

- (1) Area bounded by the curve $y = f(x)$, X-axis and lines $x = a, x = b$

$$\text{Area} = \left| \int_a^b y \, dx \right| = \left| \int_a^b f(x) \, dx \right|$$

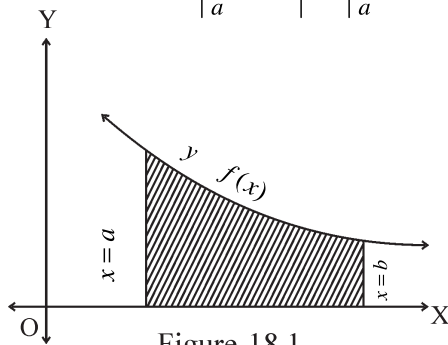


Figure 18.1

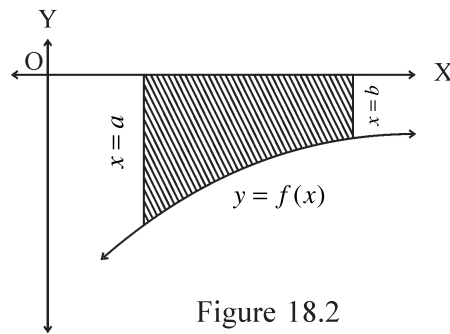


Figure 18.2

- (2) Area bounded by the curve $x = g(y)$, Y-axis and the lines $y = c, y = d$

$$\text{Area} = \left| \int_c^d x \, dy \right| = \left| \int_c^d g(y) \, dy \right|$$

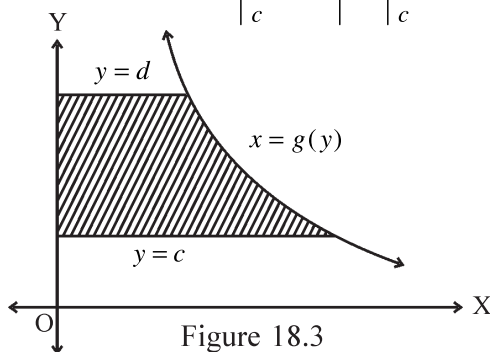


Figure 18.3

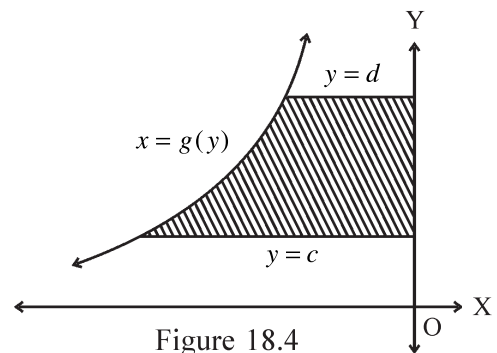


Figure 18.4

- (3) If the curve $y = f(x)$ intersects X-axis at $x = c$ and $x = d$ between the lines $x = a$ and $x = b$ ($c, d \in (a, b)$), then the area of the curve bounded by X-axis and the lines $x = a$ and $x = b$ is, ($a < c < d < b$)

$$\text{Area} = \left| \int_a^c f(x) \, dx \right| + \left| \int_c^d f(x) \, dx \right| + \left| \int_d^b f(x) \, dx \right|$$

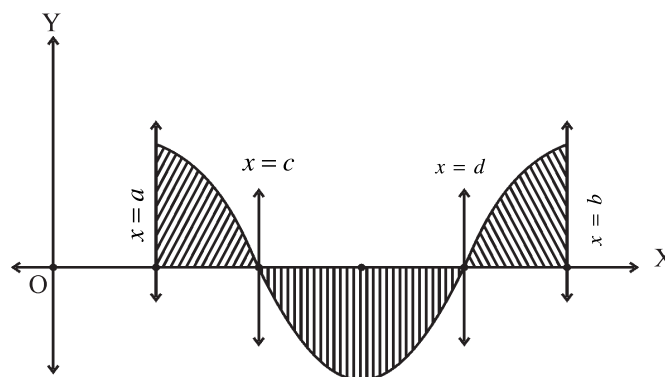


Figure 18.5

- (4) Area bounded by the curves $y = f_1(x)$ and $y = f_2(x)$ between the lines $x = a$ and $x = b$,

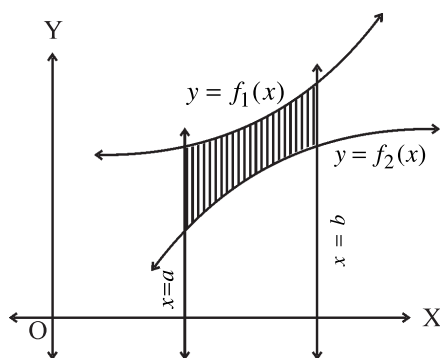


Figure 18.6

$$\text{Area } A = \left| \int_a^b [f_1(x) - f_2(x)] dx \right|$$

- (5) Area bounded by the curves $x = g_1(y)$ and $x = g_2(y)$ between the lines $y = c$ and $y = d$

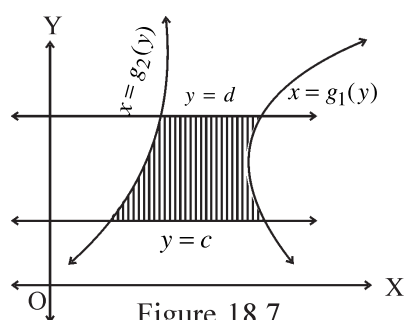


Figure 18.7

$$\text{Area } A = \left| \int_c^d [g_1(y) - g_2(y)] dy \right|$$

- (6) If a curve is symmetric about X-axis and if one region of area is above X-axis and the other below X-axis, then find the area in one region and double it to get the required area.

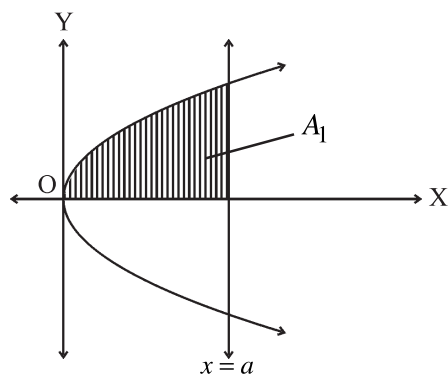


Figure 18.8

$$A_1 = \left| \int_0^a y dx \right|$$

$$A = 2A_1$$

- (7) If a curve is symmetric about both the axes, then find the area in the first quadrant and multiply it by four to get the required area.

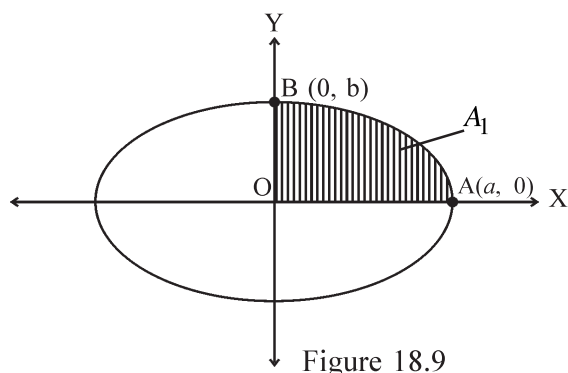


Figure 18.9

$$A_1 = \left| \int_0^a y dx \right|$$

$$A = 4A_1$$

: Multiple Choice Questions :

- (1) The area bounded by the curve $xy = 16$ and the lines $x = 4$ and $x = 8$ is
 (A) $2 \log_e 16$ (B) $16 \log_e 2$ (C) $\log_e 4$ (D) $\log_e 16$

Sol. : Here the given curve is $xy = 16$

If $x > 0$, then $y > 0$ and if $x < 0$, then $y < 0$.

$\therefore y > 0$ for $4 \leq x \leq 8$. The curve lies above X-axis.

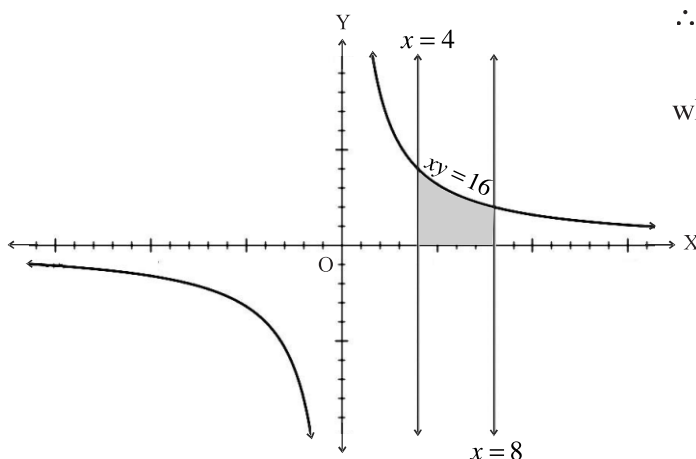


Figure 18.10

$\therefore$ Required area $A = |I|$

$$\text{where } I = \int_4^8 y \, dx$$

$$I = \int_4^8 \frac{16}{x} \, dx$$

$$= 16 \left[\log |x| \right]_4^8$$

$$= 16 [\log 8 - \log 4]$$

$$= 16 \log \left(\frac{8}{4} \right)$$

$$= 16 \log_e 2$$

Ans. : (B)

- (2) The area bounded by the curve $y = x^3$, X-axis and the lines $x = -2$ and $x = 1$ is
 (A) -9 (B) $\frac{15}{4}$ (C) $\frac{17}{4}$ (D) $\frac{13}{4}$

Sol. : Here the given curve is $y = x^3$. So, if $x < 0$, then $y < 0$ and if $x > 0$, then $y > 0$.

$\therefore$ if $-2 \leq x \leq 0$, then $y \leq 0$ and if $0 < x \leq 1$, then $y > 0$.

$\therefore$ Required area $A = |I_1| + |I_2|$

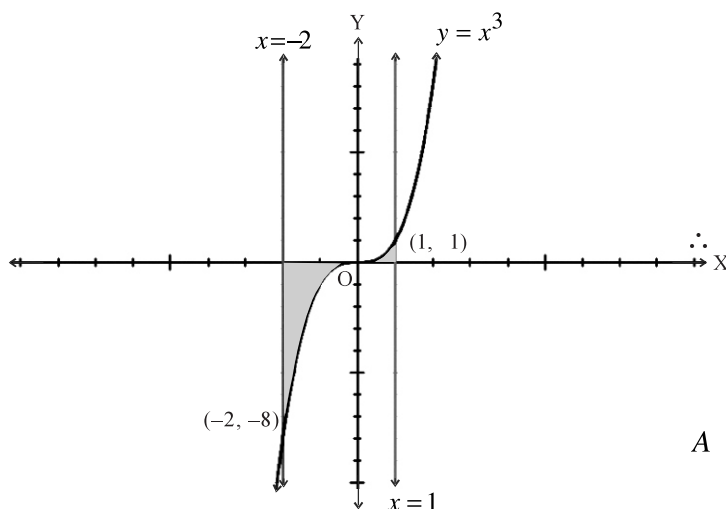


Figure 18.11

$$I_1 = \int_{-2}^0 y \, dx$$

$$= \int_{-2}^0 x^3 \, dx$$

$$= \left[\frac{x^4}{4} \right]_{-2}^0$$

$$= 0 - 4$$

$$I_1 = -4$$

$$I_2 = \int_0^1 y \, dx$$

$$= \int_0^1 x^3 \, dx$$

$$= \left[\frac{x^4}{4} \right]_0^1$$

$$I_2 = \frac{1}{4}$$

$$A = |I_1| + |I_2|$$

$$= |-4| + \left| \frac{1}{4} \right|$$

$$= 4 + \frac{1}{4} = \frac{17}{4}$$

Ans. : (C)

- (3) The area bounded by the curve $y = |x-2|$, X -axis and the lines $x=1$ and $x=3$ is
 (A) 1 (B) 2 (C) 3 (D) 4

(AIEEE : 2004)

