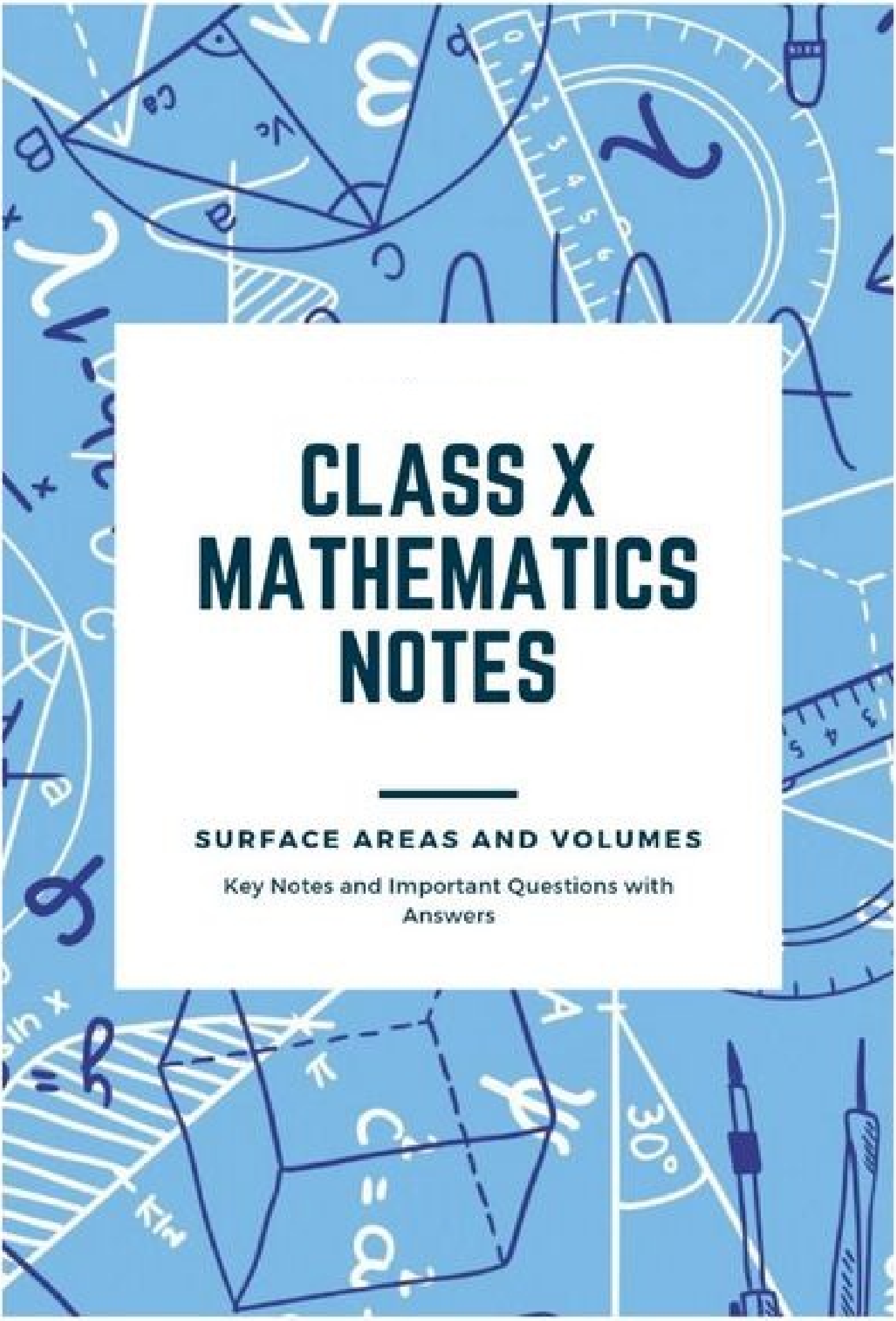


CLASS X MATHEMATICS NOTES

SURFACE AREAS AND VOLUMES

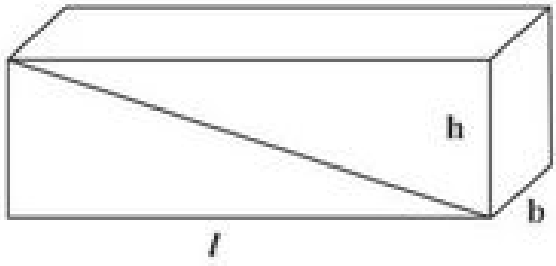
Key Notes and Important Questions with Answers

13
Chapter

Surface Areas and Volumes

Key Points

1. **Cuboid:** 3-D shapes like a book, a metch box , an almirah, a room etc. are called Cuboid.



