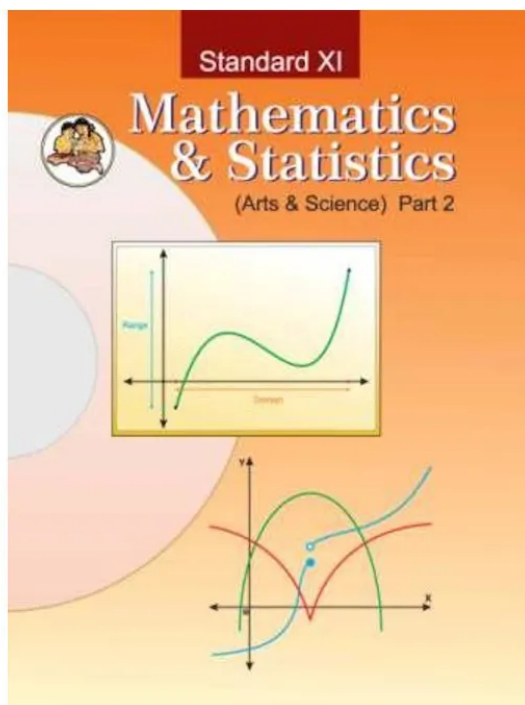


Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 8- Continuity

Class 11 - Chapter 8 Continuity



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Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 8- Continuity

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Maharashtra Board Solutions Class 11-Arts & Science Maths (Part 2): Chapter 8- Continuity

Maharashtra Board 11th Maths Chapter 8, Class 11 Maths Chapter 8 solutions

Ex 8.1

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Question 1.

Examine the continuity of

(i) $f(x) = x^3 + 2x^2 - x - 2$ at $x = -2$

Solution:

Given, $f(x) = x^3 + 2x^2 - x - 2$

 $f(x)$ is a polynomial function and hence it is continuous for all $x \in \mathbb{R}$. $\therefore f(x)$ is continuous at $x = -2$.

(ii) $f(x) = \sin x$, for $x \leq \frac{\pi}{4}$
 $= \cos x$, for $x > \frac{\pi}{4}$, at $x = \frac{\pi}{4}$

Solution:

$$f(x) = \sin x ; \quad x \leq \frac{\pi}{4}$$

$$= \cos x ; \quad x > \frac{\pi}{4}$$

$$\lim_{x \rightarrow \frac{\pi}{4}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{4}^-} (\sin x)$$

$$= \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\lim_{x \rightarrow \frac{\pi}{4}^+} f(x) = \lim_{x \rightarrow \frac{\pi}{4}^+} (\cos x)$$

$$= \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\text{Also, } f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}}$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{4}^-} f(x) = \lim_{x \rightarrow \frac{\pi}{4}^+} f(x) = f\left(\frac{\pi}{4}\right)$$

$$\therefore f(x) \text{ is continuous at } x = \frac{\pi}{4}.$$

$$(iii) f(x) = \frac{x^2 - 9}{x - 3}, \text{ for } x \neq 3$$

$$= 8 \text{ for } x = 3, \text{ at } x = 3.$$

Solution:

$$f(3) = 8 \dots (\text{given})$$

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

$$= \lim_{x \rightarrow 3} \frac{(x + 3)(x - 3)}{x - 3}$$

$$= \lim_{x \rightarrow 3} (x + 3)$$

$$\dots [\because x \rightarrow 3, x \neq 3 \therefore x - 3 \neq 0]$$

$$= 3 + 3 = 6$$

$$\lim_{x \rightarrow 3} f(x) \neq f(3)$$

$\therefore f(x)$ is discontinuous at $x = 3$.

Question 2.

Examine whether the function is continuous at the points indicated against them.

$$(i) f(x) = x^3 - 2x + 1, \text{ if } x \leq 2$$

$$= 3x - 2, \text{ if } x > 2, \text{ at } x = 2.$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^-} (x^3 - 2x + 1) \\ &= (2)^3 - 2(2) + 1 \end{aligned}$$

$$\begin{aligned} &= 5 \\ \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} (3x - 2) \\ &= 3(2) - 2 \\ &= 4 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$$

$\therefore f(x)$ is discontinuous at $x = 2$.

(ii) $f(x) = \frac{x^2 + 18x - 19}{x - 1}$, for $x \neq 1$
 $= 20$, for $x = 1$, at $x = 1$.

Solution:

$$f(1) = 20 \quad \dots(\text{given})$$

$$\begin{aligned} \lim_{x \rightarrow 1} f(x) &= \lim_{x \rightarrow 1} \frac{x^2 + 18x - 19}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{x^2 + 19x - x - 19}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{x(x + 19) - 1(x + 19)}{(x - 1)} \\ &= \lim_{x \rightarrow 1} \frac{(x - 1)(x + 19)}{(x - 1)} \\ &= \lim_{x \rightarrow 1} (x + 19) \quad \dots \left[\begin{array}{l} \because x \rightarrow 1, x \neq 1, \\ \therefore x - 1 \neq 0 \end{array} \right] \\ &= 1 + 19 = 20 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 1} f(x) = f(1)$$

$\therefore f(x)$ is continuous at $x = 1$.

(iii) $f(x) = \frac{x}{\tan 3x} + 2$, for $x < 0$
 $= \frac{7}{3}$, for $x \geq 0$, at $x = 0$.

Solution:

$$\lim_{x \rightarrow 0^+} f(x) = \frac{7}{3} \text{ and } f(0) = \frac{7}{3} \quad \dots(\text{given})$$

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} \left(\frac{x}{\tan 3x} + 2 \right) \\ &= \lim_{x \rightarrow 0^-} \frac{x}{\tan 3x} + \lim_{x \rightarrow 0^-} 2 \\ &= \lim_{x \rightarrow 0^-} \frac{1}{\frac{\tan 3x}{x}} + \lim_{x \rightarrow 0^-} 2 \\ &= \lim_{x \rightarrow 0^-} \left[\frac{1}{\frac{\tan 3x}{3x} \times 3} \right] + \lim_{x \rightarrow 0^-} 2 \\ &= \frac{\lim_{x \rightarrow 0^-} 1}{3 \lim_{x \rightarrow 0^-} \left(\frac{\tan 3x}{3x} \right)} + \lim_{x \rightarrow 0^-} 2 \\ &= \frac{1}{3(1)} + 2 = \frac{7}{3} \end{aligned}$$

$$\therefore \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = f(0)$$

$\therefore f(x)$ is continuous at $x = 0$.

Question 3.

Find all the points of discontinuities of $f(x) = [x]$ on the interval $(-3, 2)$.

Solution:

$$f(x) = [x], x \in (-3, 2)$$

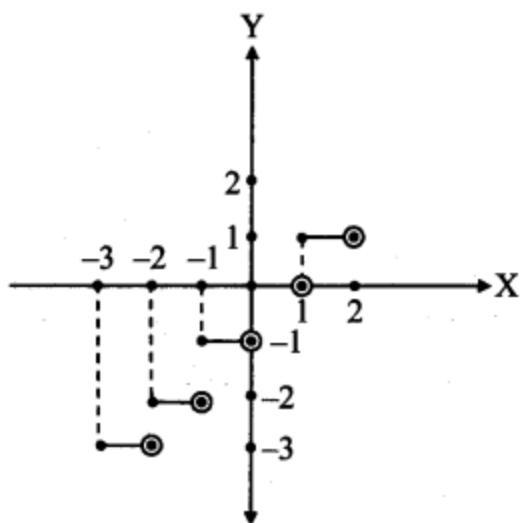
$$\text{i.e., } f(x) = -3, x \in (-3, -2)$$

$$= -2, x \in [-2, -1)$$

$$= -1, x \in [-1, 0)$$

$$= 0, x \in [0, 1)$$

$$= 1, x \in [1, 2)$$



At $x = -2$,

$$\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^-} [x] = -3$$

$$\lim_{x \rightarrow -2^+} f(x) = \lim_{x \rightarrow -2^+} [x] = -2$$

$\therefore f(x)$ is discontinuous at $x = -2$.

Similarly, $f(x)$ is discontinuous at the points $x = -1, x = 0, x = 1$.

Thus all the integer values of x in the interval $(-3, 2)$,

i.e., the points $x = -2, x = -1, x = 0$ and $x = 1$ are the required points of discontinuities.

$$= 0$$

$$\therefore \lim_{x \rightarrow \frac{-3}{2}^-} f(x) = \lim_{x \rightarrow \frac{-3}{2}^+} f(x) = f\left(\frac{-3}{2}\right)$$

$$\therefore f(x) \text{ is continuous at } x = \frac{-3}{2}.$$

Question 5.