Sol. : The curve is defined as follows :

$$y = |x-2| = \begin{cases} x-2, & x \geq 2 \\ 2-x, & x \leq 2 \end{cases}$$

$$y_2 = x-2, \quad x \geq 2 \quad \text{and} \quad y_1 = 2-x, \quad x \leq 2$$

x	2	3	4
y	0	1	2

x	2	1	-1
y	0	2	1

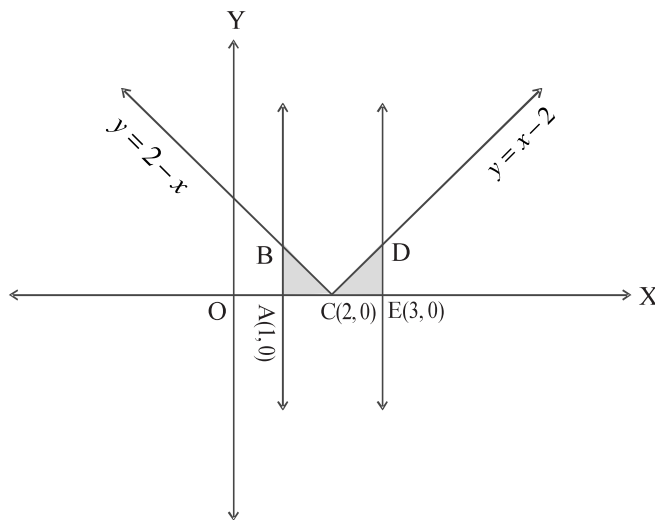


Figure 18.12

$$\therefore \text{Area} = I_1 + I_2$$

$$= \int_1^2 y_1 dx + \int_2^3 y_2 dx$$

$$= \int_1^2 (2-x) dx + \int_2^3 (x-2) dx$$

$$= \left[2x - \frac{x^2}{2} \right]_1^2 + \left[\frac{x^2}{2} - 2x \right]_2^3$$

$$= \left[(4-2) - \left(2 - \frac{1}{2} \right) \right] + \left[\left(\frac{9}{2} - 6 \right) - (2-4) \right]$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$(\text{Area} = 2 \text{ Area of } \triangle ABC = 2 \left(\frac{1}{2} \times b \times h \right))$$

$$= 2 \times \frac{1}{2} \times 1 \times 1 = 1).$$

Ans. : (A)

- (4) The area bounded by the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$ is

(A) $\frac{4}{3}(4\pi - \sqrt{3})$ (B) $\frac{2}{3}(4\pi + \sqrt{3})$ (C) $\frac{4}{3}(4\pi + \sqrt{3})$ (D) $\frac{4}{3}(\sqrt{3} + 2\pi)$

Sol. : Here the circle is $x^2 + y^2 = 16$ and

the parabola is $y^2 = 6x$.

$$\text{Solving, } x^2 + 6x = 16$$

$$\therefore x^2 + 6x - 16 = 0$$

$$\therefore (x+8)(x-2) = 0$$

$$\therefore x = -8 \text{ or } x = 2$$

For $x = -8$ we have $y^2 = -48 < 0$, which is not possible.

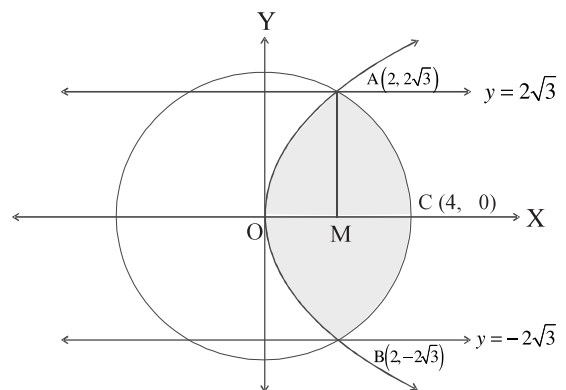


Figure 18.13

The points of intersection are, $A(2, 2\sqrt{3})$, $B(2, -2\sqrt{3})$

$$\text{Now, } x_1 = \sqrt{16 - y^2}, \quad x_2 = \frac{y^2}{6}$$

Required area = Area of OACBO

$$= 2 (\text{Area of OACO})$$

$$= 2 \int_0^{2\sqrt{3}} (x_1 - x_2) dy$$

$$= 2 \int_0^{2\sqrt{3}} \left(\sqrt{16 - y^2} - \frac{y^2}{6} \right) dy$$

$$= 2 \left[\frac{y}{2} \sqrt{16 - y^2} + 8 \sin^{-1} \frac{y}{4} - \frac{y^3}{18} \right]_0^{2\sqrt{3}}$$

$$= 2 \left[\frac{2\sqrt{3}}{2} \sqrt{16 - 12} + 8 \sin^{-1} \frac{2\sqrt{3}}{4} - \frac{24\sqrt{3}}{18} \right]$$

$$= 2 \left[\frac{2\sqrt{3}}{2} \cdot 2 + 8 \sin^{-1} \frac{\sqrt{3}}{2} - \frac{4\sqrt{3}}{3} \right]$$

$$= 2 \left[2\sqrt{3} + \frac{8\pi}{3} - \frac{4}{3} \sqrt{3} \right]$$

$$= 2 \left[\frac{2\sqrt{3} + 8\pi}{3} \right]$$

$$= \frac{4}{3} (\sqrt{3} + 4\pi)$$

Ans. : (C)

Second method :

Area of OACO = Area of OAM + Area of ACM

$$= \int_0^2 \sqrt{6x} dx + \int_2^4 \sqrt{16 - x^2} dx$$

$$= \sqrt{6} \left[\frac{2x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^2 + \left[\frac{x}{2} \sqrt{16 - x^2} + 8 \sin^{-1} \frac{x}{4} \right]_2^4$$

$$= \frac{2}{3} \sqrt{6} (2\sqrt{2}) + \left[8 \sin^{-1} 1 - 2\sqrt{3} - 8 \sin^{-1} \frac{1}{2} \right]$$

$$= \frac{8}{3} \sqrt{3} - 2\sqrt{3} + \frac{8\pi}{2} - \frac{8\pi}{6}$$

$$= \frac{2}{3} \sqrt{3} + \frac{8\pi}{3}$$

$$\therefore \text{ Required area } = \frac{4}{3} (\sqrt{3} + 4\pi)$$

Ans. : (C)

- (5) The area bounded by the curve $y = x \sin x$, X -axis and the lines $x = 0$ and $x = 2\pi$ is
 (A) 2π (B) 3π (C) 4π (D) π

Sol. : Here the given curve is $y = x \sin x$

If $\pi \leq x \leq 2\pi$, then $y \leq 0$. If $0 \leq x \leq \pi$, then $y \geq 0$

$$\begin{aligned} \therefore \text{Required area } A &= \int_0^{\pi} y \, dx + \left| \int_{\pi}^{2\pi} y \, dx \right| \\ &= \int_0^{\pi} x \sin x \, dx + \left| \int_{\pi}^{2\pi} x \sin x \, dx \right| \\ &= [-x \cos x + \sin x]_0^{\pi} + \left| [-x \cos x + \sin x]_{\pi}^{2\pi} \right| \\ &= \pi + \left| \{(-2\pi) - (\pi)\} \right| = \pi + |-3\pi| = 4\pi \quad \text{Ans. : (C)} \end{aligned}$$

- (6) The area of the region bounded by the parabola $ay = 3(a^2 - x^2)$ and X -axis is ($a > 0$)
 (A) $2a^2$ (B) $4a^2$ (C) $\frac{3}{4}a^2$ (D) $\frac{5}{4}a^2$

Sol. : Here given curve is $ay = 3(a^2 - x^2)$.

Taking $y = 0$ we get, $3(a^2 - x^2) = 0$. So, $x = \pm a$.

The curve intersects X -axis at $x = a$ and $x = -a$.

Also, if $-a \leq x \leq a$, then $y \geq 0$

The area bounded by the parabola $ay = 3(a^2 - x^2)$ and X -axis is

$$\text{Area } A = \int_{-a}^a y \, dx.$$

$$\begin{aligned} \therefore A &= \int_{-a}^a \frac{3(a^2 - x^2)}{a} \, dx \\ &= \frac{3}{a} \int_{-a}^a (a^2 - x^2) \, dx \\ &= \frac{6}{a} \int_0^a (a^2 - x^2) \, dx \\ &= \frac{6}{a} \left[a^2 x - \frac{x^3}{3} \right]_0^a \\ &= \frac{6}{a} \left[a^3 - \frac{a^3}{3} \right] = \frac{6}{a} \left(\frac{2a^3}{3} \right) = 4a^2 \end{aligned}$$

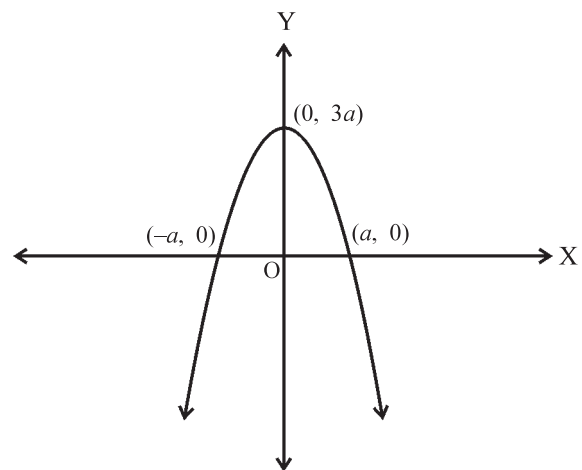


Figure 18.14

Ans. : (B)

(7) The area bounded by the curves $y=1-x^2$ and $y=x^2-1$ is

- (A) $\frac{4}{3}$ (B) $\frac{2}{3}$ (C) $\frac{8}{3}$ (D) $\frac{16}{3}$

Sol. : Here the curve $y=1-x^2$ i.e. $x^2=(1-y)$ is a parabola having vertex at $A(0,1)$ and $y=x^2-1$ i.e. $x^2=1+y$ is a parabola having vertex $B(0,-1)$.

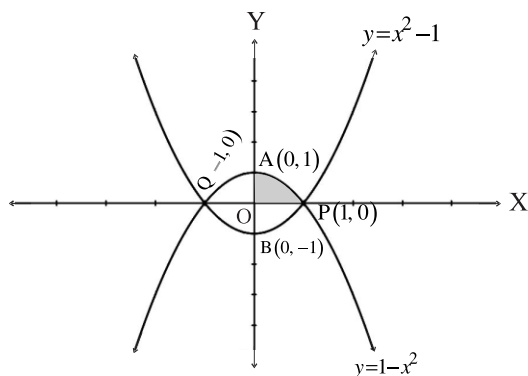


Figure 18.15

Solving we get,

$$1-x^2 = x^2-1$$

$$\therefore 2x^2 = 2$$

$$\therefore x^2 = 1$$

$$\therefore x = \pm 1$$

$$\therefore x=1 \Rightarrow y=0 \text{ and } x=-1 \Rightarrow y=0$$

$\therefore$ The points of intersection with X-axis are

$P(1, 0)$ and $Q(-1, 0)$.

$\therefore$ Required area Δ = (Area of region PAQBP)

$$= 4(\text{Area of region PAOP})$$

$$= 4 \int_0^1 (1-x^2) dx$$

$$= 4 \left[x - \frac{x^3}{3} \right]_0^1$$

$$= 4 \left(1 - \frac{1}{3} \right) = 4 \left(\frac{2}{3} \right) = \frac{8}{3}$$

Ans. : (C)

(8) The area bounded by the curves $y=|x-1|$ and $y=1$ is

(IIT : 1994)

- (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$

Sol. : The curve is defined as follows :

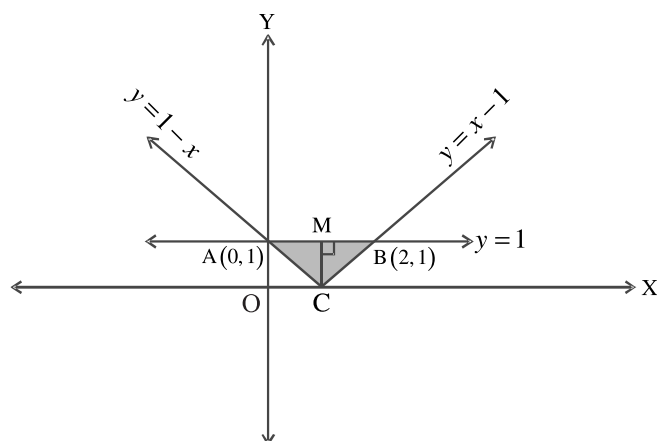


Figure 18.16

$$y = |x-1| = \begin{cases} 1-x, & x < 1 \\ x-1, & x \geq 1 \end{cases}$$

$$y = 1-x, x < 1 \quad , \quad y = x-1, x \geq 1$$

x	0	-1
y	1	2

x	1	2	3
y	0	1	2

The line $y=1$ intersects the curve $y=1-x$ at $A(0,1)$ and the line $y=1$ intersects the curve $y=x-1$ at $B(2,1)$.

$$\therefore \text{ Required area} = \text{Area of } \Delta ABC = \frac{1}{2} |D|$$

$$D = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 1(0) - 0 + 1(2) = 2$$

$$\Delta = \frac{1}{2}|D| = \frac{1}{2}|2| = 1$$

Ans. : (B)

Second method (Using integration) :

$$\begin{aligned} \Delta &= -\int_0^1 (1-y)dy + \int_0^1 (y+1)dy \\ &= -\left[y - \frac{y^2}{2}\right]_0^1 + \left[\frac{y^2}{2} + y\right]_0^1 = -\left(1 - \frac{1}{2}\right) + \frac{3}{2} = 1 \quad \text{or} \quad \int_0^1 2y dy = \left[y^2\right]_0^1 = 1 \end{aligned}$$

Third method (Geometrically)

$$\text{Area} = \frac{1}{2} AB \cdot CM = \frac{1}{2} (2-0) \cdot 1 = 1$$

(9) The area bounded by the curve $y = \log_e (x+e)$ and the coordinate axes is

(A) 2 (B) 1 (C) 3 (D) 4 (AIEEE : 2005)

Sol. : The curve $y = \log_e (x+e)$ is defined if $(x+e) > 0$ i.e. $x > -e$.

$$\text{Also, } y = 0 \Rightarrow \log_e (x+e) = 0 \Rightarrow x+e = 1 \Rightarrow x = 1-e$$

$\therefore$ The curve $y = \log_e (x+e)$, intersects X-axis at $A(1-e, 0)$.

$$\text{Also, when } x = 0 \Rightarrow y = \log_e e = 1$$

$\therefore$ The curve intersects Y-axis at $B(0, 1)$.

