quad \text{Ans. : (C)}$$

- (25) The sum of all five digit numbers formed by permuting the digits of the number 2354768 is

(A) 9333240 (B) 279997200 (C) 5833275 (D) 139998600

Sol. : n = total number of digits = 7, m = 5

Required sum $= {}_{(n-1)}P_{(m-1)}$ (the sum of all the digits) (11111)

$$= {}_6P_4 \times (2 + 3 + 4 + 5 + 6 + 7 + 8) (11111)$$

$$= 360 \times 35 \times 11111 = 139998600$$

Ans. : (D)

- (26) The sum of all the 4 digit numbers formed by using the digits of the number 23456 is
 (repetition is allowed).

(A) 2777500 (B) 53328 (C) 3777500 (D) 277500

Sol. : Here $n = 5$, $m = 4$

Required sum $= n^{m-1} \times (\text{sum of all the digits}) (1111)$

$$= 5^{4-1} \times (2 + 3 + 4 + 5 + 6) \times (1111) = 125 \times 20 \times 1111 = 2777500 \quad \text{Ans. : (A)}$$

(27) Four digit numbers are to be formed by using all the digits of the number 123456. The sum of unit digit and hundred's digit of all these numbers is (repetition is allowed).

- (A) 8712 (B) 9072 (C) 252 (D) 90720

Sol. : Here, $n = 6, m = 4$

The sum of unit's digits = The sum of hundred's digits

$$\therefore \text{Required sum} = 2 \times n^{m-1} \times (1+2+3+4+5+6) (1)$$

$$= 2 \times 6^{4-1} \times (21) = 9072$$

Ans. : (B)

(28) The sum of all two digit numbers formed by using the digits of the number 12304 only once is

- (A) 700 (B) 430 (C) 200 (D) 2200

Sol. : Here 0 does not occur at ten's position. So we get 4 numbers with unit digit 0 and three numbers with unit digit non-zero.

$$\begin{aligned} \therefore \text{The sum of unit's digits of all two digit numbers} &= 0 \times 4 + 3 \times (1+2+3+4) \\ &= 3 \times 10 = 30 \end{aligned}$$

$$\begin{aligned} \text{and the sum of ten's digit of all two digit numbers} &= 4 \times (1+2+3+4) \\ &= 4 \times 10 = 40 \end{aligned}$$

$$\therefore \text{Required sum} = \text{The sum of unit's digits} \times 1 + \text{the sum of ten's digit} \times 10$$

$$= 30 + 40 \times 10 = 30 + 400 = 430$$

Ans. : (B)

(29) If 6^k is a factor of $100!$, then maximum value of k is

- (A) 48 (B) 97 (C) 47 (D) 46

$$\begin{aligned} \text{Sol. : The largest exponent of 2 in } 100! &= \left[\frac{100}{2} \right] + \left[\frac{100}{4} \right] + \left[\frac{100}{8} \right] + \left[\frac{100}{16} \right] + \left[\frac{100}{32} \right] + \left[\frac{100}{64} \right] + \left[\frac{100}{128} \right] \\ &= 50 + 25 + 12 + 6 + 3 + 1 + 0 = 97 \text{ and} \end{aligned}$$

$$\text{The largest exponent of 3 in } 100! = \left[\frac{100}{3} \right] + \left[\frac{100}{9} \right] + \left[\frac{100}{27} \right] + \left[\frac{100}{81} \right] = 33 + 11 + 3 + 1 = 48$$

$$\therefore 100! = 2^{97} \times 3^{48} \times k \text{ where } k \neq 2n, k \neq 3n, n \in \mathbb{N}$$

$$= (2 \times 3)^{48} \times 2^{49} \times k$$

$$= 6^{48} \times 2^{49} k$$

$$\therefore \text{The largest exponent of 6 in } (100)! = 48$$

Ans. : (A)

Note : Here the largest exponent of 6 = the largest exponent of 3. So, there is no need to find the largest exponent of 2.

$$(30) {}_{100}P_7 + 7 \times {}_{100}P_6 = \dots\dots$$

- (A) ${}_{100}P_7$ (B) ${}_{101}C_7$ (C) ${}_{100}P_8$ (D) ${}_{101}P_7$

$$\text{Sol. : } {}_{100}P_7 + 7 \times {}_{100}P_6 = \frac{100!}{93!} + 7 \times \frac{100!}{94!}$$

$$= \frac{100!}{93!} \left(1 + \frac{7}{94} \right) = \frac{100!}{93!} \times \frac{101}{94} = \frac{101!}{94!} = {}_{101}P_7$$

Ans. : (D)

- (31) 10 animals of a circus have to be placed in 10 cages with one in each cage. If 3 of the cages are too small for 6 of the animals, then the number of ways of placing the animals in cages is
 (A) 30240 (B) 120960 (C) 168 (D) 5064

Sol. : 6 large animals can be placed in 7 large cages in ${}_7P_6 = 7! = 5040$ ways.

A small animal can be placed in a large cage.

$\therefore$ Remaining 4 animals can be placed in remaining 4 cages in $4! = 24$ ways.

$\therefore$ The total number of ways of this arrangement = $5040 \times 24 = 120960$ **Ans. : (B)**

- (32) AAAA BB CCC DDD E F can be arranged in a row in ways, so that two B's occupy adjacent positions.

(A) $\frac{12!}{4!3!3!} \times \binom{13}{2}$ (B) $\frac{12!}{4!3!3!} \times {}_{13}P_2$ (C) $\frac{14!}{4!3!3!2!} - 2!$ (D) $\frac{12!}{4!3!3!} \times 13^2$

Sol. : Except B's, the remaining 12 letters can be arranged in a row in $\frac{12!}{4!3!3!}$ ways.

Two B's can be arranged in 13 places in $\binom{13}{2}$ ways.

$\therefore$ The total number of ways of this arrangement = $\frac{12!}{4!3!3!} \times \binom{13}{2}$ **Ans. : (A)**

- (33) four digit odd numbers can be formed by using the digits 0, 1 and 2.

(A) 9 (B) 18 (C) 27 (D) 8

Sol. : We have to place 1 at unit place to form an odd number.

$\therefore$ The total number of ways of filling unit place = 1.

To make a four digit number; place the digits 1 or 2 at thousand's place.

Any digit from 3 given digits is possible at ten's and hundred's places.

$\therefore$ The total number of ways to decide thousand's place = 2

The total number of ways to decide ten's place = 3

The total number of ways to decide hundred's place = 3

$\therefore$ The total number of four digit numbers = $1 \times 2 \times 3 \times 3 = 18$ **Ans. : (B)**

- (34) Product of first n odd natural numbers is

(A) n^2 (B) $\left(\frac{1}{2}\right)^n {}_{2n}P_n$ (C) $2^n \times {}_{2n}P_n$ (D) $\left(\frac{1}{2}\right)^n \binom{2n}{n}$

Sol. : Here, $1 \times 3 \times 5 \times \dots \times (2n-1)$

$$= \frac{1 \times 2 \times 3 \times 4 \times 5 \times \dots \times (2n-1) \times 2n}{2 \times 4 \times 6 \times \dots \times 2n}$$

$$= \frac{(2n)!}{2^n \times n!}$$

$$= \left(\frac{1}{2}\right)^n {}_{2n}P_n$$

Ans. : (B)

- (35) By permuting the digits of the number 35679, new five digit numbers less than 56000 can be formed.

(A) 30 (B) 28 (C) 143 (D) 29

Sol. : A number of type 3 x x x x is less than 56000 and can be chosen in $4! = 24$ ways.

A number of type 5 3 x x x is less than 56000 and can be chosen by $3! = 6$ ways.

Other numbers formed are greater than 56000 and one such number is given.

Hence, the total number of numbers is $24 + 6 - 1 = 29$

Ans. : (D)

- (36) In $\square^{m}ABCD$, A – E – C and B, E, D are non-collinear. rays can be drawn by taking the points A, B, C, D and E as initial points.

(A) 14 (B) 16 (C) 20 (D) 18

Sol. : Two rays are possible with the help of two distinct points.

Here $\vec{AE} = \vec{AC}$ and $\vec{CE} = \vec{CA}$

$$\begin{aligned}\therefore \text{The total number of rays} &= 2 \binom{5}{2} - 2 \\ &= 2(10) - 2 \\ &= 18\end{aligned}$$

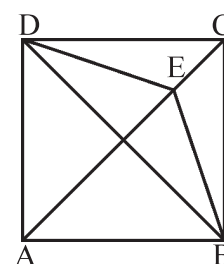


Figure 5.1

Note : $2 \binom{5}{2}$ choices because $\vec{AB} \neq \vec{BA}$

Ans. : (D)

- (37) The number of subsets of $\{1, 2, 3, 4, \dots, n\}$ containing 1 but not containing 2 and having at least 4 elements and at most five elements is ($n \geq 6$)

(A) $\binom{n-1}{4}$ (B) $\binom{n-1}{5}$ (C) $\binom{n-2}{4} \times \binom{n-2}{3}$ (D) $\binom{n}{5}$

Sol. : The number of subsets having 4 elements $= \binom{1}{1} \binom{1}{0} \binom{n-2}{3} = \binom{n-2}{3}$

The number of subsets having 5 elements $= \binom{1}{1} \binom{1}{0} \binom{n-2}{4} = \binom{n-2}{4}$

$\therefore$ The total number of required subsets $= \binom{n-2}{3} + \binom{n-2}{4} = \binom{n-1}{4}$ **Ans. :** (A)

- (38) If $m = {}_nC_2$, then ${}_mC_2 = \dots$

(A) $3 \times n + 1 P_4$ (B) $\frac{1}{2} \times \binom{n+1}{4}$ (C) $3 \times \binom{n+1}{4}$ (D) $\frac{1}{8} \times \binom{n+1}{4}$

Sol. : Here, ${}_mC_2 = \frac{m(m-1)}{2} = \frac{\frac{n(n-1)}{2} \left\{ \frac{n(n-1)}{2} - 1 \right\}}{2}$

$$\begin{aligned}
&= \frac{n(n-1)(n^2-n-2)}{8} \\
&= \frac{n(n-1)(n-2)(n+1)}{8} \\
&= 3 \frac{(n+1)n(n-1)(n-2)}{24} = 3 \binom{n+1}{4}
\end{aligned}$$

Ans. : (C)

(39) three digit even numbers can be formed without repetition of the digits.

- (A) 360 (B) 320 (C) 328 (D) 348

Sol. : The total number of three digit numbers = $9 \times 9 \times 8 = 648$

$\therefore$ The total number of three digit odd numbers = $5 \times 8 \times 8 = 320$ (last digit 1,3,5,7,9)

$\therefore$ The total number of three digit even numbers =

The total number of numbers – The total number of odd numbers = $648 - 320 = 328$

Ans. : (C)

(40) seven digit numbers can be formed using the digits 1, 2 and 3 such that the sum of the digits is 10.

- (A) 42 (B) 77 (C) 35 (D) 90

Sol. : The total number of numbers formed using the digits 1, 1, 1, 1, 1, 2, 3 = $\frac{7!}{5!} = \frac{7 \times 6 \times 5!}{5!} = 42$

The total number of numbers formed using the digits 1, 1, 1, 1, 2, 2, 2 = $\frac{7!}{4!3!} = \frac{7 \times 6 \times 120}{24 \times 6} = 35$

The total number of seven digit numbers = $42 + 35 = 77$

Ans. : (B)

Note : If the digit three is to occur two times, then the minimum sum of the digits

$$= 3 + 3 + 1 + 1 + 1 + 1 + 1 = 11.$$

So, 3 is not taken twice; if the digit 3 is taken once, then the digit 2 cannot be taken twice, because the minimum sum of the digits = $3 + 2 + 2 + 1 + 1 + 1 + 1 = 11$.

Hence, only two cases 1,1,1,1,1,2,3 or 1,1,1,1,2,2,2 are possible.

(41) If from A, B, C, D, E and F atleast one alphabet can be repeated, then words can be formed using four alphabets are possible.

- (A) 936 (B) 1296 (C) 360 (D) 1281

Sol. : The number of required words

= The total number of words with repetition allowed –

The number of words in which no alphabet is repeated

$$= 6^4 - {}_6P_4$$

$$= 1296 - 360 = 936$$

Ans. : (A)

(42) All the alphabets of the word DAUGHTER are to be arranged without repetition. words of eight alphabets can be formed such that all the vowels come at the place of consonants.

- (A) 720 (B) 3600 (C) 39600 (D) 7200

Sol. : Here the word contain 8 alphabets. In these 8 alphabets, A, U and E are three vowels and D, G, H, T, R are five consonants.

