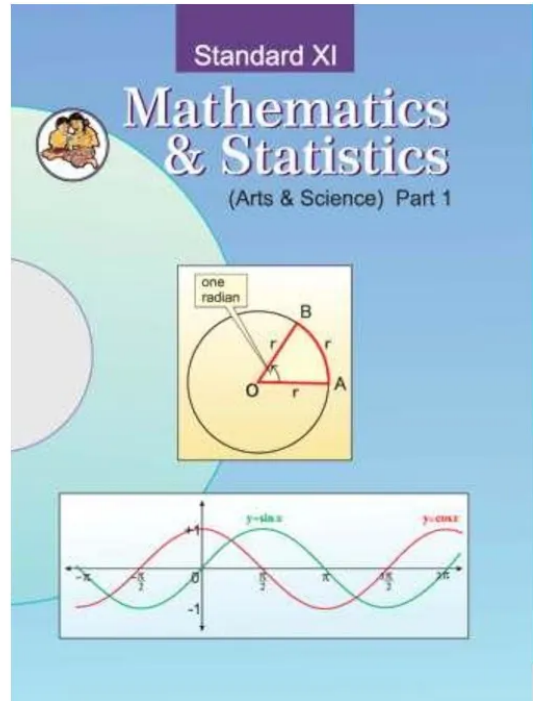


Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 1): Chapter 3- Trigonometry – II

Class 11 - Chapter 3 Trigonometry - II



For any clarifications or questions you can write to info@indcareer.com

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Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 1): Chapter 3- Trigonometry – II

Class 11: Maths Chapter 3 solutions. Complete Class 11 Maths Chapter 3 Notes.

Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 1): Chapter 3- Trigonometry – II

Maharashtra Board 11th Maths Chapter 3, Class 11 Maths Chapter 3 solutions

Ex 3.1

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Question 1.

Find the values of:

i. $\sin 150^\circ$

ii. $\cos 75^\circ$

iii. $\tan 105^\circ$

iv. $\cot 225^\circ$

Solution:

i. $\sin 15^\circ = \sin (45^\circ - 30^\circ)$

$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$$\left(\frac{1}{\sqrt{2}}\right) \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{2}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

[Note: Answer given in the textbook is $\frac{\sqrt{3}+1}{2\sqrt{2}}$ However, as per our calculation it is $\frac{\sqrt{3}-1}{2\sqrt{2}}$



$$\begin{aligned}\text{ii. } \cos 75^\circ &= \cos (45^\circ + 30^\circ) \\ &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\ &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3}-1}{2\sqrt{2}}\end{aligned}$$

$$\begin{aligned}\text{iii. } \tan 105^\circ &= \tan (60^\circ + 45^\circ) \\ &= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ} \\ &= \frac{\sqrt{3}+1}{1-(\sqrt{3})(1)} \\ &= \frac{\sqrt{3}+1}{1-\sqrt{3}}\end{aligned}$$

$$\begin{aligned}\text{iv. } \cot 225^\circ &= \frac{1}{\tan 225^\circ} = \frac{1}{\tan (180^\circ + 45^\circ)} \\ &= \frac{1}{\left(\frac{\tan 180^\circ + \tan 45^\circ}{1 - \tan 180^\circ \tan 45^\circ}\right)} \\ &= \frac{1}{\left(\frac{0+1}{1-0(1)}\right)}\end{aligned}$$

$$= \frac{1}{\left(\frac{1}{1}\right)}$$

$$= 1$$

Question 2.

Prove the following:

i. $\cos\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - y\right) - \sin\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} - y\right) = -\cos(x + y)$

Solution:

L.H.S

$$= \cos\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - y\right)$$

$$- \sin\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} - y\right)$$

$$= \sin x \sin y - \cos x \cos y$$

$$\dots \left[\begin{array}{l} \because \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta, \\ \sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \end{array} \right]$$

$$= -(\cos x \cos y - \sin x \sin y)$$

$$= -\cos(x + y)$$

$$= \text{R.H.S}$$

ii. $\tan\left(\frac{\pi}{4} + \theta\right) = \frac{1 + \tan \theta}{1 - \tan \theta}$

L.H.S $= \tan\left(\frac{\pi}{4} + \theta\right)$

$$\begin{aligned} &= \frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta} \\ &= \frac{1 + \tan \theta}{1 - (1) \tan \theta} \\ &= \frac{1 + \tan \theta}{1 - \tan \theta} \end{aligned}$$

R.H.S.

[Note : The question has been modified.]

$$\text{iii. } \left(\frac{1 + \tan x}{1 - \tan x} \right)^2 = \frac{\tan \left(\frac{\pi}{4} + x \right)}{\tan \left(\frac{\pi}{4} - x \right)}$$

Solution:

$$\begin{aligned} \text{R.H.S.} &= \frac{\tan \left(\frac{\pi}{4} + x \right)}{\tan \left(\frac{\pi}{4} - x \right)} \\ &= \frac{\left(\frac{\tan \frac{\pi}{4} + \tan x}{1 - \tan \frac{\pi}{4} \tan x} \right)}{\left(\frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x} \right)} = \frac{\left(\frac{1 + \tan x}{1 - (1) \tan x} \right)}{\left(\frac{1 - \tan x}{1 + (1) \tan x} \right)} \end{aligned}$$

$$= \left(\frac{1 + \tan x}{1 - \tan x} \right) \times \left(\frac{1 + \tan x}{1 - \tan x} \right)$$

$$= \left(\frac{1 + \tan x}{1 - \tan x} \right)^2 = \text{L.H.S.}$$

iv. $\sin [(n+1)A] \cdot \sin [(n+2)A] + \cos [(n+1)A] \cdot \cos [(n+2)A] = \cos A$

Solution:

$$\text{L.H.S.} = \sin [(n+1)A] \cdot \sin [(n+2)A] + \cos [(n+1)A] \cdot \cos [(n+2)A]$$

$$= \cos [(n+2)A] \cdot \cos [(n+1)A] + \sin [(n+2)A] \cdot \sin [(n+1)A]$$

Let $(n+2)A$ as a and $(n+1)A$ as b ... (i)

$$\therefore \text{L.H.S.} = \cos a \cdot \cos b + \sin a \cdot \sin b$$

$$= \cos (a - b)$$

$$= \cos [(n+2)A - (n+1)A]$$

... [From (i)]

$$\cos [(n+2 - n - 1)A]$$

$$= \cos A$$

$$= \text{R.H.S.}$$

v. $\sqrt{2} \cos \left(\frac{\pi}{4} - A \right) = \cos A + \sin A$

Solution:

$$\text{L.H.S.} = \sqrt{2} \cos \left(\frac{\pi}{4} - A \right)$$

$$= \sqrt{2} \left(\cos \frac{\pi}{4} \cos A + \sin \frac{\pi}{4} \sin A \right)$$

$$= \sqrt{2} \left(\frac{1}{\sqrt{2}} \cos A + \frac{1}{\sqrt{2}} \sin A \right)$$

$$= \frac{\sqrt{2}}{\sqrt{2}} (\cos A + \sin A)$$

$$= \cos A + \sin A$$

$$= \text{R.H.S.}$$

vi. $\frac{\cos(x-y)}{\cos(x+y)} = \frac{\cot x \cot y + 1}{\cot x \cot y - 1}$

Solution:

$$\begin{aligned}\text{L.H.S.} &= \frac{\cos(x-y)}{\cos(x+y)} \\ &= \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y - \sin x \sin y}\end{aligned}$$

Dividing numerator and denominator by $\sin x \sin y$, we get

$$\begin{aligned}\text{L.H.S.} &= \frac{\left(\frac{\cos x \cos y}{\sin x \sin y} + 1\right)}{\left(\frac{\cos x \cos y}{\sin x \sin y} - 1\right)} \\ &= \frac{\cot x \cot y + 1}{\cot x \cot y - 1} \\ &= \text{R.H.S.}\end{aligned}$$

vii. $\cos(x+y) \cdot \cos(x-y) = \cos^2 y - \sin^2 x$

Solution:

$$\begin{aligned}\text{L.H.S.} &= \cos(x+y) \cdot \cos(x-y) \\ &= (\cos x \cos y - \sin x \sin y) \cdot (\cos x \cos y + \sin x \sin y) \\ &= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y \\ &\dots [\because (a-b)(a+b) = a^2 - b^2] \\ &= (1 - \sin^2 x) \cos^2 y - \sin^2 x (1 - \cos^2 y) \\ &\dots [\because \sin^2 x + \cos^2 x = 1] \\ &= \cos^2 y - \cos^2 y \sin^2 x - \sin^2 x + \sin^2 x \cos^2 y \\ &= \cos^2 y - \sin^2 x \\ &= \text{R.H.S.}\end{aligned}$$

viii. $\frac{\tan 5A - \tan 3A}{\tan 5A + \tan 3A} = \frac{\sin 2A}{\sin 8A}$

Solution:

$$\begin{aligned} \text{L.H.S.} &= \frac{\tan 5A - \tan 3A}{\tan 5A + \tan 3A} \\ &= \frac{\frac{\sin 5A}{\cos 5A} - \frac{\sin 3A}{\cos 3A}}{\frac{\sin 5A}{\cos 5A} + \frac{\sin 3A}{\cos 3A}} \\ &= \frac{\left(\frac{\sin 5A \cos 3A - \sin 3A \cos 5A}{\cos 5A \cos 3A} \right)}{\left(\frac{\sin 5A \cos 3A + \sin 3A \cos 5A}{\cos 5A \cos 3A} \right)} \\ &= \frac{\sin 5A \cos 3A - \cos 5A \sin 3A}{\sin 5A \cos 3A + \cos 5A \sin 3A} \\ &= \frac{\sin(5A - 3A)}{\sin(5A + 3A)} \\ &= \frac{\sin 2A}{\sin 8A} \\ &= \text{R.H.S.} \end{aligned}$$

ix. $\tan 8\theta - \tan 5\theta - \tan 3\theta = \tan 8\theta \tan 5\theta \tan 3\theta$