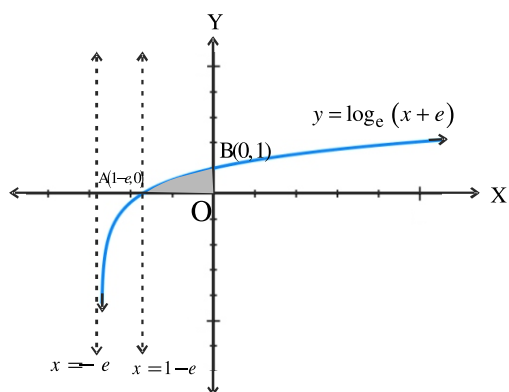


Figure 18.17

$$\therefore \text{ Required area } \Delta = \int_{1-e}^0 y dx$$

$$= \int_{1-e}^0 \log(x+e) dx$$

$$\text{Let, } t = x+e$$

$$\therefore dx = dt$$

$$\text{Also, } x = 1-e \Rightarrow t = 1$$

$$x = 0 \Rightarrow t = e$$

$$\therefore \Delta = \int_1^e \log t dt$$

$$= [t \cdot \log t - t]_1^e$$

$$= (e \log e - e) - (-1) = 1$$

Ans. : (B)

- (10) If the area bounded by the curve $y = 2^{kx}$ and the lines $x = 0$ and $x = 2$ above the X-axis is $\frac{3}{\log 2}$, then $k = \dots$

(A) -1 (B) $\frac{1}{2}$ (C) 1 (D) 2

Sol. : Here the curve is $y = 2^{kx}$. We have $y > 0$ for any value of x and k .

So the curve $y = 2^{kx}$ is lying above X-axis for $0 \leq x \leq 2$.

$$\begin{aligned} \text{Required area, } A &= \int_0^2 y \, dx = \int_0^2 2^{kx} \, dx \\ &= \frac{1}{k} \left[\frac{2^{kx}}{\log 2} \right]_0^2 \\ &= \frac{1}{k \log 2} (2^{2k} - 1) \end{aligned}$$

$$\text{But } A = \frac{3}{\log 2}$$

$$\therefore \frac{3}{\log 2} = \frac{(2^{2k} - 1)}{k \cdot \log 2}$$

$$\therefore 2^{2k} - 1 = 3k$$

$$\therefore k = 1$$

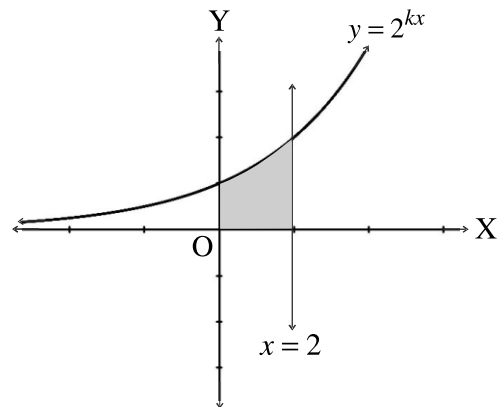


Figure 18.18

Ans. : (C)

- (11) The area bounded by the curve $y = 4 + 3x - x^2$ and X-axis is

(A) $\frac{125}{3}$ (B) $\frac{125}{4}$ (C) $\frac{125}{6}$ (D) None of these

Sol. : Given curve $y = 4 + 3x - x^2$

$$y = -\left(x^2 - 3x + \frac{9}{4}\right) + \frac{25}{4}$$

$$\therefore \left(x - \frac{3}{2}\right)^2 = -\left(y - \frac{25}{4}\right)$$

$\therefore$ The given curve is a parabola having vertex $P\left(\frac{3}{2}, \frac{25}{4}\right)$.

$$\text{Taking } y = 0, \quad 4 + 3x - x^2 = 0$$

$$x^2 - 3x - 4 = 0$$

$$\therefore (x - 4)(x + 1) = 0$$

$$\therefore x = 4 \quad \text{or} \quad x = -1$$

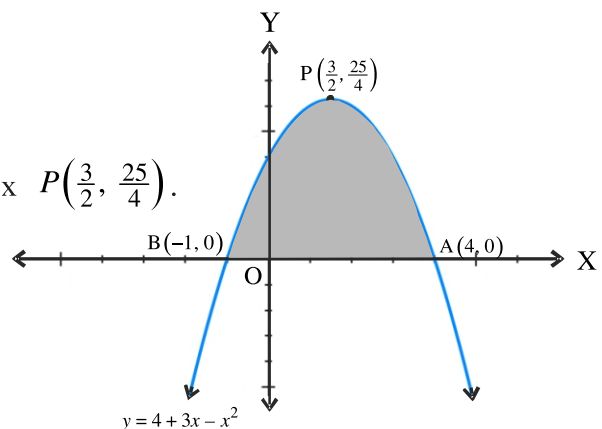


Figure 18.19

$\therefore$ The curve intersects X-axis at $A(4, 0)$ and $B(-1, 0)$.

$$\therefore \text{ Required area } \Delta = \int_{-1}^4 y \, dx$$

$$\Delta = \int_{-1}^4 (4 + 3x - x^2) \, dx$$

$$= \left[4x + \frac{3x^2}{2} - \frac{x^3}{3} \right]_{-1}^4$$

$$= \left\{ \left(16 + 24 - \frac{64}{3} \right) - \left(-4 + \frac{3}{2} + \frac{1}{3} \right) \right\}$$

$$= \frac{125}{6}$$

Ans. : (C)

(12) The area bounded by the curve $x^2 + y^2 = 2ax$ is

(A) $\frac{1}{2} \pi a^2$

(B) πa^2

(C) $2\pi a^2$

(D) $4\pi a^2$

Sol. : The given curve is $x^2 + y^2 = 2ax$

$$\therefore x^2 - 2ax + a^2 + y^2 = a^2$$

$$\therefore (x-a)^2 + (y-0)^2 = a^2$$

$\therefore$ The given curve is a circle having centre $C(a, 0)$ and radius a .

Taking $y = 0$, $x^2 = 2ax$

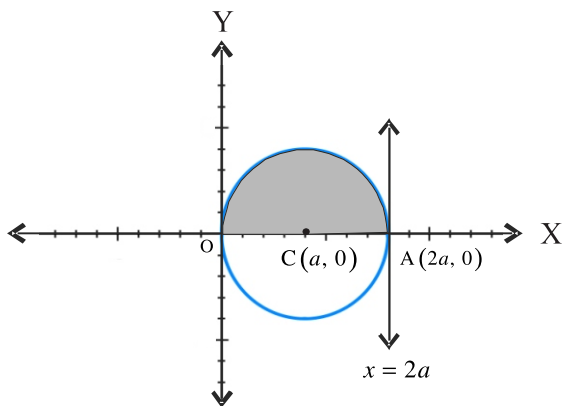


Figure 18.20

$$\therefore x^2 - 2ax = 0$$

$$\therefore x(x - 2a) = 0$$

$$\therefore x = 0 \text{ or } x = 2a$$

The given circle intersects X-axis at $O(0, 0)$ and $A(2a, 0)$.

$$\therefore \text{ Required area } = 2 \int_0^{2a} \sqrt{2ax - x^2} \, dx$$

$$= 2 \int_0^{2a} \sqrt{a^2 - (x-a)^2} \, dx$$

$$= 2 \left[\frac{x-a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x-a}{a} \right) \right]_0^{2a}$$

$$= 2 \left[\frac{\pi}{2} \times \frac{a^2}{2} - \left(-\frac{\pi}{2} \times \frac{a^2}{2} \right) \right]$$

$$= \pi a^2$$

Note : The area of a circle having radius a is πa^2 .

Ans. : (B)

- (13) The area bounded by the curve $y = (x-1)(x-2)(x-3)$, X -axis and the lines $x=0$ and $x=3$ is

- (A) $\frac{9}{4}$ (B) $\frac{11}{4}$ (C) $\frac{11}{2}$ (D) $\frac{9}{2}$

Sol. : The given curve is $y = (x-1)(x-2)(x-3)$

x	0	1	2	3
y	-6	0	0	0

When $0 \leq x \leq 1$, $y \leq 0$

When $1 \leq x \leq 2$, then $y \geq 0$

When $2 \leq x \leq 3$, then $y \leq 0$. Rough sketch of the curve is drawn.

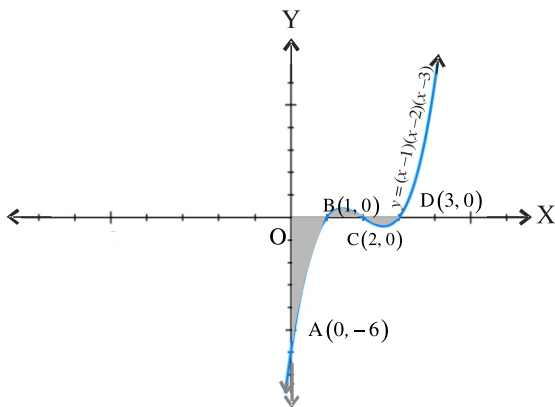


Figure 18.21

The area bounded by the curve, X -axis and the lines $x=0$ and $x=3$ is

$$\Delta = \left| \int_0^1 y \, dx \right| + \left| \int_1^2 y \, dx \right| + \left| \int_2^3 y \, dx \right|$$

where $y = f(x) = (x-1)(x-2)(x-3)$

$$= (x^3 - 6x^2 + 11x - 6)$$

$$\therefore F(x) = \int (x^3 - 6x^2 + 11x - 6) \, dx$$

$$= \left(\frac{x^4}{4} - 2x^3 + \frac{11}{2}x^2 - 6x \right)$$

$$\therefore F(0) = 0, F(1) = \left(\frac{1}{4} - 2 + \frac{11}{2} - 6 \right) = -\frac{9}{4}$$

$$F(2) = (4 - 16 + 22 - 12) = -2, F(3) = \left(\frac{81}{4} - 54 + \frac{99}{2} - 18 \right) = -\frac{9}{4}$$

$$\therefore \Delta = |F(1) - F(0)| + |F(2) - F(1)| + |F(3) - F(2)|$$

$$= \left| -\frac{9}{4} - 0 \right| + \left| -2 - \left(-\frac{9}{4} \right) \right| + \left| -\frac{9}{4} - (-2) \right| = \frac{9}{4} + \frac{1}{4} + \frac{1}{4} = \frac{11}{4} \quad \text{Ans. : (B)}$$

- (14) The area bounded by the curve $x = y^2 + 4y + 3$, Y -axis and the lines $y=2$, $y=-2$ is

- (A) 18 (B) $\frac{56}{3}$ (C) $\frac{52}{3}$ (D) $\frac{100}{3}$

Sol. : Here, $x = y^2 + 4y + 3$

$$\therefore x + 1 = y^2 + 4y + 4$$

$$\therefore (x + 1) = (y + 2)^2$$

This is parabola with vertex A(-1, -2).