The total number of ways to arrange three vowels in 5 places = ${}_5P_3 = 5 \times 4 \times 3 = 60$

Now, remaining 5 alphabets can be arranged in 5 places in $5! = 120$ ways.

So, the total number of arrangements in 8 places can be done in $60 \times 120 = 7200$ ways.

Ans. : (D)

- (43) words of 7 alphabets can be formed by permuting alphabets of the word ENGLISH, such that vowels occupy odd positions.

(A) 1440

(B) 719

(C) 2880

(D) 132

Sol. : Here, out of 7 alphabets; E and I are two vowels.



The total number of ways of arranging two vowels in 4 odd positions = ${}_4P_2 = 4 \times 3 = 12$

Remaining 5 alphabets can be arranged in 5 places in $5! = 120$ ways

The total numbers of required words = $12 \times 120 = 1440$

Ans. : (A)

- (44) If new words of 6 alphabets are to be formed by permuting all the alphabets of the word SCHOOL, then the position of the word SCHOOL is (in dictionary order)

(A) 302

(B) 243

(C) 304

(D) 303

Sol. : Here, dictionary order, is C H L O O S.

The number of words starting with C = $1 \times \frac{5!}{2!} = 60$

The number of words starting with H = 60

The number of words starting with L = 60

The number of words starting with O = $5! = 120$

The number of words with SCHL = $\frac{2!}{2} = 1$

The number of words with SCHOL = 1

SCHOOL itself = 1

Required position = $60 + 60 + 60 + 120 + 1 + 1 + 1 = 303$

Ans. : (D)

- (45) $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + 99 \cdot 99! = \dots$

(A) $100!$

(B) $100! - 1$

(C) $100! + 1$

(D) $99! + 1$

Sol. : Required sum = $\sum_{r=1}^{99} r(r!)$

$$= \sum_{r=1}^{99} \{(r+1) - 1\} r!$$

$$= \sum_{r=1}^{99} \{(r+1)! - r!\}$$

$$= (2! - 1!) + (3! - 2!) + (4! - 3!) + \dots + (100! - 99!)$$

$$= 100! - 1$$

Ans. : (B)

(46) $42!$ is divisible by

(A) 12^{20}

(B) 12^{19}

(C) 2^{40}

(D) 3^{24}

Sol. : The largest exponent of 2 in $42! = \left[\frac{42}{2} \right] + \left[\frac{42}{4} \right] + \left[\frac{42}{8} \right] + \left[\frac{42}{16} \right] + \left[\frac{42}{32} \right]$
 $= 21 + 10 + 5 + 2 + 1 = 39$

The largest exponent of 3 in $42! = \left[\frac{42}{3} \right] + \left[\frac{42}{9} \right] + \left[\frac{42}{27} \right]$
 $= 14 + 4 + 1 = 19$

$\therefore 2^{39} \times 3^{19}$ is a factor of $42!$.

$\therefore (2^2)^{19} \times 2 \times 3^{19}$ is a factor of $42!$.

$\therefore (2 \times 2 \times 3)^{19} \times 2$ is a factor of $42!$.

$\therefore 42!$ is divisible by 12^{19} .

and $42!$ is not divisible by $12^{20} = 2^{40} \times 3^{20}$.

Also $42!$ is not divisible by 2^{40} and 3^{24} .

Ans. : (B)

(47) $\binom{2n}{0} + \binom{2n+1}{1} + \binom{2n+2}{2} + \dots + \binom{2n+25}{25} = \dots$

(A) $\binom{2n+26}{25}$

(B) $\binom{2n+25}{26}$

(C) $\binom{2n+26}{26}$

(D) $\binom{2n+26}{2n-1}$

Sol. : $\binom{2n}{0} + \binom{2n+1}{1} + \binom{2n+2}{2} + \dots + \binom{2n+25}{25}$
 $= \binom{2n+1}{0} + \binom{2n+1}{1} + \binom{2n+2}{2} + \dots + \binom{2n+25}{25}$
 $= \binom{2n+2}{1} + \binom{2n+2}{2} + \dots + \binom{2n+25}{25}$
 $= \binom{2n+3}{2} + \binom{2n+3}{3} + \dots + \binom{2n+25}{25}$
 $= \dots$
 $= \dots$
 $= \binom{2n+26}{25}$

Ans. : (A)

(48) 7 lines and 5 circles intersect in at most points.

(A) 70

(B) 111

(C) 35

(D) 66

Sol. : Two lines intersect in at most one point. Two circles intersect in at most two points and one line and one circle intersect in at most two points.

$\therefore$ The maximum number of points of intersections.

$= \binom{7}{2}(1) + \binom{5}{2}(2) + \binom{7}{1} \binom{5}{1}(2) = 21 + 20 + 70 = 111$

Ans. : (B)

- (49) Three digit numbers are to be formed with the help of the digits 1, 2, 3, 4, 5 and 6. Then numbers from them are divisible by 4.

(A) 36 (B) 48 (C) 54 (D) 32

Sol. : Here, 12, 16, 24, 32, 36, 44, 52, 56, 64 are two digit numbers divisible by 4.

$\therefore$ The total number of 3 digit numbers which are divisible by four = $9 \times 6 = 54$

Here any digit can be repeated because we have to find the total number of numbers.

If last two digits of any number form a number which is divisible by 4, then the original number is also divisible by 4. **Ans. : (C)**

- (50) number of numbers with maximum nine digits can be formed using digits 1 to 9 (repetition is allowed).

(A) $\frac{1}{8} (3^{20} - 9)$ (B) $\sum_{r=1}^9 {}_9C_r$ (C) $\frac{9^{10} - 1}{8}$ (D) $\sum_{r=1}^9 {}_9P_r$

Sol. : The total number of one digit numbers = 9. The total number of two digit numbers = $9 \times 9 = 9^2$

The total number of three digit numbers = $9 \times 9 \times 9 = 9^3$

Similarly, the total number of nine digit numbers = 9^9

$\therefore$ The total number of numbers = $9 + 9^2 + 9^3 + \dots + 9^9$

$$= \frac{9(9^9 - 1)}{9 - 1} = \frac{1}{8} (9^{10} - 9) = \frac{1}{8} (3^{20} - 9) \quad \text{Ans. : (A)}$$

- (51) There are zeros at the end of 200!.

(A) 48 (B) 50 (C) 51 (D) 49

Sol. : In the prime factorization of $n!$; the largest exponent of 2 $\geq$ the largest exponent of 5

$\therefore$ The number of zeros at the end of 200! = The largest exponent of 5 in 200!

$$= \left[\frac{200}{5} \right] + \left[\frac{200}{25} \right] + \left[\frac{200}{125} \right] = 40 + 8 + 1 = 49$$

Ans. : (D)

- (52) Nine points lie on each of two parallel lines. If each point on one of the lines is joined to another point of the other line, then these line-segments intersect in n distinct points. Then n is not greater than

(A) 1296 (B) 1926 (C) 3060 (D) 5184

Sol. : With the help of a pair of two points on both the lines; we get a point of intersection,

$$\begin{aligned} \therefore \text{The maximum number of points of intersection} &= \binom{9}{2} \times \binom{9}{2} \\ &= 36 \times 36 \\ &= 1296 \end{aligned}$$

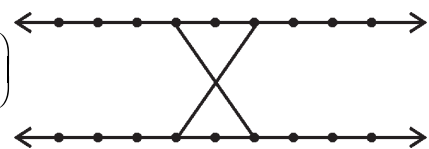


Figure 5.2

Out of 1296 points; some points may be same.

$\therefore n \leq 1296$ So, $n \neq 1296$

Ans. : (A)

(53) The number of all the divisors of $8!$ is

(A) 14

(B) 96

(C) 15

(D) 11

Sol. : Here, $8! = 2^7 \times 3^2 \times 5^1 \times 7^1$

Each divisor of $8!$ is of the type $2^a 3^b 5^c 7^d$

where $a \in \{1, 2, \dots, 7, 0\}$, $b \in \{0, 1, 2\}$

$c \in \{0, 1\}$ and $d \in \{0, 1\}$

$\therefore$ The number of divisors of $8! = 8 \times 3 \times 2 \times 2$
 $= 96$

Second method :

$8! =$ The number of divisors of $2^7 \times 3^2 \times 5^1 \times 7^1$

$= (a+1)(b+1)(c+1)\dots$ (formula)

$= (7+1)(2+1)(1+1)(1+1)$

$= 8 \times 3 \times 2 \times 2$

$= 96$

Ans. : (B)

(54) If five letters are to be placed in five addressed envelopes, then all the letters are placed in incorrect envelopes in ways.

(A) 96

(B) 24

(C) 45

(D) 44

Sol. : The total number of required ways

$$= {}_5P_5 - {}_5P_4 + {}_5P_3 - {}_5P_2 + {}_5P_1 - {}_5P_0$$

$$= 120 - 120 + 60 - 20 + 5 - 1 = 44$$

Ans. : (D)

(55) If n is an integer between 0 and 31, then the least value of $n! (31 - n)!$

(A) $14! \times 15!$

(B) $15! \times 16!$

(C) $31!$

(D) $16! \times 17!$

Sol. : If $n! (31 - n)!$ is minimum, then $\binom{31}{n} = \frac{31!}{n!(31-n)!}$ will be maximum.

$$\therefore n = \frac{31-1}{2} \quad \text{or} \quad \frac{31+1}{2} \quad \text{So, } n = 15 \text{ or } 16$$

$\therefore$ The smallest value of $n! (31 - n)! = 15! \times 16!$

Ans. : (B)

(56) If the total number of selections of atleast one thing and atmost n things out of $2n + 1$ things is 1023, then $n =$

(A) 3

(B) 4

(C) 5

(D) 6

Sol. : Here $\binom{2n+1}{1} + \binom{2n+1}{2} + \binom{2n+1}{3} + \dots + \binom{2n+1}{n} = 1023$ (1)

$$\therefore \binom{2n+1}{0} + \binom{2n+1}{1} + \binom{2n+1}{2} + \dots + \binom{2n+1}{n} +$$

$$\left\{ \binom{2n+1}{n+1} + \binom{2n+1}{n+2} + \dots + \binom{2n+1}{2n+1} \right\}$$

$$= 1023 + \binom{2n+1}{0} + \left\{ \binom{2n+1}{n+1} + \binom{2n+1}{n+2} + \dots + \binom{2n+1}{2n+1} \right\}$$

$$\begin{aligned}
\therefore 2^{2n+1} &= 1023 + 1 + \left\{ \binom{2n+1}{n} + \binom{2n+1}{n-1} + \dots + \binom{2n+1}{0} \right\} \\
&= 1024 + \left\{ \binom{2n+1}{1} + \binom{2n+1}{2} + \dots + \binom{2n+1}{n} \right\} + \binom{2n+1}{0} \\
&= 1024 + 1023 + 1 \quad \text{From (1)} \\
&= 2048 \\
&= 2^{11}
\end{aligned}$$

$$\therefore 2n+1 = 11 \quad \text{So, } n = 5 \quad \text{Ans. : (C)}$$

(57) If all the alphabets of the word FENY are permuted, then is the 7th word in the dictionary order.

- (A) EYNF (B) ENYF (C) FEYN (D) FENY

Sol. : All the alphabets of the word FENY are to be arranged in dictionary order.

$\therefore$ The number of words starting with E are $3! = 6$

Seventh word will be FENY. Ans. : (D)

(58) If all the alphabets of the word LETTERS are permuted, then words of seven alphabets are possible so that same alphabets do not occupy adjacent positions.

- (A) 36 (B) 420 (C) 540 (D) 660

Sol. : Here, given alphabets are E, E, T, T, L, R, S.

The number of required numbers = The total number of words using all alphabets once

– the number of words where two E's are together

– the number of words where two T's are together

+ the number of words where two E's and two T's are together

$$= \frac{7!}{2!2!} - \frac{6!}{2!} - \frac{6!}{2!} + 5! = \frac{7 \times 720}{4} - \frac{720}{2} - \frac{720}{2} + 120 = 1260 - 720 + 120 = 660 \quad \text{Ans. : (D)}$$

(59) The number of numbers lying between 99 and 999 with only two adjacent digits equal is

- (A) 162 (B) 153 (C) 90 (D) 161

Sol. : numbers of the type $aab = 9 \times 9 = 81$

because a is any digit from the digits 1 to 9 and b is any digit except a from the remaining 9 digits (including zero).

Similarly, numbers of the type $baa = 9 \times 9 = 81$

because b is any digit from the digits 1 to 9 and a is any digit except b from the remaining 9 digits (including zero).