Test the continuity of the following functions at the points or intervals ind

$$\begin{aligned} \text{(i) } f(x) &= \frac{\sqrt{x-1} - (x-1)^{\frac{1}{3}}}{x-2}, \quad \text{for } x \neq 2 \\ &= \frac{1}{5}, \quad \text{for } x = 2 \\ &\text{at } x = 2 \end{aligned}$$

$$\begin{aligned} \text{(ii) } f(x) &= \frac{x^3 - 8}{\sqrt{x+2} - \sqrt{3x-2}} \quad \text{for } x \neq 2 \\ &= -24 \quad \text{for } x = 2 \text{ at } x = 2 \end{aligned}$$

$$\text{(iii) } f(x) = 4x + 1, \quad \text{for } x \leq 8/3.$$

$$= \frac{59-9x}{3}, \text{ for } x > 8/3, \text{ at } x = 8/3.$$

$$\begin{aligned} \text{(iv) } f(x) &= \frac{(27-2x)^{\frac{1}{3}}-3}{9-3(243+5x)^{\frac{1}{5}}}, \text{ for } x \neq 0 \\ &= 2 \quad \text{for } x = 0, \text{ at } x = 0 \end{aligned}$$

$$\begin{aligned} \text{(v) } f(x) &= \frac{x^2+8x-20}{2x^2-9x+10} \quad \text{for } 0 < x < 3; x \neq 2 \\ &= 12, \text{ for } x = 2 \\ &= \frac{2-2x-x^2}{x-4} \quad \text{for } 3 \leq x < 4 \\ &\text{at } x = 2 \end{aligned}$$

Solution:

$$\text{i. } f(2) = \frac{1}{5} \quad \dots(\text{given})$$

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{\sqrt{x-1} - (x-1)^{\frac{1}{3}}}{x-2}$$

$$\text{Put } x-1 = y$$

$$\therefore x = 1 + y$$

$$\text{As } x \rightarrow 2, y \rightarrow 1$$

$$\therefore \lim_{x \rightarrow 2} f(x) = \lim_{y \rightarrow 1} \frac{\sqrt{y} - y^{\frac{1}{3}}}{1+y-2}$$

$$\begin{aligned}
 &= \lim_{y \rightarrow 1} \frac{\left(y^{\frac{1}{2}} - 1\right) - \left(y^{\frac{1}{3}} - 1\right)}{y - 1} \\
 &= \lim_{y \rightarrow 1} \left(\frac{y^{\frac{1}{2}} - 1}{y - 1} - \frac{y^{\frac{1}{3}} - 1}{y - 1} \right) \\
 &= \lim_{y \rightarrow 1} \frac{y^{\frac{1}{2}} - 1^{\frac{1}{2}}}{y - 1} - \lim_{y \rightarrow 1} \frac{y^{\frac{1}{3}} - 1^{\frac{1}{3}}}{y - 1} \\
 &= \frac{1}{2} (1)^{-\frac{1}{2}} - \frac{1}{3} (1)^{-\frac{2}{3}} \\
 &\quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \right] \\
 &= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}
 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 2} f(x) \neq f(2)$$

$\therefore f(x)$ is discontinuous at $x = 2$.

ii. $f(2) = -24$... (given)

$$\begin{aligned}
 \lim_{x \rightarrow 2} f(x) &= \lim_{x \rightarrow 2} \frac{x^3 - 8}{\sqrt{x+2} - \sqrt{3x-2}} \\
 &= \lim_{x \rightarrow 2} \frac{x^3 - 8}{\sqrt{x+2} - \sqrt{3x-2}} \times \frac{\sqrt{x+2} + \sqrt{3x-2}}{\sqrt{x+2} + \sqrt{3x-2}} \\
 &= \lim_{x \rightarrow 2} \frac{(x^3 - 8)(\sqrt{x+2} + \sqrt{3x-2})}{(x+2) - (3x-2)}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 2} \frac{(x^3 - 2^3)(\sqrt{x+2} + \sqrt{3x-2})}{-2x+4} \\
 &= \lim_{x \rightarrow 2} \frac{(x-2)(x^2+2x+4)(\sqrt{x+2} + \sqrt{3x-2})}{-2(x-2)} \\
 &= \lim_{x \rightarrow 2} \frac{(x^2+2x+4)(\sqrt{x+2} + \sqrt{3x-2})}{-2} \\
 &\quad \dots \left[\begin{array}{l} \because x \rightarrow 2, x \neq 2 \\ \therefore x-2 \neq 0 \end{array} \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{-1}{2} \lim_{x \rightarrow 2} (x^2+2x+4)(\sqrt{x+2} + \sqrt{3x-2}) \\
 &= \frac{-1}{2} \lim_{x \rightarrow 2} (x^2+2x+4) \lim_{x \rightarrow 2} (\sqrt{x+2} + \sqrt{3x-2}) \\
 &= \frac{-1}{2} \times [2^2+2(2)+4] \times (\sqrt{2+2} + \sqrt{3(2)-2}) \\
 &= \frac{-1}{2} \times 12 \times (2+2) \\
 &= -24
 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 2} f(x) = f(2)$$

$\therefore f(x)$ is continuous at $x = 2$.

$$\begin{aligned}
 \text{iii. } \lim_{x \rightarrow \frac{8}{3}} f(x) &= \lim_{x \rightarrow \frac{8}{3}} (4x+1) \\
 &= 4 \left(\frac{8}{3} \right) + 1
 \end{aligned}$$

$$= \frac{35}{3}$$

$$\lim_{x \rightarrow \frac{8}{3}^+} f(x) = \lim_{x \rightarrow \frac{8}{3}^+} \frac{59 - 9x}{3}$$

$$= \frac{59 - 9\left(\frac{8}{3}\right)}{3}$$

$$= \frac{59 - 24}{3} = \frac{35}{3}$$

$$f(x) = 4x + 1$$

$$\therefore f\left(\frac{8}{3}\right) = 4\left(\frac{8}{3}\right) + 1 = \frac{35}{3}$$

$$\therefore \lim_{x \rightarrow \frac{8}{3}^-} f(x) = \lim_{x \rightarrow \frac{8}{3}^+} f(x) = f\left(\frac{8}{3}\right)$$

$$\therefore f(x) \text{ is continuous at } x = \frac{8}{3}.$$

$$\text{iv. } f(0) = 2 \quad \dots(\text{given})$$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{(27 - 2x)^{\frac{1}{3}} - 3}{9 - 3(243 + 5x)^{\frac{1}{5}}}$$

$$= \lim_{x \rightarrow 0} \frac{(27 - 2x)^{\frac{1}{3}} - 3}{-3 \left[(243 + 5x)^{\frac{1}{5}} - 3 \right]}$$

$$\begin{aligned}
 &= \frac{-1}{3} \lim_{x \rightarrow 0} \frac{(27 - 2x)^{\frac{1}{3}} - (27)^{\frac{1}{3}}}{(243 + 5x)^{\frac{1}{5}} - (243)^{\frac{1}{5}}} \\
 &= \frac{-1}{3} \lim_{x \rightarrow 0} \frac{\frac{(27 - 2x)^{\frac{1}{3}} - 27^{\frac{1}{3}}}{(27 - 2x) - 27} \times [(27 - 2x) - 27]}{\frac{(243 + 5x)^{\frac{1}{5}} - (243)^{\frac{1}{5}}}{(243 + 5x) - 243} \times [(243 + 5x) - 243]} \\
 &\quad \dots \left[\begin{array}{l} \because x \rightarrow 0, -2x \rightarrow 0 \text{ and } 5x \rightarrow 0 \\ \therefore (27 - 2x) - 27 \rightarrow 0 \text{ and } (243 + 5x) - 243 \rightarrow 0 \\ \therefore (27 - 2x) - 27 \neq 0 \text{ and } (243 + 5x) - 243 \neq 0 \end{array} \right] \\
 &= \frac{-1}{3} \cdot \frac{\lim_{x \rightarrow 0} \frac{(27 - 2x)^{\frac{1}{3}} - 27^{\frac{1}{3}}}{(27 - 2x) - 27} \times (-2x)}{\lim_{x \rightarrow 0} \frac{(243 + 5x)^{\frac{1}{5}} - (243)^{\frac{1}{5}}}{(243 + 5x) - 243} \times (5x)} \\
 &= \frac{-1}{3} \times \frac{-2}{5} \times \frac{\lim_{x \rightarrow 0} \frac{(27 - 2x)^{\frac{1}{3}} - 27^{\frac{1}{3}}}{(27 - 2x) - 27}}{\lim_{x \rightarrow 0} \frac{(243 + 5x)^{\frac{1}{5}} - (243)^{\frac{1}{5}}}{(243 + 5x) - 243}} \\
 &\quad \dots [\because x \rightarrow 0, x \neq 0] \\
 &= \frac{2}{15} \times \frac{\frac{1}{3} (27)^{-\frac{2}{3}}}{\frac{1}{5} (243)^{-\frac{4}{5}}} \quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1} \right]
 \end{aligned}$$

$$\begin{aligned} &= \frac{2}{15} \times \frac{5}{3} \times \frac{(3^3)^{-2}}{(3^5)^{-4}} \\ &= \frac{2}{9} \times \frac{(3)^{-2}}{(3)^{-4}} = \frac{2}{9} \times (3)^2 \\ &= 2 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