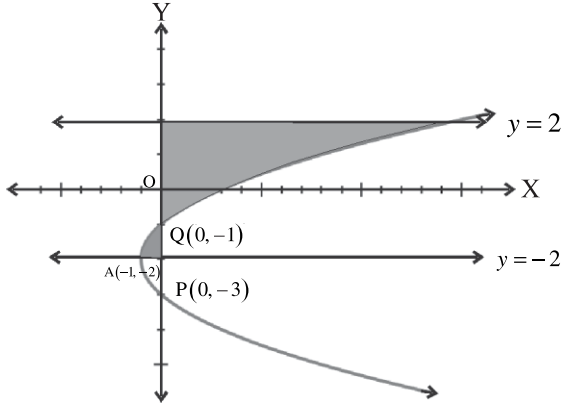


Figure 18.22

Let $x = 0$ for the intersection of $x = y^2 + 4y + 3$ with Y-axis. We get

$$y^2 + 4y + 3 = 0$$

$$\therefore (y + 3)(y + 1) = 0$$

$$\therefore y = -3 \text{ or } y = -1.$$

The parabola intersects Y-axis at $P(0, -3)$ and $Q(0, -1)$.

$$\therefore \text{ Required area } \Delta = \left| \int_{-2}^{-1} x \, dy \right| + \left| \int_{-1}^2 x \, dy \right|$$

$$\Delta = |I_1| + |I_2|$$

$$I_1 = \int_{-2}^{-1} (y^2 + 4y + 3) \, dy$$

$$= \left[\frac{y^3}{3} + \frac{4y^2}{2} + 3y \right]_{-2}^{-1}$$

$$= \left(\frac{-1}{3} + 2 - 3 \right) - \left(\frac{-8}{3} + 8 - 6 \right) = \frac{-2}{3}$$

$$\therefore |I_1| = \frac{2}{3}$$

$$I_2 = \int_{-1}^2 (y^2 + 4y + 3) \, dy$$

$$= \left[\frac{y^3}{3} + \frac{4y^2}{2} + 3y \right]_{-1}^2$$

$$= \left(\frac{8}{3} + 8 + 6 \right) - \left(\frac{-1}{3} + 2 - 3 \right) = 18$$

$$\Delta = |I_1| + |I_2| = \frac{2}{3} + 18 = \frac{56}{3}$$

Ans. : (B)

(15) The area bounded by the curves $x^2 = 4by$ and $y^2 = 4ax$ is ($a > 0, b > 0$)

- (A) $16ab$ (B) $16\pi ab$ (C) $\frac{16ab}{3}$ (D) $16\pi^2 ab$

Sol. : The curves $x^2 = 4by$ and $y^2 = 4ax$ are parabolas symmetric about Y-axis and X-axis respectively and having vertex at the origin.

Solving, $\frac{x^2}{4b} = y$ and $y^2 = 4ax$, we get

$$\left(\frac{x^2}{4b}\right)^2 = 4ax$$

$$\therefore x^4 - 64ab^2x = 0 \Rightarrow x(x^3 - 64ab^2) = 0$$

$$\therefore x = 0 \text{ or } x = 4 \cdot a^{\frac{1}{3}} b^{\frac{2}{3}}$$

Taking $x = 0$ we get $y = 0$ and taking $x = 4 a^{\frac{1}{3}} b^{\frac{2}{3}}$ we get $y = 4 a^{\frac{2}{3}} b^{\frac{1}{3}}$

$\therefore$ The points of intersection of given curves are $A(0, 0)$ and $B\left(4 a^{\frac{1}{3}} b^{\frac{2}{3}}, 4 a^{\frac{2}{3}} b^{\frac{1}{3}}\right)$.

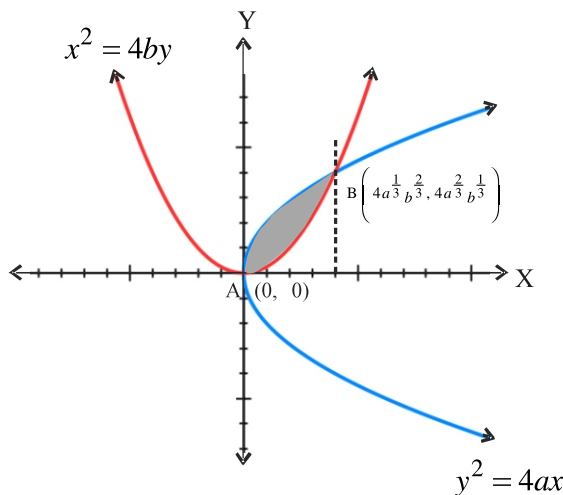


Figure 18.23

$$\text{Required area} = \Delta = \int_0^{4 a^{\frac{1}{3}} b^{\frac{2}{3}}} (y_1 - y_2) dx$$

$$= \int_0^{4 a^{\frac{1}{3}} b^{\frac{2}{3}}} \left(2\sqrt{ax} - \frac{x^2}{4b}\right) dx$$

$$= 2\sqrt{a} \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^{4 a^{\frac{1}{3}} b^{\frac{2}{3}}} - \left[\frac{x^3}{12b} \right]_0^{4 a^{\frac{1}{3}} b^{\frac{2}{3}}}$$

$$= 2a^{\frac{1}{2}} \left(\frac{2}{3} \cdot 8 a^{\frac{1}{2}} b \right) - \frac{1}{4b} \left(\frac{64ab^2}{3} \right)$$

$$= \frac{32}{3} ab - \frac{16}{3} ab$$

$$= \frac{16}{3} ab$$

Ans. : (C)

(16) If the area bounded by the curves $x = ay^2$ and $y = ax^2$ is 1, then $a = \dots\dots$. ($a > 0$)

- (A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 3

(IIT :2004)

Sol. : Solving equations of given curves $x = ay^2$ and $y = ax^2$, we get

$$\begin{aligned}
 x &= (ax^2)^2 \\
 \therefore x &= a^3 x^4 \\
 \therefore x(1 - a^3 x^3) &= 0 \\
 \therefore x &= 0 \quad \text{or} \quad x = \frac{1}{a}
 \end{aligned}$$

If $x=0$, then $y=0$ and $x=\frac{1}{a}$, then $y=\frac{1}{a}$

$\therefore$ The points of intersection of given curves are $O(0,0)$ and $B(\frac{1}{a}, \frac{1}{a})$.

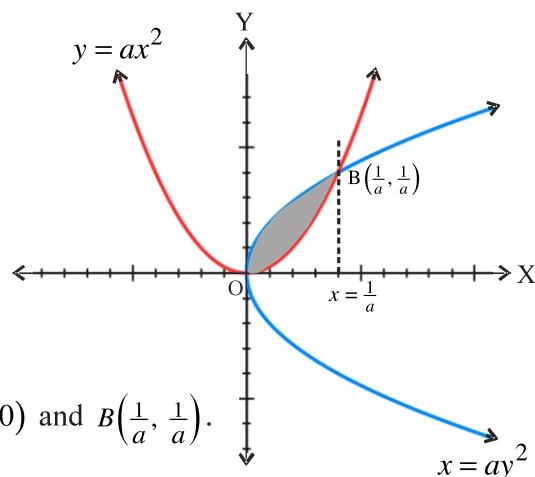


Figure 18.24

$$\text{Required area } A = \int_0^{\frac{1}{a}} (y_1 - y_2) dx$$

$$= \int_0^{\frac{1}{a}} \left(\sqrt{\frac{x}{a}} - ax^2 \right) dx$$

$$= \left[\frac{1}{\sqrt{a}} \left\{ \frac{2}{3} x^{\frac{3}{2}} \right\} - \frac{ax^3}{3} \right]_0^{\frac{1}{a}}$$

$$= \left[\frac{2 \cdot x^{\frac{3}{2}}}{3\sqrt{a}} - \frac{ax^3}{3} \right]_0^{\frac{1}{a}}$$

$$= \frac{2}{3a^2} - \frac{1}{3a^2} = \frac{1}{3a^2}$$

Now $A=1$

$$\therefore \frac{1}{3a^2} = 1. \quad \text{So, } a^2 = \frac{1}{3}. \quad \text{Hence } a = \frac{1}{\sqrt{3}}. \quad \text{Ans. : (A)}$$

(17) The area bounded by the common region of the curves $x^2 + y^2 = 4$ and $x^2 + y^2 = 4y$ is

(A) $\frac{8\pi}{3} - 2\sqrt{3}$ (B) $\frac{4\pi}{3} - \sqrt{3}$ (C) $2\sqrt{3} - \frac{4\pi}{3}$ (D) $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

Sol. : The curve $x^2 + y^2 = 4$ represents a circle having centre $(0,0)$ and radius 2,

$$x^2 + y^2 = 4y \Rightarrow x^2 + y^2 - 4y + 4 = 4$$

$$\Rightarrow x^2 + (y-2)^2 = 2^2$$

represents a circle having centre $(0,2)$ and radius 2.

Solving $x^2 + y^2 = 4$ and $x^2 + y^2 = 4y$, we get

$$4y = 4. \quad \text{So, } y = 1$$

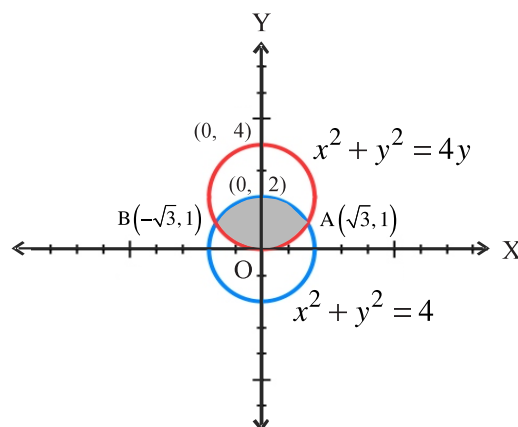


Figure 18.25

$$y=1 \Rightarrow x^2+1=4 \Rightarrow x^2=3$$

$$\therefore x = \pm\sqrt{3}$$

The points of intersection of given two circles are $A(\sqrt{3}, 1)$ and $B(-\sqrt{3}, 1)$.

$$\text{Now, } x^2 + y^2 = 4.$$

$$\text{So, } y^2 = 4 - x^2$$

$$\therefore y = \sqrt{4 - x^2}.$$

$$\text{Hence } y_1 = \sqrt{4 - x^2}$$

$$\text{Also, } x^2 + (y-2)^2 = 4$$

$$(y-2)^2 = 4 - x^2$$

$$\therefore y-2 = \pm\sqrt{4 - x^2}$$

From the figure $y < 2$.

So, $(y-2) < 0$ (Also, $y \neq 2$ for $x^2 + y^2 = 4$)

$$\therefore y = 2 - \sqrt{4 - x^2}.$$

$$\text{Hence } y_2 = 2 - \sqrt{4 - x^2}$$

$$\therefore y_1 > y_2$$

$$\text{Required area } A = \int_{-\sqrt{3}}^{\sqrt{3}} (y_1 - y_2) dx$$

$$= \int_{-\sqrt{3}}^{\sqrt{3}} \left(\sqrt{4 - x^2} - 2 + \sqrt{4 - x^2} \right) dx$$

$$= \int_{-\sqrt{3}}^{\sqrt{3}} \left(2\sqrt{4 - x^2} - 2 \right) dx$$

$$= 4 \int_0^{\sqrt{3}} \left(\sqrt{4 - x^2} - 1 \right) dx \quad (\text{even function})$$

$$= 4 \left[\frac{x}{2} \sqrt{4 - x^2} + 2 \sin^{-1} \frac{x}{2} - x \right]_0^{\sqrt{3}}$$

$$= 4 \left[\frac{\sqrt{3}}{2} (1) + 2 \sin^{-1} \frac{\sqrt{3}}{2} - \sqrt{3} - 0 \right]$$

$$= 4 \left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right]$$

$$= \frac{8\pi}{3} - 2\sqrt{3}.$$

Ans. : (A)

(18) The area bounded by the curves $y=2^x$, $y=2x-x^2$ and the lines $x=0$, $x=2$ is

- (A) $\frac{4}{3}-\frac{1}{\log 2}$ (B) $\frac{3}{\log 2}-\frac{4}{3}$ (C) $\frac{3}{\log 2}+\frac{4}{3}$ (D) $\frac{4}{\log 2}-1$

Sol. : A curve is $y=2x-x^2$

$$\therefore -y=x^2-2x$$

$$-(y-1)=(x-1)^2$$

It is a parabola having vertex (1, 1).