Solution:

Since, $8\theta = 5\theta + 3\theta$

$\therefore \tan 8\theta = \tan(5\theta + 3\theta)$

$\therefore \tan 8\theta = \frac{\tan 5\theta + \tan 3\theta}{1 - \tan 5\theta \tan 3\theta}$

$\therefore \tan 8\theta (1 - \tan 5\theta \tan 3\theta) = \tan 5\theta + \tan 3\theta$

$\therefore \tan 8\theta - \tan 8\theta \tan 5\theta \tan 3\theta = \tan 5\theta + \tan 3\theta$

$\therefore \tan 8\theta - \tan 5\theta - \tan 3\theta = \tan 8\theta \tan 5\theta \tan 3\theta$

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$$x. \tan 50^\circ = \tan 40^\circ + 2 \tan 10^\circ$$

Solution:

$$\text{Since, } 50^\circ = 10^\circ + 40^\circ$$

$$\therefore \tan 50^\circ = \tan (10^\circ + 40^\circ)$$

$$\therefore \frac{\tan 10^\circ + \tan 40^\circ}{1 - \tan 10^\circ \tan 40^\circ}$$

$$\therefore \tan 50^\circ (1 - \tan 10^\circ \tan 40^\circ) = \tan 10^\circ + \tan 40^\circ$$

$$\therefore \tan 50^\circ - \tan 10^\circ \tan 40^\circ \tan 50^\circ = \tan 10^\circ + \tan 40^\circ$$

$$\therefore \tan 50^\circ - \tan 10^\circ \tan 40^\circ \tan (90^\circ - 40^\circ) = \tan 10^\circ + \tan 40^\circ$$

$$\therefore \tan 50^\circ - \tan 10^\circ \tan 40^\circ \cot 40^\circ$$

$$= \tan 10^\circ + \tan 40^\circ \dots [\because \tan (90^\circ - \theta) = \cot \theta]$$

$$\therefore \tan 50^\circ - \tan 10^\circ \tan 40^\circ \cdot \frac{1}{\tan 40^\circ} = \tan 10^\circ + \tan 40^\circ$$

$$\therefore \tan 50^\circ - \tan 10^\circ \cdot 1 = \tan 10^\circ + \tan 40^\circ$$

$$\therefore \tan 50^\circ = \tan 40^\circ + 2 \tan 10^\circ$$

$$xi. \frac{\cos 27^\circ + \sin 27^\circ}{\cos 27^\circ - \sin 27^\circ} = \tan 72^\circ$$

Solution:

$$\frac{\cos 27^\circ + \sin 27^\circ}{\cos 27^\circ - \sin 27^\circ}$$

Dividing numerator and denominator by $\cos 27^\circ$, we get

$$\begin{aligned} \text{L.H.S.} &= \frac{1 + \frac{\sin 27^\circ}{\cos 27^\circ}}{1 - \frac{\sin 27^\circ}{\cos 27^\circ}} \\ &= \frac{1 + \tan 27^\circ}{1 - \tan 27^\circ} \end{aligned}$$

$$= \frac{\tan 45^\circ + \tan 27^\circ}{1 - \tan 45^\circ \tan 27^\circ} \dots [\because \tan 45^\circ = 1]$$

$$= \tan (45^\circ + 27^\circ)$$

$$= \tan 72^\circ = \text{R.H.S.}$$

$$= \tan (45^\circ + 27^\circ)$$

$$= \tan 72^\circ = \text{R.H.S.}$$

xii. $\frac{\cos 27^\circ + \sin 27^\circ}{\cos 27^\circ - \sin 27^\circ} = \tan 72^\circ$

Solution:

Since $45^\circ = 10^\circ + 35^\circ$,

$\tan 45^\circ = \tan (10^\circ + 35^\circ)$

$\therefore \frac{\tan 10^\circ + \tan 35^\circ}{1 - \tan 10^\circ \tan 35^\circ}$

$\therefore 1 - \tan 10^\circ \tan 35^\circ = \tan 10^\circ + \tan 35^\circ$

$\therefore \tan 10^\circ + \tan 35^\circ + \tan 10^\circ \tan 35^\circ = 1$

xiii. $\tan 10^\circ + \tan 35^\circ + \tan 10^\circ \cdot \tan 35^\circ = 1$

Solution:

$$\begin{aligned} \text{L.H.S.} &= \frac{\cot A \cot 4A + 1}{\cot A \cot 4A - 1} \\ &= \frac{\frac{\cos A}{\sin A} \cdot \frac{\cos 4A}{\sin 4A} + 1}{\frac{\cos A}{\sin A} \cdot \frac{\cos 4A}{\sin 4A} - 1} \\ &= \frac{\cos A \cos 4A + \sin A \sin 4A}{\cos A \cos 4A - \sin A \sin 4A} \\ &= \frac{\cos(4A - A)}{\cos(4A + A)} \\ &= \frac{\cos 3A}{\cos 5A} = \text{R.H.S.} \end{aligned}$$

$$\text{xiv. } \frac{\cos 15^\circ - \sin 15^\circ}{\cos 15^\circ + \sin 15^\circ} = \frac{1}{\sqrt{3}}$$

Solution:

Dividing numerator and denominator by $\cos 15^\circ$, we get

$$\begin{aligned}\text{L.H.S.} &= \frac{1 - \frac{\sin 15^\circ}{\cos 15^\circ}}{1 + \frac{\sin 15^\circ}{\cos 15^\circ}} \\&= \frac{1 - \tan 15^\circ}{1 + \tan 15^\circ} \\&= \frac{\tan 45^\circ - \tan 15^\circ}{1 + (\tan 45^\circ)(\tan 15^\circ)} \dots [\because \tan 45^\circ = 1] \\&= \tan (45^\circ - 15^\circ) \\&= \tan 30^\circ = \frac{1}{\sqrt{3}} = \text{R.H.S.}\end{aligned}$$

$$= \tan (45^\circ + 15^\circ)$$

$$= \tan 30^\circ = \frac{1}{\sqrt{3}} = \text{R.H.S.}$$

Question 3.

If $\sin A = -\frac{5}{13}$, $n < A < \frac{3\pi}{2}$ and $\cos B = \frac{3}{5}$, $\frac{3\pi}{2} < B < 2\pi$, find

i. $\sin (A+B)$

ii. $\cos (A-B)$

iii. $\tan (A + B)$

Solution:

$$\text{Given, } \sin A = -\frac{5}{13}$$

We know that,

$$\cos^2 A = 1 - \sin^2 A = 1 - \left(-\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\therefore \cos A = \pm \frac{12}{13}$$

$$\text{Since, } \pi < A < \frac{3\pi}{2}$$

$\therefore$ 'A' lies in the 3rd quadrant.

$$\therefore \cos A < 0$$

$$\cos A = -\frac{12}{13}$$

$$\text{Also, } \cos B = \frac{3}{5}$$

$$\therefore \sin^2 B = 1 - \cos^2 B = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\therefore \sin B = \pm \frac{4}{5}$$

$$\text{Since, } \frac{3\pi}{2} < B < 2\pi$$

$\therefore$ 'B' lies in the 4th quadrant.

$$\therefore \sin B < 0$$

$$\sin B = -\frac{4}{5}$$

$$\text{i. } \sin (A + B) = \sin A \cos B + \cos A \sin B$$

$$= \left(-\frac{5}{13}\right)\left(\frac{3}{5}\right) + \left(-\frac{12}{13}\right)\left(-\frac{4}{5}\right)$$

$$= -\frac{15}{65} + \frac{48}{65} = \frac{33}{65}$$

$$\text{ii. } \cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$= \left(-\frac{12}{13}\right)\left(\frac{3}{5}\right) + \left(-\frac{5}{13}\right)\left(-\frac{4}{5}\right)$$

$$= -\frac{36}{65} + \frac{20}{65} = -\frac{16}{65}$$

iii.

$$\tan A = \frac{\sin A}{\cos A} = \frac{\left(-\frac{5}{13}\right)}{\left(-\frac{12}{13}\right)} = \frac{5}{12}$$

$$\tan B = \frac{\sin B}{\cos B} = \frac{\left(-\frac{4}{5}\right)}{\left(\frac{3}{5}\right)} = -\frac{4}{3}$$

$$\begin{aligned}\tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{5}{12} - \frac{4}{3}}{1 - \left(\frac{5}{12}\right)\left(-\frac{4}{3}\right)} \\ &= \frac{\left(-\frac{33}{36}\right)}{\left(\frac{56}{36}\right)} = -\frac{33}{56}\end{aligned}$$

Question 4.

If $\tan A = \frac{5}{6}$, $\tan B = \frac{1}{11}$ prove that $A + B = \frac{\pi}{4}$

Solution:

Given $\tan A = \frac{5}{6}$, $\tan B = \frac{1}{11}$

$$\begin{aligned}\tan (A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{5}{6} + \frac{1}{11}}{1 - \left(\frac{5}{6}\right)\left(\frac{1}{11}\right)} \\ &= \frac{\left(\frac{61}{66}\right)}{\left(\frac{61}{66}\right)} = 1\end{aligned}$$

$$\therefore \tan (A+B) = \tan \frac{\pi}{4}$$

$$\therefore A+B = \frac{\pi}{4}$$

Ex 3.2

Question 1.