$\therefore f(x)$ is continuous at $x = 0$.

v. $f(2) = 12$... (given)

$$\begin{aligned} \lim_{x \rightarrow 2} f(x) &= \lim_{x \rightarrow 2} \frac{x^2 + 8x - 20}{2x^2 - 9x + 10} \\ &= \lim_{x \rightarrow 2} \frac{(x + 10)(x - 2)}{(2x - 5)(x - 2)} \\ &= \lim_{x \rightarrow 2} \frac{x + 10}{2x - 5} \quad \dots \left[\begin{array}{l} \because x \rightarrow 2, x \neq 2, \\ \therefore x - 2 \neq 0 \end{array} \right] \\ &= \frac{\lim_{x \rightarrow 2} (x + 10)}{\lim_{x \rightarrow 2} (2x - 5)} = \frac{2 + 10}{2(2) - 5} \\ &= \frac{12}{-1} = -12 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 2} f(x) \neq f(2)$$

$\therefore f(x)$ is discontinuous at $x = 2$.

Question 6.

Identify discontinuities for the following functions as either a jump or a removable discontinuity.

$$(i) f(x) = \frac{x^2 - 10x + 21}{x - 7}$$

Solution:

$$\text{Given, } f(x) = \frac{x^2 - 10x + 21}{x - 7}$$

It is a rational function and is discontinuous if

$$x - 7 = 0, \text{ i.e., } x = 7$$

$\therefore f(x)$ is continuous for all $x \in \mathbb{R}$, except at $x = 7$.

$\therefore f(7)$ is not defined.

$$\begin{aligned} \text{Now, } \lim_{x \rightarrow 7} f(x) &= \lim_{x \rightarrow 7} \frac{x^2 - 10x + 21}{x - 7} \\ &= \lim_{x \rightarrow 7} \frac{(x - 7)(x - 3)}{x - 7} \\ &= \lim_{x \rightarrow 7} (x - 3) \dots \left[\begin{array}{l} \because x \rightarrow 7, x \neq 7, \\ \therefore x - 7 \neq 0 \end{array} \right] \\ &= 7 - 3 \\ &= 4 \end{aligned}$$

Thus, $\lim_{x \rightarrow 7} f(x)$ exist but $f(7)$ is not defined.

$\therefore f(x)$ has a removable discontinuity.

(ii) $f(x) = x^2 + 3x - 2$, for $x \leq 4$
 $= 5x + 3$, for $x > 4$.

Solution:

$$f(x) = x^2 + 3x - 2, x \leq 4$$

$$= 5x + 3, x > 4$$

$f(x)$ is a polynomial function for both the intervals.

$\therefore f(x)$ is continuous for both the given intervals.

Let us test the continuity at $x = 4$.

$$\begin{aligned}\lim_{x \rightarrow 4^-} f(x) &= \lim_{x \rightarrow 4^-} (x^2 + 3x - 2) \\ &= (4)^2 + 3(4) - 2 \\ &= 26\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 4^+} f(x) &= \lim_{x \rightarrow 4^+} (5x + 3) \\ &= 5(4) + 3 \\ &= 23\end{aligned}$$

$$\therefore \lim_{x \rightarrow 4^-} f(x) \neq \lim_{x \rightarrow 4^+} f(x)$$

$$\therefore \lim_{x \rightarrow 4} f(x) \text{ does not exist.}$$

$\therefore f(x)$ is discontinuous at $x = 4$.

$\therefore f(x)$ has a jump discontinuity at $x = 4$.

(iii) $f(x) = x^2 - 3x - 2$, for $x < -3$ $= 3 + 8x$, for $x > -3$.

Solution:

$$f(x) = x^2 - 3x - 2, x < -3 = 3 + 8x, x > -3$$

$f(x)$ is a polynomial function for both the intervals.

$\therefore f(x)$ is continuous for both the given intervals.

Let us test the continuity at $x = -3$.

$$\begin{aligned}\lim_{x \rightarrow -3^-} f(x) &= \lim_{x \rightarrow -3^-} (x^2 - 3x - 2) \\ &= (-3)^2 - 3(-3) - 2 \\ &= 9 + 9 - 2 \\ &= 16\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow -3^+} f(x) &= \lim_{x \rightarrow -3^+} (3 + 8x) \\ &= 3 + 8(-3) \\ &= -21\end{aligned}$$

$$\therefore \lim_{x \rightarrow -3^-} f(x) \neq \lim_{x \rightarrow -3^+} f(x)$$

$$\therefore \lim_{x \rightarrow -3} f(x) \text{ does not exist.}$$

$\therefore f(x)$ is discontinuous at $x = -3$.

$\therefore f(x)$ has a jump discontinuity at $x = -3$

(iv) $f(x) = 4 + \sin x$, for $x < \pi$ $= 3 - \cos x$ for $x > \pi$.

Solution:

$$f(x) = 4 + \sin x, x < \pi = 3 - \cos x, x > \pi$$

$\sin x$ and $\cos x$ are continuous for all $x \in \mathbb{R}$.

4 and 3 are constant functions.

$\therefore 4 + \sin x$ and $3 - \cos x$ are continuous for all $x \in \mathbb{R}$.

$\therefore f(x)$ is continuous for both the given intervals.

Let us test the continuity at $x = \pi$.

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$$\begin{aligned}\lim_{x \rightarrow \pi^-} f(x) &= \lim_{x \rightarrow \pi^-} (4 + \sin x) \\ &= 4 + \sin \pi \\ &= 4 + 0 \\ &= 4\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow \pi^+} f(x) &= \lim_{x \rightarrow \pi^+} (3 - \cos x) \\ &= 3 - \cos \pi \\ &= 3 - (-1) \\ &= 4\end{aligned}$$

$$\therefore \lim_{x \rightarrow \pi^-} f(x) = \lim_{x \rightarrow \pi^+} f(x)$$

$$\therefore \lim_{x \rightarrow \pi} f(x) = 4$$

But $f(\pi)$ is not defined.

$\therefore f(x)$ has a removable discontinuity at $x = \pi$.

Question 7.

Show that the following functions have a continuous extension to the point where $f(x)$ is not defined. Also, find the extension.

(i) $f(x) = \frac{1 - \cos 2x}{\sin x}$, for $x \neq 0$.

Solution:

$$f(x) = \frac{1 - \cos 2x}{\sin x}, \text{ for } x \neq 0$$

Here, $f(0)$ is not defined.

Consider,

$$\begin{aligned}\lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{\sin x} \\&= \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{\sin x} \\&= 2 \lim_{x \rightarrow 0} (\sin x) \quad \dots \left[\begin{array}{l} \because x \rightarrow 0, x \neq 0 \\ \therefore \sin x \neq 0 \end{array} \right] \\&= 2(\sin 0) = 2 \times 0 \\&= 0\end{aligned}$$

$\lim_{x \rightarrow 0} f(x)$ exists.

But $f(0)$ is not defined.

$\therefore f(x)$ has a removable discontinuity at $x = 0$.

$\therefore$ The extension of the original function is

$$f(x) = \frac{1 - \cos 2x}{\sin x} \text{ for } x \neq 0$$

$$= 0 \text{ for } x = 0$$

$\therefore f(x)$ is continuous at $x = 0$.

$$(ii) f(x) = \frac{3 \sin^2 x + 2 \cos x (1 - \cos 2x)}{2(1 - \cos^2 x)}, \text{ for } x \neq 0.$$

Solution:

$$f(x) = \frac{3 \sin^2 x + 2 \cos x (1 - \cos 2x)}{2(1 - \cos^2 x)}, \text{ for } x \neq 0$$

Here, $f(0)$ is not defined.

