C 2825 (Pages : 16) Name.....

Reg. No.....

U.G./P.G. ENTRANCE EXAMINATION, APRIL 2021

STATISTICS

Time : Two Hours Maximum : 100 Marks

Part A (1–50)

Answer **all** *questions.*

Each question carries 1 *mark.*

1. The CSO was established in :

   (a) May 1961. (b) June 1961.

   (c) May 1951. (d) June 1951.

2. Which of the following is an example of ordinal variable ?

   (a) Caste. (b) Nationality.

   (c) Annual income. (d) Credit rating of a bank.

3. The equations $2x + 5 = 5, x + 3y = 5, x - 2y = 0$ have ———— number of solutions.

   (a) Zero. (b) One.

   (c) Two. (d) Many.

4. The most widely used sampling plans are given by which standard :

   (a) MIL-STD-100A. (b) MIL-STD-105E.

   (c) MIL-STD-105F. (d) MIL-STD-105H.

5. Consider the following statements :

   I. If S is closed and δ is admissible, then every risk function is convex.

   II. If S is closed, then S will not have a limit point.

   III. If S is closed from below, then $S \notin E_k$.

   Which of the above is correct ?

   (a) Only I is correct. (b) Only II is correct.

   (c) Both I and II is correct. (d) Both II and III is correct.

6. In the analysis of RBD with b blocks and v treatments, the error degrees of freedom are :

(a) $b(v-1)$. (b) $v(b-1)$.

(c) $(b-1)(v-1)$. (d) $b(v-1)$.

7. Consider the following statements :

I. A complete class of decision rules contains only admissible decision rules.

II. A minimal complete class of decision rule contains only admissible decision rules.

III. A minimal complete class of decision rule is always complete.

Which of the above is correct ?

(a) Only I is correct. (b) Only II is correct.

(c) Both I and II is correct. (d) Both II and III is correct.

8. The special case of Birth-Death process with $\lambda_n = n\lambda$ and there is no death is called the _____ process.

(a) Immigration. (b) Poisson Process.

(c) Linear growth. (d) Yule-Furry.

9. Wishart distribution is a generalization of :

(a) Normal distribution. (b) t-distribution.

(c) Chi-square distribution. (d) Beta distribution.

10. Let $\{X_n, Y_n\}, n = 1, 2, \ldots\ldots,$ be a sequence of random variables. Then $[X_n - Y_n] \xrightarrow{P} 0$ and $Y_n \xrightarrow{L} Y$ implies :

(a) $X_n \xrightarrow{L} X$. (b) $X_n \xrightarrow{L} Y$.

(c) $Y_n \xrightarrow{L} X$. (d) $X_n Y_n \xrightarrow{L} Y$.

11. The UCL and LCL of a basic mean chart is given as 12 and 8 respectively. If variance of the process is given as 16, then what is the sample size of the process ?

(a) 36. (b) 28.

(c) 25. (d) 18.

12. If $X \sim \text{Poisson}(4)$ and $Y \sim \text{Poisson}(3)$, and X and Y are independent. What is the value of $E[X|(X+Y)]$, if $n = 10$?

(a) 3.45. (b) 4.32.

(c) 5.23. (d) 5.71.

13. The value of m so that the vector $(m, 3, 1)$ is a linear combination of the vectors $(3, 2, 1)$ and $(2, 1, 0)$ is:

(a) 1. (b) 3.

(c) 5. (d) 7.

14. Given the following statements about a one parameter exponential family of distribution :

I. It always admits sufficient statistics.

II. The moment estimator $\hat{\theta}$ based on sufficient statistics is CAN for θ.

III. The asymptotic variance zero every time.

Which of the above are correct ?

(a) Only I and II are correct. (b) Only I and III are correct.

(c) Only II and III are correct. (d) All are correct.

15. If the mean value function of a renewal process is $m(t) = 2t, t \geq 0$, what is the value of M (T)?

(a) 2. (b) t.

(c) t^2. (d) $2t^2$.

16. The following statements given in respect of Maximum Likelihood Estimation (MLE) :

I. MLE's are always unique.

II. MLE's are not necessarily unbiased.

III. MLE's satisfies invariance property, provided the transformation is one-to-one.

Which of the above are correct ?

(a) Only I and II are correct. (b) Only I and III are correct.

(c) Only II and III are correct. (d) All are correct.