$\therefore$ The total number of numbers = $81 + 81 = 162$ Ans. : (A)

(60) six digit numbers are divisible by 4 (repetition is not allowed).

- (A) 33600 (B) 87360 (C) 36300 (D) 32340

Sol. : If the last two digits are 04, 08, 20, 40, 60 or 80, then the number is divisible by 4.

The total number of the numbers of this type $= 6 \times {}_8P_4 = 6 \times 8 \times 7 \times 6 \times 5 = 10080$

Now, if the last two digits are 12, 16, 24, 28, 32, 36, 48, 52, 56, 64, 68, 72, 76, 84, 92, 96, then the total number of numbers of this type of numbers

$$= 16 \times 7 \times {}_7P_3 = 112 \times 7 \times 6 \times 5 = 112 \times 210 = 23520$$

The total number of numbers $= 10080 + 23520 = 33600$

Ans. : (A)

- (61) Two numbers from 1 to 100 can be selected such that the product of them is divisible by 3, in ways.

(A) $\binom{100}{2} - \binom{67}{2}$ (B) $\binom{100}{2} - \binom{33}{2}$ (C) $\binom{67}{2}$ (D) $\binom{33}{2}$

Sol. : From 1 to 100 any two numbers can be selected in $\binom{100}{2}$ ways.

From selected two numbers, if any number is not divisible by 3, then their product is also not divisible by 3.

Here, 3, 6, 9 ... 99 (i.e., 33 numbers) are divisible by 3.

$\therefore 100 - 33 = 67$ numbers are not divisible by 3. So, if any two numbers are selected from these 67 numbers, then their product is not divisible by 3

$\therefore$ The total number of ways of required type $= \binom{100}{2} - \binom{67}{2}$

Ans. : (A)

- (62) n balls can be distributed among two friends in ways so that one of the friends gets m balls and the other friend gets $n - m$ balls.

(A) $\binom{n}{m}$ (B) $2\binom{n}{m}$ (C) $2\frac{n!}{m!}$ (D) $2 \cdot {}_nP_m$

The total number of ways of distribution $= \binom{n}{m}\binom{n-m}{n-m} + \binom{n}{n-m}\binom{m}{m}$

$$= \binom{n}{m}(1) + \binom{n}{n-m}(1)$$

$$= \binom{n}{m} + \binom{n}{m}$$

$$= 2\binom{n}{m}$$

Ans. : (B)

- (63) If A, B, C, D, E and F are to be arranged in a row, then there are ways in which this arrangement can be done so that A always precedes B.

(A) 120 (B) 360 (C) 90 (D) 345

Sol. : Six letters can be arranged in a straight line in $6! = 720$ ways.

In half of these ways, A is placed before B and in remaining half number of ways, A follows B.

$\therefore$ If A precedes B, then the total number of ways of this type of arrangements $= \frac{6!}{2} = \frac{720}{2} = 360$

Ans. : (B)

(64) If A, B, C, D, E and F are to be arranged in a straight line, then this arrangement can be done in ways such that B is placed next to A.

- (A) 120 (B) 360 (C) 60 (D) 90

Sol. : Here, B is placed next to A, so assume that AB is a one letter.

Thus, five letters (AB), C, D, E and F can be arranged in a straight line in $5! = 120$ ways

Ans. : (A)

(65) If the number of diagonals is 52 more than the number of sides in a convex polygon, then the number of vertices of this polygon is

- (A) 16 (B) 15 (C) 14 (D) 13

Sol. : Suppose the convex polygon has n vertices.

$$\therefore \text{The number of diagonals} = \binom{n}{2} - n$$

Now, the number of diagonals = the number of sides + 52

$$\therefore \binom{n}{2} - n = n + 52$$

$$\therefore \frac{n(n-1)}{2} = 2n + 52$$

$$\therefore n^2 - n = 4n + 104$$

$$\therefore n^2 - 5n - 104 = 0$$

$$\therefore (n + 8) (n - 13) = 0$$

$$\therefore n = -8 \text{ or } n = 13$$

But $n = -8$ is impossible.

$$\therefore n = 13$$

Ans. : (D)

(66) four digit numbers formed without repetition of the digits are multiples of 5.

- (A) 840 (B) 952 (C) 672 (D) 1008

Sol. : The total number of ways of getting the numbers with unit digit 0 = $1 \times {}_9P_3 = 9 \times 8 \times 7 = 504$

If the unit digit is 5, then 0 is not possible at thousand's place.

$\therefore$ Thousand's place is to be decided in 8 ways.

Hundred's place is to be decided in 8 ways.

Ten's place is to be decided in 7 ways.

$$\therefore \text{The total number of ways to form the numbers with unit digit 5} = 1 \times 8 \times 8 \times 7 = 448$$

$$\therefore \text{The total number of required numbers} = 504 + 448 = 952$$

Ans. : (B)

Second method : For a multiple of 5, unit digit is 0 or 5.

$$\therefore \text{The total number of ways} = {}_9P_3 \times 2 = 1008$$

But a number of the type 0 a b 5 is not admissible.

$$\text{and the total number of ways of type } 0 a b 5 = {}_8P_2 = 56$$

$$\therefore \text{The total number of required numbers} = 1008 - 56 = 952$$

Ans. : (B)

(67) four digit numbers are divisible by 9 in the rearrangement of the digits of the number 27456, (repetition is not allowed.).

- (A) 18 (B) 20 (C) 24 (D) 120

Sol. : If the sum of the digits of a number is divisible by 9, then the number is divisible by 9.

Here, maximum sum = $2 + 7 + 5 + 6 + 4 = 24$ and minimum sum = $2 + 4 + 5 + 6 = 17$.

Hence the sum of four digits will not be 9 or 27.

But the sum = $2 + 7 + 4 + 5 = 18$ is possible.

$\therefore$ A four digit number can be made by using the digits 2, 7, 4 and 5 only once, such that it is divisible by 9.

$\therefore$ The total number of 4 digit numbers divisible by 9 = $4! = 24$ **Ans. : (C)**

(68) triangles have one side in common with the polygon in a convex polygon of 10 sides.

- (A) 270 (B) 80 (C) 60 (D) 61

Sol. : Select any one side of a polygon. The number of ways of this selection = $\binom{10}{1} = 10$

Select any one vertex except two end points of the selected side and two adjacent vertices of this side. Now, by joining this point with end points of the selected side, then we will get a triangle of required type. So, here we are left with $10 - 4 = 6$ vertices corresponding to each selected side.

$\therefore$ The number of required triangles = $10 \times \binom{6}{1} = 60$ **Ans. : (C)**

(69) If $n(A) = 51$, then the number of subsets of A with at least 25 members is

- (A) $\binom{51}{25} + 2^{51}$ (B) 2^{51} (C) 2^{50} (D) $\binom{51}{26} + 2^{50}$

Sol. : The numbers of subsets = $\binom{51}{25} + \binom{51}{26} + \dots + \binom{51}{51}$

$$= \binom{51}{51} + \binom{51}{50} + \dots + \binom{51}{26} + \binom{51}{25}$$

$$= \binom{51}{0} + \binom{51}{1} + \dots + \binom{51}{25} + \binom{51}{25}$$

$$= \binom{50}{0} + \left\{ \binom{50}{0} + \binom{50}{1} \right\} + \left\{ \binom{50}{1} + \binom{50}{2} \right\} + \dots + \left\{ \binom{50}{24} + \binom{50}{25} \right\} + \binom{51}{25}$$

$$= \binom{50}{0} + \binom{50}{50} + \binom{50}{1} + \binom{50}{49} + \binom{50}{2} + \dots + \binom{50}{26} + \binom{50}{25} + \binom{51}{25}$$

$$= \binom{50}{0} + \binom{50}{1} + \binom{50}{2} + \dots + \binom{50}{50} + \binom{51}{25}$$

$$= 2^{50} + \binom{51}{25}$$

$$= \binom{51}{26} + 2^{50}$$

Ans. : (D)

- (70) We have 20 lines in a plane out of which 10 lines intersect in A and 4 lines intersect in B. No two lines are parallel and out of the other lines, no three of them are concurrent. These 20 lines intersect in points.

(A) 141 (B) 187 (C) 139 (D) 137

Sol. : 20 lines can intersect in $\binom{20}{2}$ points. But instead of $\binom{10}{2}$ points of intersection, we get only one point A as point of intersection and instead of $\binom{4}{2}$ points of intersection, we get only one point B as point of intersection.

$$\therefore \text{The number of points of intersection} = \binom{20}{2} - \binom{10}{2} - \binom{4}{2} + 2$$

$$= 190 - 45 - 6 + 2 = 141$$

Ans. : (A)

- (71) There are two parallel lines l_1 and l_2 in plane. l_1 contains 5 distinct points and l_2 contains 4 distinct points. convex quadrilaterals are possible with these as vertices.

(A) 180 (B) 60 (C) 90 (D) 720

Sol. : The number of convex quadrilaterals = $\binom{5}{2} \binom{4}{2}$
 $= 10 \times 6$
 $= 60$

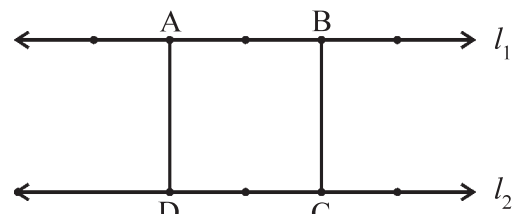


Figure 5.3

Ans. : (B)

- (72) Using the digits 0 to 9 only once four digit numbers divisible by 20 can be formed.

(A) 112 (B) 288 (C) 224 (D) 256

Sol. : If a number is divisible by 20, then its unit digit is 0 and ten's digit is an even integer.

Thousand	Hundred	Ten	Unit
		2, 4, 6, 8	0

Total number of ways to decide the unit's digit = 1

Total number of ways to decide ten's digit = 4

Total number of ways of the arrangement of remaining 8 digits at hundred's and thousand's place
 $= {}_8P_2 = 56$

$\therefore$ Total number of required numbers = $1 \times 4 \times 56 = 224$

Ans. : (C)

(73) triangles have no side in common, where vertices of the triangles are vertices of a heptagon.

- (A) 7 (B) 8 (C) 9 (D) 10

Sol. : The number of required triangles = The total number of triangles – The total number of triangles which have one side in common with the polygon – The number of triangles which have two sides in common with polygon.

Note : In the last type, 7 triangles (i.e. like $\Delta A_1 A_2 A_3$) are possible.

$$\begin{aligned} &= \binom{7}{3} - \binom{7}{1} \binom{7-4}{1} - 7 \\ &= \frac{7 \times 6 \times 5}{6} - 7 \times 3 - 7 \times 1 \\ &= 35 - 21 - 7 \\ &= 7 \end{aligned}$$

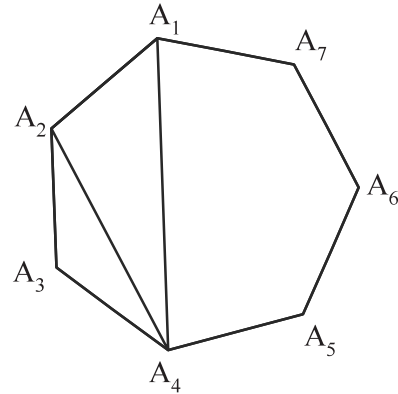


Figure 5.4

Ans. : (A)

Explanation :

Out of 7 vertices of the heptagon, $\binom{7}{3}$ triangles can be formed by joining any three vertices.

Select a point except two end points of a selected side of the polygon and two adjacent points of this side. By joining this point with the end points of the selected side we have a triangle with one side in common with polygon.

$$\text{The total number of triangles of this type} = \binom{7}{1} \binom{7-4}{1}$$

Now one vertex can be selected from 7 vertices in $\binom{7}{1}$ ways. Now select two adjacent points of this vertex. By joining these three points we get a triangle having two sides in common with polygon.

$$\text{The total number of triangles of this type} = \binom{7}{1} \binom{2}{2} = 7.$$

$\therefore$ The number of triangles having no side in common with polygon

$$= \binom{7}{3} - \binom{7}{1} \binom{3}{1} - 7 = 35 - 21 - 7 = 7$$

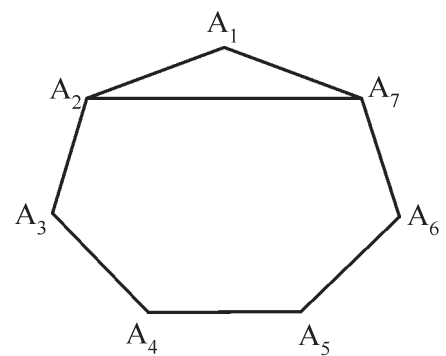


Figure 5.5

(74) If a convex polygon has 65 diagonals, then convex quadrilaterals can be formed by joining the vertices of this convex polygon.