Consider,

$$\begin{aligned} \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{3 \sin^2 x + 2 \cos x (1 - \cos 2x)}{2(1 - \cos^2 x)} \\ &= \lim_{x \rightarrow 0} \frac{3 \sin^2 x + 2 \cos x \cdot (2 \sin^2 x)}{2 \sin^2 x} \\ &= \lim_{x \rightarrow 0} \frac{\sin^2 x (3 + 4 \cos x)}{2 \sin^2 x} \\ &= \lim_{x \rightarrow 0} \frac{3 + 4 \cos x}{2} \\ &\quad \dots \left[\begin{array}{l} \because x \rightarrow 0, x \neq 0, \sin x \neq 0 \\ \therefore \sin^2 x \neq 0 \end{array} \right] \\ &= \frac{1}{2} \lim_{x \rightarrow 0} (3 + 4 \cos x) = \frac{1}{2} (3 + 4 \cos 0) \\ &= \frac{1}{2} (3 + 4) = \frac{7}{2} \end{aligned}$$

$\lim_{x \rightarrow 0} f(x)$ exists.

But $f(0)$ is not defined.

$\therefore f(x)$ has a removable discontinuity at $x = 0$.

$\therefore$ The extension of the original function is

$$f(x) = \frac{3 \sin^2 x + 2 \cos x (1 - \cos 2x)}{2(1 - \cos^2 x)}, x \neq 0$$

$$= \frac{7}{2}, x = 0$$

$\therefore f(x)$ is continuous at $x = 0$.

(iii) $f(x) = \frac{x^2-1}{x^3+1}$, for $x \neq -1$

Solution:

$$f(x) = \frac{x^2-1}{x^3+1}, \text{ for } x \neq -1$$

Here, $f(-1)$ is not defined.

Consider,

$$\begin{aligned}\lim_{x \rightarrow -1} f(x) &= \lim_{x \rightarrow -1} \left(\frac{x^2-1}{x^3+1} \right) \\&= \lim_{x \rightarrow -1} \frac{(x+1)(x-1)}{(x+1)(x^2-x+1)} \\&= \lim_{x \rightarrow -1} \frac{x-1}{x^2-x+1} \quad \dots \left[\begin{array}{l} \because x \rightarrow -1, x \neq -1 \\ \therefore x+1 \neq 0 \end{array} \right] \\&= \frac{-1-1}{(-1)^2 - (-1) + 1} = -\frac{2}{3}\end{aligned}$$

But $f(-1)$ is not defined.

$\therefore f(x)$ has a removable discontinuity at $x = -1$.

$\therefore$ The extension of the original function is

$$f(x) = \frac{x^2-1}{x^3+1}, x \neq -1$$

$$= -\frac{2}{3}, x = -1$$

$\therefore f(x)$ is continuous at $x = -\frac{2}{3}$

Question 8.

Discuss the continuity of the following functions at the points indicated against them.

$$\begin{aligned}\text{(i) } f(x) &= \frac{\sqrt{3} - \tan x}{\pi - 3x}, x \neq \frac{\pi}{3} \\ &= \frac{3}{4}, \quad \text{for } x = \frac{\pi}{3}, \text{ at } x = \frac{\pi}{3}.\end{aligned}$$

$$\begin{aligned}\text{(ii) } f(x) &= \frac{e^{1/x} - 1}{e^{1/x} + 1}, \quad \text{for } x \neq 0 \\ &= 1, \quad \text{for } x = 0, \text{ at } x = 0.\end{aligned}$$

$$\begin{aligned}\text{(iii) } f(x) &= \frac{4^x - 2^{x+1} + 1}{1 - \cos 2x}, \quad \text{for } x \neq 0 \\ &= \frac{(\log 2)^2}{2}, \quad \text{for } x = 0, \text{ at } x = 0.\end{aligned}$$

Solution:

$$\text{i. } f\left(\frac{\pi}{3}\right) = \frac{3}{4} \quad \dots(\text{given})$$

$$\lim_{x \rightarrow \frac{\pi}{3}} f(x) = \lim_{x \rightarrow \frac{\pi}{3}} \frac{\sqrt{3} - \tan x}{\pi - 3x}$$

$$\text{Put } \frac{\pi}{3} - x = h,$$

$$\therefore x = \frac{\pi}{3} - h$$

$$\text{As } x \rightarrow \frac{\pi}{3}, h \rightarrow 0$$

$$\begin{aligned}
 \therefore \lim_{x \rightarrow \frac{\pi}{3}} f(x) &= \lim_{h \rightarrow 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3} - h\right)}{\pi - 3\left(\frac{\pi}{3} - h\right)} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{3} - \frac{\tan \frac{\pi}{3} - \tan h}{1 + \tan \frac{\pi}{3} \tan h}}{3h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{3} - \frac{\sqrt{3} - \tan h}{1 + \sqrt{3} \tan h}}{3h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{3}(1 + \sqrt{3} \tan h) - (\sqrt{3} - \tan h)}{3h(1 + \sqrt{3} \tan h)} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{3} + 3 \tan h - \sqrt{3} + \tan h}{3h(1 + \sqrt{3} \tan h)} \\
 &= \lim_{h \rightarrow 0} \frac{4 \tan h}{3h(1 + \sqrt{3} \tan h)} \\
 &= \lim_{h \rightarrow 0} \frac{4}{3(1 + \sqrt{3} \tan h)} \times \frac{\tan h}{h} \\
 &= \frac{4}{3} \left(\lim_{h \rightarrow 0} \frac{1}{1 + \sqrt{3} \tan h} \right) \left(\lim_{h \rightarrow 0} \frac{\tan h}{h} \right) \\
 &= \frac{4}{3} \left[\frac{1}{1 + \sqrt{3}(0)} \right] (1)
 \end{aligned}$$

$$= \frac{4}{3}$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{3}} f(x) \neq f\left(\frac{\pi}{3}\right)$$

$$\therefore f(x) \text{ is discontinuous at } x = \frac{\pi}{3}.$$

$$\text{ii. } f(0) = 1 \quad \dots(\text{given})$$

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{h \rightarrow 0} \frac{e^{\frac{1}{(0-h)}} - 1}{e^{\frac{1}{(0-h)}} + 1} \\ &= \lim_{h \rightarrow 0} \frac{e^{-\frac{1}{h}} - 1}{e^{-\frac{1}{h}} + 1} \\ &= \lim_{h \rightarrow 0} \frac{\frac{1}{e^{\frac{1}{h}}} - 1}{\frac{1}{e^{\frac{1}{h}}} + 1} \\ &= \frac{0 - 1}{0 + 1} \\ &= -1 \end{aligned}$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} \frac{e^{\frac{1}{(0+h)}} - 1}{e^{\frac{1}{(0+h)}} + 1}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{e^{\frac{1}{h}} - 1}{e^{\frac{1}{h}} + 1} \\
 &= \lim_{h \rightarrow 0} \frac{e^{\frac{1}{h}} \left(1 - \frac{1}{e^{\frac{1}{h}}} \right)}{e^{\frac{1}{h}} \left(1 + \frac{1}{e^{\frac{1}{h}}} \right)} \\
 &= \frac{1-0}{1+0} = 1
 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

$\therefore f(x)$ is discontinuous at $x = 0$.

$$\text{iii. } f(0) = \frac{(\log 2)^2}{2} \quad \dots(\text{given})$$

$$\begin{aligned}
 \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{4^x - 2^{x+1} + 1}{1 - \cos 2x} \\
 &= \lim_{x \rightarrow 0} \frac{(2^2)^x - 2^x \cdot 2^1 + 1}{2 \sin^2 x} \\
 &= \lim_{x \rightarrow 0} \frac{(2^x)^2 - 2(2^x) + 1}{2 \sin^2 x} \\
 &= \lim_{x \rightarrow 0} \frac{(2^x - 1)^2}{2 \sin^2 x}
 \end{aligned}$$

$$\dots [\because a^2 - 2ab + b^2 = (a - b)^2]$$

$$= \lim_{x \rightarrow 0} \frac{(2^x - 1)^2}{2 \frac{x^2}{\sin^2 x}}$$

$$\dots \left[\begin{array}{l} \because x \rightarrow 0, x \neq 0, \\ \therefore x^2 \neq 0 \end{array} \right]$$

$$= \frac{\lim_{x \rightarrow 0} \left(\frac{2^x - 1}{x} \right)^2}{2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2}$$

$$= \frac{(\log 2)^2}{2(1)^2} = \frac{(\log 2)^2}{2}$$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

$\therefore f(x)$ is continuous at $x = 0$.

Question 9.

Which of the following functions has a removable discontinuity? If it has a removable discontinuity, redefine the function so that it becomes continuous.