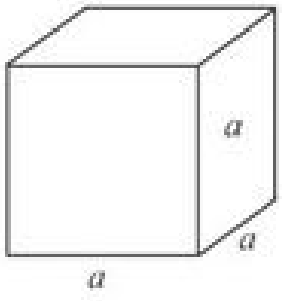
For cuboid length $= l$, breadth = b, height = h

Volume $= l \times b \times h$

Lateral surface area of solid cuboid $= 2h(l + b)$

Total surface area of solid cuboid $= 2(lb + bh + hl)$

2. **Cube:** 3-D shapes like ice-cubes, dice, etc. are called cube.



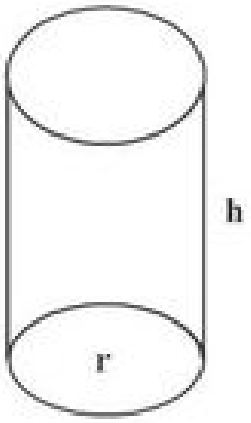
In cube, length = breadth = height = a

Volume $= a^3$

Lateral surface area of solid cube $= 4a^2$

Total surface area of solid cube $= 6a^2$

3. **Cylinder**: 3-D shapes like jars, circular pillars, circular pipes, rood rollers etc. are called cylinder.



(a) For right circular cylinder solid, base radius = r, height = h

Volume = $\pi r^2 h$

Lateral surface area of solid cylinder = $2\pi rh$

Total surface area of solid cylinder = $2\pi r\,(r + h)$

(b) For right circular cylinder (Hollow)

external radius = R

internal radius = r

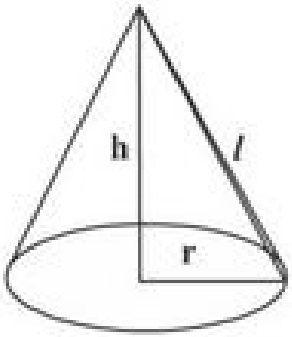
height = h

Volume = $\pi(R^2 - r^2)h$

Curved surface area = $2\pi(R + r)h$

Total surface area = $2\pi(R + r)\,h + 2\pi(R^2 - r^2)$

4. **Cone:** 3-D shapes like conical tents, ice-cream cone are called Cone.



For right circular cone,

base radius = r

height = h

slant height = l

$l = \sqrt{h^2 + r^2}$

Volume $= \frac{1}{3}\pi r^2 h$

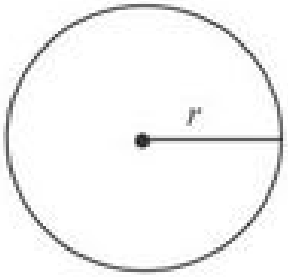
Curved surface area of solid cone $= \pi rl$

Total surface area of solid cone $= \pi r\,(r + l)$

It may be noted that if radius and height of a cone and cylinder are same then

3 × volume of a cone = volume of right circular cylinder

5. **Sphere**: 3-D shapes like cricket balls, footballs etc. are called sphere.

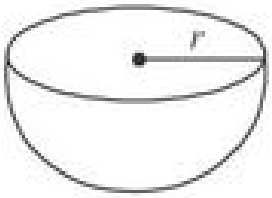


(a) For sphere : Radius = r

Volume $= \frac{4}{3}\pi r^3$

surface area $= 4\pi r^2$

(b) For Hemisphere (solid): Radius = r

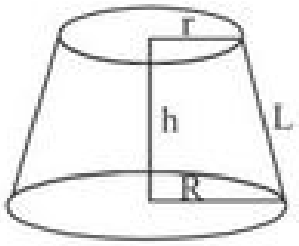


Volume $= \frac{2}{3}\pi r^3$

Curved surface area $= 2\pi r^2$

Total surface area $= 3\pi r^2$

6. **Frustum**: When a cone is cut by a plane parallel to the base of the cone, then the portion between the plane and the base is called the frustum of the cone.



Example = Turkish Cap

For a frustum of cone:

Base radius = R

Top radius = r

Height = h

slant height = l

$$l = \sqrt{h^2 + (R - r)^2}$$

$$\text{volume} = \frac{1}{3}\pi h(r^2 + R^2 + Rr)$$

Curved surface area (solid frustum) = $\pi l(R + r)$

Total surface area (solid frustum) = $\pi l(R + r) + \pi(R^2 + r^2)$

Important Questions with Solutions

VERY SHORT ANSWER TYPE QUESTIONS

1. What geometrical shapes is a "FUNNEL" combination of?

2. What geometrical shapes is a "SURAHI" combination of?

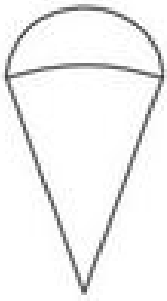
3. What geometrical shapes is a cylindrical "PENCIL" sharped at one edge combination of?