- (A) $\binom{13}{4} \times 3!$ (B) $\binom{10}{4}$ (C) $\binom{13}{4}$ (D) $\binom{13}{4} \times 4!$

Sol. : Let a convex polygon has n vertices.

$$\therefore \text{The number of diagonals of the polygon} = \binom{n}{2} - n = 65$$

$$\therefore \frac{n(n-1)}{2} - n - 65 = 0$$

$$\therefore n^2 - 3n - 130 = 0$$

$$\therefore (n + 10)(n - 13) = 0$$

$$\therefore n = 13 \quad (n = -10 \text{ is impossible})$$

$$\therefore \text{The number of quadrilaterals} = \binom{13}{4} \quad \text{Ans. : (C)}$$

(75) The number of the four digit numbers containing 2 at most twice is (repetition is allowed)

- (A) 8966 (B) 8965 (C) 8964 (D) 873

Sol. : The total number of 4 digit numbers = $9 \times 10 \times 10 \times 10 = 9000$

The numbers of the type X 2 2 2 = 8

The numbers of the type 2 X 2 2 = 9

The numbers of the type 2 2 X 2 = 9

The numbers of the type 2 2 2 X = 9

The number of the type 2 2 2 2 = 1

Required numbers of 4 digit numbers

= The total number of 4 digit numbers –

The total number of numbers containing 2 at least thrice

$$= 9000 - (8 + 9 + 9 + 9 + 1)$$

$$= 9000 - 36$$

$$= 8964$$

Ans. : (C)

(76) If $\sum_{k=1}^{50} k!$ is divided by 14, then the remainder is

- (A) 4 (B) 6 (C) 5 (D) 7

Sol. : $7! = 7 \times 6 \times 120 = 14 \times 360$ is divisible by 14.

So, $8!, 9! \dots 50!$ are divisible by 14.

$$\therefore \sum_{k=7}^{50} k! \text{ is divisible by 14.}$$

Now, $1! + 2! + 3! + 4! + 5! + 6!$

$$= 1 + 2 + 6 + 24 + 120 + 720$$

$$= 873$$

$$= 14 \times 62 + 5$$

$\therefore$ If $\sum_{k=1}^{50} k!$ is divided by 14, then the remainder is 5.

Ans. : (C)

Note : $\sum_{k=1}^{50} k! \equiv 1 + 2 + 6 + 10 + 8 + 6 + 0 + 0 + \dots \pmod{14}$

$$\equiv 33 \pmod{14}$$

$$\equiv 5 \pmod{14}$$

$\therefore$ If $\sum_{k=1}^{50} k!$ is divided by 14, then the remainder is 5.

Ans. : (C)

(77) If $\sum_{k=4}^{200} (k!)^2$ is divided by 100, then the remainder is

(A) 17

(B) 76

(C) 24

(D) 6

Sol. : $(5!)^2 = (120)^2 = 14400$ is divisible by 100.

So, $(6!)^2, (7!)^2, \dots, (200!)^2$ are divisible by 100.

$\therefore \sum_{k=5}^{200} (k!)^2$ is divisible by 100.

Now, for $k = 4$, $(4!)^2 = (24)^2 = 576$ when divided by 100 leaves remainder = 76.

Ans. : (B)

(78) 2 occurs times in numbers 1 to 9999.

(A) 4000

(B) 3001

(C) 3000

(D) 5000

Sol. : If 2 is at unit place, then the number is of the type $abc2$, where a, b, c are any digits from 0 to 9.

$\therefore$ The total number of numbers of this type = $1 \times 10 \times 10 \times 10 = 1000$

Thus 2 comes 1000 times at unit's place.

Similarly, 2 also comes 1000 times at ten's, hundred's and thousand's place.

$\therefore$ 2 occurs 4000 times in numbers 1 to 9999.

Ans. : (A)

(79) If all the alphabets of the word MONDAY are permuted, then words of six alphabets, can be formed such that vowels always come in dictionary order.

(A) 360

(B) 120

(C) 240

(D) 390

Sol. : Here, O and A are two vowels in the given word.

The total number of six alphabet words = $6! = 720$

Half of them means that in 360 words A precedes O.

$\therefore$ The numbers of required type of words = 360

Ans. : (A)

(80) At least one alphabet can be selected from A, A, A, A, B, B, B, C, C, D in ways.

(A) 23

(B) 24

(C) 120

(D) 119

Sol. : Here A occurs 4 times; B occurs 3 times; C occurs two times and D occurs once.

$\therefore n_1 = 4, n_2 = 3, n_3 = 2, n_4 = 1$

The total number of ways of selection of atleast one alphabet

$$\begin{aligned}
 &= (n_1 + 1) (n_2 + 1) (n_3 + 1) (n_4 + 1) - 1 \\
 &= (4 + 1) (3 + 1) (2 + 1) (1 + 1) - 1 \\
 &= 5 \times 4 \times 3 \times 2 - 1 \\
 &= 119
 \end{aligned}$$

Ans. : (D)

- (81) At least one ball of each colour can be selected from five distinct red colour balls; four distinct yellow colour balls and three distinct blue colour balls in ways.

(A) 4095 (B) 3254 (C) 3720 (D) 3255

Sol. : Here, the total number of ways of selection of at least one red colour ball

$$\begin{aligned}
 &= \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5} \\
 &= 2^5 - 1 = 31
 \end{aligned}$$

Similarly,

The total number of ways of selection of at least one yellow colour ball = $2^4 - 1 = 15$ and

The total number of ways of selection of at least one blue colour ball = $2^3 - 1 = 7$

$\therefore$ The total number of ways of required type of selection = $31 \times 15 \times 7$

$$= 3255 \quad \text{Ans. : (D)}$$

- (82) triangles can be formed with the help of four line-segments of 2, 3, 5 and 7 units length. Any three line-segments from given four line-segments do not lie on any one line.

(A) 4 (B) 3 (C) 2 (D) 1

Sol. : A triangle can be formed by any three line-segments which do not lie on any one line.

The total number of ways of selection of three line-segments from four line-segments = $\binom{4}{3} = 4$

But the sum of the measurements of any two sides of a triangle is greater than the measure of its third side.

Here, $3 + 5 > 7$. So the measure of the sides of the triangle are 3, 5 and 7 units length, we get only one triangle with this measure.

Ans. : (D)

- (83) 30! is divisible by

(A) $3^{19} \times 5^{17}$ (B) 45^8 (C) 37×15^7 (D) 45^7

Sol. : $\left[\frac{30}{3} \right] + \left[\frac{30}{9} \right] + \left[\frac{30}{27} \right] = 10 + 3 + 1 = 14$. So, 30! is divisible by 3^{14}

$$\left[\frac{30}{5} \right] + \left[\frac{30}{25} \right] = 6 + 1 = 7. \text{ So, 30! is divisible by } 5^7.$$

$\therefore 3^{14} \times 5^7 = (3^2)^7 \times (5)^7 = (9 \times 5)^7$. So, $30!$ is divisible by 45^7 .

$\therefore 30!$ is not divisible by $3^{19} \times 5^{17}$ and $30!$ is not divisible by 45^8 . Also $30!$ is not divisible by 37 because $\left\lfloor \frac{30}{37} \right\rfloor = 0$.

$\therefore 30!$ is not divisible by 37×15^7 .

$\therefore 30!$ is divisible by 45^7 .

Ans. : (D)

(84) 50 objects can be divided into 5 groups of 10, 10, 10, 15 and 5 objects in ways.

(A) $\frac{50!}{(10!)^3 (15!) (5!) (3!)}$ (B) $\frac{50!}{(10!)^3 (15!) (5!)}$ (C) $\frac{50! \times 5}{(10!)^3 (15!) (5!) (3!)}$ (D) $\frac{50!}{(10!)^3 (15!) (3!)}$

Sol. : Here, $n = 50$, $p = 10$, $l = 3$, $q = 15$, $m = 1$, $r = 5$, $s = 1$

$$\text{The total number of ways} = \frac{n!}{(p!)^l (q!)^m (r!)^s l! m! s!}$$

$$= \frac{50!}{(10!)^3 (15!)^1 (5!)^1 3! 1! 1!}$$

$$= \frac{50!}{(10!)^3 (15!) (5!) (3!)}$$

Ans. : (A)

(85) 5 distinct things are to be distributed among three persons, such that each gets at least one thing. Then the total number of ways = (JEE Advanced : 2012)

(A) 300 (B) 150 (C) 25 (D) 120

Sol. : 1, 1, 3 or 2, 2, 1 things are to be distributed among three persons. Then,

$$\text{The total number of ways} = \frac{5!}{(1!)^2 \times 3! \times 2!} \times 3! + \frac{5! \times 3!}{(2!)^2 (1!) 2!} = \frac{120}{2} + \frac{120 \times 6}{8} = 60 + 90 = 150$$

Ans. : (B)

→ **Second method :**

1, 1, 3 or 2, 2, 1 things are to be distributed among three persons.

$$\text{The total number of ways} = \frac{\binom{5}{1} \times \binom{4}{1} \times \binom{3}{3} \times 3!}{2!} + \frac{\binom{5}{2} \times \binom{3}{2} \times \binom{1}{1}}{2!} \times 3! = 60 + 90 = 150$$

Ans. : (B)

(86) There are four balls of different colours and four boxes of colours, same as those of the balls. The numbers of ways in which the balls, one each in a box, could be placed such that a ball is not placed in a box of its own colour is (IIT - 1992)

(A) 3 (B) 6 (C) 9 (D) 12

Sol. : First ball is to be placed in a box of different colour other than the colour of that ball. Total number of ways of this type = 3

After placing a ball in a box; a ball of the same colour as the box can be placed in any box out of remaining three boxes.

∴ Total number of ways of placing the second ball = 3

Total number of ways of putting remaining two balls = 1

∴ Total number of ways = $3 \times 3 \times 1 \times 1 = 9$

Second method :

Total number of ways = ${}_4P_4 - {}_4P_3 + {}_4P_2 - {}_4P_1 + {}_4P_0 = 24 - 24 + 12 - 4 + 1 = 9$ **Ans. : (C)**

- (87) The alphabets of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in a dictionary. The number of words that precede the word COCHIN is (IIT : 2007)

(A) 360 (B) 192 (C) 96 (D) 48

Sol. : Dictionary order of the alphabets of the word COCHIN is C, C, H, I, N, O.

The numbers starting with the alphabets CC = $1 \times 1 \times 4! = 24$

The numbers starting with the alphabets CH = $1 \times 1 \times 4! = 24$

The numbers starting with the alphabets CI = $1 \times 1 \times 4! = 24$

The numbers starting with the alphabets CN = $1 \times 1 \times 4! = 24$

In the words, starting with alphabets CO; the first word is COCHIN.

∴ There are $24 + 24 + 24 + 24 = 96$ words preceding the word COCHIN. **Ans. : (C)**

- (88) Ten different alphabets are given. Words with 5 alphabets are formed from these given alphabets. Then the number of words which have at least one alphabet repeated is (IIT : 1980)

(A) 69760 (B) 30240 (C) 99748 (D) None of these

Sol. : The number of required words = the number of words having 5 alphabets.

The number of words which have at least one alphabet repeated is $10^5 - {}_{10}P_5 = 69760$ **Ans. : (A)**

- (89) Consider all possible permutations of the alphabets of the word ENDEANOEL. Then the number of permutations containing the word ENDEA is (IIT : 2008)

(A) 5! (B) $2 \times 5!$ (C) $7 \times 5!$ (D) $21 \times 5!$

Sol. : Here, consider ENDEA as one alphabet, total number of permutations of five alphabets ENDEA, N, O, E, L = 5! **Ans. : (A)**

- (90) Consider all possible permutations of the alphabets of the word ENDEANOEL. Then the number of permutations in which the alphabet E occurs in the first and last positions is (IIT : 2008)

(A) 5! (B) $2 \times 5!$ (C) $7 \times 5!$ (D) $21 \times 5!$

Sol. : E occurs in the first and last positions in $1 \times 1 = 1$ way.

Seven alphabets N, N, D, A, O, E, L are to be arranged between two E's in $\frac{7!}{2!} = 21 \times 5!$ ways.