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-p-art-2-chapter-8-continuity/>

$$(ii) f(x) = \log_{(1+3x)} (1+5x) \text{ for } x > 0$$

$$= \frac{32^x - 1}{8^x - 1}, \quad \text{for } x < 0, \text{ at } x = 0.$$

$$(iii) f(x) = \left(\frac{3-8x}{3-2x} \right)^{\frac{1}{x}}, \text{ for } x \neq 0.$$

$$(iv) f(x) = 3x + 2, \quad \text{for } -4 \leq x \leq -2$$
$$= 2x - 3, \quad \text{for } -2 < x \leq 6.$$

$$(v) f(x) = \frac{x^3 - 8}{x^2 - 4}, \quad \text{for } x > 2$$
$$= 3, \quad \text{for } x = 2$$

$$(i) f(x) = \frac{e^{5\sin x} - e^{2x}}{5 \tan x - 3x}, \text{ for } x \neq 0$$
$$= 3/4, \quad \text{for } x = 0, \text{ at } x = 0.$$

$$= \frac{e^{3(x-2)^2} - 1}{2(x-2)^2}, \quad \text{for } x < 2$$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-11-arts-science-maths-part-2-chapter-8-continuity/>

i. $f(0) = \frac{3}{4}$... (given)

$$\begin{aligned}
 \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \frac{e^{5\sin x} - e^{2x}}{5\tan x - 3x} \\
 &= \lim_{x \rightarrow 0} \frac{(e^{5\sin x} - 1) - (e^{2x} - 1)}{5\tan x - 3x} \\
 &= \lim_{x \rightarrow 0} \left[\frac{\frac{(e^{5\sin x} - 1) - (e^{2x} - 1)}{x}}{\frac{5\tan x - 3x}{x}} \right] \dots [\because x \rightarrow 0, x \neq 0] \\
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{e^{5\sin x} - 1}{x} - \frac{e^{2x} - 1}{x} \right)}{\lim_{x \rightarrow 0} \left(\frac{5\tan x}{x} - 3 \right)} \\
 &= \frac{\lim_{x \rightarrow 0} \left(\frac{e^{5\sin x} - 1}{5\sin x} \times \frac{5\sin x}{x} \right) - \lim_{x \rightarrow 0} \left(\frac{e^{2x} - 1}{2x} \times 2 \right)}{\lim_{x \rightarrow 0} \frac{5\tan x}{x} - \lim_{x \rightarrow 0} 3} \\
 &= \frac{5 \lim_{x \rightarrow 0} \left(\frac{e^{5\sin x} - 1}{5\sin x} \right) \cdot \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) - 2 \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{2x}}{5 \lim_{x \rightarrow 0} \frac{\tan x}{x} - \lim_{x \rightarrow 0} (3)} \\
 &= \frac{5(1)(1) - 2(1)}{5(1) - 3} \\
 &\quad \dots \left[\because x \rightarrow 0, 2x \rightarrow 0, \sin x \rightarrow 0, 5\sin x \rightarrow 0 \right. \\
 &\quad \left. \text{and } \lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right) = 1 \right] \\
 &= \frac{3}{2}
 \end{aligned}$$

$\therefore \lim_{x \rightarrow 0} f(x) \neq f(0)$

$\therefore f(x)$ is discontinuous at $x = 0$.

∴ $f(x)$ has a removable discontinuity at $x = 0$.

This discontinuity can be removed by

redefining $f(0) = \frac{3}{2}$.

∴ $f(x)$ can be redefined as

$$f(x) = \frac{e^{5\sin x} - e^{2x}}{5\tan x - 3x}, \quad x \neq 0$$

$$= \frac{3}{2}, \quad x = 0$$

ii. $f(x) = \log_{(1+3x)}(1+5x), \quad x > 0$

$$= \frac{32^x - 1}{8^x - 1}, \quad x < 0$$

Here, $f(0)$ is not defined.

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \log_{(1+3x)}(1+5x)$$

$$= \lim_{x \rightarrow 0^+} \frac{\log(1+5x)}{\log(1+3x)}$$

$$= \lim_{x \rightarrow 0^+} \left[\frac{\frac{\log(1+5x)}{x}}{\frac{\log(1+3x)}{x}} \right]$$

...[∵ $x \rightarrow 0, x \neq 0$]

$$= \frac{\lim_{x \rightarrow 0^+} \frac{\log(1+5x)}{5x} \times 5}{\lim_{x \rightarrow 0^+} \frac{\log(1+3x)}{3x} \times 3}$$

$$= \frac{1 \times 5}{1 \times 3} \quad \dots \left[\begin{array}{l} \because x \rightarrow 0, 3x \rightarrow 0, 5x \rightarrow 0 \\ \text{and } \lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1 \end{array} \right]$$

$$= \frac{5}{3}$$

$$\begin{aligned}
 \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} \frac{32^x - 1}{8^x - 1} \\
 &= \lim_{x \rightarrow 0^-} \left[\frac{\frac{32^x - 1}{x}}{\frac{8^x - 1}{x}} \right] \quad \dots [\because x \rightarrow 0, x \neq 0] \\
 &= \frac{\lim_{x \rightarrow 0^-} \frac{32^x - 1}{x}}{\lim_{x \rightarrow 0^-} \frac{8^x - 1}{x}} \\
 &= \frac{\log 32}{\log 8} \quad \dots \left[\because \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a \right] \\
 &= \frac{\log (2)^5}{\log (2)^3} \\
 &= \frac{5 \log 2}{3 \log 2} \\
 &= \frac{5}{3}
 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x)$$

$$\therefore \lim_{x \rightarrow 0} f(x) \text{ exists.}$$

But $f(0)$ is not defined.

$\therefore f(x)$ has a removable discontinuity at $x = 0$.

This discontinuity can be removed by defining

$$f(0) = \frac{5}{3}.$$

$\therefore f(x)$ can be redefined as

$$f(x) = \log_{(1+3x)} (1 + 5x) \quad ; x > 0$$

$$= \frac{5}{3} \quad ; x = 0$$

$$= \frac{32^x - 1}{8^x - 1} \quad ; x < 0$$

iii. $f(x) = \left(\frac{3-8x}{3-2x} \right)^{\frac{1}{x}} ; x \neq 0$

Here, $f(0)$ is not defined.

Consider, $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \left(\frac{3-8x}{3-2x} \right)^{\frac{1}{x}}$

$$= \lim_{x \rightarrow 0} \left[\frac{3 \left(1 - \frac{8x}{3} \right)}{3 \left(1 - \frac{2x}{3} \right)} \right]^{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0} \frac{\left(1 - \frac{8x}{3} \right)^{\frac{1}{x}}}{\left(1 - \frac{2x}{3} \right)^{\frac{1}{x}}}$$

$$= \frac{\lim_{x \rightarrow 0} \left[\left(1 - \frac{8x}{3} \right)^{\frac{-3}{8x}} \right]^{\frac{-8}{3}}}{\lim_{x \rightarrow 0} \left[\left(1 - \frac{2x}{3} \right)^{\frac{-3}{2x}} \right]^{\frac{-2}{3}}}$$

$$= \frac{e^{\frac{-8}{3}}}{e^{\frac{-2}{3}}} \dots \left[\begin{array}{l} \because x \rightarrow 0, \frac{-8x}{3} \rightarrow 0, \frac{-2x}{3} \rightarrow 0 \\ \text{and } \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e \end{array} \right]$$

$$= e^{\frac{-6}{3}} = e^{-2}$$

$\therefore \lim_{x \rightarrow 0} f(x)$ exists.

But $f(0)$ is not defined.

$\therefore f(x)$ has a removable discontinuity at $x = 0$.

This discontinuity can be removed by defining

$$f(0) = e^{-2}.$$

$\therefore f(x)$ can be redefined as

$$f(x) = \left(\frac{3-8x}{3-2x} \right)^{\frac{1}{x}} ; x \neq 0$$

$$= e^{-2} ; x = 0$$

iv. $f(x) = 3x + 2$, for $-4 \leq x \leq -2$
 $= 2x - 3$, for $-2 < x \leq 6$.

$$\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^-} (3x + 2)$$

$$= 3(-2) + 2 = -4$$

$$\lim_{x \rightarrow -2^+} f(x) = \lim_{x \rightarrow -2^+} (2x - 3)$$

$$= 2(-2) - 3$$

$$= -7$$

$$\therefore \lim_{x \rightarrow -2^-} f(x) \neq \lim_{x \rightarrow -2^+} f(x)$$

$\therefore \lim_{x \rightarrow -2} f(x)$ does not exist.

$\therefore f(x)$ is discontinuous at $x = -2$.