17. Consider the following statements :

I. Least square estimators are unbiased for all general linear models.

II. Under fairly general conditions, the estimates obtained by method of moments will have asymptotically normal distribution for large n.

III. The minimum Chi-square estimators are not necessarily consistent.

Which of the above are correct ?

(a) Only I and II are correct.

(b) Only I and III are correct.

(c) Only II and III are correct.

(d) All are correct.

18. The matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ is :

(a) Positive definite.

(b) Negative definite.

(c) Positive semidefinite.

(d) Negative semidefinite.

19. Let N (t) be a Poisson process with constant intensity function on R. What is the covariance of N (s) and N (t) ?

(a) λs, if $s < t$.

(b) $\lambda(t-s)$, if $s < t$.

(c) $\lambda(s-t)$, if $(t < s)$.

(d) $\lambda(s+t)$.

20. A bag contains 5 black, 6 red and 3 white balls. If a ball is drawn at random, what is the probability that it is not a white ball ?

(a) 11/14.

(b) 13/14.

(c) 38/55.

(d) 27/35.

21. Every sequence $\{X_n\}$ of independent random variables with uniformly bounded variances obeys :

(a) Borel-Cantelli lemma.

(b) Cauchy's criterion.

(c) WLLN.

(d) SLLN.

22. If V be a collection of vectors, then V is said to be subspace, if :

(a) V is closed under multiplication.

(b) V is closed under multiplication and addition.

(c) V is closed under scalar multiplication.

(d) V is closed under addition and scalar multiplication.

23. Let X be a random variable with pgf P (S). Then the pgf of 3X – 1 is :

(a) $SP(S)$. (b) $S/P(S)$.

(c) $P(S)/S$. (d) $P(S^3)/S$.

24. The AQL of a process is the :

(a) Lowest fraction defective that is acceptable to the customer.

(b) Lowest fraction defective that is unacceptable to the customer.

(c) Highest fraction defective that is unacceptable to the customer.

(d) Highest fraction defective that is acceptable to the customer.

25. With the usual notations, find p for a binomial random variable X, if $n = 6$ and if $9\,P(X=1) = P(X=2)$.

(a) 12/15. (b) 18/23.

(c) 19/27. (d) 9/14.

26. The quadratic form $6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_2x_3 + 18x_1x_3 + 4x_1x_2$ is :

(a) Negative definite.

(b) Positive definite.

(c) Positive semidefinite.

(d) Negative semidefinite.

27. Let T be CAN for θ so that $T \sim AN\left(\theta, \sigma_T^2(\theta)/a_n^2\right)$ and let ψ be a differentiable function such that $\frac{d\Psi}{d\theta}$ is continuous and non-vanishing then $\Psi(T)$ is CAN for $\Psi(\theta)$ with asymptotic variance :

(a) $\left(\frac{d\Psi}{d\theta}\right)^2 \sigma_T^2(\theta)$.

(b) $\left(\frac{d\Psi}{d\theta}\right)^2 a_n^2 \sigma_T^2(\theta)$.

(c) $\left(\frac{d\Psi}{d\theta}\right)^2 \frac{\sigma_T^2(\theta)}{a_n^2}$.

(d) $\left(\frac{d\Psi}{d\theta}\right)^2 \frac{\sigma_T^4(\theta)}{a_n^4}$.

28. If the percent of trend for a year in a time series is greater than 100 %, it indicates that :

(a) The actual time series value lies below the trend line and the relative cyclical residual is positive.

(b) The actual time series value lies below the trend line and the relative cyclical residual is negative.

(c) The actual time series value lies above the trend line and the relative cyclical residual is negative.

(d) The actual time series value lies above the trend line and the relative cyclical residual is positive.

29. If 3, 8, 5, 4 and 10 are exponential samples with mean θ. The Fisher information function evaluated at $\theta = 2$ is :

(a) 0.50. (b) 0.80.

(c) 1.20. (d) 1.25.

30. The ratio of number of replication required in CRD and RBD for the same amount of information is :

(a) 3 : 2. (b) 5 : 3.

(c) 5 : 4. (d) 3 : 5.

31. Which one is the improper prior for p, the probability of success in Bernoulli distribution ?

(a) $g(p)=1$. (b) $g(p)=\frac{1}{\beta(a,b)}p^{a-1}(1-p)^{b-1}, a>0, b>0$.

(c) $g(p)=2p$. (d) $g(p)=c, c\neq 1$.