Find the values of:

- i. $\sin 690^\circ$
- ii. $\sin 495^\circ$
- iii. $\cos 315^\circ$
- iv. $\cos 600^\circ$
- v. $\tan 225^\circ$
- vi. $\tan (-690^\circ)$
- vii. $\sec 240^\circ$
- viii. $\sec (-855^\circ)$
- ix. $\operatorname{cosec} 780^\circ$
- x. $\cot (-1110^\circ)$

Solution:

$$\text{i. } \sin 690^\circ = \sin (720^\circ - 30^\circ)$$

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Solution:

$$\text{i. } \sin 690^\circ = \sin (720^\circ - 30^\circ)$$

$$= \sin (2 \times 360^\circ - 30^\circ)$$

$$= -\sin 30^\circ$$

$$= -12$$

$$\begin{aligned}\text{ii. } \sin 495^\circ &= \sin (360^\circ + 135^\circ) \\ &= \sin (135^\circ) \\ &= \sin (90^\circ + 45^\circ) \\ &= \cos 45^\circ \\ &= \frac{1}{\sqrt{2}}\end{aligned}$$

$$\begin{aligned}\text{iii. } \cos 315^\circ &= \cos (270^\circ + 45^\circ) \\ \sin 45^\circ &= \frac{1}{\sqrt{2}}\end{aligned}$$

$$\begin{aligned}\text{iv. } \cos 600^\circ &= \cos (360^\circ + 240^\circ) \\ &= \cos 240^\circ \\ &= \cos (180^\circ + 60^\circ) \\ &= -\cos 60^\circ \\ &= -\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{v. } \tan 225^\circ &= \tan (180^\circ + 45^\circ) \\ &= \tan 45^\circ \\ &= 1.\end{aligned}$$

$$\begin{aligned}\text{vi. } \tan (-690^\circ) &= -\tan 690^\circ \\ &= -\tan (720^\circ - 30^\circ) \\ &= -\tan (2 \times 360^\circ - 30^\circ) \\ &= -(-\tan 30^\circ) \\ &= \tan 30^\circ \\ &= \frac{1}{\sqrt{3}}\end{aligned}$$

$$\begin{aligned}\text{vii. } \sec 240^\circ &= \sec (180^\circ + 60^\circ) \\ &= -\sec 60^\circ \\ &= -2\end{aligned}$$

$$\begin{aligned}\text{viii. } \sec (-855^\circ) &= \sec (855^\circ) \\ &= \sec (720^\circ + 135^\circ) \\ &= \sec (2 \times 360^\circ + 135^\circ) = \sec 135^\circ \\ &= \sec (90^\circ + 45^\circ) \\ &= -\operatorname{cosec} 45^\circ \\ &= -\sqrt{2}\end{aligned}$$

$$\begin{aligned}\text{ix. } \operatorname{cosec} 780^\circ &= \operatorname{cosec} (720^\circ + 60^\circ) \\ &= \operatorname{cosec} (2 \times 360^\circ + 60^\circ) \\ &= \operatorname{cosec} 60^\circ \\ &= \frac{2}{\sqrt{3}}\end{aligned}$$

$$\begin{aligned}x. \cot(-1110^\circ) &= -\cot(1110^\circ) \\&= -\cot(1080^\circ + 30^\circ) \\&= -\cot(3 \times 360^\circ + 30^\circ) \\&= -\cot 30^\circ \\&= -\sqrt{3}\end{aligned}$$

Question 2.

Prove the following:

- i. $\frac{\cos(\pi+x) \cos(-x)}{\sin(\pi-x) \cos\left(\frac{\pi}{2}+x\right)} = \cot^2 x$
- ii. $\cos\left(\frac{3\pi}{2} + x\right) \cos(2\pi + x) [\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)]$
- iii. $\sec 840^\circ \cot(-945^\circ) + \sin 600^\circ \tan(-690^\circ) = 3/2$
- iv. $\frac{\operatorname{cosec}(90^\circ - x) \sin(180^\circ - x) \cot(360^\circ - x)}{\sec(180^\circ + x) \tan(90^\circ + x) \sin(-x)} = 1$
- v. $\frac{\sin^3(\pi+x) \sec^2(\pi-x) \tan(2\pi-x)}{\cos^2\left(\frac{\pi}{2}+x\right) \sin(\pi-x) \operatorname{cosec}^2(-x)} = \tan^3 x$
- vi. $\cos \theta + \sin(270^\circ + \theta) - \sin(270^\circ - \theta) + \cos(180^\circ + \theta) = 0$

Solution:

i.

$$\begin{aligned}\text{L.H.S.} &= \frac{\cos(\pi+x) \cos(-x)}{\sin(\pi-x) \cos\left(\frac{\pi}{2}+x\right)} \\&= \frac{(-\cos x)(\cos x)}{(\sin x)(-\sin x)}\end{aligned}$$

$$\begin{aligned} &= \frac{\cos^2 x}{\sin^2 x} \\ &= \cot^2 x \\ &= \text{R.H.S.} \end{aligned}$$

ii. L.H.S.

$$\begin{aligned} &= \cos \left(\frac{3\pi}{2} + x \right) \cos (2n + x) \cdot [\cot (-x) + (2n + x)] \\ &= (\sin x)(\cos x) (\tan x + \cot x) \\ &= \sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) \\ &= \sin x \cos x \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right) \\ &= \sin x \cos x \left(\frac{1}{\sin x \cos x} \right) \\ &= 1 = \text{R.H.S} \end{aligned}$$

$$\begin{aligned}\text{iii. } \sec 840^\circ &= \sec (720^\circ + 120^\circ) \\ &= \sec (2 \times 360^\circ + 120^\circ) \\ &= \sec (120^\circ) \\ &= \sec (90^\circ + 30^\circ) \\ &= -\operatorname{cosec} 30^\circ \\ &= -2\end{aligned}$$

$$\begin{aligned}\cot(-945^\circ) &= -\cot 945^\circ \\ &= -\cot (720^\circ + 225^\circ) \\ &= -\cot (2 \times 360^\circ + 225^\circ) \\ &= -\cot (225^\circ) \\ &= -\cot (180^\circ + 45^\circ) \\ &= -\cot 45^\circ \\ &= -1\end{aligned}$$

$$\begin{aligned}\sin 600^\circ &= \sin (360^\circ + 240^\circ) \\ &= \sin (240^\circ) \\ &= \sin (180^\circ + 60^\circ) \\ &= -\sin 60^\circ = -\frac{\sqrt{3}}{2}\end{aligned}$$

$$\begin{aligned}\tan (-690^\circ) &= -\tan 690^\circ \\ &= -\tan (360^\circ + 330^\circ) \\ &= -\tan (330^\circ) \\ &= -\tan (360^\circ - 30^\circ) \\ &= -(-\tan 30^\circ) \\ &= \tan 30^\circ = \frac{1}{\sqrt{3}}\end{aligned}$$

$$\begin{aligned}\text{L.H.S.} &= \sec 840^\circ \cot (-945^\circ) + \sin 600^\circ \tan (-690^\circ) \\ &= (-2)(-1) + \left(-\frac{\sqrt{3}}{2}\right) \left(\frac{1}{\sqrt{3}}\right) \\ &= 2 - \frac{1}{2} = \frac{3}{2} \\ &= \text{R. H. S.}\end{aligned}$$

iv.

$$\begin{aligned}\text{L.H.S.} &= \frac{\operatorname{cosec}(90^\circ - x) \cdot \sin(180^\circ - x) \cdot \cot(360^\circ - x)}{\sec(180^\circ + x) \cdot \tan(90^\circ + x) \cdot \sin(-x)} \\ &= \frac{\sec x \cdot \sin x \cdot (-\cot x)}{(-\sec x) \cdot (-\cot x) \cdot (-\sin x)} \\ &= 1 \\ &= \text{R.H.S.}\end{aligned}$$

v.

$$\begin{aligned}\text{L.H.S.} &= \frac{\sin^3(\pi + x) \sec^2(\pi - x) \tan(2\pi - x)}{\cos^2\left(\frac{\pi}{2} + x\right) \sin(\pi - x) \operatorname{cosec}^2(-x)} \\ &= \frac{[\sin(\pi + x)]^3 [\sec(\pi - x)]^2 \tan(2\pi - x)}{\left[\cos\left(\frac{\pi}{2} + x\right)\right]^2 \sin(\pi - x) \cdot (-\operatorname{cosec} x)^2}\end{aligned}$$

$$\begin{aligned} &= \frac{(-\sin x)^3 (-\sec x)^2 (-\tan x)}{(-\sin x)^2 \cdot \sin x \cdot \operatorname{cosec}^2 x} \\ &= \frac{(-\sin^3 x) \cdot \sec^2 x \cdot (-\tan x)}{\sin^2 x \cdot \sin x \cdot \frac{1}{\sin^2 x}} \\ &= \frac{\sin^3 x \cdot \sec^2 x \cdot \tan x}{\sin x} \\ &= \sin^2 x \cdot \frac{1}{\cos^2 x} \cdot \tan x \\ &= \tan^2 x \cdot \tan x \\ &= \tan^3 x \\ &= \text{R.H.S.} \end{aligned}$$

$$\begin{aligned} \text{vi. L.H.S.} &= \cos \theta + \sin (270^\circ + \theta) - \sin (270^\circ - \theta) + \cos (180^\circ + \theta) \\ &= \cos \theta + (-\cos \theta) - (-\cos \theta) - \cos \theta \\ &= \cos \theta - \cos \theta + \cos \theta - \cos \theta \\ &= 0 \\ &= \text{R.H.S.} \end{aligned}$$

Ex 3.3

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-1-chapter-3-trigonometry-ii/>

Question 1.