This discontinuity is irremovable.

v. $f(2) = 3$... (given)

$$\begin{aligned}\lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} \frac{x^3 - 8}{x^2 - 4} = \lim_{x \rightarrow 2^+} \frac{x^3 - 2^3}{x^2 - 2^2} \\&= \lim_{x \rightarrow 2^+} \frac{(x - 2)(x^2 + 2x + 4)}{(x - 2)(x + 2)} \\&= \lim_{x \rightarrow 2^+} \frac{x^2 + 2x + 4}{x + 2} \quad \dots \left[\begin{array}{l} \because x \rightarrow 2, x \neq 2 \\ \therefore x - 2 \neq 0 \end{array} \right] \\&= \frac{\lim_{x \rightarrow 2^+} (x^2 + 2x + 4)}{\lim_{x \rightarrow 2^+} (x + 2)} \\&= \frac{(2)^2 + 2(2) + 4}{2 + 2} \\&= \frac{12}{4} \\&= 3\end{aligned}$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{e^{3(x-2)^2} - 1}{2(x-2)^2}$$

Put $x - 2 = h$

$$\therefore x = 2 + h$$

As $x \rightarrow 2$, $h \rightarrow 0$

$$\begin{aligned}\therefore \lim_{x \rightarrow 2^-} f(x) &= \lim_{h \rightarrow 0} \frac{e^{3h^2} - 1}{2h^2} \\ &= \frac{1}{2} \lim_{h \rightarrow 0} \frac{e^{3h^2} - 1}{3h^2} \times 3 \\ &= \frac{1}{2} \times 1 \times 3 \quad \dots \left[\begin{array}{l} \because h \rightarrow 0, 3h^2 \rightarrow 0 \\ \text{and } \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \end{array} \right] \\ &= \frac{3}{2}\end{aligned}$$

$$\therefore \lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$$

$\therefore \lim_{x \rightarrow 2} f(x)$ does not exist.

$\therefore f(x)$ is discontinuous at $x = 2$.

This discontinuity is irremovable.

Question 10.

(i) If $f(x) = \frac{\sqrt{2+\sin x} - \sqrt{3}}{\cos^2 x}$, for $x \neq \frac{\pi}{2}$, is continuous at $x = \frac{\pi}{2}$ then find $f(\frac{\pi}{2})$.

Solution:

$f(x)$ is continuous at $x = \frac{\pi}{2}$,(given)

$$\begin{aligned}
 f\left(\frac{\pi}{2}\right) &= \lim_{x \rightarrow \frac{\pi}{2}} f(x) \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sqrt{2+\sin x} - \sqrt{3}}{\cos^2 x} \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \left[\frac{\sqrt{2+\sin x} - \sqrt{3}}{1 - \sin^2 x} \times \frac{\sqrt{2+\sin x} + \sqrt{3}}{\sqrt{2+\sin x} + \sqrt{3}} \right] \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{2 + \sin x - 3}{(1 - \sin x)(1 + \sin x)(\sqrt{2+\sin x} + \sqrt{3})} \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x - 1}{-(\sin x - 1)(1 + \sin x)(\sqrt{2+\sin x} + \sqrt{3})} \\
 &= \lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{-(1 + \sin x)(\sqrt{2+\sin x} + \sqrt{3})} \\
 &\quad \dots \left[\because x \rightarrow \frac{\pi}{2}, \sin x \rightarrow 1, \right. \\
 &\quad \left. \sin x \neq 1, \therefore \sin x - 1 \neq 0 \right] \\
 &= \frac{-1}{\lim_{x \rightarrow \frac{\pi}{2}} (1 + \sin x)(\sqrt{2+\sin x} + \sqrt{3})}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{-1}{\lim_{x \rightarrow \frac{\pi}{2}} (1 + \sin x) \cdot \lim_{x \rightarrow \frac{\pi}{2}} (\sqrt{2 + \sin x} + \sqrt{3})} \\
 &= \frac{-1}{(1+1)(\sqrt{2+1} + \sqrt{3})} = \frac{-1}{2 \times 2\sqrt{3}} \\
 f\left(\frac{\pi}{2}\right) &= \frac{-1}{4\sqrt{3}}
 \end{aligned}$$

(ii) If $f(x) = \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{3x^2 + 1} - 1}$ for $x \neq 0$, is continuous at $x = 0$ then find $f(0)$.

Solution:

$f(x)$ is continuous at $x = 0$,(given)

$$\begin{aligned}
 f(0) &= \lim_{x \rightarrow 0} f(x) \\
 &= \lim_{x \rightarrow 0} \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{3x^2 + 1} - 1} \\
 &= \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\sqrt{3x^2 + 1} - 1} \times \frac{\sqrt{3x^2 + 1} + 1}{\sqrt{3x^2 + 1} + 1} \\
 &= \lim_{x \rightarrow 0} \frac{-(1 - \cos 2x)(\sqrt{3x^2 + 1} + 1)}{(3x^2 + 1) - 1}
 \end{aligned}$$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{-2 \sin^2 x \cdot (\sqrt{3x^2 + 1} + 1)}{3x^2} \\ &= \frac{-2}{3} \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} (\sqrt{3x^2 + 1} + 1) \\ &= \frac{-2}{3} \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 \times \lim_{x \rightarrow 0} (\sqrt{3x^2 + 1} + 1) \\ &= \frac{-2}{3} (1)^2 \times (\sqrt{3(0) + 1} + 1) \\ &= \frac{-2}{3} \times (1 + 1) \\ f(0) &= \frac{-4}{3} \end{aligned}$$

(iii) If $f(x) = \frac{4^{x-\pi} + 4^{\pi-x} - 2}{(x-\pi)^2}$ for $x \neq \pi$, is continuous at $x = \pi$, then find $f(\pi)$.

Solution:

$f(x)$ is continuous at $x = \pi$,(given)

$$\therefore f(\pi) = \lim_{x \rightarrow \pi} f(x) = \lim_{x \rightarrow \pi} \frac{4^{x-\pi} + 4^{\pi-x} - 2}{(x-\pi)^2}$$

Put $x - \pi = h$

As $x \rightarrow \pi$, $h \rightarrow 0$

$$\therefore f(\pi) = \lim_{h \rightarrow 0} \frac{4^h + 4^{-h} - 2}{h^2}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{4^h + \frac{1}{4^h} - 2}{h^2} \\
 &= \lim_{h \rightarrow 0} \frac{(4^h)^2 + 1 - 2(4^h)}{4^h \cdot (h^2)} \\
 &= \lim_{h \rightarrow 0} \frac{(4^h - 1)^2}{4^h \cdot h^2} \\
 &\quad \dots [\because a^2 - 2ab + b^2 = (a - b)^2] \\
 &= \lim_{h \rightarrow 0} \left(\frac{4^h - 1}{h} \right)^2 \times \frac{1}{4^h} \\
 &= \lim_{h \rightarrow 0} \left(\frac{4^h - 1}{h} \right)^2 \times \lim_{h \rightarrow 0} \frac{1}{4^h} \\
 &= (\log 4)^2 \times \frac{1}{4^0} \quad \dots \left[\because \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a \right] \\
 &= (\log 2^2)^2 \times \frac{1}{1} = (2 \log 2)^2 \\
 \therefore f(\pi) &= 4(\log 2)^2
 \end{aligned}$$

Question 11.

(i) If $f(x) = \frac{24^x - 8^x - 3^x + 1}{12^x - 4^x - 3^x + 1}$, for $x \neq 0$

$= k$, for $x = 0$

is continuous at $x = 0$, then find k .

Solution:

$f(x)$ is continuous at $x = 0$ (given)

$$\therefore f(0) = \lim_{x \rightarrow 0} f(x)$$

$$\therefore k = \lim_{x \rightarrow 0} \frac{24^x - 8^x - 3^x + 1}{12^x - 4^x - 3^x + 1}$$

$$= \lim_{x \rightarrow 0} \frac{8^x \cdot 3^x - 8^x - 3^x + 1}{4^x \cdot 3^x - 4^x - 3^x + 1}$$

$$= \lim_{x \rightarrow 0} \frac{8^x(3^x - 1) - 1(3^x - 1)}{4^x(3^x - 1) - 1(3^x - 1)}$$

$$= \lim_{x \rightarrow 0} \frac{(3^x - 1)(8^x - 1)}{(3^x - 1)(4^x - 1)}$$

$$= \lim_{x \rightarrow 0} \frac{8^x - 1}{4^x - 1} \quad \dots \left[\begin{array}{l} \text{As } x \rightarrow 0, 3^x \rightarrow 3^0, \\ 3^x \rightarrow 1, 3^x \neq 1 \\ \therefore 3^x - 1 \neq 0 \end{array} \right]$$

$$= \lim_{x \rightarrow 0} \left(\frac{\frac{8^x - 1}{x}}{\frac{4^x - 1}{x}} \right) \quad \dots [\because x \rightarrow 0, x \neq 0]$$

$$= \frac{\lim_{x \rightarrow 0} \frac{8^x - 1}{x}}{\lim_{x \rightarrow 0} \frac{4^x - 1}{x}}$$

$$= \frac{\log 8}{\log 4} \quad \dots \left[\because \lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x} \right) = \log a \right]$$

$$= \frac{\log(2)^3}{\log(2)^2}$$

$$= \frac{3 \log 2}{2 \log 2}$$

$$k = \frac{3}{2}$$

(ii) If $f(x) = \frac{5^x + 5^{-x} - 2}{x^2}$, for $x \neq 0$

$= k$, for $x = 0$

is continuous at $x = 0$, then find k .