Taking $y=0$ we get, $2x-x^2=0$

$$\therefore x(2-x)=0$$

$$\therefore x=0 \text{ or } x=2$$

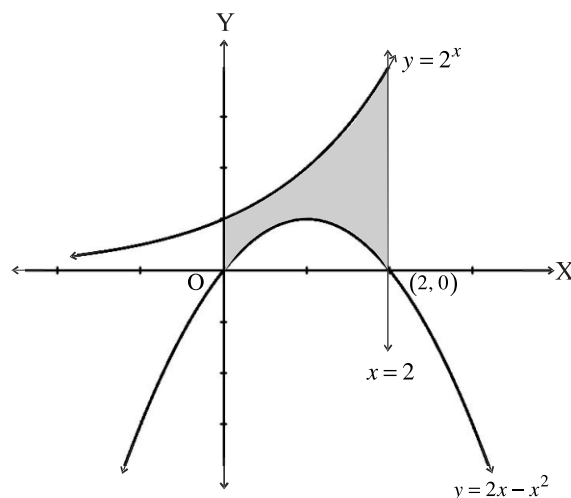


Figure 18.26

Thus it intersect X-axis at the points O(0, 0) and B(2, 0).

$$\text{Required area } A = \int_0^2 [2^x - (2x - x^2)] dx$$

$$= \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3} \right]_0^2$$

$$= \frac{4}{\log 2} - 4 + \frac{8}{3} - \frac{1}{\log 2}$$

$$A = \frac{3}{\log 2} - \frac{4}{3}$$

Ans. : (B)

(19) X-axis divides the area bounded by parabolas $y=4x-x^2$ and $y=x^2-x$ in the ratio

- (A) $\frac{128}{5}$ (B) $\frac{125}{4}$ (C) $\frac{52}{3}$ (D) $\frac{15}{4}$ (IIT : 1994)

Sol. : Here, $y=4x-x^2$

$\therefore (x-2)^2 = -(y-4)$ is a parabola having vertex P(2, 4).

Taking $y=0$ we get, $4x-x^2=0 \Rightarrow x(4-x)=0 \Rightarrow x=0$ or $x=4$.

$$\therefore x=0 \Rightarrow y=0 \text{ and } x=4 \Rightarrow y=0$$

$\therefore$ The points of intersection of parabola $y=4x-x^2$ with X-axis are O(0, 0) and B(4, 0).

Also, $y = x^2 - x$

$\left(y + \frac{1}{4}\right) = \left(x - \frac{1}{2}\right)^2$ which is a parabola having vertex $Q\left(\frac{1}{2}, -\frac{1}{4}\right)$.

Taking $y = 0$ we get $x^2 - x = 0 \Rightarrow x(x-1) = 0$

$$\Rightarrow x = 0 \text{ or } x = 1$$

The points of intersection of parabola $y = x^2 - x$ with X-axis are $O(0, 0)$ and $C(1, 0)$.

Now, we find points of intersection of given curves.

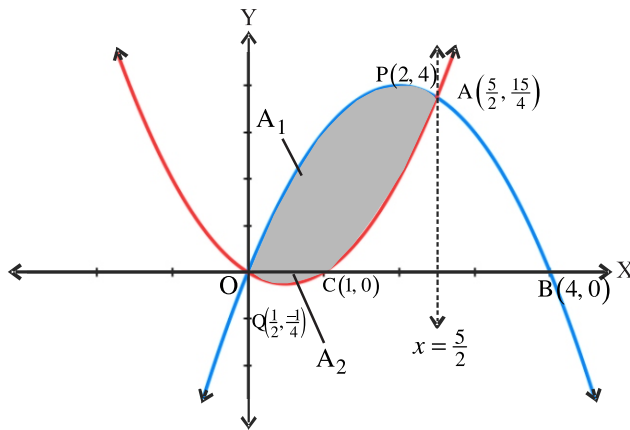


Figure 18.27

$$y = 4x - x^2 = x^2 - x$$

$$\therefore 2x^2 - 5x = 0$$

$$\therefore x(2x - 5) = 0$$

$$\therefore x = 0 \text{ or } x = \frac{5}{2}$$

$O(0, 0)$ and $A\left(\frac{5}{2}, \frac{15}{4}\right)$ are the points of intersection.

X-axis divides the region between two curves into two regions A_1 and A_2 . Part A_1 is lying above the X-axis and A_2 is lying below it.

$$A_1 = \int_0^{\frac{5}{2}} (y_1 - y_2) dx$$

$$= \int_0^{\frac{5}{2}} \left[(4x - x^2) - (x^2 - x) \right] dx$$

$$= \left[2x^2 - \frac{x^3}{3} - \frac{x^3}{3} + \frac{x^2}{2} \right]_0^{\frac{5}{2}}$$

$$= \left[\frac{5}{2} \cdot x^2 - \frac{2}{3} x^3 \right]_0^{\frac{5}{2}}$$

$$= \frac{5}{2} \times \frac{25}{4} - \frac{2}{3} \cdot \frac{125}{8} = \frac{125}{8} \left[1 - \frac{2}{3} \right] = \frac{125}{8} \times \frac{1}{3} = \frac{125}{24}$$

$$A_2 = \left| \int_0^1 (x^2 - x) dx \right|$$

$$= \left| \left[\frac{x^3}{3} - \frac{x^2}{2} \right]_0^1 \right|$$

$$= \left| \frac{1}{3} - \frac{1}{2} \right| = \frac{1}{6}$$

$$\therefore \text{ Required ratio} = \frac{A_1}{A_2} = \frac{125}{4}$$

Ans. : (B)

- (20) If a tangent to the curve $f(x) = x^2 + bx - b$ at $(1, 1)$ forms a triangle of area 2 in the first quadrant, then the value of $b = \dots\dots$. (IIT : 2001)

- (A) -3 (B) -1 (C) 1 (D) 3

Sol. : The curve is $f(x) = x^2 + bx - b$.

$$\therefore f'(x) = 2x + b$$

$$\therefore [f'(x)]_{(1,1)} = 2 + b$$

The equation of the tangent passing through $(1, 1)$ and having slope $m = 2 + b$

$$(y - 1) = (2 + b)(x - 1)$$

$$\therefore y = (2 + b)x - (b + 1) \quad (1)$$

Taking $y = 0$ in equation (1), we get

$$x = \frac{b+1}{b+2}. \text{ So, the tangent intersects X-axis at } P\left(\frac{b+1}{b+2}, 0\right).$$

Taking $x = 0$ in equation (1) we get $y = -b - 1$.

The tangent intersects Y-axis at $Q(0, -b - 1)$.

Required area $A = \text{Area of } \triangle OPQ$

$$= \frac{1}{2} \times OP \times OQ$$

$$= \frac{1}{2} \times \frac{b+1}{b+2} \times (-b-1)$$

Now, we are given $A = 2$.

$$\therefore \frac{-1(b+1)^2}{2(b+2)} = 2$$

$$\therefore b^2 + 2b + 1 = -4b - 8$$

$$\therefore b^2 + 6b + 9 = 0$$

$$\therefore (b+3)^2 = 0$$

$$\therefore b = -3$$

Ans. : (A)

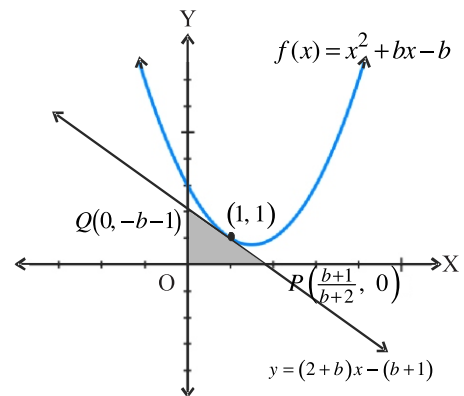


Figure 18.28

- (21) A curve passes through (1, 2) and the slope of the tangent at $(x, f(x))$ is $(2x+1)$. The area bounded by this curve, X-axis and the line $x = 1$ is

(A) $\frac{5}{6}$ (B) $\frac{6}{5}$ (C) $\frac{1}{6}$ (D) 6 (IIT : 1995)

Sol. : The slope of the tangent to the curve $\frac{dy}{dx} = (2x+1)$

$$\therefore \int dy = \int (2x+1) dx$$

$$\therefore y = x^2 + x + c$$

The curve passes through the point B(1, 2),

$$\therefore 2 = 1 + 1 + c$$

$$\therefore c = 0$$

$\therefore$ The equation of the curve is $y = x^2 + x$

$$\therefore \left(y + \frac{1}{4}\right) = \left(x + \frac{1}{2}\right)^2.$$

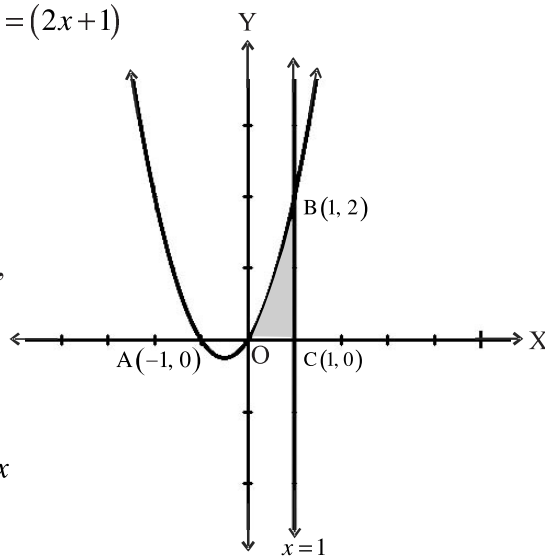


Figure 18.29

This represents a parabola having vertex $P\left(-\frac{1}{2}, -\frac{1}{4}\right)$.

Taking $y = 0$ in the equation, we get $x^2 + x = 0$.

$\therefore x(x+1) = 0$. So, $x = 0$ or $x = -1$.

$\therefore$ The curve intersects, X-axis at $O(0, 0)$ and $A(-1, 0)$.

Taking $x = 1$ in the equation of the curve we get $y = 2$.

$\therefore$ The line $x = 1$ intersects the curve at B(1, 2).

$$\text{Required Area } \Delta = \int_0^1 y dx = \int_0^1 (x^2 + x) dx = \left[\frac{x^3}{3} + \frac{x^2}{2} \right]_0^1 = \frac{5}{6} \quad \text{Ans. : (A)}$$

- (22) Area of the smaller part bounded by the curve $x^2 + y^2 = 9$ and the line $x = 1$ is

(A) $(9 \sec^{-1} 3 - \sqrt{8})$ (B) $\frac{1}{2} (9 \sec^{-1} 3 - \sqrt{8})$
 (C) $(\sqrt{8} - 9 \sec^{-1} 3)$ (D) None of these

Sol. : Here curve $x^2 + y^2 = 9$ is a circle with centre (0, 0) and radius 3.

Taking $x = 1$ we get, $1 + y^2 = 9 \Rightarrow y = \pm\sqrt{8}$

$\therefore$ The line $x = 1$ intersects the given circle $x^2 + y^2 = 9$ at $P(1, 2\sqrt{2})$ and $Q(1, -2\sqrt{2})$.

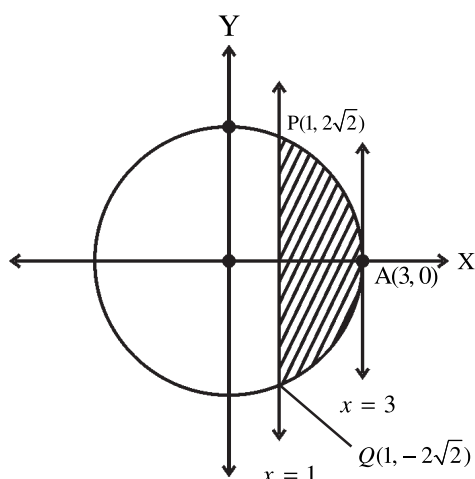


Figure 18.30

$$\begin{aligned}
 \text{Required area } \Delta &= 2 \int_1^3 y \, dx \\
 &= 2 \int_1^3 \sqrt{9-x^2} \, dx \\
 &= 2 \left[\frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \left(\frac{x}{3} \right) \right]_1^3 \\
 &= 2 \left[\frac{9\pi}{4} - \left(\frac{\sqrt{8}}{2} + \frac{9}{2} \sin^{-1} \left(\frac{1}{3} \right) \right) \right] \\
 &= 9 \left[\frac{\pi}{2} - \sin^{-1} \left(\frac{1}{3} \right) \right] - \sqrt{8} \\
 &= 9 \cos^{-1} \left(\frac{1}{3} \right) - \sqrt{8} \\
 &= 9 \sec^{-1} 3 - \sqrt{8} \quad \text{Ans. : (A)}
 \end{aligned}$$

(23) The curve $y = a\sqrt{x} + bx$ is passing through point (1, 2). If the area bounded by this curve, X-axis and the line $x = 4$ is 8, then

- (A) $a = 3, b = -1$ (B) $a = 3, b = 1$ (C) $a = -3, b = 1$ (D) $a = -3, b = -1$

Sol. : The curve $y = a\sqrt{x} + bx$ is passing through the point (1, 2).

$$\therefore 2 = a + b \quad (1)$$

The area bounded by the curve $y = a\sqrt{x} + bx$, X-axis and the line $x = 4$ is 8.

$$\therefore \text{Required area } A = \int_0^4 (a\sqrt{x} + bx) \, dx$$

$$\therefore 8 = \left[a \cdot \frac{(x)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + \frac{bx^2}{2} \right]_0^4$$

$$\therefore a \times \frac{2}{3} \times 8 + 8b = 8. \quad \text{So, } 2a + 3b = 3 \quad (2)$$