∴ The total number of ways of required permutations = $1 \times 21 \times 5! = 21 \times 5!$ **Ans. : (D)**

- (91) Consider all possible permutations of the alphabets of the word ENDEANOEL. Then the number of permutations in which none of the alphabets D, L, N, N occur in the last five positions is (IIT : 2008)

(A) 5! (B) $2 \times 5!$ (C) $7 \times 5!$ (D) $21 \times 5!$

Sol. : The total number of ways of the arrangement of the alphabets D, L, N, N at first four places and remaining alphabets in last five places = $\frac{4!}{2!} \times \frac{5!}{3!} = 12 \times 20 = 240 = 2 \times 5!$ **Ans. : (B)**

- (92) The number of numbers greater than 1000 but not greater than 4000 formed using the digits 0, 1, 2, 3 and 4 is (AIEEE : 2002)
 (A) 576 (B) 450 (C) 375 (D) 350

Sol. : Here, we have to find the numbers between 1000 and 4000. So that the required numbers are of 4 digits.

Thousandth place can be filled in by the digits 1, 2 and 3 and the remaining places can be filled in by the digits 0, 1, 2, 3, 4. Then the total number of numbers = $3 \times 5 \times 5 \times 5 = 375$.

In this 1000 is included but 4000 is not included. But in the calculation, we have to take the number 4000 and not 1000.

$$\therefore \text{Required number of numbers} = 375 - 1 + 1 = 375 \quad \text{Ans. : (C)}$$

- (93) The total number of four digit odd numbers that can be formed using the digits 0, 1, 2, 3, 5 and 7 is (AIEEE : 2002)
 (A) 400 (B) 720 (C) 375 (D) 216

Sol. : To make a four digit odd number; unit place can be filled in by the digits 1, 3, 5 and 7 i.e. in 4 ways, thousandth place can be filled in by the digits 1, 2, 3, 5 and 7 i.e. in 5 ways and the remaining two places can be filled in by the digits 0, 1, 2, 3, 5 and 7 in 6 ways.

$$\therefore \text{Required number of numbers} = 4 \times 5 \times 6 \times 6 = 720 \quad \text{Ans. : (B)}$$

- (94) If ${}_nC_r$ denotes, the combination of r objects out of n objects, then ${}_nC_{(r+1)} + {}_nC_{(r-1)} + 2 \times {}_nC_r =$ (AIEEE : 2003)
 (A) ${}_{(n+2)}C_r$ (B) ${}_{(n+2)}C_{(r+1)}$ (C) ${}_{(n+1)}C_r$ (D) ${}_{(n+1)}C_{(r+1)}$

Sol. : Here, ${}_nC_{(r+1)} + {}_nC_{(r-1)} + 2 \times {}_nC_r$

$$= {}_nC_r + {}_nC_{(r+1)} + {}_nC_{(r-1)} + {}_nC_r$$

$$= {}_{(n+1)}C_{(r+1)} + {}_{(n+1)}C_r$$

$$= {}_{(n+2)}C_{(r+1)} \quad \text{Ans. : (B)}$$

- (95) If a student has to answer atleast four questions out of first five questions, he can answer 10 questions out of 13 questions in ways. (AIEEE : 2003)
 (A) 196 (B) 280 (C) 346 (D) 140

Sol. : The total number of ways = $\binom{5}{4} \binom{8}{6} + \binom{5}{5} \binom{8}{5}$

$$= 5 \times \frac{8 \times 7}{2} + 1 \times \frac{8 \times 7 \times 6}{6}$$

$$= 5 \times 28 + 56 = 140 + 56 = 196 \quad \text{Ans. : (A)}$$

- (96) The total number of ways of distributing 8 identical balls in 3 different boxes in such a way that no box remains empty = (AIEEE : 2004)
 (A) 5 (B) 21 (C) 3^8 (D) ${}_8C_3$

Sol. : Total number of ways of the arrangement of n identical things in r distinct places such that no place is empty = ${}_{(n-1)}C_{(r-1)}$

$$= {}_7C_2 \quad (n = 8, r = 3)$$

$$= 21 \quad \text{Ans. : (B)}$$

→ **Second method :**

8 identical balls are distributed into three parts by (1, 1, 6), (2, 2, 4), (3, 3, 2), (1, 2, 5), (1, 3, 4).

Now, the total number of ways of the arrangement of 1, 1, 6 balls in 3 boxes = 3.

Similarly, the arrangement of 2, 2, 4 balls and 3, 3, 2 balls in three boxes can be done in 3 ways.

1, 2 and 5 balls can be arranged in 3 boxes in $3! = 6$ ways.

1, 3 and 4 balls can be arranged in 3 boxes in $3! = 6$ ways.

∴ 8 identical balls can be arranged in 3 boxes in $3 \times 3 + 2 \times 6 = 9 + 12 = 21$ ways.

- (97) In an election a voter may vote for any number of candidates not greater than the number to be elected. 4 candidates are to be elected from 10 candidates. If a voter votes for at least one candidate, then the number of ways in which he/she can vote is (AIEEE : 2006)

(A) 6210 (B) 1110 (C) 385 (D) 5040

Sol. : The total number of ways of voting $= \binom{10}{1} + \binom{10}{2} + \binom{10}{3} + \binom{10}{4}$
 $= 10 + 45 + 120 + 210 = 385$

Ans. : (C)

- (98) If the set $S = \{1, 2, 3, \dots, 12\}$ is divided into subsets A, B and C having equal number of elements such that $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \emptyset$, then this can be done in ways.

(AIEEE : 2007)

(A) $\frac{12!}{(4!)^4}$ (B) $\frac{12!}{(4!)^3}$ (C) $\frac{12!}{3(4!)^3}$ (D) $\frac{12!}{3(4!)^4}$

Sol. : There are 12 elements in the set S, so that each of A, B and C will have $\frac{12}{3} = 4$ elements.

The total number of ways $= \binom{12}{4} \times \binom{8}{4} \times \binom{4}{4} \frac{3!}{3!} = \frac{12!}{4!8!} \times \frac{8!}{4!4!} \times \frac{3!}{3!}$
 $= \frac{12!}{(4!)^3}$

Ans. : (B)

- (99) There are 10 points in a plane out of which 6 are collinear. If N triangles are formed using all these points, then (AIEEE : 2011)

(A) $N \leq 100$ (B) $100 < N \leq 140$ (C) $140 < N \leq 190$ (D) $N > 190$

Sol. : Maximum $N = \binom{10}{3} - \binom{6}{3} = \frac{10 \times 9 \times 8}{6} - \frac{6 \times 5 \times 4}{6} = 120 - 20 = 100$

∴ $N \leq 100$

Ans. : (A)

- (100) different words can be formed by permuting the alphabets of the word MISSISSIPPI such that no two S are in adjacent positions. (AIEEE : 2008)

(A) $8 \cdot {}^6C_4 \cdot {}^7C_4$ (B) $6 \cdot 7 \cdot {}^8C_4$ (C) $6 \cdot 8 \cdot {}^7C_4$ (D) $7 \cdot {}^6C_4 \cdot {}^8C_4$

Sol. : The alphabets M, I, I, I, I, P, P (other than S, S, S, S) can be arranged in a straight line in

$$\frac{7!}{4!2!} = 7 \times 5 \times 3 = 105 \text{ ways.}$$

S, S, S, S can be arranged in 8 places

shown in 8C_4 ways.

$$\begin{aligned} \therefore \text{The number of required words} &= 7 \times 5 \times 3 \times {}^8C_4 \\ &= 7 \times \frac{6 \times 5}{2} \times {}^8C_4 \\ &= 7 \times {}^6C_2 \times {}^8C_4 \\ &= 7 \times {}^6C_4 \times {}^8C_4 \end{aligned}$$

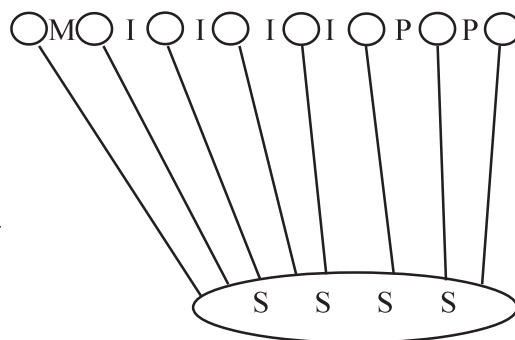


Figure 5.6

Ans. : (D)

- (101) 4 novels and 1 dictionary are to be selected from 6 different novels and 3 different dictionaries and arranged linearly on a shelf so that dictionary is always in the centre. Then the number of such arrangements is (AIEEE : 2009)

- (A) Less than 500 (B) atleast 500 and less than 750
(C) atleast 750 but less than 1000 (D) atleast 1000

Sol. : The total number of ways of the selection of 1 dictionary out of 3 dictionaries = ${}^3C_1 = 3$

The total number of ways of the selection of 4 novels out of 6 novels = ${}^6C_4 = 15$

Now, the total number of ways of the arrangement on a shelf such that a dictionary is in the middle and two novels are on the both sides of the dictionary = $3 \times 15 \times 4! = 3 \times 15 \times 24 = 1080 \geq 1000$

$\therefore$ The total number of ways of this arrangement is atleast 1000.

Ans. : (D)

- (102) If the L.C.M. of p and q is $r^2 t^4 s^2$ where r, s, t are primes. If p and q are positive integers, then the number of ordered pairs (p, q) is (IIT : 2006)

- (A) 225 (B) 252 (C) 254 (D) 224

Sol. : r^2 is a factor of L.C.M. so that there are five possibilities of the exponent of r in (p, q) i.e. $(r^0, r^2), (r^2, r^0), (r^1, r^2), (r^2, r^1), (r^2, r^2)$.

Similarly, there are 9 possibilities for the exponent of t . i.e. $(t^0, t^4), (t^4, t^0), (t^1, t^4), (t^4, t^1), (t^2, t^4), (t^4, t^2), (t^3, t^4), (t^4, t^3), (t^4, t^4)$ and also there are 5 possibilities for the exponent of s in the same manner.

$\therefore$ The number of ordered pairs $(p, q) = 5 \times 9 \times 5 = 225$

Ans. : (A)

- (103) Suppose that all balls are similar except for colour. Out of all balls; one or more of 10 white, 9 green and 7 black balls can be selected in ways. (AIEEE : 2012)

- (A) 629 (B) 630 (C) 880 (D) 879

Sol. : The total number of ways of the selection of at least one thing from a group of p identical things, q identical things and r identical things = $(p+1)(q+1)(r+1) - 1$

$$= (10+1)(9+1)(7+1) - 1$$

$$= 11 \times 10 \times 8 - 1 = 880 - 1 = 879$$

(The total number of ways of the selection of 0, 1, 2, ..., 10 white balls out of 10 white balls = $10 + 1 = 11$ etc.)

Ans. : (D)

(104) Range of the function $f(x) = {}_{(7-x)}P_{(x-3)}$ is (AIEEE : 2004)

- (A) $\{1, 2, 3, 4\}$ (B) $\{1, 2, 3, 4, 5, 6\}$ (C) $\{1, 2, 3\}$ (D) $\{1, 2, 3, 4, 5\}$

Sol. : Here, $x - 3 \geq 0$ and $7 - x \geq x - 3$

$$\therefore x \geq 3 \text{ and } 10 \geq 2x$$

$$\therefore 3 \leq x \text{ and } x \leq 5$$

$$\therefore 3 \leq x \leq 5$$

$$\therefore x = 3, 4, \text{ or } 5$$

$$\therefore f(3) = {}_4P_0 = 1 \text{ and } f(4) = {}_3P_1 = 3 \text{ and } f(5) = {}_2P_2 = 2! = 2$$

$$\therefore \text{Range of } f = \{1, 2, 3\}$$

Ans. : (C)

(105) If $\binom{n-1}{r} = (k^2 - 3) \binom{n}{r+1}$, then $k \in$ (IIT : 2004)

(A) $[-2, -\sqrt{3}] \cup [\sqrt{3}, 2]$

(B) $(-2, -\sqrt{3}) \cup (\sqrt{3}, 2)$

(C) $(-2, -\sqrt{3}) \cup [\sqrt{3}, 2]$

(D) $[-2, -\sqrt{3}) \cup (\sqrt{3}, 2]$

Sol. : Here, $\frac{(n-1)!}{(n-r-1)!r!} = (k^2 - 3) \frac{n!}{(n-r-1)!(r+1)!}$

$$\therefore \frac{r+1}{n} = k^2 - 3$$

$$\text{Now, } 0 \leq r \leq n-1 \Rightarrow 1 \leq r+1 \leq n$$

$$\Rightarrow \frac{1}{n} \leq \frac{r+1}{n} \leq 1$$

$$\Rightarrow 0 < \frac{r+1}{n} \leq 1$$

$$\Rightarrow 0 < k^2 - 3 \leq 1$$

$$\Rightarrow 3 < k^2 \leq 4$$

$$\Rightarrow k \in [-2, -\sqrt{3}) \cup (\sqrt{3}, 2]$$

Ans. : (D)

(106) If $n(A) = 4$ and $n(B) = 2$, then the number of subsets of $A \times B$ having at least three elements is (JEE 2013-15)

(A) 219

(B) 256

(C) 275

(D) 510

Sol. : Here, $n(A) = 4$ and $n(B) = 2$, so, $n(A \times B) = 4 \times 2 = 8$

$$\therefore \text{The number of subsets of } A \times B = 2^8 = 256$$

$$\text{and the number of subsets of } A \times B \text{ with 0 element} = \binom{8}{0} = 1$$

The number of subsets of $A \times B$ with 1 element $= \binom{8}{1} = 8$

The number of subsets of $A \times B$ with 2 elements $= \binom{8}{2} = 28$

$\therefore$ The number of subsets of $A \times B$ with atleast 3 elements $= 256 - 1 - 8 - 28 = 219$ **Ans. : (A)**

- (107) positive integers greater than 6000 can be formed using the digits 3, 5, 6, 7 and 8, if repetition is not allowed. (JEE : 2015)

(A) 216 (B) 192 (C) 120 (D) 72

Sol. : Case 1 : The number of positive integers of five digits greater than 6000 $= 5! = 120$

Case 2 : For a four digit number, 6, 7 or 8 are possible at thousandth place.