Find the values of:

i. $\sin \frac{\pi}{8}$

ii. $\frac{\pi}{8}$

Solution:

We know that $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$ [/*atex*] Substituting $\theta = [latex] \frac{\pi}{8}$, we get

$$\sin^2 \frac{\pi}{8} = \frac{1 - \cos \frac{\pi}{4}}{2} = \frac{1 - \frac{1}{\sqrt{2}}}{2} = \frac{\sqrt{2} - 1}{2\sqrt{2}}$$

$$\therefore \sin \frac{\pi}{8} = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}} \quad \dots \left[\because \sin \frac{\pi}{8} \text{ is positive} \right]$$

$$\begin{aligned} \therefore \sin \frac{\pi}{8} &= \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \sqrt{\frac{2 - \sqrt{2}}{4}} \end{aligned}$$

$$\therefore \sin \frac{\pi}{8} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

ii. We know that, $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

Substituting $\theta = \frac{\pi}{8}$, we get

$$\begin{aligned}\cos^2 \frac{\pi}{8} &= \frac{1 + \cos \frac{\pi}{4}}{2} \\&= \frac{1 + \frac{1}{\sqrt{2}}}{2} = \frac{\sqrt{2} + 1}{2\sqrt{2}} \\ \therefore \cos \frac{\pi}{8} &= \sqrt{\frac{\sqrt{2} + 1}{2\sqrt{2}}} \quad \dots \left[\because \cos \frac{\pi}{8} \text{ is positive} \right] \\ \therefore \cos \frac{\pi}{8} &= \sqrt{\frac{\sqrt{2} + 1}{2\sqrt{2}}} \times \frac{\sqrt{2}}{\sqrt{2}} \\&= \sqrt{\frac{2 + \sqrt{2}}{4}} \\ \therefore \cos \frac{\pi}{8} &= \frac{\sqrt{2 + \sqrt{2}}}{2}\end{aligned}$$

Question 2.

Find $\sin 2x$, $\cos 2x$, $\tan 2x$ if $\sec x = -\frac{13}{5}$, $\frac{\pi}{2} < x < \pi$

Solution:

$$\sec x = -\frac{13}{5}, \frac{\pi}{2} < x < \pi$$

We know that

$$\sec^2 x = 1 + \tan^2 x$$

$$\tan^2 x = \frac{169}{25} - 1 = \frac{144}{25}$$

$$\tan x = \pm \frac{12}{5}$$

$$\text{Since } \frac{\pi}{2} < x < \pi$$

x lies in the 2nd quadrant.

$$\tan x < 0$$

$$\therefore \tan x = -\frac{12}{5}$$

$$\begin{aligned}\sin 2x &= \frac{2 \tan x}{1 + \tan^2 x} \\&= \frac{2\left(-\frac{12}{5}\right)}{1 + \left(-\frac{12}{5}\right)^2} \\&= \frac{-\frac{24}{5}}{1 + \frac{144}{25}} = \frac{-\frac{24}{5}}{\frac{169}{25}} \\&= \frac{-24}{5} \times \frac{25}{169} = \frac{-120}{169}\end{aligned}$$

$$\begin{aligned}\cos 2x &= \frac{1 - \tan^2 x}{1 + \tan^2 x} \\&= \frac{1 - \left(-\frac{12}{5}\right)^2}{1 + \left(-\frac{12}{5}\right)^2} \\&= \frac{1 - \frac{144}{25}}{1 + \frac{144}{25}} = \frac{25 - 144}{25 + 144} \\&= -\frac{119}{169}\end{aligned}$$

$$\begin{aligned}\tan 2x &= \frac{2 \tan x}{1 - \tan^2 x} \\&= \frac{2\left(-\frac{12}{5}\right)}{1 - \left(-\frac{12}{5}\right)^2} \\&= \frac{-\frac{24}{5}}{1 - \frac{144}{25}} \\&= \frac{-\frac{24}{5}}{-\frac{119}{25}} = \frac{24}{5} \times \frac{25}{119} \\&= \frac{120}{119}\end{aligned}$$

Question 3.

i. $= \tan^2 \theta$

Solution:

$$\text{L. H. S.} = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$= \frac{2 \sin^2 \theta}{2 \cos^2 \theta}$$

$$= 2 \tan^2 \theta$$

$$= \text{R.H.S.}$$

ii. $(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$

Solution:

$$\text{L.H.S.} = (\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x$$

$$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$$

$$= (\cos 3x \cos x + \sin 3x \sin x)$$

$$- (\cos^2 x - \sin^2 x)$$

$$= \cos (3x - x) - \cos 2x$$

$$= \cos 2x - \cos 2x$$

$$= 0$$

$$= \text{R.H.S.}$$

iii. $(\cos x + \cos y)^2 + (\sin x + \sin y)^2 = 4 \cos^2 \left(\frac{x-y}{2} \right)$

Solution:

$$\text{L.H.S.} = (\cos x + \cos y)^2 + (\sin x + \sin y)^2$$

$$= \cos^2 x + \cos^2 y + 2 \cos x \cos y + \sin^2 x + \sin^2 y + 2 \sin x \sin y$$

$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x \cos y + \sin x \sin y)$$

$$\begin{aligned} &= 1 + 1 + 2\cos(x - y) \\ &= 2 + 2\cos(x - y) \\ &= 2[1 + \cos(x - y)] \\ &= 2\left[2\cos^2\left(\frac{x-y}{2}\right)\right] \dots [\because 1 + \cos \theta = 2\cos^2 \frac{\theta}{2}] \\ &= 4\cos^2\left(\frac{x-y}{2}\right) \\ &= \text{R.H.S.} \end{aligned}$$

[Note: The question has been modified]

$$\text{iv. } (\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x-y}{2}\right)$$

Solution:

$$\begin{aligned} \text{L.H.S.} &= (\cos x - \cos y)^2 + (\sin x - \sin y)^2 \\ &= \cos^2 x + \cos^2 y + 2\cos x \cdot \cos y + \sin^2 x + \sin^2 y + 2\sin x \cdot \sin y \\ &= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2(\cos x \cdot \cos y + \sin x \cdot \sin y) \\ &= 1 + 1 - 2\cos(x - y) \\ &= 2 - 2\cos(x - y) \\ &= 2[1 - \cos(x - y)] \\ &= 2\left[2\sin^2\left(\frac{x-y}{2}\right)\right] \dots [\because 1 - \cos \theta = 2\sin^2 \frac{\theta}{2}] \\ &= 4\sin^2\left(\frac{x-y}{2}\right) \\ &= \text{R.H.S.} \end{aligned}$$

$$\text{v. } \tan x + \cot x = 2 \operatorname{cosec} 2x$$

Solution:

$$\text{L.H.S.} = \tan x + \cot x$$

$$\begin{aligned}
 &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \\
 &= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \\
 &= \frac{1}{\sin x \cos x} = \frac{2}{2 \sin x \cos x} \\
 &= \frac{2}{\sin 2x} \\
 &= 2 \operatorname{cosec} 2x = \text{R.H.S.}
 \end{aligned}$$

vi. $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2 \tan 2x$

Solution:

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} \\
 &= \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)} \\
 &= \frac{(\cos^2 x + \sin^2 x + 2 \sin x \cos x) - (\cos^2 x + \sin^2 x - 2 \sin x \cos x)}{\cos^2 x - \sin^2 x} \\
 &= \frac{1 + 2 \sin x \cos x - 1 + 2 \sin x \cos x}{\cos^2 x - \sin^2 x} \\
 &= \frac{2(2 \sin x \cos x)}{\cos^2 x - \sin^2 x} \\
 &= \frac{2 \sin 2x}{\cos 2x}
 \end{aligned}$$

$$= 2 \tan 2x$$

$$= \text{R.H.S.}$$

$$\text{vii. } \sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 8x}}} = 2 \cos x$$

Solution:

$$\begin{aligned} \text{L.H.S.} &= \sqrt{2 + \sqrt{2 + \sqrt{2 + 2 \cos 8x}}} \\ &= \sqrt{2 + \sqrt{2 + \sqrt{2[1 + \cos 2(4x)]}}} \\ &= \sqrt{2 + \sqrt{2 + \sqrt{2 \times 2 \cos^2 4x}}} \\ &\quad \dots [\because 1 + \cos 2\theta = 2 \cos^2 \theta] \\ &= \sqrt{2 + \sqrt{2 + 2 \cos 4x}} \\ &= \sqrt{2 + \sqrt{2[1 + \cos 2(2x)]}} \\ &= \sqrt{2 + \sqrt{2 \times 2 \cos^2 2x}} \\ &= \sqrt{2 + 2 \cos 2x} = \sqrt{2(1 + \cos 2x)} \\ &= \sqrt{2 \times 2 \cos^2 x} \end{aligned}$$

$$= 2 \cos x$$

$$= \text{R.H.S.}$$

[Note : The question has been modified.]

$$\text{viii. } 16 \sin \theta \cos \theta \cos 2\theta \cos 4\theta \cos 8\theta = \sin 16\theta$$

Solution:

$$\text{L.H.S.} = 16 \sin \theta \cos \theta \cos 2\theta \cos 4\theta \cos 8\theta$$

$$= 8(2 \sin \theta \cos \theta) \cos 2\theta \cos 4\theta \cos 8\theta$$

$$= 8 \sin 2\theta \cos 2\theta \cos 4\theta \cos 8\theta$$

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$$= 4(2\sin 2\theta \cos 2\theta) \cos 4\theta \cos 8\theta$$

$$= 4\sin 4\theta \cos 4\theta \cos 8\theta$$

$$= 2(2\sin 4\theta \cos 4\theta) \cos 8\theta$$

$$= 2\sin 8\theta \cos 8\theta$$

$$= \sin 16\theta$$

$$= \text{R.H.S.}$$

$$\text{ix. } \cot x = 2 \cot 2x$$

Solution:

$$\begin{aligned} \text{L.H.S.} &= \frac{\sin 3x}{\cos x} + \frac{\cos 3x}{\sin x} \\ &= \frac{\sin 3x \sin x + \cos 3x \cos x}{\sin x \cos x} \\ &= \frac{\cos(3x - x)}{\sin x \cos x} \\ &= \frac{2 \cos 2x}{2 \sin x \cos x} \\ &= \frac{2 \cos 2x}{\sin 2x} \\ &= 2 \cot 2x \\ &= \text{R.H.S.} \end{aligned}$$

$$\text{x. } \frac{\cos x}{1 + \sin x} = \frac{\cot \left(\frac{x}{2} \right) - 1}{\cot \left(\frac{x}{2} \right) + 1}$$

Solution:

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$$\begin{aligned}
 \text{L.H.S.} &= \frac{\cos x}{1 + \sin x} \\
 &= \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\left(\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} \right) + 2 \sin \frac{x}{2} \cos \frac{x}{2}} \\
 &= \frac{\left(\cos \frac{x}{2} - \sin \frac{x}{2} \right) \left(\cos \frac{x}{2} + \sin \frac{x}{2} \right)}{\left(\cos \frac{x}{2} + \sin \frac{x}{2} \right)^2} \\
 &= \frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}}
 \end{aligned}$$

Dividing numerator and denominator by $\sin \frac{x}{2}$,

we get

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} - 1}{\frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} + 1} \\
 &= \frac{\cot \frac{x}{2} - 1}{\cot \frac{x}{2} + 1} \\
 &= \text{R.H.S.}
 \end{aligned}$$

xi. $\frac{\tan\left(\frac{\theta}{2}\right) + \cot\left(\frac{\theta}{2}\right)}{\cot\left(\frac{\theta}{2}\right) - \tan\left(\frac{\theta}{2}\right)} = \sec \theta$

Solution:

$$\begin{aligned} \text{L.H.S.} &= \frac{\tan \frac{\theta}{2} + \cot \frac{\theta}{2}}{\cot \frac{\theta}{2} - \tan \frac{\theta}{2}} \\ &= \frac{\frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} + \frac{\cos \frac{\theta}{2}}{\sin \frac{\theta}{2}}}{\frac{\cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} - \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}}} \\ &= \frac{\frac{\sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2}}{\cos \frac{\theta}{2} \sin \frac{\theta}{2}}}{\frac{\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}}{\cos \frac{\theta}{2} \sin \frac{\theta}{2}}} \\ &= \frac{1}{\cos \theta} \\ &= \sec \theta = \text{R.H.S.} \end{aligned}$$

xii. $\frac{1}{\tan 3A - \tan A} - \frac{1}{\cot 3A - \cot A} = \cot 2A$

Solution:

$$\text{L.H.S.} = \frac{1}{\tan 3A - \tan A} - \frac{1}{\cot 3A - \cot A}$$

$$\begin{aligned}
 &= \frac{1}{\tan 3A - \tan A} - \frac{1}{\frac{1}{\tan 3A} - \frac{1}{\tan A}} \\
 &= \frac{1}{\tan 3A - \tan A} - \frac{\tan 3A \cdot \tan A}{\tan A - \tan 3A} \\
 &= \frac{1}{\tan 3A - \tan A} + \frac{\tan 3A \cdot \tan A}{\tan 3A - \tan A} \\
 &= \frac{1 + \tan 3A \cdot \tan A}{\tan 3A - \tan A} \\
 &= \frac{1}{\frac{\tan 3A - \tan A}{1 + \tan 3A \cdot \tan A}} \\
 &= \frac{1}{\tan(3A - A)} \\
 &= \frac{1}{\tan 2A} \\
 &= \cot 2A = \text{R.H.S.}
 \end{aligned}$$

xiii. $\cos 7^\circ \cos 14^\circ \cos 28^\circ \cos 56^\circ \frac{\sin 68^\circ}{16 \cos 83^\circ}$

Solution:

$$\begin{aligned}
 \text{L.H.S.} &= \cos 7^\circ \cos 14^\circ \cos 28^\circ \cos 56^\circ \\
 &= \frac{1}{2 \sin 7^\circ} (2 \sin 7^\circ \cos 7^\circ) \cos 14^\circ \cos 28^\circ \cos 56^\circ \\
 &\quad \quad \quad 1
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2 \sin 7^\circ} (\sin 14^\circ \cos 14^\circ \cos 28^\circ \cos 56^\circ) \\
 &\dots [\because 2 \sin \theta \cos \theta = \sin 2\theta] \\
 &= \frac{1}{2 \sin 7^\circ} (2 \sin 14^\circ \cos 14^\circ) \cos 28^\circ \cos 56^\circ \\
 &= \frac{1}{4 \sin 7^\circ} (\sin 28^\circ \cos 28^\circ \cos 56^\circ) \\
 &= \frac{1}{2(4 \sin 7^\circ)} (2 \sin 28^\circ \cos 28^\circ) \cos 56^\circ \\
 &= \frac{1}{8 \sin 7^\circ} (\sin 56^\circ \cos 56^\circ) \\
 &= \frac{1}{8 \sin 7^\circ} (2 \sin 56^\circ \cos 56^\circ) \\
 &= \frac{1}{16 \sin 7^\circ} (\sin 112^\circ) \\
 &= \frac{\sin(180^\circ - 68^\circ)}{16 \sin 7^\circ} \\
 &= \frac{16 \sin(90^\circ - 83^\circ)}{\sin 68^\circ} \\
 &= \frac{16 \cos 83^\circ}{16 \cos 83^\circ} \\
 &= \text{R.H.S.}
 \end{aligned}$$

$$\text{xiv.} = \frac{\sin^2(-160^\circ)}{\sin^2 70^\circ} + \frac{\sin(180^\circ - \theta)}{\sin \theta} = \sec^2 20^\circ$$

Solution:

$$\begin{aligned}
 \text{L.H.S.} &= \frac{\sin^2(-160^\circ)}{\sin^2 70^\circ} + \frac{\sin(180^\circ - \theta)}{\sin \theta} \\
 &= \frac{(-\sin 160^\circ)^2}{\sin^2 70^\circ} + \frac{\sin \theta}{\sin \theta} \\
 &= \frac{\sin^2 160^\circ}{\sin^2 70^\circ} + 1 \\
 &= 1 + \frac{(\sin 160^\circ)^2}{(\sin 70^\circ)^2}
 \end{aligned}$$

$$\begin{aligned} &= 1 + \frac{[\sin(180^\circ - 20^\circ)]^2}{[\sin(90^\circ - 20^\circ)]^2} \\ &= 1 + \frac{\sin^2 20^\circ}{\cos^2 20^\circ} \\ &= 1 + \tan^2 20^\circ \\ &= \sec^2 20^\circ = \text{R.H.S.} \end{aligned}$$

xv. $\frac{2 \cos 4x + 1}{2 \cos x + 1} = (2 \cos x - 1)(2 \cos 2x - 1)$

Solution:

$$\begin{aligned} \text{L.H.S.} &= \frac{2 \cos 4x + 1}{2 \cos x + 1} \\ &= \frac{2[2 \cos^2(2x) - 1] + 1}{2 \cos x + 1} \\ &\quad \dots [\because \cos 2\theta = 2 \cos^2 \theta - 1] \\ &= \frac{4 \cos^2 2x - 2 + 1}{2 \cos x + 1} \\ &= \frac{(2 \cos 2x)^2 - (1)^2}{2 \cos x + 1} \\ &= \frac{(2 \cos 2x + 1)(2 \cos 2x - 1)}{2 \cos x + 1} \\ &= \frac{[2(2 \cos^2 x - 1) + 1](2 \cos 2x - 1)}{2 \cos x + 1} \end{aligned}$$

$$\begin{aligned}
 &= \frac{[2(2\cos^2 x - 1) + 1](2\cos 2x - 1)}{2\cos x + 1} \\
 &= \frac{(4\cos^2 x - 2 + 1)(2\cos 2x - 1)}{2\cos x + 1} \\
 &= \frac{[(2\cos x)^2 - (1)^2](2\cos 2x - 1)}{2\cos x + 1} \\
 &= \frac{(2\cos x + 1)(2\cos x - 1)(2\cos 2x - 1)}{2\cos x + 1} \\
 &= (2\cos x - 1)(2\cos 2x - 1) \\
 &= \text{R.H.S.}
 \end{aligned}$$

$$\text{xvi.} = \cos^2 x + \cos^2 (x + 120^\circ) + \cos^2 (x - 120^\circ) = \frac{3}{2}$$

Solution:

$$\text{L.H.S} = \cos^2 x + \cos^2 (x + 120^\circ) + \cos^2 (x - 120^\circ) =$$

$$\begin{aligned}
 &= \frac{1 + \cos 2x}{2} + \frac{1 + \cos 2(x + 120^\circ)}{2} \\
 &\quad + \frac{1 + \cos 2(x - 120^\circ)}{2} \\
 &\quad \dots \left[\because \cos^2 \theta = \frac{1 + \cos 2\theta}{2} \right]
 \end{aligned}$$

$$\begin{aligned}
 &\frac{3}{2} + \frac{1}{2} [\cos 2x + \cos(2x + 240^\circ) + \cos(2x - 240^\circ)] \\
 &= \frac{3}{2} + \frac{1}{2} (\cos 2x + \cos 2x \cos 240^\circ - \sin 2x \sin 240^\circ + \cos 2x \cos 240^\circ + \sin 2x \sin 240^\circ) \\
 &= \frac{3}{2} + \frac{1}{2} (\cos 2x + 2 \cos 2x \cos 240^\circ) \\
 &= \frac{3}{2} + \frac{1}{2} [\cos 2x + 2 \cos 2x \cos (180^\circ + 60^\circ)]
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{3}{2} + \frac{1}{2} [\cos 2x + 2 \cos 2x (-\cos 60^\circ)] \\
 &= \frac{3}{2} + \frac{1}{2} [\cos 2x - 2 \cos 2x (\frac{1}{2})] \\
 &= \frac{3}{2} + \frac{1}{2} (\cos 2x - \cos 2x) \\
 &= \frac{3}{2} + \frac{1}{2} (0) \\
 &= \frac{3}{2} = \text{R.H.S.}
 \end{aligned}$$

xvii. $2 \operatorname{cosec} 2x + \operatorname{cosec} x = \sec \cot \frac{x}{2}$