Solving equations (1) and (2) we get, $a = 3$ and $b = -1$. **Ans. : (A)**

(24) If $f(x) = x^2 - 3x + 2$, then the area bounded by $f(|x|)$ above the X-axis is

- (A) $\frac{5}{3}$ (B) $\frac{2}{5}$ (C) $\frac{5}{2}$ (D) $\frac{3}{5}$

Sol. : $f(|x|) = f(x) = x^2 - 3x + 2, x \geq 0$

$$\therefore y = x^2 - 3x + \frac{9}{4} - \frac{1}{4} \quad (1)$$

$$\therefore y + \frac{1}{4} = \left(x - \frac{3}{2}\right)^2 \text{ is a part of parabola having vertex } \left(\frac{3}{2}, \frac{-1}{4}\right). (x > 0)$$

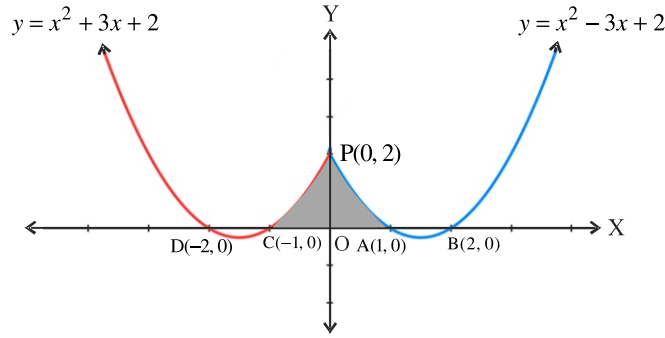


Figure 18.31

Also, $f(|x|) = f(-x), x \leq 0 = x^2 + 3x + 2$

$$\therefore y = x^2 + 3x + 2 \quad (2)$$

$$\therefore y + \frac{1}{4} = \left(x + \frac{3}{2}\right)^2 \text{ is a part of parabola having vertex } \left(\frac{-3}{2}, \frac{-1}{4}\right)$$

Solving (1) and (2), we get $x^2 - 3x + 2 = x^2 + 3x + 2$

$$\therefore x = 0 \text{ and } x = 0 \Rightarrow y = 2$$

$$\therefore \text{Both the curves intersect at } P(0, 2).$$

Taking $y = 0$ in the equation of the curve $y = x^2 - 3x + 2$,

$$(x - 2)(x - 1) = 0 \Rightarrow x = 2, x = 1$$

$$\therefore \text{The curve intersects X-axis at } A(1, 0) \text{ and } B(2, 0).$$

Taking $y = 0$ in the equation of the curve $y = x^2 + 3x + 2$,

$$(x + 2)(x + 1) = 0 \Rightarrow x = -2, x = -1$$

$$\therefore \text{The curve intersects X-axis at } C(-1, 0) \text{ and } D(-2, 0).$$

$$\begin{aligned} \therefore \text{Required area } \Delta &= \text{Area of region APCOA} \\ &= 2 \times \text{Area of region APOA} \end{aligned}$$

$$= 2 \int_0^1 (x^2 - 3x + 2) dx$$

$$= 2 \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_0^1$$

$$= 2 \left[\frac{1}{3} - \frac{3}{2} + 2 \right] = 2 \left(\frac{5}{6} \right) = \frac{5}{3}$$

Ans. : (A)

- (25) The function $f(x)$ is non-negative and continuous. If the area bounded by the curve $y = f(x)$, X-axis and lines $x = \frac{\pi}{4}$, $x = \beta > \frac{\pi}{4}$ is $\left(\beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2} \beta\right)$, then $f\left(\frac{\pi}{2}\right) = \dots\dots$

(AIEEE : 2005)

- (A) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$ (B) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$ (C) $1 - \frac{\pi}{4} + \sqrt{2}$ (D) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$

Sol. : Required area $A = \left(\beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2} \beta\right)$

$$\therefore \int_{\frac{\pi}{4}}^{\beta} f(x) dx = \beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2} \beta$$

$$\therefore f(\beta) = \beta \cos \beta + \sin \beta - \frac{\pi}{4} \sin \beta + \sqrt{2}$$

(Differentiate with respect to β and if $\int_{\alpha}^t f(x) dx = F(t)$, then $f(t) = F'(t)$)

$$\begin{aligned} \therefore f\left(\frac{\pi}{2}\right) &= \frac{\pi}{2} \cos \frac{\pi}{2} + \sin \frac{\pi}{2} - \frac{\pi}{4} \sin \frac{\pi}{2} + \sqrt{2} \\ &= 1 - \frac{\pi}{4} + \sqrt{2} \end{aligned}$$

Ans. : (C)

- (26) A square is formed by the lines $x = 4$, $y = 4$ and coordinate axes. Parabola $y^2 = 4x$ and $x^2 = 4y$ divide this square into three parts S_1 , S_2 and S_3 . Then $S_1 : S_2 : S_3 = \dots\dots$

- (A) 1:2:3 (B) 1:2:1 (C) 1:1:1 (D) 2:1:2

(AIEEE : 2005)

Sol. : Parabola $y^2 = 4x$ and $x^2 = 4y$ are having their vertex at the origin and symmetric about X-axis and Y-axis respectively.

For the points of intersection of $y^2 = 4x$ and $y = \frac{x^2}{4}$, we have

$$\therefore \left(\frac{x^2}{4}\right)^2 = 4x$$

$$\therefore x(x^3 - 64) = 0$$

$$\therefore x = 0 \text{ or } x = 4$$

The points of intersection are $O(0, 0)$ and $A(4, 4)$.

$$S_1 = \int_0^4 x dy = \int_0^4 \frac{y^2}{4} dy = \frac{1}{4} \left[\frac{y^3}{3} \right]_0^4 = \frac{1}{12} \times 4^3 = \frac{16}{3}$$

$$S_2 = \int_0^4 (y_1 - y_2) dx = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4} \right) dx$$

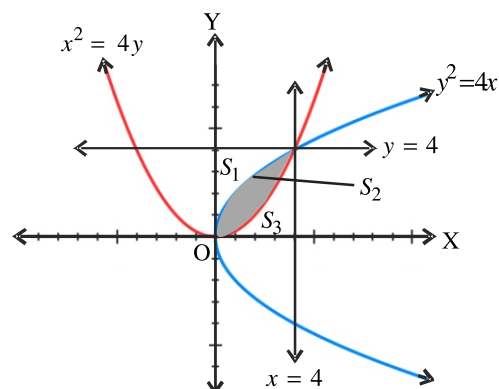


Figure 18.32

$$= \left[2 \cdot \frac{2}{3} \cdot x^{\frac{3}{2}} - \frac{x^3}{12} \right]_0^4 = \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$

$$S_3 = \int_0^4 y \, dx = \int_0^4 \frac{x^2}{4} \, dx = \frac{1}{4} \left[\frac{x^3}{3} \right]_0^4 = \frac{16}{3}$$

$$S_1 : S_2 : S_3 = \frac{16}{3} : \frac{16}{3} : \frac{16}{3}$$

$$S_1 : S_2 : S_3 = 1 : 1 : 1$$

Ans. : (C)

(27) The area bounded by the parabola $y = -x^2 + 2$ and the line $x + y = 0$ is

(A) $\frac{9}{2}$

(B) 9

(C) $\frac{19}{2}$

(D) $\frac{17}{6}$

Sol. : Parabola $y = -x^2 + 2$ and line $x + y = 0$ are given.

Parabola $x^2 = -(y - 2)$ has vertex $P(0, 2)$.

Putting $y = 0$ in the equation, we get, $2 - x^2 = 0 \Rightarrow x = \pm\sqrt{2}$

$\therefore$ The parabola intersects, X-axis at $C(\sqrt{2}, 0)$ and $D(-\sqrt{2}, 0)$.

For the points of intersection of both curves we have $2 - x^2 = -x$.

$$\therefore x^2 - x - 2 = 0$$

$$\therefore (x - 2)(x + 1) = 0$$

$$\therefore x = 2 \text{ or } -1$$

Thus both curves intersect at $A(2, -2)$ and $B(-1, 1)$.

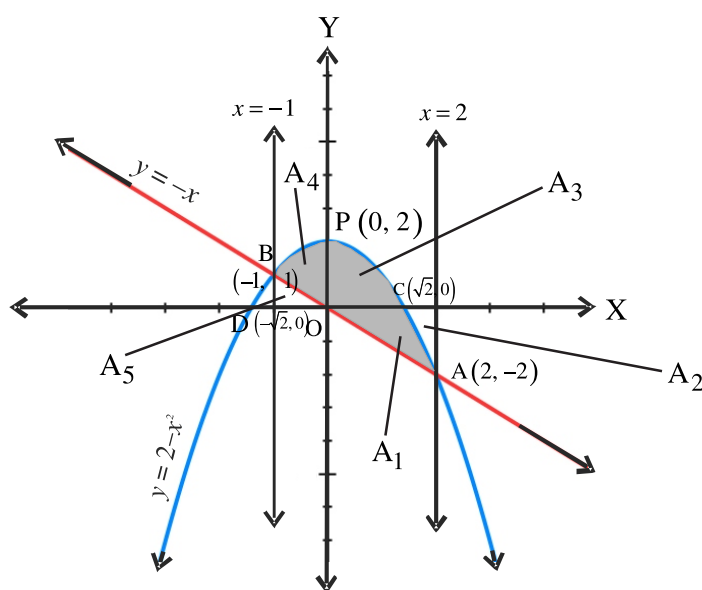


Figure 18.33

Required area $\Delta = \text{Area I} + \text{Area II} + \text{Area III}$

$$\begin{aligned}
 &= \int_{-1}^0 (y_1 - y_2) dx + \int_0^{\sqrt{2}} y_1 dx + \left| \int_0^2 y_2 dx - \int_{\sqrt{2}}^2 y_1 dx \right| \\
 &= \int_{-1}^0 \left\{ (2-x^2) - (-x) \right\} dx + \int_0^{\sqrt{2}} (2-x^2) dx + \left| \int_0^2 (-x) dx - \int_{\sqrt{2}}^2 (2-x^2) dx \right| \\
 &= \left[2x - \frac{x^3}{3} + \frac{x^2}{2} \right]_{-1}^0 + \left[2x - \frac{x^3}{3} \right]_0^{\sqrt{2}} + \left| \left[-\frac{x^2}{2} \right]_0^2 - \left[2x - \frac{x^3}{3} \right]_{\sqrt{2}}^2 \right| \\
 &= \left[0 - \left\{ -2 + \frac{1}{3} + \frac{1}{2} \right\} \right] + \left[2\sqrt{2} - \frac{2\sqrt{2}}{3} \right] + \left| (-2) - \left\{ 4 - \frac{8}{3} - \left(2\sqrt{2} - \frac{2\sqrt{2}}{3} \right) \right\} \right| \\
 &= \frac{7}{6} + \frac{4\sqrt{2}}{3} + \left| \frac{-10}{3} + \frac{4\sqrt{2}}{3} \right| = \frac{7}{6} + \frac{4\sqrt{2}}{3} + \frac{10}{3} - \frac{4\sqrt{2}}{3} = \frac{27}{6} = \frac{9}{2}
 \end{aligned}$$

or

$$\begin{aligned}
 A &= \int_{-1}^2 (y_1 - y_2) dx = \int_{-1}^2 \left[(2-x^2) - (-x) \right] dx \quad (\text{Explain !}) \\
 &= \int_{-1}^2 (2-x^2+x) dx = \left[2x - \frac{x^3}{3} + \frac{x^2}{2} \right]_{-1}^2 = \left(4 - \frac{8}{3} + 2 \right) - \left(-2 + \frac{1}{3} + \frac{1}{2} \right) \\
 &= 6 - \frac{8}{3} + 2 - \frac{1}{3} - \frac{1}{2} = \frac{9}{2} \quad \text{Ans. : (A)}
 \end{aligned}$$

- (28) The area bounded in the first quadrant by the curve $y = \sqrt{x}$ and the line $2y+3=x$ is
 (A) $\frac{27}{4}$ (B) 9 (C) 18 (D) 36 (IIT : 2003)

Sol. : The curve is $y = \sqrt{x}$. So, $y^2 = x$, $x \geq 0$, $y \geq 0$.