Solution:

$f(x)$ is continuous at $x = 0$ (given)

$$\therefore f(0) = \lim_{x \rightarrow 0} f(x)$$

$$= \lim_{x \rightarrow 0} \frac{5^x + 5^{-x} - 2}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{5^x + \frac{1}{5^x} - 2}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(5^x)^2 + 1 - 2(5^x)}{5^x \cdot x^2}$$

$$= \lim_{x \rightarrow 0} \frac{(5^x - 1)^2}{5^x \cdot x^2}$$

$$\dots [\because a^2 - 2ab + b^2 = (a - b)^2]$$

$$= \lim_{x \rightarrow 0} \left(\frac{5^x - 1}{x} \right)^2 \cdot \frac{1}{5^x}$$

$$= \lim_{x \rightarrow 0} \left(\frac{5^x - 1}{x} \right)^2 \times \lim_{x \rightarrow 0} \frac{1}{5^x}$$

$$= (\log 5)^2 \times \frac{1}{5^0} \quad \dots \left[\because \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a \right]$$

$$\therefore k = (\log 5)^2$$

(iii) If $f(x) = \frac{\sin 2x}{5x} - a$, for $x > 0$

$= 4$ for $x = 0$

$= x^2 + b - 3$, for $x < 0$

is continuous at $x = 0$, find a and b .

Solution:

$f(x)$ is continuous at $x = 0$ (given)

$$\therefore \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\therefore \lim_{x \rightarrow 0^+} \left(\frac{\sin 2x}{5x} - a \right) = 4$$

$$\therefore \lim_{x \rightarrow 0^+} \frac{\sin 2x}{5x} - \lim_{x \rightarrow 0^+} a = 4$$

$$\therefore \frac{1}{5} \lim_{x \rightarrow 0^+} \frac{\sin 2x}{2x} \times (2) - \lim_{x \rightarrow 0^+} a = 4$$

$$\therefore \frac{1}{5} (1) (2) - a = 4 \quad \dots \left[\begin{array}{l} \because x \rightarrow 0, 2x \rightarrow 0 \\ \text{and } \lim_{\theta \rightarrow 0^+} \frac{\sin \theta}{\theta} = 1 \end{array} \right]$$

$$\therefore \frac{2}{5} - a = 4$$

$$\therefore \frac{2}{5} - 4 = a$$

$$\therefore a = -\frac{18}{5}$$

Also, $\lim_{x \rightarrow 0^-} f(x) = f(0)$

$$\therefore \lim_{x \rightarrow 0^-} (x^2 + b - 3) = 4$$

$$\therefore b - 3 = 4$$

$$\therefore b = 7$$

$$\therefore a = \frac{-18}{5} \text{ and } b = 7$$

(iv) For what values of a and b is the function

$$f(x) = ax + 2b + 18, \text{ for } x \leq 0$$

$$= x^2 + 3a - b, \text{ for } 0 < x \leq 2 = 8x - 2, \text{ for } x > 2,$$

continuous for every x?

Solution:

$f(x)$ is continuous for every x(given)

$\therefore f(x)$ is continuous at $x = 0$ and $x = 2$.

As $f(x)$ is continuous at $x = 0$,

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$$

$$\therefore \lim_{x \rightarrow 0^-} (ax + 2b + 18) = \lim_{x \rightarrow 0^+} (x^2 + 3a - b)$$

$$\therefore a(0) + 2b + 18 = (0)^2 + 3a - b$$

$$\therefore 3a - 3b = 18$$

$$\therefore a - b = 6 \quad \dots(i)$$

As $f(x)$ is continuous at $x = 2$,

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$$

$$\therefore \lim_{x \rightarrow 2^-} (x^2 + 3a - b) = \lim_{x \rightarrow 2^+} (8x - 2)$$

$$\therefore (2)^2 + 3a - b = 8(2) - 2$$

$$\therefore 4 + 3a - b = 14$$

$$\therefore 3a - b = 10 \dots\dots(ii)$$

Subtracting (i) from (ii), we get

$$2a = 4$$

$$\therefore a = 2$$

Substituting $a = 2$ in (i), we get

$$2 - b = 6$$

$$\therefore b = -4$$

$$\therefore a = 2 \text{ and } b = -4$$

(v) For what values of a and b is the function

$$f(x) = \frac{x^2 - 4}{x - 2}, \text{ for } x < 2$$

$$= ax^2 - bx + 3, \text{ for } 2 \leq x < 3$$

$$= 2x - a + b, \text{ for } x \geq 3$$

continuous for every x on $\mathbb{R}$?

Solution:

$f(x)$ is continuous for every x on $\mathbb{R}$ (given)

$\therefore f(x)$ is continuous at $x = 2$ and $x = 3$.

As $f(x)$ is continuous at $x = 2$,

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$$

$$\therefore \lim_{x \rightarrow 2^-} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2^+} (ax^2 - bx + 3)$$

$$\therefore \lim_{x \rightarrow 2^-} \frac{(x-2)(x+2)}{x-2} = \lim_{x \rightarrow 2^+} (ax^2 - bx + 3)$$

$$\therefore \lim_{x \rightarrow 2^-} (x+2) = \lim_{x \rightarrow 2^+} (ax^2 - bx + 3) \dots \left[\begin{array}{l} \because x \rightarrow 2, x \neq 2 \\ \therefore x-2 \neq 0 \end{array} \right]$$

$$\therefore 2+2 = a(2)^2 - b(2) + 3$$

$$\therefore 4a - 2b + 3 = 4$$

$$\therefore 4a - 2b = 1 \quad \dots(i)$$

As $f(x)$ is continuous at $x = 3$,

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x)$$

$$\therefore \lim_{x \rightarrow 3^-} (ax^2 - bx + 3) = \lim_{x \rightarrow 3^+} (2x - a + b)$$

$$\therefore a(3)^2 - b(3) + 3 = 2(3) - a + b$$

$$\therefore 9a - 3b + 3 = 6 - a + b$$

$$\therefore 10a - 4b = 3 \quad \dots(ii)$$

Multiplying (i) by 2, we get

$$8a - 4b = 2 \quad \dots(iii)$$

Subtracting (ii) from (iii), we get

$$-2a = -1$$

$$\therefore a = \frac{1}{2}$$

Substituting $a = \frac{1}{2}$ in (i), we get

$$4\left(\frac{1}{2}\right) - 2b = 1$$

$$\therefore 2 - 2b = 1$$

$$\therefore 1 = 2b$$

$$\therefore b = \frac{1}{2}$$

$$\therefore a = \frac{1}{2} \text{ and } b = \frac{1}{2}$$

Question 12.

Discuss the continuity of f on its domain, where

$$f(x) = |x + 1|, \text{ for } -3 \leq x \leq 2$$

$$= |x - 5|, \text{ for } 2 < x \leq 7$$

Solution:

$$\begin{aligned}\lim_{x \rightarrow 2^-} f(x) &= \lim_{x \rightarrow 2^-} |x + 1| \\ &= |2 + 1| \\ &= |3| \\ &= 3\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} |x - 5| \\ &= |2 - 5| \\ &= |-3| \\ &= 3\end{aligned}$$

$$\begin{aligned}f(2) &= |2 + 1| \\ &= 3\end{aligned}$$

$$\therefore \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$$

$\therefore f(x)$ is continuous at $x = 2$.

Question 13.