$\therefore$ The number of ways to fill in thousand's place $= 3$

The number of ways to fill in hundred's place $= 4$

The number of ways to fill in ten's place $= 3$

The number of ways to fill in unit's place $= 2$

The total number of 4 digit numbers greater than 6000 $= 3 \times 4 \times 3 \times 2 = 72$

The total number of numbers greater than 6000 $= 120 + 72 = 192$ **Ans. : (B)**

- (108) Let T_n be the number of all possible triangles formed by joining the vertices of n sided convex polygon. If $T_{n+1} - T_n = 10$, then the value of n is (JEE : 2013)

(A) 7 (B) 5 (C) 10 (D) 8

Sol. : Here $T_{n+1} - T_n = 10$

$$\therefore \binom{n+1}{3} - \binom{n}{3} = 10$$

$$\therefore \frac{(n+1)n(n-1)}{6} - \frac{n(n-1)(n-2)}{6} = 10$$

$$\therefore n(n-1)(n+1-n+2) = 60$$

$$\therefore n(n-1)3 = 60$$

$$\therefore n(n-1) = 20 = 5 \times 4$$

$$\therefore n = 5$$

Ans. : (B)

- (109) If the prime factorisation of $\binom{100}{50}$ is $2^a 3^b 5^c 7^d \dots$, then from the following is true.
(where $a, b, c, d \dots \in \mathbb{N}$)

(A) $a < b$ (B) $a < d$ (C) $a + d = b + c + 1$ (D) $a + c = b + d$

Sol. : The largest exponent of 2 in $100!$ $= \left[\frac{100}{2} \right] + \left[\frac{100}{4} \right] + \left[\frac{100}{8} \right] + \left[\frac{100}{16} \right] + \left[\frac{100}{32} \right] + \left[\frac{100}{64} \right] \dots$
 $= 50 + 25 + 12 + 6 + 3 + 1 = 97$

$$\begin{aligned}\text{The largest exponent of 2 in } 50! &= \left[\frac{50}{2} \right] + \left[\frac{50}{4} \right] + \left[\frac{50}{8} \right] + \left[\frac{50}{16} \right] + \left[\frac{50}{32} \right] \\ &= 25 + 12 + 6 + 3 + 1 = 47\end{aligned}$$

$$\therefore \binom{100}{50} = \frac{100!}{(50!)^2} = \frac{2^{97}}{2^{47} \times 2^{47}} \times k = 2^3 \times k \quad (k \neq 2n, n \in \mathbb{N})$$

$$\therefore a = 3$$

$$\begin{aligned}b &= \left[\frac{100}{3} \right] + \left[\frac{100}{9} \right] + \left[\frac{100}{27} \right] + \left[\frac{100}{81} \right] - 2 \left[\left[\frac{50}{3} \right] + \left[\frac{50}{9} \right] + \left[\frac{50}{27} \right] \right] \\ &= 33 + 11 + 3 + 1 - 2(16 + 5 + 1) = 4\end{aligned}$$

$$\therefore a < b \text{ is true.}$$

Ans. : (A)

$$c = \left[\frac{100}{5} \right] + \left[\frac{100}{25} \right] - 2 \left[\left[\frac{50}{5} \right] + \left[\frac{50}{25} \right] \right] = 20 + 4 - 2(10 + 2) = 0$$

$$d = \left[\frac{100}{7} \right] + \left[\frac{100}{49} \right] - 2 \left[\left[\frac{50}{7} \right] + \left[\frac{50}{49} \right] \right] = 14 + 2 - 2(7 + 1) = 0$$

$$\therefore \text{(B), (C) and (D) are not true. So. Ans. (A).}$$

Write the answers from the following paragraph :

(110) By using the digits 0, 1 or both,

- (i) The number of integers of n digits such that two 0 do not occur in adjacent positions $= a_n$
- (ii) The number of integers of n digits such that two 0 do not occur in adjacent positions and the last digit is 1 $= b_n$ and
- (iii) The number of integers of n digits such that two 0 do not occur in adjacent positions and the last digit is 0 $= c_n$.

(1) Which one is true from the following ?

$$(A) a_{17} = a_{16} + a_{15} \quad (B) c_{17} \neq c_{16} + c_{15} \quad (C) b_{17} = b_{16} + c_{16} \quad (D) a_{17} = c_{17} + b_{16}$$

Sol. : (I) Here one digit numbers are 1 and 1 and none

$$\therefore a_1 = 1, b_1 = 1, c_1 = 0$$

Two digit numbers are 11, 10 and 11 and 10.

$$\therefore a_2 = 2, b_2 = 1, c_2 = 1$$

Three digits numbers are 111, 101, 110. Out of them last digit is 1 in 2 numbers and if last digit is 0, then there is one number

$$\therefore a_3 = 3, b_3 = 2, c_3 = 1$$

Four digit numbers are 1111, 1011, 1101, 1110, 1010. Out of them last digit is 1 in 3 numbers i.e. 1111, 1011, 1101 and last digit is 0 in 2 numbers i.e. 1110 and 1010.

$$\therefore a_4 = 5, b_4 = 3, c_4 = 2$$

Thus, $a_n + a_{n+1} = a_{n+2}$, $b_n + b_{n+1} = b_{n+2}$ and

$$c_n + c_{n+1} = c_{n+2} \text{ and } b_n + c_n = a_n \quad (n \in \mathbb{N})$$

∴ For the option (A), $a_{17} = a_{16} + a_{15}$ is true.

$c_{17} \neq c_{16} + c_{15}$ is not true because $c_n + c_{n+1} = c_{n+2}$.

For the option (C), $b_{16} + c_{16} = a_{16} \neq b_{17}$ is not true.

For the option (D), $a_{17} = b_{17} + c_{17} = c_{17} + b_{16}$ because $b_{17} = b_{16}$ which is not true.

(B) is not true.

∴ Correct answer is $a_{17} = a_{16} + a_{15}$.

Ans. : (A)

(II) Value of b_6 is

(A) 7

(B) 8

(C) 9

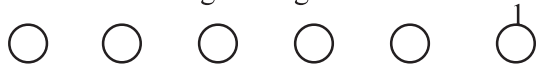
(D) 11

$$b_6 = b_4 + b_5 = b_4 + b_4 + b_3 = 2b_4 + b_3 = 8$$

Ans. : (B)

Second method :

A number having last digit 1 is formed as follows :



If 0 does not lie in place out of 4 places in the middle, then total number of numbers of this type = 1

If 0 is placed only once, then the total number of numbers = 4 and

If 1 is placed twice, then the total number of numbers = 3 (0101, 1010, 0110)

∴ The total number of numbers = 8

(Here, for n digit integers, it is compulsory that 1 is at first position.)

Ans. : (B)

(111) There are six letters and six envelopes. The numbers 1 to 6 are written on each of them. A letter numbered 1 is placed in the envelope numbered 2 and if no letter is placed in the same numbered envelope, then only one letter can be placed in each envelope in ways. (JEE Advance : 2014)

(A) 264

(B) 265

(C) 53

(D) 67

Sol. : If any letter cannot be placed in the same numbered envelope, then total number of ways

$$\begin{aligned} &= {}_6P_6 - {}_6P_5 + {}_6P_4 - {}_6P_3 + {}_6P_2 - {}_6P_1 + {}_6P_0 \\ &= 720 - 720 + 360 - 120 + 30 - 6 + 1 \\ &= 265 \end{aligned}$$

The letter numbered 1 can be placed in the other numbered envelopes in 5 ways. From it, if the letter numbered 1 is placed in the envelope numbered 2, then total number of ways = $\frac{265}{5} = 53$

Ans. : (C)

(112) n distinct points lying on a circle are joined by line-segments. Colour each line-segment formed with two adjacent points by blue colour and other line-segments by red colour. If the number of blue and red line-segments are equal, then $n = \dots$ (where $n \geq 2, n \in \mathbb{N}$)

(A) 4

(B) 5

(C) 6

(D) 7

Sol. : Here a red line-segment becomes a diagonal of the polygon and a blue line-segment becomes

a side of the polygon. So the number of diagonals = $\binom{n}{2} - n$ and the number of sides = n .

$$\therefore \binom{n}{2} - n = n$$

$$\therefore n(n-1) = 4n$$

$$\therefore n^2 - 5n = 0$$

$$\therefore n(n-5) = 0$$

$$\therefore n = 0 \text{ or } n = 5$$

$$\therefore n = 5$$

Ans. : (B)

- (113) The number of distinct arrangements, of five positive integers (n_1, n_2, n_3, n_4, n_5) such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$ are, where, $n_1 < n_2 < n_3 < n_4 < n_5$ (JEE Advanced : 2014)
 (A) 12 (B) 9 (C) 8 (D) 7

Sol. : Here, the arrangement is possible in the following seven ways. i.e.

$$(1, 2, 3, 4, 10), (1, 2, 3, 5, 9), (1, 2, 3, 6, 8)$$

$$(1, 2, 4, 5, 8), (1, 2, 4, 6, 7), (1, 3, 4, 5, 7) \quad (2, 3, 4, 5, 6)$$

Ans. : (D)

- (114) There are n lines in a plane. No two of them are parallel or coincident and no three lines are concurrent. By joining the points of intersection of these lines new lines can be formed.
 (A) $\frac{1}{8} n(n-1)(n-2)(n-3)$ (B) $\frac{1}{8} n(n-1)(n-2)(n-3) + n$
 (C) $\frac{1}{8} n(n-1)(n-2)(n-3) - n$ (D) $\frac{1}{8} n(n^2-1)(n-2) - n$

Sol. : The number of points of intersection of given lines = ${}_nC_2 = N$. (Say)

Every line intersects remaining $(n-1)$ lines in $(n-1)$ points. These $(n-1)$ points are collinear. Only one line is obtained by joining any two of these $(n-1)$ points.

$\therefore$ The number of new lines

$$= {}_NC_2 - {}_nC_1 \times {}_{(n-1)}C_2$$

$$= \frac{1}{2} N(N-1) - n \times \frac{(n-1)(n-2)}{2}$$

$$= \frac{1}{2} \frac{n(n-1)}{2} \left\{ \frac{n(n-1)}{2} - 1 \right\} - \frac{n(n-1)(n-2)}{2} \quad N = \frac{n(n-1)}{2}$$

$$= \frac{1}{8} n(n-1)(n^2 - n - 2) - \frac{1}{2} n(n-1)(n-2)$$

$$= \frac{1}{8} n(n-1)(n^2 - n - 2 - 4n + 8)$$

$$= \frac{1}{8} n(n-1)(n^2 - 5n + 6)$$

$$= \frac{1}{8} n(n-1)(n-2)(n-3)$$

Ans. : (A)

- (115) There are n points in a plane. 4 of them are collinear. distinct quadrilaterals can be formed by taking these points as the vertices.

$$(A) \binom{n}{4} - 1 \quad (B) \binom{n}{4} - 4n + 15 \quad (C) \binom{n-4}{4} \quad (D) \binom{n}{4} - 5$$

Sol. : A quadrilateral can be formed using 4 distinct points. But from these four points, it is necessary that more than two points are non-collinear.