For points of intersection of $y^2 = x$ and the line $y = \frac{x-3}{2}$, we have

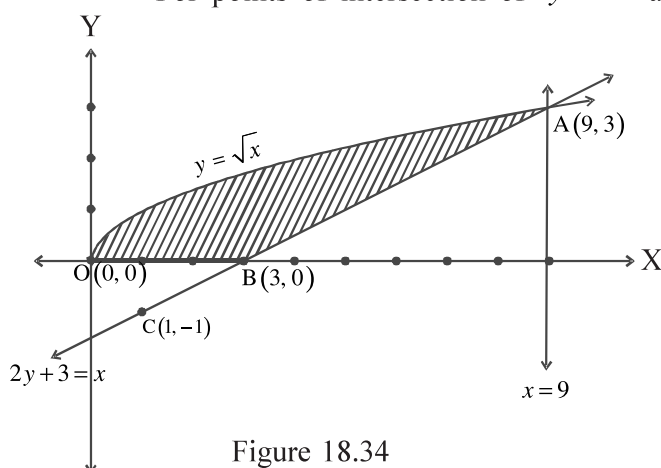


Figure 18.34

$$\left(\frac{x-3}{2} \right)^2 = x \Rightarrow x^2 - 10x + 9 = 0$$

$$\therefore (x-1)(x-9) = 0$$

$$\therefore x = 1 \text{ or } x = 9$$

Both the curves $y^2 = x$ and $2y+3=x$, intersect at $A(9,3)$ and $C(1,-1)$.

Taking $y = 0$ in $2y+3=x$ we get, $x = 3$.

∴ The line $2y+3=x$, intersects X-axis at $B(3, 0)$.

$$\begin{aligned}\text{Required area } \Delta &= \int_0^9 \sqrt{x} \, dx - \int_3^9 \left(\frac{x-3}{2}\right) dx \\ &= \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^9 - \left[\frac{x^2}{4} - \frac{3}{2}x \right]_3^9 \\ &= 18 - \left[\left(\frac{81}{4} - \frac{27}{2} \right) - \left(\frac{9}{4} - \frac{9}{2} \right) \right] = 9\end{aligned}$$

Ans. : (B)

(29) The area bounded by the curves $y^2 = x$ and $y = |x|$ is

- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 1 (AIEEE : 2007)

Sol. : $y^2 = x$ is a parabola having vertex at $O(0, 0)$ and symmetric about X-axis.

$$y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x \leq 0 \end{cases}$$

For $x \geq 0$ solving $y^2 = x$ and $y = x$,

$$x^2 = x \Rightarrow x(x-1) = 0 \Rightarrow x = 0 \text{ or } x = 1$$

∴ The curves intersect at $O(0, 0)$ and $A(1, 1)$.

For $y^2 = x$ it is not possible that $x < 0$.

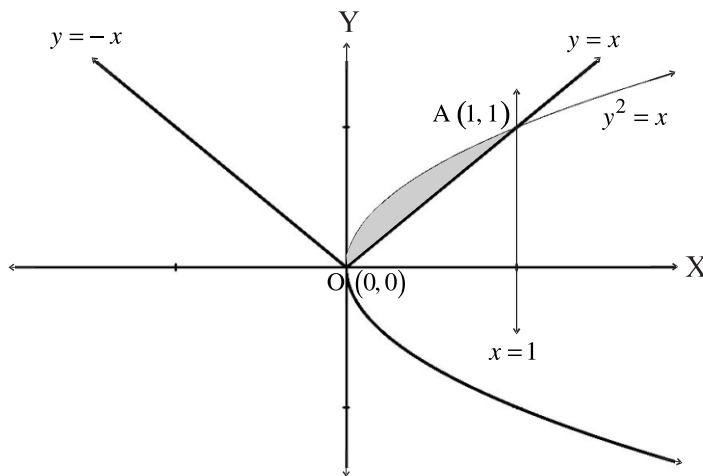


Figure 18.35

$$\begin{aligned}\text{Required area } \Delta &= \int_0^1 (y_1 - y_2) \, dx \\ &= \int_0^1 (\sqrt{x} - x) \, dx \\ &= \left[\frac{2}{3} x^{\frac{3}{2}} - \frac{x^2}{2} \right]_0^1 \\ &= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}\end{aligned}$$

Ans. : (A)

(30) The area bounded by the curves $x+2y^2=0$ and $x+3y^2=1$ is

- (A) $\frac{4}{3}$ (B) $\frac{5}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$ (AIEEE : 2008)

Sol. : The curve $x+2y^2=0 \Rightarrow 2y^2=-x \Rightarrow y^2=-\frac{1}{2}x$ is a parabola having vertex origin (1)

and $x+3y^2=1 \Rightarrow 3y^2=1-x \Rightarrow y^2=-\frac{1}{3}(x-1)$ (2)

is a parabola having vertex $P(1, 0)$.

Solving (1) and (2) we get,

$$-\frac{1}{2}x = -\frac{1}{3}(x-1) \Rightarrow 3x = 2x - 2 \Rightarrow x = -2$$

Taking $x = -2$ we get $y = \pm 1$.

$\therefore$ Both the curves intersect at $A(-2, 1)$ and $B(-2, -1)$.

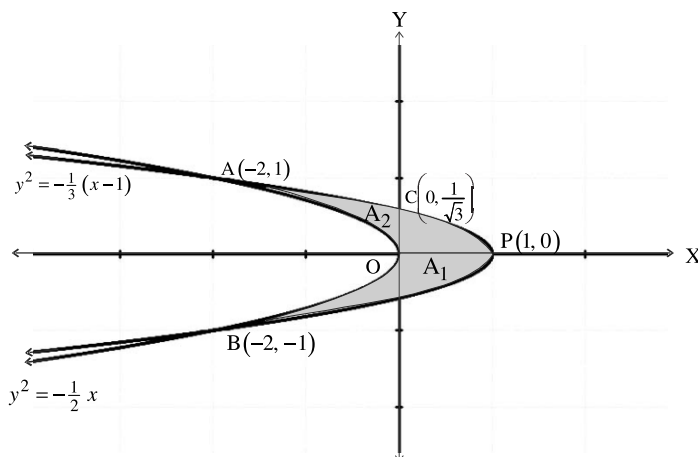


Figure 18.36

$$\begin{aligned}
 \text{Required area } \Delta &= \text{Area of } APBOA \\
 &= 2 \text{ Area of } (APOA) \\
 &= 2 \int_0^1 (x_1 - x_2) dy \\
 &= 2 \int_0^1 \left((1 - 3y^2) - (-2y^2) \right) dy \\
 &= 2 \int_0^1 (1 - 3y^2 + 2y^2) dy \\
 &= 2 \int_0^1 (1 - y^2) dy \\
 &= 2 \left[y - \frac{y^3}{3} \right]_0^1 = 2 \left[1 - \frac{1}{3} \right] = 2 \left(\frac{2}{3} \right)
 \end{aligned}$$

$$\therefore \text{ Required area } \Delta = \frac{4}{3}$$

Ans. : (A)

$$\text{Infact, } A_1 = 2 \int_0^{\frac{1}{\sqrt{3}}} (1 - 3y^2) dy$$

$$A_2 = -2 \left(\int_0^1 -2y^2 dy - \int_{\frac{1}{\sqrt{3}}}^1 (1 - 3y^2) dy \right) = -2 \int_0^1 -2y^2 dy + 2 \int_{\frac{1}{\sqrt{3}}}^1 (1 - 3y^2) dy$$

$$\begin{aligned}
 A = A_1 + A_2 &= -2 \int_0^1 -2y^2 dy + 2 \int_0^1 (1 - 3y^2) dy \\
 &= 2 \int_0^1 (1 - y^2) dy
 \end{aligned}$$

(31) The area bounded by the curves $x^2 + y^2 = 9$ and $y^2 = 8x$ is ($0 \leq x \leq 3$).

(A) 16π

(B) $\frac{2\sqrt{2}}{3} + \frac{9\pi}{2} - 9\sin^{-1}\frac{1}{3}$

(C) $\frac{8\sqrt{2}}{3} + \frac{3\sqrt{3}\pi}{2} - 6\sin^{-1}\left(\frac{1}{12}\right)$

(D) $\frac{4\sqrt{2}}{3} + \frac{9\sqrt{3}\pi}{2} - 12\sin^{-1}\left(\frac{1}{6}\right)$

Sol. : $x^2 + y^2 = 9$ is circle having centre (0, 0) and radius 3. (1)

$y^2 = 8x$ is a parabola having vertex (0, 0) and symmetric with respect to X-axis (2)

Solving (1) and (2), we get $x^2 + 8x = 9$

$$\therefore x^2 + 8x - 9 = 0$$

$$\therefore (x+9)(x-1) = 0$$

$$\therefore x = -9 \text{ or } x = 1.$$

Since $0 \leq x \leq 3$, taking $x = 1$ we get $y = \pm 2\sqrt{2}$

$\therefore$ The curves intersect at $A(1, 2\sqrt{2})$ and $B(1, -2\sqrt{2})$.

Required area = Area of region OACBO

Both the curves are symmetric about X-axis,

Area = 2(Area of region OACDO)

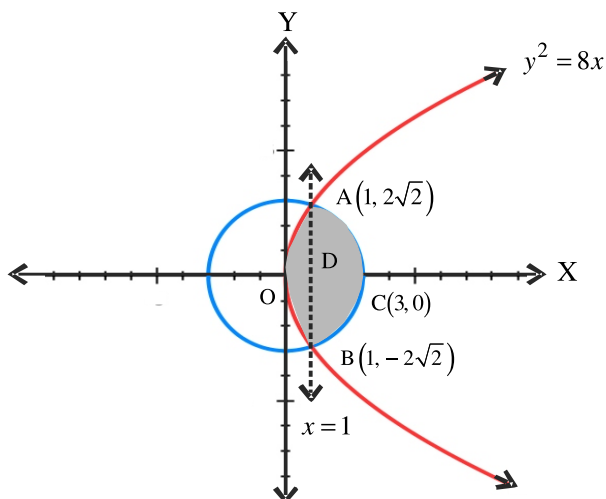


Figure 18.37

$$= 2 (\text{Area of region OADO} + \text{Area of region DACD})$$

$$= 2 \left[\int_0^1 \sqrt{8x} \, dx + \int_1^3 \sqrt{9-x^2} \, dx \right]$$

$$= 2 \left\{ \left[\sqrt{8} \cdot \frac{2}{3} \cdot x^{\frac{3}{2}} \right]_0^1 + \left[\frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_1^3 \right\}$$

$$= 2 \left\{ \left[\frac{4\sqrt{2}}{3} \right] + \left[\frac{9\pi}{4} - \left(\sqrt{2} + \frac{9}{2} \sin^{-1} \left(\frac{1}{3} \right) \right) \right] \right\}$$

$$= \frac{2\sqrt{2}}{3} + \frac{9\pi}{2} - 9\sin^{-1}\left(\frac{1}{3}\right)$$

Ans. : (B)

(32) The area of the region bounded by the circle $x^2 + y^2 = 4$, the line $x - \sqrt{3}y = 0$ and X-axis in the first quadrant is

(A) $\sqrt{3} - \frac{\pi}{3}$

(B) $\sqrt{3} + \frac{\pi}{3}$

(C) $\sqrt{3}$

(D) $\frac{\pi}{3}$

Sol. :

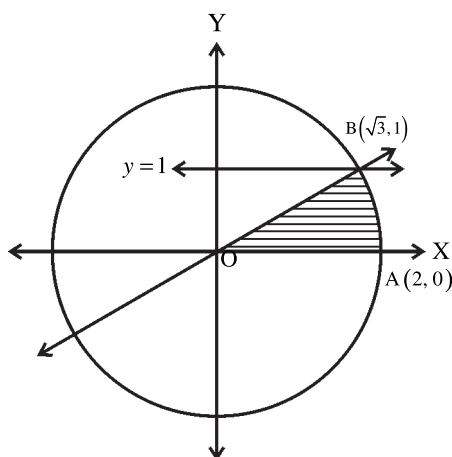


Figure 18.38

Given curves are circle $x^2 + y^2 = 4$ and the line $x - \sqrt{3}y = 0$.

Taking $y = \frac{x}{\sqrt{3}}$ in $x^2 + y^2 = 4$ we get, $x^2 + \frac{x^2}{3} = 4$

$\therefore x^2 = 3$. So, $x = \pm\sqrt{3}$.

In the first quadrant $x = \sqrt{3}$. Hence $y = \frac{x}{\sqrt{3}} = 1$.

$\therefore$ The circle and line intersect at $B(\sqrt{3}, 1)$ in the first quadrant.

$$\begin{aligned}\Delta &= \int_0^1 (x_1 - x_2) dy \\ &= \int_0^1 \left(\sqrt{4 - y^2} - \sqrt{3}y \right) dy \\ &= \left[\frac{y}{2} \sqrt{4 - y^2} + \frac{4}{2} \sin^{-1} \frac{y}{2} - \frac{\sqrt{3}}{2} y^2 \right]_0^1 \\ &= \frac{\sqrt{3}}{2} + 2 \sin^{-1} \frac{1}{2} - \frac{\sqrt{3}}{2} = 2 \times \frac{\pi}{6} = \frac{\pi}{3}\end{aligned}$$

$\therefore$ The area = $\frac{\pi}{3}$.