32. If $A=\begin{bmatrix} 3 & -7 \\ -4 & 1 \end{bmatrix}$, then the determinant of A^3+3A^2+12A is :

(a) – 2900. (b) – 29500.

(c) – 3000. (d) – 39500.

33. The Cramer-Rao lower bound for $\psi(\theta)=e^{-\theta}$ in Poisson distribution with parameter θ is :

(a) $\frac{\theta}{n}e^{-2\theta}$. (b) $\frac{n}{\theta}e^{-2\theta}$.

(c) $\frac{2\theta}{n}e^{-2\theta}$. (d) $\frac{\theta^2}{n}e^{-2\theta}$.

34. Let $X_1, X_2, \ldots\ldots, X_n$ be i.i.d. Poisson (μ). Then the UMVUE of $P(X=0)$ is :

(a) $\left(1-\frac{1}{\bar{x}}\right)^n$. (b) $\left(1-\frac{n}{\bar{x}}\right)^n$.

(c) $\left(1-\frac{1}{n}\right)^{\Sigma x}$. (d) $\left(1-\frac{1}{n}\right)^{\bar{x}}$.

35. If 3, 8, 5, 4 and 10 are exponential samples with mean θ. Then the score function evaluated at $\theta = 4$ is :

(a) 0.36. (b) 0.40.

(c) 0.56. (d) 0.68.

36. Consider the following statements :

I. For an estimator to be consistent, the unbiasedness of the estimator is necessary.

II. If the variance of an estimator attains the Crammer-Rao lower bound, the estimator is consistent.

III. A UMVUE is unique, if it exists.

Which of the above are correct ?

(a) Only I is correct. (b) Only II is correct.

(c) Only III is correct. (d) None is correct.

37. If 12, 18, 8, 22 and 15 are random samples from $N(\mu, \sigma^2)$. An unbiased estimator of σ^2 is :

Given by

(a) 20. (b) 25.

(c) 29. (d) 33.

38. Basu's theorem is useful in determining the statistic V which is :

(a) Independent of sufficient statistic T.

(b) Linear function of U and T.

(c) Monotone in U for fixed t.

(d) None of these.

39. The non-parametric test equivalent of a one-way ANOVA is :

(a) Wilcoxon Signed Rank test.

(b) Wilcoson Rank Sum Test.

(c) Kruskall-Wallis Test.

(d) Ansari-Bradley test.

40. The summary of two variables are given as follows :

$\sum x_i = 53, \sum x_i^2 = 297, \sum y_i = 277.5, \sum y_i^2 = 9941.25, \sum x_i\, y_i = 1630,$ and $n = 12$.

What is the value of slope ?

(a) 2.5. (b) 3.6.

(c) 4.8. (d) 6.4.

41. If $n = 15, \sum x = 480, \sum x^2$, then the standard deviation of $y = 5x - 10$ is :

(a) 100. (b) 96.82.

(c) 47.56. (d) 112.88.

42. Let $X_1, X_2, \ldots\ldots$ be i.i.d. Poisson (λ) random variables. If $S_n = \sum_{k=1}^{n} X_k$. If $\lambda = 1$ and $n = 64$, then.

The value of $P\{50 < S_n < 80\}$ is approximately :

(a) 0.7329. (b) 0.8321.

(c) 0.7884. (d) 0.9348.

43. Consider a discrete classification with n_1, n_2, n_3, n_4 as the number of observations in each cell such that $\sum_{i=1}^{3} n_i = n$. The cell probabilities are respectively given as $\theta^2, \theta(1-\theta), \theta(1-\theta)$ and $\theta(1-\theta)^2$. What is the MLE of θ is :

(a) $\frac{n^4}{2n}$. (b) $\frac{2n_1 + n_2}{n_1 + n_3}$.

(c) $\frac{2n_1 + n_3}{n_1 + n_2 + n_4}$. (d) $\frac{2n_1 + n_2 + n_3}{2n}$.

44. Let X be a random variable having the probability function :

$$f(x,\theta)=\binom{n}{x}\theta^{x}(1-\theta)^{n-x},\ x=0,1,2,\ldots\ldots,n.$$

If $d(x)=\frac{x}{n}$, then the risk function $R(\theta,d)$ under squared error loss function is :

(a) $\frac{\theta(\theta-1)}{n}$.

(b) $\frac{\theta(\theta+1)}{n}$.

(c) $\frac{\theta(1-\theta)}{n}$.