The total number of ways of the selection of 4 points out of n points = $\binom{n}{4}$ and

The total number of ways of the selection of 3 points out of 4 collinear points and 1 point from the remaining points $= \binom{4}{3} \binom{n-4}{1} = 4(n-4) = 4n-16$

Total number of ways of selection of 4 points out of 4 collinear points $= \binom{4}{4} = 1$

$$\begin{aligned} \therefore \text{The number of required quadrilaterals} &= \binom{n}{4} - \binom{4}{3} \binom{n-4}{1} - \binom{4}{4} \\ &= \binom{n}{4} - (4n-16) - 1 \\ &= \binom{n}{4} - 4n + 15 \end{aligned} \quad \text{Ans. : (B)}$$

- (116) The alphabets of the word MATHEMATICS which are not repeated are placed at odd positions and the alphabets which are repeated are placed at even positions, then the number of words of 6 alphabets formed in this way is

(A) 720 (B) 540 (C) 2520 (D) 102

Sol. : To form a word of 6 alphabets, we have 3 odd and 3 even places.

Non-repeated 5 alphabets H, E, I, C, S are to be arranged in 3 odd places in ${}_5P_3 = 60$ ways.

Now, even places can be filled in two ways,

- (1) Two identical and one distinct alphabet are selected. Out of 3 groups (MM, AA, TT) one

group can be selected in $\binom{3}{1}$ ways and third different alphabet can be selected from remaining two groups in $\binom{2}{1}$ ways.

Now, the selected three alphabets can be arranged in $3!$ ways.

- (2) All distinct alphabets are selected. This arrangement can be done in $3!$ ways.

$\therefore$ Even places can be filled in $\binom{3}{1} \binom{2}{1} 3! + 3! = 36 + 6 = 42$ ways.

$\therefore$ The total number of words of 6 alphabets $= 60 \times 42 = 2520$ **Ans. : (C)**

- (117) Six persons A, B, C, D, E and F are seated on a circular table such that the person B or C is seated to the right side of person A and person C or D is seated to the right side of person B. This arrangement is possible in ways.

(A) 18 (B) 72 (C) 20 (D) 16

Sol. : Allot number 1 at the position of person A and give numbers 2, 3, 4, 5 and 6 in anticlockwise direction at the position of remaining 5 persons.

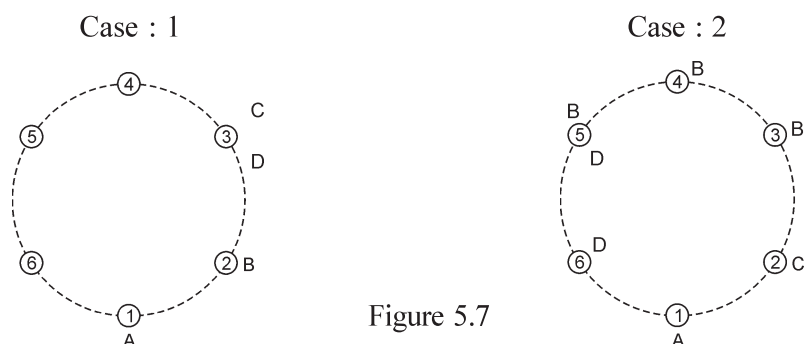


Figure 5.7

- (1) If B is at second position, then C or D is at third position and the remaining three persons can be seated in three positions in $3!$ ways.

$$\therefore \text{In this case the total number of ways of this arrangement} = 1 \times 1 \times \binom{2}{1} \times 3! = 2 \times 6 = 12$$

- (2) If C is at second position, then the person B can be seated in $\binom{3}{1}$ ways in third or fourth or fifth position. Person D can be seated in one way to the right side of person B and in remaining two positions two persons can be seated in $2! = 2$ ways.

Here, the persons B and D can be seated at 3, 4 or 4, 5 or 5, 6 positions.

$$\begin{aligned} \text{In this case, the total number of ways of this arrangement} &= 1 \times 1 \times \binom{3}{1} \times \binom{1}{1} \times 2! \\ &= 3 \times 2 = 6 \end{aligned}$$

$$\therefore \text{The total number of ways of the arrangement} = 12 + 6 = 18 \quad \text{Ans. : (A)}$$

- (118) If 5 boys and 5 girls are seated in a line in n ways such that 5 girls are in adjacent positions and 5 boys and 5 girls are seated in a line in m ways such that only 4 girls are in adjacent positions, then $m:n = \dots\dots\dots$. (JEE-Advanced : 2015)

Sol. : Five girls can be seated in a line in $5!$ ways.

Now, the group of 5 girls and 5 boys can be seated in a line in $6!$ ways.

$$\therefore \text{The total number of ways of the arrangement of five girls in adjacent positions} = n = 5! \times 6!$$



The group of 4 girls

Now, 4 out of 5 girls are to be arranged in a line in ${}_5P_4$ ways.

Consider the group of 4 girls as one person. This one person and 5 boys i.e. 6 persons can be arranged in a row in $6!$ ways.

Then the remaining girl can be seated in ${}_5P_1 = 5$ ways (i.e. $\square$ = square) such that she does not come to the adjacent position of the group of 4 girls.

Thus, $m =$ The total number of ways of the arrangements of 5 boys and 5 girls such that only 4 girls are in adjacent position $= {}_5P_4 \times 6! \times 5$

$$\therefore \frac{m}{n} = \frac{{}_5P_4 \times 6! \times 5}{6! \times 5!} = 5$$

$$\therefore m : n = 5$$

- (119) In the expansion of $(1+x)(1+x^2)(1+x^3)\dots(1+x^{100})$ the coefficient of x^9 is

(JEE Advanced : 2015)

Sol. :

- (1) In $(1+x^9)$; the coefficient of $x^9 = 1$
- (2) In $(1+x)(1+x^8)$; the coefficient of $x^9 = 1$
- In $(1+x^2)(1+x^7)$; the coefficient of $x^9 = 1$
- In $(1+x^3)(1+x^6)$; the coefficient of $x^9 = 1$
- In $(1+x^4)(1+x^5)$; the coefficient of $x^9 = 1$

- (3) In $(1+x)(1+x^2)(1+x^6)$; the coefficient of $x^9 = 1$
 In $(1+x)(1+x^3)(1+x^5)$; the coefficient of $x^9 = 1$
 In $(1+x^2)(1+x^3)(1+x^4)$; the coefficient of $x^9 = 1$
 $\therefore$ Coefficient of $x^9 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 8$

Note : $1 + 8, 2 + 7, 3 + 6, 4 + 5, 1 + 2 + 6, 1 + 3 + 5, 2 + 3 + 4$

Ans. : 8

- (120) The set of eight vectors $V = \{a\hat{i} + b\hat{j} + c\hat{k} \mid a, b, c \in \{-1, 1\}\}$ is given. If three non-coplanar vectors are to be selected from V in 2^p ways, then $p = \dots$

Sol. : Here, the 8 vectors can be grouped as follows in group 1 and group 2

Group 1

Group 2

$$\hat{i} + \hat{j} + \hat{k}$$

$$-\hat{i} - \hat{j} - \hat{k}$$

$$-\hat{i} + \hat{j} + \hat{k}$$

$$\hat{i} - \hat{j} - \hat{k}$$

$$\hat{i} - \hat{j} + \hat{k}$$

$$-\hat{i} + \hat{j} - \hat{k}$$

$$-\hat{i} - \hat{j} + \hat{k}$$

$$\hat{i} + \hat{j} - \hat{k}$$

$2^p =$ The total number of ways of the selection of three non-coplanar vectors.

$$= \binom{4}{2} \times \binom{2}{1} + \binom{4}{1} \binom{3}{2} + \binom{4}{3} \binom{4}{0} + \binom{4}{0} \binom{4}{3}$$

$$= 12 + 12 + 4 + 4$$

$$= 32$$

$$\therefore p = 5$$

Ans. : 5

- (121) There are n cards in a heap marked with integers 1 to n . From this heap, two cards labelled consecutive integers are withdrawn. Then the sum of the integers on the remaining cards is 1224. If the least integer on a card from the cards withdrawn is k , then $k - 20 = \dots$

(JEE Advanced : 2013)

Sol. : The sum of n integers $= \frac{n(n+1)}{2}$

If the numbers of cards withdrawn are k and $k + 1$, then the sum of the integers of remaining cards =

$$\frac{n(n+1)}{2} - k - (k+1) = 1224$$

$$\therefore n(n+1) = 2450 + 4k$$

$$\therefore n(n+1) = 2550 - 100 + 4k$$

$$\therefore n(n+1) = 50 \times 51 - 100 + 4k$$

$$\therefore n = 50 \text{ and } k = 25$$

$$\therefore k - 20 = 5$$

Ans. : 5

- (122) In the expansion of $(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$; the coefficient of x^{11} is $\dots$

(JEE Advanced : 2014)]

(A) 1051

(B) 1106

(C) 1113

(D) 1120

Sol. : In the multiplication; x^{11} occurs as follows :

$$\begin{aligned} x^{11} &= x^0 x^3 x^8 = (x^2)^0 (x^3)^1 (x^4)^2, \\ &= x^8 x^3 x^0 = (x^2)^4 (x^3)^1 (x^4)^0 \\ &= x^2 x^9 x^0 = (x^2)^1 (x^3)^3 (x^4)^0, \\ &= x^4 x^3 x^4 = (x^2)^2 (x^3)^1 (x^4)^1 \end{aligned}$$

$$\begin{aligned} \therefore \text{The coefficient of } x^{11} &= \binom{4}{0} \binom{7}{1} \binom{12}{2} + \binom{4}{4} \binom{7}{1} \binom{12}{0} + \binom{4}{1} \binom{7}{3} \binom{12}{0} + \binom{4}{2} \binom{7}{1} \binom{12}{1} \\ &= (1 \times 7 \times 66) + (1 \times 7 \times 1) + (4 \times 35 \times 1) + (6 \times 7 \times 12) \\ &= 462 + 7 + 140 + 504 \\ &= 1113 \end{aligned}$$

Ans. : (C)

(123) four digit numbers greater than 4000 can be formed using the digits 3, 3, 5, 5, 5, 8, 8, 8 and 8.

(A) 48

(B) 60

(C) 51

(D) 87

Sol. :

Case 1 : If 8 is at thousand's place then,

⑧ ○ ○ ○
3, 3, 5, 5, 5, 8, 8, 8

The total number of numbers with three distinct digits or three identical digits or two

$$\text{identical digits (one pair) and one different digit} = 3! + 2 + \binom{3}{1} \binom{2}{1} \frac{3!}{2!} = 6 + 2 + 18 = 26$$

Case 2 : 5 is at thousand's place then

⑤ ○ ○ ○
3, 3, 5, 5, 8, 8, 8, 8

The total number of numbers with three distinct digits or three identical digits or two identical

$$\text{digits (one pair) and one different digit} = 3! + 1 + \binom{3}{1} \binom{2}{1} \frac{3!}{2!} = 6 + 1 + 18 = 25$$

$$\text{The total number of required 4 digit numbers} = 26 + 25 = 51$$

Ans. : (C)

(124) Using the alphabets of the word GUJARAT four alphabet words can be formed such that the letter G always occur in that word.

(A) 300

(B) 296

(C) 336

(D) 288

Sol. :

Sr.No.	Type	Number of Selections	Number of Arrangements
1	All the 4 alphabets are distinct	$\binom{1}{1} \binom{5}{3} = 10$	$10 \times 4! = 240$
2	2 A and 2 distinct selected alphabets	$\binom{1}{1} \binom{2}{2} \binom{4}{1} = 4$	$4 \times \frac{4!}{2!} = 48$

$$\text{The total number of words} = 240 + 48 = 288$$

Ans. : (D)

(125) five alphabet words can be formed using 5A, 4B, 3C and 2D.