4. What geometrical 3-D shapes is a "GLASS (tumbler)"?

5. What geometrical shapes is a "SHUTTLE COCK" in badminton game combination of?

6. What geometrical shapes is a "GILLI" in gilli-danda game combination of?

7. What geometrical shapes is a "PLUMBLINE" (SAHUL) use by masons combination of?



8. A solid shape is converted from one form to another. What is the change in its volume?
9. What cross-section is made by a cone when it is cut parallel to its base?
10. Find total surface area of a solid hemi-sphere of radius 7cm.
11. Volume of two spheres is in the ratio 64 : 125. Find the ratio of their surface areas.

12. A right circular cylinder of radius r cm and height h cm ($h > 2r$) just encloses a sphere. Find diameter of the sphere.
13. A cylinder and a cone are of same base radius and of same height. Find the ratio of the volumes of cylinder to that of the cone.
14. A solid sphere of radius r is melted and recast into the shape of a solid cone of height r. Find radius of the base of the cone.
15. Find the total surface area of a solid hemi-sphere of radius r.
16. If the volume and the surface area of a sphere are numerically equal, then find the radius of the sphere.
17. A cylinder, a cone and a hemisphere are of same base and have the same height. What is the ratio of their volumes?
18. If two solid hemi-spheres of same base radius r are joined together along their base, then find the total surface area of this new solid.
19. If the volume of a cube is 1331 cm^3, then find the length of its edge.
20. What does the "CAPACITY" for a hollow cylinder means?

SHORT ANSWER TYPE QUESTION (TYPE-I)

21. How many cubes of side 2 cm can be cut from a cuboid measuring (16cm×12cm×10cm).
22. Find the height of largest right circular cone that can be cut out of a cube whose volume is 729 cm^3.
23. Two identical cubes each of volume 64 cm^3 are joined together end to end. What is the surface area of the resulting cuboid?
24. Twelve solid spheres of the same sizes are made by melting a solid metallic cylinder of base diameter 2 cm and height 16cm. Find the radius of each sphere.
25. The diameters of the two circular ends of the bucket are 44 cm and 24 cm. The height of the bucket is 35cm. Find the volume of the bucket.

SHORT ANSWER TYPE QUESTION (TYPE-II)

26. Find the length of the longest rod that can be put in a room of 10m×10m×5m dimensions.
27. Find surface area of a cube whose volume is 1000 cm^3.

28. The volume of two hemi-sphere are in the ratio 8:27. Find the ratio of their radii.

29. Find the curved surface area and the total surface area of a solid cone whose height is 28 cm and radius is 21 cm.

30. A bucket is in the form of a frustum of a cone and hold 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.

31. Three cubes of a metal whose edge are in the ratio 3:4:5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edge of three cubes.

32. Find the depth of a cylindrical tank of radius 10.5 cm, if its capacity is equal to that of a rectangular tank of size 15 cm × 11 cm × 10.5 cm.

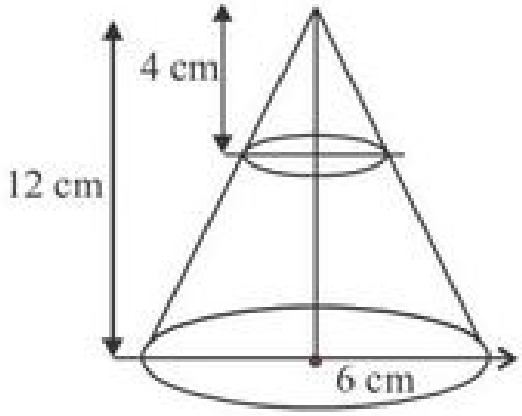
33. A cone of radius 8cm and height 12cm is divided into two parts by a plane through the mid-point of its axis parallel to its base. Find the ratio of the volumes of the two parts.

34. A petrol tank is a cylinder of base diameter 28cm and length 24cm filted with conical ends each of axis length 9cm. Determine the capacity of the tank.

LONG ANSWER TYPE QUESTIONS

35. In the given figure, from the top of a solid cone of height 12cm and base radius 6cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid.