(d) $\frac{\theta^2}{n}$.

45. Let $X_1, X_2, \ldots$ be i.i.d. Bernoulli with parameter (λ). If apriori it is known that $\lambda \in [1/4, 3/4]$.

If $\overline{X} \geq \frac{3}{4}$. What is the MLE of λ ?

(a) 1/4.

(b) 1/2.

(c) 3/4.

(d) 1.

46. An inspection of 10 samples of size 400 each from 10 lots revealed the following defective units : 17, 15, 14, 26, 9, 4, 19, 12, 9, 15

The upper control limit for number of defective is :

(a) 18.95.

(b) 21.45.

(c) 23.32.

(d) 25.03.

47. The measure of Kurtosis of t-distribution is :

(a) $\frac{n-2}{n-3}$.

(b) $\frac{3(n-2)}{n-4}$.

(c) $\frac{3(n-2)}{n+4}$.

(d) $\frac{n+2}{n+4}$.

48. If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$ is an idempotent matrix, then the value of x is :

(a) -1.
(b) -3.
(c) -5.
(d) 3.

49. Let $X_1, X_2, \ldots X_n$ be i.i.d. with $f(x) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0$. Then the Cramer-Rao Lower Bound for estimating θ is :

(a) $n\theta$.
(b) $\frac{\theta}{n}$.
(c) $\frac{\theta^2}{n}$.
(d) $\frac{\theta^2}{n^2}$.

50. For a queuing model of M/M/1(∞)/FCFS model, the probability of n^{th} realization is obtained as :

(a) $\rho^n(1-\rho)$.
(b) $\rho(1-\rho)$.
(c) $\rho(1-\rho)^n$.
(d) $\rho^n(1-\rho)^{n-1}$.

(50 × 1 = 50 marks)

Part B (51–75)

Answer **all** *questions.*
Each question carries 2 *marks.*

51. Degrees of freedom for Chi-square in case of contingency table of order (4×3) are :

(a) 12.
(b) 9.
(c) 8.
(d) 6.

52. A small sample has been taken from a normally distributed population and the sample mean has been found to be 62. The upper limit of a 95 percent confidence interval for population mean is 81.60. The population variance is known to be 2,400. What is the sample size ?

(a) 24. (b) 30.

(c) 36. (d) 64.

53. Consider the following results on a correlation study :

Regression equations : $6y = 5x + 90$ and $15x = 8y + 130$ and Variance of $X = 4$. What is the co-efficient of correlation between X and Y ?

(a) 0.45. (b) 0.67.

(c) 0.78. (d) 0.88.

54. A magazine claims that 25 % of its readers are college students. Of a random sample of 200 readers, 42 are college students. It is to be tested at a 0.10 level of significance whether the proportion of college students among all the readers of the magazine is not equal to 0.25. What is the conclusion ?

(a) The proportion of college students among the readers of the magazine is 0.25.

(b) The sample data are incorrect.

(c) The proportion of college students among the readers of the magazine is less than 0.25.

(d) The proportion of college students among the readers of the magazine is more than 0.25.

55. The variance of Hypergeometric distribution with N = 20, n = 5 and M = 12 is given by :

(a) 1.34. (b) 1.28.

(c) 1.02. (d) 0.95.

56. A random sample of 100 articles are taken from a batch of 2000 articles shows that the average diameter of the articles is 0.354 and standard deviation 0.048. What is the 95 % confidence interval for the average diameter of the batch ?

(a) (0.2934, 0.4235). (b) (0.3448, 0.3632).

(c) (0.3021, 0.3824). (d) (0.3923, 0.4212).

57. A multiple regression relationship contains two independent variables. The standard error of estimate is 4.8 and error sum of squares is 576. What is the sample size ?

(a) 24. (b) 25.

(c) 26. (d) 28.

58. Given the following joint density function :

$f(x, y) = \lambda^2 e^{-\lambda y}, 0 \le x \le y < \infty$

What is $E(Y|X)$?

(a) $x + 1/\lambda$.

(b) $x - 1/\lambda$.

(c) $2x + \lambda$.

(d) $2x - \lambda$.

59. Let X has the distribution function :

$$F(x) = \begin{cases} 0, x < 0 \\ x/2, 0 \le x \le 2, \\ 1, x > 2. \end{cases}$$

Let $Y = X^2$, then what is the value of $P(X \le 2Y)$?

(a) 1/2.

(b) 2/3.