(A) 901

(B) 900

(C) 41

(D) 910

Sol. : If all 5 alphabets are same, then total number of words = 1 (i.e. AAAAA)

The total number of words with 4 same alphabets and one distinct alphabet

$$= \binom{2}{1} \times \binom{3}{1} \times \frac{5!}{4!} = 2 \times 3 \times 5 = 30$$

(e.g. AAAAB, BBBBD, BCBBB)

The total number of words with 3 same alphabets and 2 different but identical alphabets

$$= \binom{3}{1} \binom{3}{1} \times \frac{5!}{3!2!} = 3 \times 3 \times 10 = 90 \text{ (e.g. AAADD)}$$

The total number of words with 3 same alphabets and 2 distinct alphabets

$$= \binom{3}{1} \times \binom{3}{2} \times \frac{5!}{3!} = 3 \times 3 \times 20 = 180 \text{ (e.g. AAABC)}$$

The total number of words with 2 pairs of 2 alphabets and 1 different alphabet

$$= \binom{4}{2} \times \binom{2}{1} \times \frac{5!}{2!2!} = 6 \times 2 \times 30 = 360 \text{ (e.g. AABBC)}$$

The total number of words with 1 pair of 2 alphabets and 3 distinct alphabets

$$= \binom{4}{1} \times \binom{3}{3} \times \frac{5!}{2!} = 4 \times 60 = 240 \text{ (e.g. AABCD)}$$

The total number of required words = 1 + 30 + 90 + 180 + 360 + 240 = 901

Ans. : (A)

: Assertion and Reason Type Questions :

Explanation : Each of these questions contains statement 1 (Assertion) and statement 2 (Reason). Each question has four choices, option (A), (B), (C) and (D), which are as follows. You have to select the correct choice.

(A) Statement 1 is true, statement 2 is true. Statement 2 is the correct explanation for statement 1.

(B) Statement 1 is true, statement 2 is true. Statement 2 is not a correct explanation for statement 1.

(C) Statement 1 is true, statement 2 is false.

(D) Statement 1 is false, statement 2 is true.

(126) There are five types of ice-creams available in a shop. A child buys six ice-creams.

Statement 1 : The number of different ways the child can buy six ice-creams is ${}_{10}C_5$.

Statement 2 : The number of different ways the child can buy six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row. (AIEEE : 2008)

Sol. : 6 A's and 4 B's are to be arranged in a row in $= \frac{10!}{6!4!} = {}_{10}C_6$ ways

= The number of selections of 6 out of 10 ice-creams $\neq {}_{10}C_5$

$\therefore$ Statement 2 is true and statement 1 is false.

Ans. : (D)

- (127) **Statement 1** : 512 matrices of the order 3×3 are to be formed with the help of digits 0 and 1.
Statement 2 : If repetition is allowed, then n distinct things can be arranged in m places in n^m ways.

Sol. : Here, each object can be arranged at any places out of m place in n ways.

$\therefore$ The total number of ways of the arrangement = $n \times n \times n \times \dots m \text{ times} = n^m$

$\therefore$ The total number of ways of the arrangement of 0, 1 in 9 places is $2^9 = 512$.

$\therefore$ Statements 1 and 2 both are true and statement 2 is the correct explanation of statement 1.

Ans. : (A)

- (128) **Statement 1** : The number of zeros at the end of $70!$ is 16.

Statement 2 : In the prime factorization of each positive integer n ; the largest exponent of 10 and the largest exponent of 5 are same.

Sol. : The largest exponent of 5 in $70! = \left\lfloor \frac{70}{5} \right\rfloor + \left\lfloor \frac{70}{25} \right\rfloor + \left\lfloor \frac{70}{125} \right\rfloor = 14 + 2 + 0 = 16$

$\therefore$ The number of zeros at the end of $70!$ is 16.

$\therefore$ Statement 1 is true.

Statement 2 is false because, the largest exponent of 5 in $125 = 3$ and the largest exponent of 10 in $125 = 0$.

$\therefore$ The largest exponent of 5 $\neq$ the largest exponent of 10.

It is clear that if n is not divisible by 2, then the largest exponent of 10 is 0.

Ans. : (C)

- (129) **Statement 1** : Product of n consecutive integers is divisible by $n!$

Statement 2 : Product of any n consecutive integers is not divisible by $(n+1)!$ ($\forall n \in \mathbb{N}$)

Sol. : Product of n consecutive integers = $n(n-1) \dots (n-r+1) = {}_n P_r = \frac{n!}{(n-r)!} \times \frac{1}{r!} \times r!$

$\therefore$ Product of r consecutive integers = $\binom{n}{r} \times r!$ is divisible by $r!$ because $\binom{n}{r} \in \mathbb{N}$

$\therefore$ Statement 1 is true.

Statement 2 is false because 24×25 is divisible by $(2+1)! = 3!$

($n=2$)

Ans. : (C)

- (130) **Statement 1** : $(n)!$ is divisible by $n^{(n-1)!}$

Statement 2 : $(kn)!$ is divisible by $(n!)^k$ ($k, n \in \mathbb{N}$)

Sol. : If kn distinct things are distributed in k equal parts, then the total number of ways to do this

$$= \binom{kn}{n} \binom{kn-n}{n} \binom{kn-2n}{n} \dots \binom{2n}{n} \binom{n}{n} \times \frac{1}{k!} = \frac{(kn)!}{(n!)^k k!} \text{ is an integer.}$$

$\therefore (kn)!$ is divisible by $(n!)^k$.

$\therefore$ Statement 2 is true.

Now, taking $k = (n-1)!$ in the statement 2,

$\{(n-1)! n\}$ is divisible by $(n!)^{(n-1)!}$.

$\therefore (n!)!$ is divisible by $(n!)^{(n-1)!}$.

$\therefore$ Statement 1 is also true and statement 1 can be proved with the help of statement 2.

$\therefore$ Statement 2 is the correct explanation of statement 1.

Ans. : (A)

: Questions with More Than One Correct Answers :

(131) $28!$ is divisible by

(A) 30^6

(B) 30^7

(C) 90^6

(D) 6×90^6

Sol. : The largest exponent of 2 in $28! = \left[\frac{28}{2} \right] + \left[\frac{28}{4} \right] + \left[\frac{28}{8} \right] + \left[\frac{28}{16} \right] = 14 + 7 + 3 + 1 = 25$

The largest exponent of 3 in $28! = \left[\frac{28}{3} \right] + \left[\frac{28}{9} \right] + \left[\frac{28}{27} \right] = 9 + 3 + 1 = 13$

The largest exponent of 5 in $28! = \left[\frac{28}{5} \right] + \left[\frac{28}{25} \right] = 5 + 1 = 6$

$\therefore 28! = 2^{25} \times 3^{13} \times 5^6 \times 7^a \times 11^b \times 13^c \times \dots$

$= 30^6 \times 2^{19} \times 3^7 \times 7^a \times 11^b \times 13^c \times \dots = 30^6 \times 3^6 \times 2^{19} \times 3 \times 7^9 \dots = 90^6 \times 2 \times 3 \times 2^{18} \times 7^9 \dots$

Hence $(28)!$ is divisible by 30^6 , 90^6 and 6×90^6 but not divisible by 30^7 because the largest exponent of 5 in $28!$ is 6.

Ans. : (A), (C), (D)

(132) $\binom{2n+1}{r}$ is maximum for $r = \dots \dots \dots (n \in \mathbb{N})$.

(A) $\left\lfloor \frac{2n+1}{2} \right\rfloor$

(B) $\left\lfloor \frac{2n+3}{2} \right\rfloor$

(C) $\left\lfloor \frac{2n+5}{2} \right\rfloor$

(D) $[n+1]$

Sol. : $\left\lfloor \frac{2n+1}{2} \right\rfloor = \left\lfloor n + \frac{1}{2} \right\rfloor = n + 1$

$\left\lfloor \frac{2n+3}{2} \right\rfloor = \left\lfloor n + \frac{3}{2} \right\rfloor = \left\lfloor (n+1) + \frac{1}{2} \right\rfloor = n + 1$

$\left\lfloor \frac{2n+5}{2} \right\rfloor = \left\lfloor (n+2) + \frac{1}{2} \right\rfloor = n + 2$

$[n+1] = n + 1$

If n is an odd then for, $r = \frac{n+1}{2}$ and $r = \frac{n-1}{2}$; $\binom{n}{r}$ is maximum.

Here, $2n+1$ is an odd. So for $r = \frac{2n+1+1}{2} = n+1$ and $r = \frac{2n+1-1}{2} = n$; $\binom{2n+1}{r}$ is maximum.

$\therefore$ For $r = n+1$ and $r = n$; $\binom{2n+1}{r}$ is maximum.

$\therefore$ Options (A), (B), (D) are correct answers.

Ans. : (A), (B), (D)

(133) The number of diagonals of a convex polygon with $2n$ sides is

- (A) $\binom{n}{2} - 2n$ (B) $2n^2 - 3n$ (C) $\binom{2n}{2}$ (D) $\binom{2n}{2} - 2n$

$$\begin{aligned}
 \text{Sol. : The number of diagonals} &= \binom{2n}{2} - 2n \\
 &= \frac{(2n)(2n-1)}{2} - 2n \\
 &= 2n \left(\frac{2n-1}{2} - 1 \right) \\
 &= 2n \frac{(2n-3)}{2} \\
 &= n(2n-3) = 2n^2 - 3n
 \end{aligned}$$

Ans. : (B), (D)

(134) $\binom{2n}{4}$, $\binom{2n}{5}$ and $\binom{2n}{6}$ are in arithmetic progression for $n = \dots\dots$.

- (A) $\frac{7}{2}$ (B) 7 (C) $\frac{9}{2}$ (D) $\frac{7}{2}$ or 7

Sol. : Suppose $2n = m$

$\binom{m}{4}$, $\binom{m}{5}$ and $\binom{m}{6}$ are in arithmetic progression.

$$\Leftrightarrow \binom{m}{4} + \binom{m}{6} = 2\binom{m}{5}$$

$$\Leftrightarrow \frac{m!}{4!(m-4)!} + \frac{m!}{6!(m-6)!} = 2 \frac{m!}{5!(m-5)!}$$

$$\Leftrightarrow \frac{1}{(m-4)(m-5)} + \frac{1}{6 \times 5} = \frac{2}{5(m-5)}$$

$$\Leftrightarrow 30 + (m-4)(m-5) = 12(m-4)$$

$$\Leftrightarrow 30 + m^2 - 9m + 20 = 12m - 48$$

$$\Leftrightarrow m^2 - 21m + 98 = 0$$

$$\Leftrightarrow (m-7)(m-14) = 0$$

$$\Leftrightarrow m = 7 \text{ or } m = 14$$

$$\Leftrightarrow 2n = 7 \text{ or } 2n = 14$$

$$\Leftrightarrow n = \frac{7}{2} \text{ or } n = 7$$

Ans. : (A), (B), (D)

(135) If n is a positive integer, then is true from the following.

- (A) $n! \leq n^{n+1}$ (B) $n! (n^n) < (2n)!$ (C) $n! (n^n) \leq (2n)!$ (D) $n! < (n^n)$

Sol. : $n! = 1 \times 2 \times 3 \times \dots \times n \leq n \times n \times n \times \dots \times n$ (n times)

$$\leq n^n$$

$$\leq n^n \times n$$

$$\leq n^{n+1}$$

$\therefore n! \leq n^{n+1}$ is true.

$$n! (n^n) = 1 \times 2 \times 3 \times \dots \times n \times (n \times n \times n \times \dots \times n \text{ } n \text{ times})$$

$$< 1 \times 2 \times 3 \times \dots \times n \times (n+1) \times (n+2) \times \dots \times (n+n)$$

$$= 1 \times 2 \times 3 \times \dots \times 2n$$

$$= (2n)!$$

$\therefore n! (n^n) < (2n)!$ is true.

But, for $n = 1$, $n! < n^n$ is not true.

Ans. : (A), (B), (C)

(136) If $x = \dots$, then $\binom{x-1}{4} - \binom{x-1}{3} < \frac{5}{4} (x-2)P_2$

(A) 5, 6, 7, 8, 9, 10

(B) 9, 10

(C) 7, 8

(D) 6

Sol. : Here, $x-1 \geq 4$, $x-1 \geq 3$, $x-2 \geq 2$

$$\therefore x \geq 5, x \geq 4$$

$$\therefore x \geq 5$$

$$\text{and } \frac{(x-1)(x-2)(x-3)(x-4)}{24} - \frac{(x-1)(x-2)(x-3)}{6} < \frac{5}{4} (x-2)(x-3)$$

$$\Leftrightarrow (x-1)(x-4) - 4(x-1) < 30$$

$$\Leftrightarrow x^2 - 5x + 4 - 4x + 4 < 30$$

$$\Leftrightarrow x^2 - 9x - 22 < 0$$

$$\Leftrightarrow (x-11)(x+2) < 0$$

$$\Leftrightarrow -2 < x < 11$$

$$\Leftrightarrow x = 5, 6, 7, 8, 9, 10$$

Here, all options are correct.

Ans. : (A), (B), (C), (D)