Solution:

$$\begin{aligned}
 \text{L.H.S.} &= 2 \operatorname{cosec} 2x + \operatorname{cosec} x \\
 &= \frac{2}{\sin 2x} + \frac{1}{\sin x} \\
 &= \frac{2}{2 \sin x \cos x} + \frac{1}{\sin x} \\
 &= \frac{1}{\sin x \cos x} + \frac{1}{\sin x} \\
 &= \frac{1 + \cos x}{\sin x \cos x} \\
 &= \frac{2 \cos^2 \frac{x}{2}}{\left(2 \sin \frac{x}{2} \cos \frac{x}{2}\right) \cos x} \\
 &= \frac{1}{\cos x} \cdot \frac{\cos \left(\frac{x}{2}\right)}{\sin \left(\frac{x}{2}\right)}
 \end{aligned}$$

$$= \sec x \cot \left(\frac{x}{2} \right)$$
$$= \text{R.H.S.}$$

xviii. $4 \cos x \cos \left(\frac{\pi}{3} + x \right) \cos \left(\frac{\pi}{3} - x \right) = \cos 3x$

Solution:

$$\begin{aligned} \text{L.H.S.} &= 4 \cos x \cdot \cos \left(\frac{\pi}{3} + x \right) \cdot \cos \left(\frac{\pi}{3} - x \right) \\ &= 4 \cos x \left(\cos \frac{\pi}{3} \cos x - \sin \frac{\pi}{3} \sin x \right) \\ &\quad \left(\cos \frac{\pi}{3} \cos x + \sin \frac{\pi}{3} \sin x \right) \\ &= 4 \cos x \left(\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x \right) \left(\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right) \\ &= 4 \cos x \left[\left(\frac{1}{2} \cos x \right)^2 - \left(\frac{\sqrt{3}}{2} \sin x \right)^2 \right] \\ &= 4 \cos x \left(\frac{1}{4} \cos^2 x - \frac{3}{4} \sin^2 x \right) \\ &= \cos^3 x - 3 \cos x \sin^2 x \\ &= \cos^3 x - 3 \cos x (1 - \cos^2 x) \\ &= \cos^3 x - 3 \cos x + 3 \cos^3 x \\ &= 4 \cos^3 x - 3 \cos x \\ &= \cos 3x = \text{R.H.S.} \end{aligned}$$

Note: The question has been modified.

$$\text{xix. } \sin x \tan \frac{x}{2} + 2\cos x = \frac{2}{1+\tan^2\left(\frac{x}{2}\right)}$$

Solution:

$$\text{L.H.S.} = \sin x \tan \left(\frac{x}{2}\right) + 2\cos x$$

$$= \left(2 \sin \frac{x}{2} \cos \frac{x}{2}\right) \left(\frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}\right) + 2\cos x$$

$$= \left(2 \sin \frac{x}{2} \cos \frac{x}{2}\right) \left(\frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}\right) + 2 \cos x$$

$$= 2\sin^2 \frac{x}{2} + 2\cos x$$

$$= 1 - \cos x + 2\cos x$$

$$= 1 + \cos x$$

$$= 2\cos^2 \frac{x}{2}$$

$$= \frac{2}{\sec^2 \frac{x}{2}} = \frac{2}{1+\tan^2 \frac{x}{2}} = \text{R.H.S.}$$

Ex 3.4

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Question 1.

Express the following as a sum or difference of two trigonometric functions.

i. $2\sin 4x \cos 2x$

ii. $2\sin \frac{2\pi}{3} \cos \frac{\pi}{2}$

iii. $2\cos 4\theta \cos 2\theta$

iv. $2\cos 35^\circ \cos 75^\circ$

Solution:

i. $2\sin 4x \cos 2x = \sin(4x + 2x) + \sin(4x - 2x)$
 $= \sin 6x + \sin 2x$

ii.

$$2\sin \frac{2\pi}{3} \cos \frac{\pi}{2} = \sin \left(\frac{2\pi}{3} + \frac{\pi}{2} \right) + \sin \left(\frac{2\pi}{3} - \frac{\pi}{2} \right)$$
$$= \sin \frac{7\pi}{6} + \sin \frac{\pi}{6}$$

[Note: Answer given in the textbook is $\sin \frac{7\pi}{12} + \sin \frac{\pi}{12}$ However, as per our calculation it is $\sin \frac{7\pi}{6} + \sin \frac{\pi}{6}$

iii. $2\cos 4\theta \cos 2\theta = \cos(4\theta + 2\theta) + \cos(4\theta - 2\theta)$
 $= \cos 6\theta + \cos 2\theta$

$$\begin{aligned}\text{iv. } & 2\cos 35^\circ \cos 75^\circ \\ &= \cos(35^\circ + 75^\circ) + \cos(35^\circ - 75^\circ) \\ &= \cos 110^\circ + \cos(-40^\circ) \\ &= \cos 110^\circ + \cos 40^\circ \dots [\because \cos(-\theta) = \cos \theta]\end{aligned}$$

Question 2.

Prove the following:

$$\text{i. } \frac{\sin 2x + \sin 2y}{\sin 2x - \sin 2y} = \frac{\tan(x+y)}{\tan(x-y)}$$

Solution:

$$\begin{aligned}\text{L.H.S.} &= \frac{\sin 2x + \sin 2y}{\sin 2x - \sin 2y} \\ &= \frac{2\sin\left(\frac{2x+2y}{2}\right)\cos\left(\frac{2x-2y}{2}\right)}{2\cos\left(\frac{2x+2y}{2}\right)\sin\left(\frac{2x-2y}{2}\right)} \\ &= \frac{\sin(x+y)\cos(x-y)}{\cos(x+y)\sin(x-y)} \\ &= \tan(x+y) \cdot \cot(x-y) \\ &= \tan(x+y) \cdot \frac{1}{\tan(x-y)} \\ &= \frac{\tan(x+y)}{\tan(x-y)} \\ &= \text{R.H.S.}\end{aligned}$$

ii. $\sin 6x + \sin 4x - \sin 2x = 4 \cos x \sin 2x \cos 3x$

Solution:

$$\begin{aligned} \text{L.H.S.} &= \sin 6x + \sin 4x - \sin 2x \\ &= 2 \sin \left(\frac{6x+4x}{2} \right) \cos \left(\frac{6x-4x}{2} \right) - 2 \sin x \cos x \\ &= 2 \sin 5x \cos x - 2 \sin x \cos x \\ &= 2 \cos x (\sin 5x - \sin x) \\ &= 2 \cos x \left[2 \cos \left(\frac{5x+x}{2} \right) \sin \left(\frac{5x-x}{2} \right) \right] \\ &= 2 \cos x (2 \cos 3x \sin 2x) \\ &= 4 \cos x \sin 2x \cos 3x \\ &= \text{R.H.S.} \end{aligned}$$

[Note: The question has been modified.]

iii. $\frac{\sin x - \sin 3x + \sin 5x - \sin 7x}{\cos x - \cos 3x - \cos 5x + \cos 7x} = \cot 2x$

Solution:

$$\begin{aligned} \text{L.H.S.} &= \frac{\sin x - \sin 3x + \sin 5x - \sin 7x}{\cos x - \cos 3x - \cos 5x + \cos 7x} \\ &= \frac{(\sin 5x + \sin x) - (\sin 7x + \sin 3x)}{(\cos x - \cos 5x) - (\cos 3x - \cos 7x)} \\ &= \frac{2 \sin \left(\frac{5x+x}{2} \right) \cdot \cos \left(\frac{5x-x}{2} \right) - 2 \sin \left(\frac{7x+3x}{2} \right) \cdot \cos \left(\frac{7x-3x}{2} \right)}{2 \sin \left(\frac{x+5x}{2} \right) \cdot \sin \left(\frac{5x-x}{2} \right) - 2 \sin \left(\frac{3x+7x}{2} \right) \cdot \sin \left(\frac{7x-3x}{2} \right)} \\ &= \frac{2 \sin 3x \cdot \cos 2x - 2 \sin 5x \cdot \cos 2x}{2 \sin 3x \cdot \sin 2x - 2 \sin 5x \cdot \sin 2x} \\ &= \frac{2 \cos 2x (\sin 3x - \sin 5x)}{2 \sin 2x (\sin 3x - \sin 5x)} \\ &= \frac{\cos 2x}{\sin 2x} = \cot 2x = \text{R.H.S.} \end{aligned}$$

iv. $\sin 18^\circ \cos 39^\circ + \sin 6^\circ \cos 15^\circ = \sin 24^\circ \cos 33^\circ$

Solution:

$$\begin{aligned}\text{L.H.S.} &= \sin 18^\circ \cdot \cos 39^\circ + \sin 6^\circ \cdot \cos 15^\circ \\&= \frac{1}{2} (2 \cos 39^\circ \sin 18^\circ + 2 \cos 15^\circ \sin 6^\circ) \\&= \frac{1}{2} [\sin(39^\circ + 18^\circ) - \sin(39^\circ - 18^\circ) + \sin(15^\circ + 6^\circ) - \sin(15^\circ - 6^\circ)] \\&= \frac{1}{2} (\sin 57^\circ - \sin 21^\circ + \sin 21^\circ - \sin 9^\circ) \\&= \frac{1}{2} (\sin 57^\circ - \sin 9^\circ) \\&= \frac{1}{2} \times 2 \cdot \cos \left(\frac{57^\circ + 9^\circ}{2} \right) \cdot \sin \left(\frac{57^\circ - 9^\circ}{2} \right) \\&= \cos 33^\circ \cdot \sin 24^\circ \\&= \sin 24^\circ \cdot \cos 33^\circ \\&= \text{R.H.S.}\end{aligned}$$

v. $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = 1/16$

Solution:

$$\begin{aligned}\text{L.H.S.} &= \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ \\&= \cos 20^\circ \cdot \cos 40^\circ \cdot \frac{1}{2} \cdot \cos 80^\circ \\&= \frac{1}{2 \times 2} (2 \cos 40^\circ \cdot \cos 20^\circ) \cdot \cos 80^\circ \\&= \frac{1}{4} [\cos(40^\circ + 20^\circ) + \cos(40^\circ - 20^\circ)] \cdot \cos 80^\circ \\&= \frac{1}{4} (\cos 60^\circ + \cos 20^\circ) \cos 80^\circ \\&= \frac{1}{4} \cos 60^\circ \cdot \cos 80^\circ + \frac{1}{4} \cos 20^\circ \cdot \cos 80^\circ \\&= \frac{1}{4} \left(\frac{1}{2} \right) \cos 80^\circ + \frac{1}{2 \times 4} (2 \cos 80^\circ \cos 20^\circ) \\&= \frac{1}{8} \cos 80^\circ + \frac{1}{8} [\cos(80^\circ + 20^\circ) + \cos(80^\circ - 20^\circ)] \\&= \frac{1}{8} \cos 80^\circ + \frac{1}{8} (\cos 100^\circ + \cos 60^\circ) \\&= \frac{1}{8} \cos 80^\circ + \frac{1}{8} \cos 100^\circ + \frac{1}{8} \cos 60^\circ\end{aligned}$$

$$= \frac{1}{8} \cos 80^\circ - \frac{1}{8} \cos 80^\circ + \frac{1}{16} \dots [\because \cos (180 - \theta) = -\cos \theta]$$

$$= \frac{1}{16} = \text{R.H.S}$$

vi. $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = 3/16$

Solution:

$$\begin{aligned} \text{L.H.S.} &= \sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ \\ &= \sin 20^\circ \cdot \sin 40^\circ \cdot \left(\frac{\sqrt{3}}{2}\right) \cdot \sin 80^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{1}{2} (2 \cdot \sin 40^\circ \cdot \sin 20^\circ) \cdot \sin 80^\circ \\ &= \frac{\sqrt{3}}{4} [\cos(40^\circ - 20^\circ) - \cos(40^\circ + 20^\circ)] \cdot \sin 80^\circ \\ &= \frac{\sqrt{3}}{4} (\cos 20^\circ - \cos 60^\circ) \sin 80^\circ \\ &= \frac{\sqrt{3}}{4} \cdot \cos 20^\circ \cdot \sin 80^\circ - \frac{\sqrt{3}}{4} \cos 60^\circ \cdot \sin 80^\circ \\ &= \frac{\sqrt{3}}{2 \times 4} (2 \sin 80^\circ \cdot \cos 20^\circ) - \frac{\sqrt{3}}{4} \left(\frac{1}{2}\right) \cdot \sin 80^\circ \\ &= \frac{\sqrt{3}}{8} [\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ)] \\ &\quad - \frac{\sqrt{3}}{8} \cdot \sin 80^\circ \\ &= \frac{\sqrt{3}}{8} (\sin 100^\circ + \sin 60^\circ) - \frac{\sqrt{3}}{8} \cdot \sin 80^\circ \end{aligned}$$

$$\begin{aligned}
 &= \frac{\sqrt{3}}{8} (\sin 100^\circ + \sin 60^\circ) - \frac{\sqrt{3}}{8} \sin 80^\circ \\
 &= \frac{\sqrt{3}}{8} \sin 100^\circ + \frac{\sqrt{3}}{8} \sin 60^\circ - \frac{\sqrt{3}}{8} \sin 80^\circ \\
 &= \frac{\sqrt{3}}{8} \sin(180^\circ - 80^\circ) + \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{8} \sin 80^\circ \\
 &= \frac{\sqrt{3}}{8} \sin 80^\circ + \frac{3}{16} - \frac{\sqrt{3}}{8} \sin 80^\circ \\
 &\quad \dots [\because \sin(180^\circ - \theta) = \sin \theta] \\
 &= \frac{3}{16} = \text{R.H.S.}
 \end{aligned}$$

Ex 3.5

Question 1.

In ΔABC , $A + B + C = \pi$, show that

$$\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$$

Solution:

$$\begin{aligned}
 \text{L.H.S.} &= \cos 2A + \cos 2B + \cos 2C \\
 &= 2 \cdot \cos\left(\frac{2A+2B}{2}\right) \cdot \cos\left(\frac{2A-2B}{2}\right) + \cos 2C \\
 &= 2 \cdot \cos(A+B) \cdot \cos(A-B) + 2\cos^2 C - 1 \\
 \text{In } \Delta ABC, A + B + C &= \pi \\
 \therefore A + B &= \pi - C \\
 \therefore \cos(A+B) &= \cos(\pi - C) \\
 \therefore \cos(A+B) &= -\cos C \dots\dots\dots(i)
 \end{aligned}$$

$$\therefore \text{L.H.S.} = -2 \cos C \cos(A-B) + 2\cos^2 C - 1 \dots [\text{From (i)}]$$

$$= -1 - 2 \cos C [\cos(A-B) - \cos C]$$

$$= -1 - 2 \cos C [\cos(A-B) + \cos(A+B)]$$

... [From (i)]

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-1-chapter-3-trigonometry-ii/>

$$= -1 - 2 \cos C (2 \cos A \cos B)$$

$$= -1 - 4 \cos A \cos B \cos C = \text{R.H.S.}$$

Question 2.

$$\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

Solution:

$$\begin{aligned} \text{L.H.S.} &= \sin A + \sin B + \sin C \\ &= 2.\sin\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right) \\ &\quad + 2.\sin\frac{C}{2}.\cos\frac{C}{2} \end{aligned}$$

$$\text{In } \Delta ABC, A + B + C = \pi$$

$$\therefore A + B = \pi - C$$

$$\begin{aligned} \therefore \sin\left(\frac{A+B}{2}\right) &= \sin\left(\frac{\pi-C}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{C}{2}\right) \\ &= \cos\frac{C}{2} \quad \dots(i) \end{aligned}$$

$$\begin{aligned} \text{and } \cos\left(\frac{A+B}{2}\right) &= \cos\left(\frac{\pi-C}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{C}{2}\right) \\ &= \sin\frac{C}{2} \quad \dots(ii) \end{aligned}$$

$$\begin{aligned} \therefore \text{L.H.S.} &= 2.\cos\frac{C}{2}.\cos\left(\frac{A-B}{2}\right) \\ &\quad + 2.\cos\left(\frac{A+B}{2}\right).\cos\frac{C}{2} \\ &\quad \dots[\text{From (i) and (ii)}] \end{aligned}$$

$$= 2.\cos\frac{C}{2}.\left[\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\right]$$

$$= 2.\cos\frac{C}{2}.2\cos\left[\frac{\frac{A+B}{2} + \frac{A-B}{2}}{2}\right]$$

$$. \cos\left[\frac{\frac{A+B}{2} - \frac{A-B}{2}}{2}\right]$$

$$= 2.\cos\frac{C}{2}.\left(2.\cos\frac{A}{2}.\cos\frac{B}{2}\right)$$

$$= 4.\cos\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2} = \text{R.H.S.}$$

Question 3.

$$\begin{aligned}\cos A + \cos B + \cos C &= 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} \\ &= 2 \cdot \cos \left(\frac{A+B}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right) - \left(1 - 2 \sin^2 \frac{C}{2} \right)\end{aligned}$$

Solution:

$$\begin{aligned}\text{L.H.S.} &= \sin A + \sin B + \sin C \\ &= 2 \cdot \cos \left(\frac{A+B}{2} \right) \cdot \cos \left(\frac{A-B}{2} \right) - \left(1 - 2 \sin^2 \frac{C}{2} \right)\end{aligned}$$

In ΔABC , $A + B + C = \pi$,

$$\therefore A + B = \pi - C$$

$$\begin{aligned}\therefore \cos \left(\frac{A+B}{2} \right) &= \cos \left(\frac{\pi-C}{2} \right) = \cos \left(\frac{\pi}{2} - \frac{C}{2} \right) \\ &= \sin \frac{C}{2} \quad \dots(i)\end{aligned}$$

$$\begin{aligned}\therefore \text{L.H.S.} &= 2 \sin \frac{C}{2} \cdot \cos \left(\frac{A-B}{2} \right) - 1 + 2 \sin^2 \frac{C}{2} \\ &\quad \dots[\text{From (i)}]\end{aligned}$$

$$= 2 \sin \frac{C}{2} \cdot \left[\cos \left(\frac{A-B}{2} \right) + \sin \frac{C}{2} \right] - 1$$

$$= 2 \sin \frac{C}{2} \cdot \left[\cos \left(\frac{A-B}{2} \right) + \cos \left(\frac{A+B}{2} \right) \right] - 1$$

$$\begin{aligned}&\quad \dots[\text{From (i)}] \\ &= 2 \sin \frac{C}{2} \cdot 2 \cos \left[\frac{\frac{A+B}{2} + \frac{A-B}{2}}{2} \right]\end{aligned}$$

$$\begin{aligned}
 & \cos \left[\frac{\frac{A+B}{2} - \frac{A-B}{2}}{2} \right] - 1 \\
 &= 2 \sin \frac{C}{2} \cdot \left(2 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \right) - 1 \\
 &= 4 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \sin \frac{C}{2} - 1 \\
 &= \text{R.H.S.}
 \end{aligned}$$

Question 4.

$$\sin^2 A + \sin^2 B - \sin^2 C = 2 \sin A \sin B \cos C$$

Solution:

$$\text{We know that, } \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

L.H.S.

$$\begin{aligned}
 &= \sin^2 A + \sin^2 B + \sin^2 C \\
 &= \frac{1 - \cos 2A}{2} + \frac{1 - \cos 2B}{2} - \sin^2 C \\
 &= \frac{1}{2} [2 - (\cos 2A + \cos 2B)] - \sin^2 C \\
 &= \frac{1}{2} \left[2 - 2 \cos \left(\frac{2A+2B}{2} \right) \cdot \cos \left(\frac{2A-2B}{2} \right) \right] \\
 &\quad - \sin^2 C
 \end{aligned}$$

$$\begin{aligned}
 &= 1 - \cos(A+B) \cdot \cos(A-B) - \sin^2 C \\
 &= (1 - \sin^2 C) - \cos(A+B) \cdot \cos(A-B)
 \end{aligned}$$

$$\begin{aligned}
 &= \cos^2 C - \cos(A+B) \cdot \cos(A-B) \\
 &\therefore \cos(A+B) = \cos(\pi - C) \\
 &\therefore \cos(A+B) = -\cos C \dots (i) \\
 &\therefore \text{L.H.S.} = \cos^2 C + \cos C \cdot \cos(A-B) \\
 &\dots [\text{From (i)}] \\
 &= \cos C [\cos C + \cos(A-B)] \\
 &= \cos C [-\cos(A+B) + \cos(A-B)] \\
 &\dots [\text{From (i)}] \\
 &= \cos C [\cos(A-B) - \cos(A+B)] \\
 &= \cos C (2 \sin A \cdot \sin B) \\
 &= 2 \sin A \cdot \sin B \cdot \cos C \\
 &= \text{R.H.S.}
 \end{aligned}$$

[Note: The question has been modified.]