Discuss the continuity of $f(x)$ at $x = \frac{\pi}{4}$ where,

$$f(x) = \frac{(\sin x + \cos x)^3 - 2\sqrt{2}}{\sin 2x - 1}, \text{ for } x \neq \frac{\pi}{4}$$
$$= \frac{3}{\sqrt{2}}, \text{ for } x = \frac{\pi}{4}$$

Solution:

$$f\left(\frac{\pi}{4}\right) = \frac{3}{\sqrt{2}}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} f(x) = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(\sin x + \cos x)^3 - 2\sqrt{2}}{\sin 2x - 1}$$

$$(\sin x + \cos x)^3 = \left[(\sin x + \cos x)^2\right]^{\frac{3}{2}}$$
$$= (1 + \sin 2x)^{\frac{3}{2}}$$

$$\therefore \lim_{x \rightarrow \frac{\pi}{4}} f(x) = \lim_{x \rightarrow \frac{\pi}{4}} \frac{(1 + \sin 2x)^{\frac{3}{2}} - 2^{\frac{3}{2}}}{\sin 2x - 1}$$

$$\text{Put } 1 + \sin 2x = t$$

$$\therefore \sin 2x = t - 1$$

$$\text{As } x \rightarrow \frac{\pi}{4}, t \rightarrow 1 + \sin 2\left(\frac{\pi}{4}\right)$$

$$\text{i.e., } t \rightarrow 1 + \sin \frac{\pi}{2}$$

$$\text{i.e., } t \rightarrow 1 + 1$$

$$\text{i.e., } t \rightarrow 2$$

$$\begin{aligned}
 \therefore \lim_{x \rightarrow \frac{\pi}{4}} f(x) &= \lim_{t \rightarrow 2} \frac{t^{\frac{3}{2}} - 2^{\frac{3}{2}}}{t - 1 - 1} \\
 &= \lim_{t \rightarrow 2} \frac{t^{\frac{3}{2}} - 2^{\frac{3}{2}}}{t - 2} \\
 &= \frac{3}{2} (2)^{\frac{1}{2}} \quad \dots \left[\because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right] \\
 &= \frac{3\sqrt{2}}{2} = \frac{3}{\sqrt{2}} \\
 \therefore \lim_{x \rightarrow \frac{\pi}{4}} f(x) &= f\left(\frac{\pi}{4}\right) \\
 \therefore f(x) &\text{ is continuous at } x = \frac{\pi}{4}.
 \end{aligned}$$

Question 14.

Determine the values of p and q such that the following function is continuous on the entire real number line.

$$f(x) = x + 1, \text{ for } 1 < x < 3$$

$$= x^2 + px + q, \text{ for } |x - 2| \geq 1.$$

Solution:

$$|x - 2| \geq 1$$

$$\therefore x - 2 \geq 1 \text{ or } x - 2 \leq -1$$

$$\therefore x \geq 3 \text{ or } x \leq 1$$

$$\therefore f(x) = x^2 + px + q \text{ for } x \geq 3 \text{ as well as } x \leq 1$$

$$\text{Thus, } f(x) = x^2 + px + q; x \leq 1$$

$$= x + 1; 1 < x < 3 = x^2 + px + q; x > 3$$

f(x) is continuous for all $x \in \mathbb{R}$.

$\therefore f(x)$ is continuous at $x = 1$ and $x = 3$.

As f(x) is continuous at $x = 1$,

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$$\begin{aligned}\lim_{x \rightarrow 1^-} f(x) &= \lim_{x \rightarrow 1^+} f(x) \\ \therefore \lim_{x \rightarrow 1^-} (x^2 + px + q) &= \lim_{x \rightarrow 1^+} (x + 1) \\ \therefore (1)^2 + p(1) + q &= 1 + 1 \\ \therefore 1 + p + q &= 2 \\ \therefore p + q &= 1 \quad \dots(i)\end{aligned}$$

$$\begin{aligned}\text{As } f(x) \text{ is continuous at } x = 3, \\ \lim_{x \rightarrow 3^-} f(x) &= \lim_{x \rightarrow 3^+} f(x) \\ \therefore \lim_{x \rightarrow 3^-} (x + 1) &= \lim_{x \rightarrow 3^+} (x^2 + px + q) \\ \therefore 3 + 1 &= (3)^2 + 3p + q \\ \therefore 3p + q + 9 &= 4 \\ \therefore 3p + q &= -5 \quad \dots(ii)\end{aligned}$$

Subtracting (i) from (ii), we get

$$2p = -6$$

$$\therefore p = -3$$

Substituting $p = -3$ in (i), we get

$$-3 + q = 1$$

$$\therefore q = 4$$

$$\therefore p = -3 \text{ and } q = 4$$

Question 15.

Show that there is a root for the equation $2x^3 - x - 16 = 0$ between 2 and 3.

Solution:

$$\text{Let } f(x) = 2x^3 - x - 16$$

$f(x)$ is a polynomial function and hence it is continuous for all $x \in \mathbb{R}$.

A root of $f(x)$ exists, if $f(x) = 0$ for at least one value of x .

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$$f(2) = 2(2)^3 - 2 - 16 = -2 < 0$$

$$f(3) = 2(3)^3 - 3 - 16 = 35 > 0$$

$$\therefore f(2) < 0 \text{ and } f(3) > 0$$

$\therefore$ By intermediate value theorem,

there has to be point 'c' between 2 and 3 such that $f(c) = 0$.

$\therefore$ There is a root of the given equation between 2 and 3.

Question 16.

Show that there is a root for the equation $x^3 - 3x = 0$ between 1 and 2.

Solution:

$$\text{Let } f(x) = x^3 - 3x$$

$f(x)$ is a polynomial function and hence it is continuous for all $x \in \mathbb{R}$.

A root of $f(x)$ exists, if $f(x) = 0$ for at least one value of x .

$$f(1) = (1)^3 - 3(1) = -2 < 0$$

$$f(2) = (2)^3 - 3(2) = 2 > 0$$

$$\therefore f(1) < 0 \text{ and } f(2) > 0$$

$\therefore$ By intermediate value theorem,

there has to be point 'c' between 1 and 2 such that $f(c) = 0$.

There is a root of the given equation between 1 and 2.

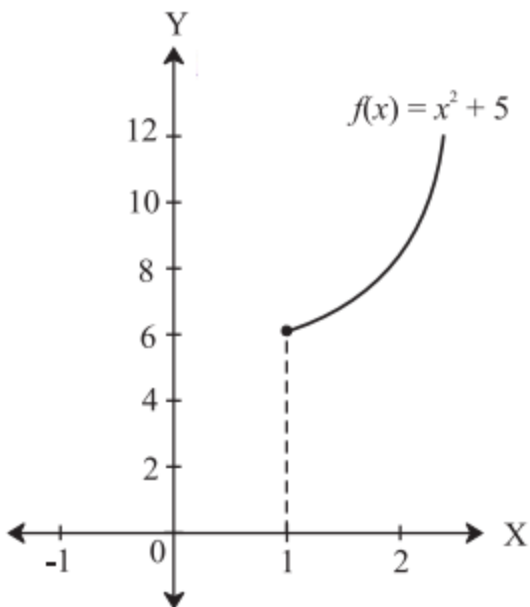
Question 17.

Let $f(x) = ax + b$ (where a and b are unknown)

$$= x^2 + 5 \text{ for } x \in \mathbb{R}$$

Find the values of a and b , so that $f(x)$ is continuous at $x = 1$.

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Solution:

$$f(x) = x^2 + 5, x \in \mathbb{R}$$

$$\therefore f(1) = 1 + 5 = 6$$

If $f(x) = ax + b$ is continuous at $x = 1$, then

$$f(1) = \lim_{x \rightarrow 1} (ax + b) = a + b$$

$$\therefore 6 = a + b \text{ where, } a, b \in \mathbb{R}$$

$\therefore$ There are infinitely many values of a and b .

Question 18.

Activity: Suppose $f(x) = px + 3$ for $a \leq x \leq b$

$$= 5x^2 - q \text{ for } b < x \leq c$$

Find the condition on p, q , so that $f(x)$ is continuous on $[a, c]$, by filling in the boxes.

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$$f(b) = \boxed{}$$

$$\lim_{x \rightarrow b^+} f(x) = \boxed{}$$

$$\therefore pb + 3 = \boxed{} - q$$

$$\therefore p = \frac{\boxed{}}{b} \text{ is the required condition.}$$

Solution:

$$f(b) = \boxed{pb + 3}$$

$$\lim_{x \rightarrow b^+} f(x) = \boxed{5b^2} - q$$

$$\therefore pb + 3 = \boxed{5b^2} - q$$

$$\therefore p = \frac{\boxed{5b^2 - q - 3}}{b} \text{ is the required condition.}$$



Maharashtra Board Solutions

Class 11 Arts & Science Maths

(Part 2)

- Chapter 1- Complex Numbers
- Chapter 2- Sequences and Series
- Chapter 3- Permutations and Combination
- Chapter 4- Methods of Induction and Binomial Theorem
- Chapter 5- Sets and Relations
- Chapter 6- Functions
- Chapter 7- Limits
- Chapter 8- Continuity
- Chapter 9- Differentiation

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The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

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The Maharashtra government established the Maharashtra State Bureau of Textbook Production and Curriculum Research, also commonly referred to as Ebalbharati, in 1967 to take up the responsibility of providing quality textbooks to students from all classes studying under the Maharashtra State Board. MSBHSE prepares and updates the curriculum to provide holistic development for students. It is designed to tackle the difficulty in understanding the concepts with simple language with simple illustrations. Every year around 10 lakh students are enrolled in schools that are affiliated with the Maharashtra State Board.

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