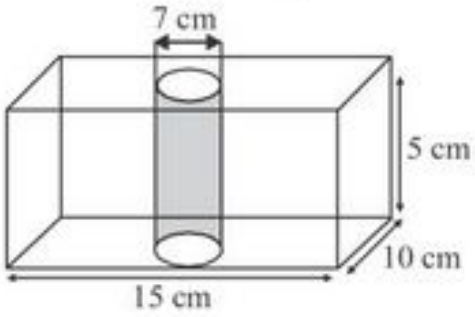
(Use $\pi = \frac{22}{7}$ and $\sqrt{5} = 2.236$)



36. A solid wooden toy is in the form of a hemi-sphere surmounted by a cone of same radius. The radius of hemi-sphere is 3.5cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm^3. Find the height of the toy. Also, find the cost of painting the hemi-spherical part of the toy at the rate of Rs. 10 per cm^2.

(use $\pi = \frac{22}{7}$).

37. In the given figure, from a cuboidal solid metalic block of dimensions 15 cm × 10 cm × 5 cm a cylindrical hole of diameter 7cm is drilled out. Find the surface area of the remaining block. (Use $\pi = \frac{22}{7}$).



38. Water is flowing at the rate of 2.52 km/hr. through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm. If the increase in the level of water in the tank, in half an hour is 3.15m, find internal diameter of the pipe.

39. A solid toy is the form of a right circular cylinder with a hemispherical shape at one end and a cone at the other end. Their diameter is 4.2 cm and the heights of the cylindrical and conical portions are 12 cm and 7 cm respectively. Find the volume of the toy.

40. A tent is in the shape of a right circular cylinder upto a height of 3m and conical above it. The total height of the tent is 13.5 m and radius of base is 14 m. Find the cost of cloth required to make the tent at the rate of ₹ 80 per sq. m.

41. The rain water from a roof 22m × 20m drains into a cylindrical vessel having diameter of base 2m and height 3.5m. If the vessel is just full, find the rainfall in cm.

42. A container, shaped like a right circular cylinder, having diameter 12cm and height 15 cm is full of ice-cream. this ice-cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

43. The difference between outer and inner curved surface areas of a hollow right circular cylinder, 14 cm long is $88cm^2$. If the volume of the metal used in making the cylinder is $176cm^3$. Find the outer and inner diameters of the cylinder.

44. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of hemispherical and comical parts are the same as that of the cylindrical part. Find the surface area of the toy if the total height of the toy is 30 cm.

45. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ th of the curved surface of the whole cone, find the ratio of the line segmens into which the altitude of the cone is divided by the plane.

ANSWERS

1. Cylinder, Frustum
2. Cylinder, Sphere
3. Cylinder, Cone
4. Frustum
5. Hemi-sphere, Frustum
6. Cylinder with Conical ends
7. Hemi-sphere, Cone
8. Remains unchanged
9. Circle
10. 462 cm^2
11. 16:25
12. $2r$ cm
13. 3:1
14. $2r$ units
15. $3\pi r^2$
16. 3 units
17. 3:1:2
18. $4\pi r^2$
19. 11 cm
20. Volume
21. 240
22. 9 cm
23. 160 cm^2
24. 1 cm
25. 32706.6 cm^3 approx
26. 15 m
27. 600 cm^2
28. 2:3
29. C.S.A = 2310 cm^2
    T.S.A = 3696 cm^2
30. 15 cm
31. 6 cm, 8 cm, 10 cm
32. 5 cm
33. 1:7 or 7:1
34. 18480 cm^3
35. 350.592 cm^2
36. h = 6 cm, Rs. 770
37. 583 cm^2
38. 4 cm
39. 218.064 cm^3
40. Rs. 82720
41. 2.5 cm
42. 10
43. 5 cm, 3 cm
44. 770 cm^2
45. 1 : 2

Practice Test

Surface Areas and Volumes

Time: 50 minutes **M.M: 20**

SECTION-A

1. What is the formula for total surface area of a solid hemi-sphere? 1
2. What geometrical shapes is a "FUNNEL" combination of? 1

SECTION-B

3. A clyindrical boiler is 2 m high and has 3.5m radius. Find its volume. 2
4. What is the formula for total surface area of a bucket? 2

SECTION-C

5. What will be the volume of the largest right circular cone that can be cut from a cube of edge 4.2 cm. 3
6. Find the volume of a frustum of a cone whose height is 4 m and radii of the ends are 7 m amd 4 m. 3

SECTION-D

7. Show that the ratio of the volumes of a cylinder, a cone and a hemi-sphere of same base and same height is 3:1:2. 4
8. Two solid metallic cubes of sides 40cm and 30cm are melted together recast into 5824 equal solid cubical dice. Determine the side of the cubical dice. 4

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