

PE-256-CV-19
M.A./M.Sc. MATHEMATICS (2nd Semester)
Examination, June 2022
COMPLEX ANALYSIS (II)

Time : Three Hours]

[Maximum Marks : 80
[Minimum Pass Marks : 29

Note : Answer from both the Sections as directed. The figures in the right-hand margin indicate marks.

Section-A

1. Answer the following questions:

1x10=10

- (a) Define Euler's Gamma function.
- (b) Write the Duplication formula.
- (c) Define Riemann zeta function.
- (d) Write the value of $\frac{1}{2\pi} \int_{-\infty}^{\infty} P_r(\theta) d\theta$
- (e) Write the Harnack's inequality.
- (f) Write the definition of genus of an entire function.
- (g) Write the statement of Bloch's theorem.
- (h) Write the statement of Monodramy theorem.
- (i) Write the expression for functional equation.
- (j) Define Univalent function.

2. Answer the following short type questions:

2x5=10

- (a) Use Schwarz's refection principle to show that $\overline{\sin z + \cos z} = \sin \bar{z} + \cos \bar{z}$
- (b) By definition of Gamma function show that $\sqrt{\frac{1}{2}} = \sqrt{\pi}$
- (c) Construct the canonical product associated with $\cos z$
- (d) Find the order of $\cos \sqrt{z}$
- (e) Find the radius of convergence of series $Z - \frac{z^2}{2} + \frac{z^3}{3} \dots \dots \dots$

Section-B

12x5=60

Answer any five of the following questions:

3. a). Prove that if $|z| < \frac{1}{2}$ then $|E_p(z)| \geq \exp \left(\frac{-2}{|z|^{p+1}} \right)$
b). Find the residues at the pole of $\bar{z}$
4. State and prove Mittag Leffler's theorem.
5. State and prove Weiersrass factorization theorem.
6. a). Let $u: G \rightarrow R$ be a continuous fuction which has mean value property then prove that u harmonic.
b). Prove that the metric spece $\text{Har}(G)$ is complete.
7. Use Hamard's factorization to show that $\sin \pi z = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2} \right)$

Hence deduce the Walli's formula

8. Let f be analyfic in $D = \{z / |z| < 1\}$ and let $f(0) = 0$, $f'(0) = 1$ and $|f(z)| \leq M$ for all Z in D , then prove that $M \geq 1$ and $f(D) \supset B \left(0, \frac{1}{6m} \right)$
9. State and prove Schoftky's theorem.
10. State and prove the Little picard theorem.
11. State and prove " $\frac{1}{4}$ theorem"
12. State and prove the Borel's theorem.