(c) 3/4.

(d) 4/7.

60. The percent of total variation of the dependent variable Y explained by the set of independent variable X is measured by :

(a) Co-efficient of correlation.

(b) Co-efficient of skewness.

(c) Co-efficient of determination.

(d) Standard error.

61. While conducting a one-way ANOVA, comparing five treatments with ten observations per treatment, let SST = 42.41 and MSE = 6.34. What is the value of F ?

(a) 42.41.

(b) 6.34.

(c) 1.67.

(d) 0.74.

62. The algebraic expression for interaction ABC in 2^3 experiment is :

(a) $\frac{1}{4}(a-1)(b+1)(c+1)$.

(b) $\frac{1}{4}(a-1)(b-1)(c+1)$.

(c) $\frac{1}{4}(a-1)(b-1)(c-1)$.

(d) None of the above.

63. Relative efficiency of LSD over RBD when rows are taken as blocks is :

(a) $\frac{S_R^2 + (t-1)S_E^2}{tS_E^2}$.

(b) $\frac{S_C^2 + (t-1)S_E^2}{tS_E^2}$.

(c) $\frac{S_C^2 + S_R^2 (t-1)S_E^2}{tS_E^2}$.

(d) None of the above.

64. An experiment is replicated more than once :

(a) To remove experimental error.

(b) To remove the effect of natural factors creating experimental error.

(c) To find the estimate of experimental error.

(d) None of the above.

65. Experimental error is due to :

(a) Experimenter's mistakes.

(b) Extraneous factors.

(c) Variation in treatment effects.

(d) None of the above.

66. Which of the following is a contrast ?

(a) $3T_1 + T_2 - 3T_3 + T_4$.

(b) $T_1 + 3T_2 - 3T_3 + T_4$.

(c) $-3T_1 - T_2 + T_3 + 3T_4$.

(d) $T_1 + T_2 + T_3 - T_4$.

67. The formula for estimating one missing value in a RED having b blocks and t treatments with usual notations :

(a) $\frac{bT' + tB' - G'}{(b-1)(t-1)}$.

(b) $\frac{bB' + bT' - G'}{(b-1)(t-1)}$.

(c) $\frac{bB' + tT' - G'}{(b-1)(t-1)}$.

(d) None of the above.

68. While analyzing the data of a $k \times k$ Latin square, the error d.f. in analysis of variance is equal to :

(a) $(k-1)(k-2)$.

(b) $k(k-1)(k-2)$.

(c) $k^2 - 2$.

(d) $k^2 - k - 2$.

69. If the responses for treatments in a factorial experiment with factors A and B each at two levels from three replications are (1) = 18, (a) = 17, (b) = 25 and (ab) = 30, the sum of squares for the interaction AB is equal to :

(a) 4.

(b) 3.

(c) 6.

(d) 7.

70. The formula for estimating one missing value in a Latin square of order k with usual notations is :

(a) $(R' + C' + T' - G') / (k-1)(k-2)$.

(b) $[k(R' + C' + T') - 2G'] / (k-1)(k-2)$.

(c) $K(R' + C' + T' - 2G') / (k^2 - 1)$.

(d) None of the above.

71. Kalman Filter is a method of updating the best estimate of the 'Signal' in a T.S. :

(a) When noise is absent.

(b) When noise is present.

(c) When random component is present.

(d) When cyclic component is present.

72. Moving Average is also a kind of 'Filtering' when the weights in the filter are chosen in such a way that their sum is :

(a) Zero.

(b) Unity.

(c) Finite.

(d) Convergent.

73. In order to determine the order of an AR process for higher order the autocorrelation function may be :

(a) A mixture of sine and cosine curve.

(b) A mixture of exponential and sinusoidal curve.

(c) A mixture of damped exponential and sinusoidal curve.

(d) A mixture of damped exponential and sine curve.

74. While constructing the 'cost of living index number using the Aggregate expenditure method the weights to be assigned to various commodities are provided by the :

(a) Quantities consumed in the base year.

(b) Quantities consumed in the current year.

(c) Quantities consumed in the previous year.

75. The mean height of 10,000 children of age 6 years is 41.26" and the standard deviation is 2.24". Then the odds against the possibility that the mean of a random sample of 100 is greater than 41.7" is :

(a) 39 : 1. (b) 1 : 39.

(c) 40 : 1. (d) 1 : 40.

(25 × 2 = 50 marks)