Question 5.

$$\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} = 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$$

Solution:

$$\text{We know that, } \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

L.H.S.

$$\begin{aligned}
 &= \sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} \\
 &= \frac{1 - \cos A}{2} + \frac{1 - \cos B}{2} - \sin^2 \frac{C}{2} \\
 &= \frac{1}{2} [2 - (\cos A + \cos B)] - \sin^2 \frac{C}{2}
 \end{aligned}$$

$$= \frac{1}{2} \left[2 - 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right] - \sin^2 \frac{C}{2}$$

$$= 1 - \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) - \sin^2 \frac{C}{2}$$

In ΔABC , $A + B + C = \pi$

$$\therefore A + B = \pi - C$$

$$\therefore \cos \left(\frac{A+B}{2} \right) = \cos \left(\frac{\pi - C}{2} \right) = \cos \left(\frac{\pi}{2} - \frac{C}{2} \right)$$

$$= \sin \frac{C}{2} \quad \dots(i)$$

$$\therefore \text{L.H.S.} = 1 - \sin \frac{C}{2} \cos \left(\frac{A-B}{2} \right) - \sin^2 \frac{C}{2}$$

$$\dots[\text{From (i)}]$$

$$= 1 - \sin \frac{C}{2} \left[\cos \left(\frac{A-B}{2} \right) + \sin \frac{C}{2} \right]$$

$$= 1 - \sin \frac{C}{2} \left[\cos \left(\frac{A-B}{2} \right) + \cos \left(\frac{A+B}{2} \right) \right]$$

$$\dots[\text{From (i)}]$$

$$= 1 - \sin \frac{C}{2} \cdot 2 \cos \left[\frac{\frac{A+B}{2} + \frac{A-B}{2}}{2} \right]$$

$$\cdot \cos \left[\frac{\frac{A+B}{2} - \frac{A-B}{2}}{2} \right]$$

$$= 1 - \sin \frac{C}{2} \cdot \left(2 \cos \frac{A}{2} \cos \frac{B}{2} \right)$$

$$= 1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$$

$$= \text{R.H.S.}$$

Question 6.

$$\tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{B}{2} \tan \frac{C}{2} \tan \frac{C}{2} \tan \frac{A}{2} = 1$$

Solution:

In $\triangle ABC$,

$$A + B + C = \pi$$

$$\therefore A + B = \pi - C$$

$$\therefore \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi-C}{2}\right)$$

$$\therefore \tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$\therefore \frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \cdot \tan \frac{B}{2}} = \cot \frac{C}{2}$$

$$\therefore \frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{1 - \tan \frac{A}{2} \cdot \tan \frac{B}{2}} = \frac{1}{\tan \frac{C}{2}}$$

$$\therefore \tan \frac{C}{2} \cdot \left(\tan \frac{A}{2} + \tan \frac{B}{2}\right) = 1 - \tan \frac{A}{2} \cdot \tan \frac{B}{2}$$

$$\therefore \tan \frac{A}{2} \cdot \tan \frac{B}{2} + \tan \frac{B}{2} \cdot \tan \frac{C}{2} + \tan \frac{C}{2} \cdot \tan \frac{A}{2} = 1$$

Question 7.

$$\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2}$$

Solution:

In ΔABC ,

$$A + B + C = \pi$$

$$\therefore A + B = \pi - C$$

$$\therefore \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi-C}{2}\right)$$

$$\therefore \tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$$

$$\therefore \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2} \cdot \tan\frac{B}{2}} = \cot\frac{C}{2}$$

$$\therefore \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2} \cdot \tan\frac{B}{2}} = \frac{1}{\tan\frac{C}{2}}$$

$$\therefore \tan\frac{C}{2} \cdot \left(\tan\frac{A}{2} + \tan\frac{B}{2}\right) = 1 - \tan\frac{A}{2} \cdot \tan\frac{B}{2}$$

$$\therefore \tan\frac{B}{2} \cdot \tan\frac{C}{2} + \tan\frac{A}{2} \cdot \tan\frac{C}{2} + \tan\frac{A}{2} \cdot \tan\frac{B}{2} = 1$$

Dividing throughout by $\tan\frac{A}{2} \cdot \tan\frac{B}{2} \cdot \tan\frac{C}{2}$,

we get

$$\frac{1}{\tan\frac{A}{2}} + \frac{1}{\tan\frac{B}{2}} + \frac{1}{\tan\frac{C}{2}} = \frac{1}{\tan\frac{A}{2} \cdot \tan\frac{B}{2} \cdot \tan\frac{C}{2}}$$

$$\therefore \cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2} \cdot \cot\frac{B}{2} \cdot \cot\frac{C}{2}$$

Question 8.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-1-chapter-3-trigonometry-ii/>

$$\tan 2A + \tan 2B + \tan 2C = \tan 2A \tan 2B + \tan 2C$$

Solution:

In ΔABC ,

$$A + B + C = \pi$$

$$\therefore 2A + 2B + 2C = 2\pi$$

$$\therefore 2A + 2B = 2\pi - 2C$$

$$\tan(2A + 2B) = \tan(2\pi - 2C)$$

$$\frac{\tan 2A + \tan 2B}{1 - \tan 2A \cdot \tan 2B} = -\tan 2C$$

$$\therefore \tan 2A + \tan 2B = -\tan 2C \cdot (1 - \tan 2A \cdot \tan 2B)$$

$$\therefore \tan 2A + \tan 2B = -\tan 2C + \tan 2A \cdot \tan 2B \cdot \tan 2C$$

$$\therefore \tan 2A + \tan 2B + \tan 2C = \tan 2A \cdot \tan 2B \cdot \tan 2C$$

Question 9.

$$\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2 \sin A \sin B \sin C$$

Solution:

$$\text{we know that } \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

L.H.S.

$$\begin{aligned} &= \cos^2 A + \cos^2 B + \cos^2 C \\ &= \frac{1 + \cos 2A}{2} + \frac{1 + \cos 2B}{2} - \cos^2 C \\ &= \frac{1}{2} [2 + (\cos 2A + \cos 2B)] - \cos^2 C \\ &= \frac{1}{2} \left[2 + 2 \cdot \cos \left(\frac{2A + 2B}{2} \right) \cdot \cos \left(\frac{2A - 2B}{2} \right) \right] \\ &\quad - \cos^2 C \end{aligned}$$

$$= 1 + \cos(A + B) \cdot \cos(A - B) - \cos^2 C$$

In $\triangle ABC$,

$$A + B + C = \pi$$

$$A + B = \pi - C$$

$$\cos(A + B) = \cos(\pi - C)$$

$$\cos(A + B) = -\cos C \dots\dots\dots (i)$$

$$\text{L.H.S.} = 1 - \cos C \cdot \cos(A - B) - \cos^2 C$$

...[From(i)]

$$= 1 - \cos C \cdot [\cos(A - B) + \cos C]$$

$$= 1 - \cos C \cdot [\cos(A - B) - \cos(A + B)]$$

..[From (i)]

$$= 1 - \cos C \cdot (2 \sin A \sin B)$$

$$= 1 - 2 \sin A \sin B \cos C$$

= R.H.S.



Maharashtra Board Solutions

Class 11 Arts & Science Maths

(Part 1)

- Chapter 1- Angle and its Measurement
- Chapter 2- Trigonometry – I
- Chapter 3- Trigonometry – II
- Chapter 4- Determinants and Matrices
- Chapter 5- Straight Line
- Chapter 6- Circle
- Chapter 7- Conic Sections
- Chapter 8- Measures of Dispersion
- Chapter 9- Probability

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The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

The Maharashtra State Board of Secondary & Higher Secondary Education (MSBSHSE), Pune is an independent body of the Maharashtra Government. There are more than 1.4 million students that appear in the examination every year. The Maha State Board conducts the board examination twice a year. This board conducts the examination for SSC and HSC.

The Maharashtra government established the Maharashtra State Bureau of Textbook Production and Curriculum Research, also commonly referred to as Ebalbharati, in 1967 to take up the responsibility of providing quality textbooks to students from all classes studying under the Maharashtra State Board. MSBHSE prepares and updates the curriculum to provide holistic development for students. It is designed to tackle the difficulty in understanding the concepts with simple language with simple illustrations. Every year around 10 lakh students are enrolled in schools that are affiliated with the Maharashtra State Board.

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Who developed the Maharashtra State board books?

As of now, the MSCERT and Balbharti are responsible for the syllabus and textbooks of Classes 1 to 8, while Classes 9 and 10 are under the Maharashtra State Board of Secondary and Higher Secondary Education (MSBSHSE).

How many state boards are there in Maharashtra?

The Maharashtra State Board of Secondary & Higher Secondary Education, conducts the HSC and SSC Examinations in the state of Maharashtra through its nine Divisional Boards located at Pune, Mumbai, Aurangabad, Nasik, Kolhapur, Amravati, Latur, Nagpur and Ratnagiri.

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