Ans. : (D)

Note : For the line $y = \frac{x}{\sqrt{3}}$ we have $m = \tan \theta = \frac{1}{\sqrt{3}}$. So, $\theta = \frac{\pi}{6}$

$\therefore$ The area of the sector = $\frac{1}{2} r^2 \theta = \frac{1}{2} \cdot 4 \cdot \frac{\pi}{6} = \frac{\pi}{3}$

Second method

$$\begin{aligned}\Delta &= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} dx + \int_{\sqrt{3}}^2 \sqrt{4 - x^2} dx \\ &= \frac{1}{2\sqrt{3}} \left[x^2 \right]_0^{\sqrt{3}} + \left[\frac{x}{2} \sqrt{4 - x^2} + 2 \sin^{-1} \frac{x}{2} \right]_{\sqrt{3}}^2 \\ &= \frac{\sqrt{3}}{2} + 2 \left(\frac{\pi}{2} \right) - \frac{\sqrt{3}}{2} - 2 \left(\frac{\pi}{3} \right) = \frac{\pi}{3}\end{aligned}$$

- (33) If the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ is $\frac{9}{2}$, then
 $m = \dots\dots\dots$ (IIT : 1999)
 (A) 2, -4 (B) -2, 4 (C) 2, 4 (D) -2, -4

Sol. : Given curves are $y = x - x^2$ and $y = mx$.

To find the points of intersection, we solve the given equations.

$$\begin{aligned} x - x^2 &= mx \\ \therefore x^2 + (m-1)x &= 0 \\ \therefore x[x + (m-1)] &= 0 \\ \therefore x = 0 \text{ or } x = 1-m \end{aligned}$$

If $x = 0$ then $y = 0$ and if $x = 1-m$ then $y = m(1-m)$.

$\therefore$ The points of intersection of given curves are $A(0, 0)$ and $B(1-m, m(1-m))$.

$$\begin{aligned} I &= \int_0^{1-m} (y_1 - y_2) dx \\ &= \int_0^{1-m} (x - x^2 - mx) dx \\ &= \left[(1-m) \frac{x^2}{2} - \frac{x^3}{3} \right]_0^{1-m} \\ &= \frac{(1-m)^3}{6} \end{aligned}$$

$$\text{Area, } A = |I|$$

$$\therefore \frac{9}{2} = \frac{|(1-m)^3|}{6}$$

$$\therefore |(1-m)^3| = 27$$

$$\therefore (1-m) = \pm 3$$

$$\therefore 1-m = 3 \text{ or } 1-m = -3$$

$$\therefore m = -2 \text{ or } m = 4.$$

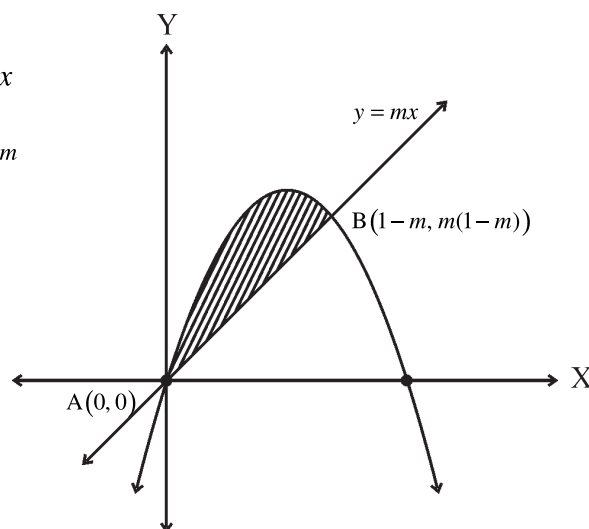


Figure 18.39

In fact the options give two figures :

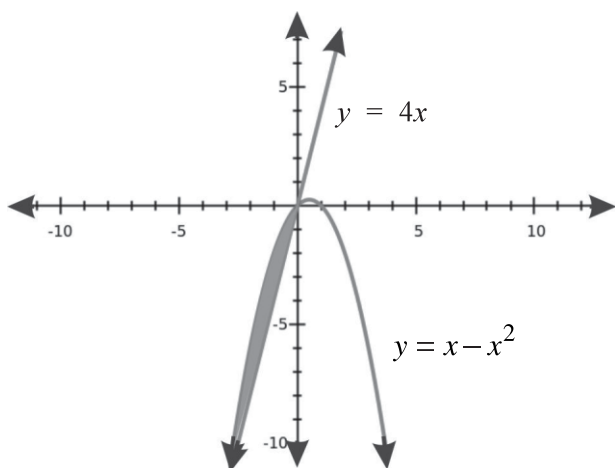


Figure 18.39(a)

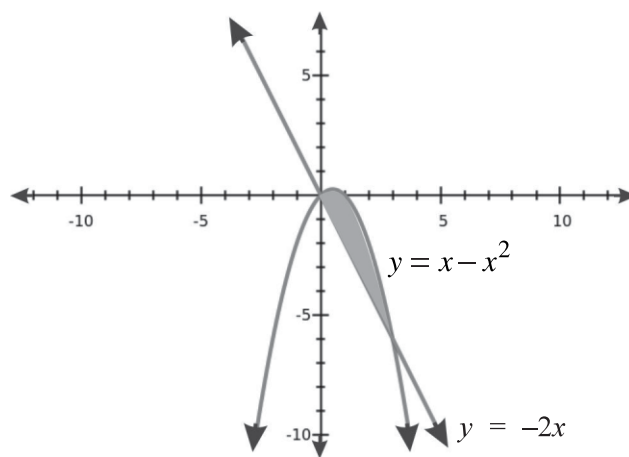


Figure 18.39(b)

Note :

$$(1) \quad A = \int_{-3}^0 [(x-x^2) - 4x] dx = \int_{-3}^0 (-3x-x^2) dx = \left[\frac{-3x^2}{2} - \frac{x^3}{3} \right]_{-3}^0 = -\left(\frac{-27}{2} + 9\right) = \frac{9}{2}$$

$$(2) \quad I_1 = \int_0^1 (x-x^2) dx = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$\begin{aligned} I_2 &= \int_0^3 -2x dx - \int_1^3 (x-x^2) dx = \left[-x^2 \right]_0^3 - \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_1^3 \\ &= -9 - \left[\left(\frac{9}{2} - 9 \right) - \left(\frac{1}{2} - \frac{1}{3} \right) \right] \\ &= -9 - \left(-\frac{9}{2} - \frac{1}{6} \right) \\ &= -\frac{9}{2} + \frac{1}{6} = -\frac{26}{6} \end{aligned}$$

$$A = |I_1| + |I_2| = \frac{1}{6} + \frac{26}{6} = \frac{27}{6} = \frac{9}{2}$$

Ans. : (B)

(34) The area bounded by the curve $y=(x+1)^2$ and $y=(x-1)^2$ and line $y=\frac{1}{4}$ is

(A) $\frac{1}{6}$

(B) $\frac{2}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$ (IIT : 2005)

Sol. : The curve $y=(x+1)^2$ is a parabola with vertex $A(-1, 0)$ and $y=(x-1)^2$ is a parabola with vertex $B(1, 0)$.

Solving both the equations, we have $(x+1)^2 = (x-1)^2$.

$$\therefore (x+1)^2 - (x-1)^2 = 0$$

$$\therefore 4x = 0$$

$$\therefore x = 0$$

Taking $x=0$ we have $y=1$.

$\therefore$ Both parabolas intersect at $P(0, 1)$.

Putting $y=\frac{1}{4}$, in $y=(x+1)^2$ we get points of intersections as $C\left(-\frac{3}{2}, \frac{1}{4}\right)$ and $D\left(-\frac{1}{2}, \frac{1}{4}\right)$

. Similarly, putting $y=\frac{1}{4}$, in $y=(x-1)^2$ we get points of intersections as $E\left(\frac{3}{2}, \frac{1}{4}\right)$ and $F\left(\frac{1}{2}, \frac{1}{4}\right)$.

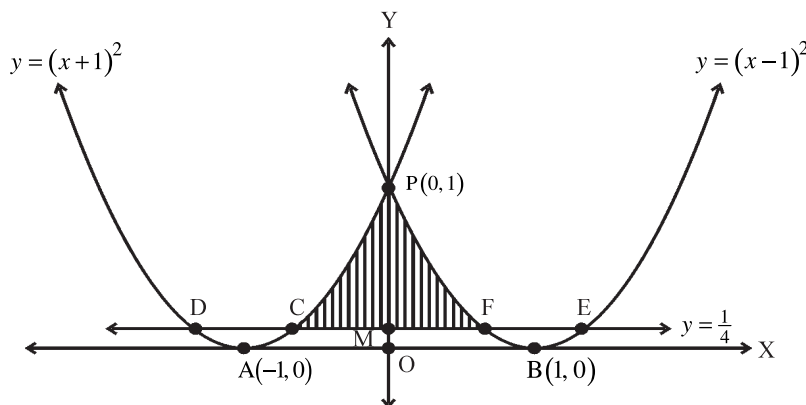


Figure 18.40

Required area = Area of CPFC

$$= 2 (\text{Area of PMF})$$

$$= 2 \int_0^{\frac{1}{2}} \left[(x-1)^2 - \frac{1}{4} \right] dx$$

$$= 2 \int_0^{\frac{1}{2}} \left(x^2 - 2x + \frac{3}{4} \right) dx$$

$$= 2 \left[\frac{x^3}{3} - x^2 + \frac{3}{4}x \right]_0^{\frac{1}{2}} = 2 \left[\frac{1}{24} - \frac{1}{4} + \frac{3}{8} \right] = \frac{1}{3} \quad \text{Ans. : (D)}$$

(35) The area of the region bounded by the curves $y = |x-1|$ and $y = 3-|x|$ is

- (A) 3 (B) 4 (C) 2 (D) 6

(AIEEE : 2003)

Sol. : Curve $y = |x-1| = \begin{cases} x-1, & x \geq 1 \\ 1-x, & x \leq 1 \end{cases}$

x	0	1	2	-1	-2	3
y	1	0	1	2	3	2

 $y = 3-|x| = \begin{cases} 3-x, & x \geq 0 \\ 3+x, & x \leq 0 \end{cases}$

x	0	1	2	-1	-2
y	3	2	1	2	1

Solving equations $y = x-1$ and $y = 3-x$ we get $A(2, 1)$ as a point of intersection.

Solving equations $y = 1-x$ and $y = 3+x$ we get $B(-1, 2)$ as a point of intersection.

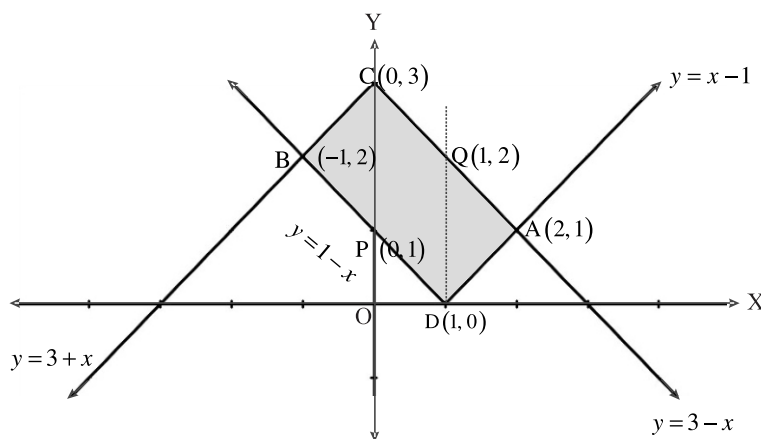


Figure 18.41

Required area A = Area of rectangle ACBD

$$= \text{Area of region BCPB} + \text{Area of region CQDPC} +$$

Area of region QADQ

$$= \int_{-1}^0 \{(3+x)-(1-x)\} dx + \int_0^1 \{(3-x)-(1-x)\} dx$$

$$+ \int_1^2 \{(3-x)-(x-1)\} dx$$

$$\begin{aligned}
 &= \int_{-1}^0 (2x+2) dx + \int_0^1 2 dx + \int_1^2 (4-2x) dx \\
 &= \left[x^2 + 2x \right]_{-1}^0 + \left[2x \right]_0^1 + \left[4x - x^2 \right]_1^2 \\
 &= [(0 - (-1 - 2))] + [2 - 0] + [(8 - 4) - (4 - 1)] \\
 &= 1 + 2 + 1 \\
 &= 4
 \end{aligned}$$

Note : Area = $BC \times BD = \sqrt{2} \times 2\sqrt{2} = 4$

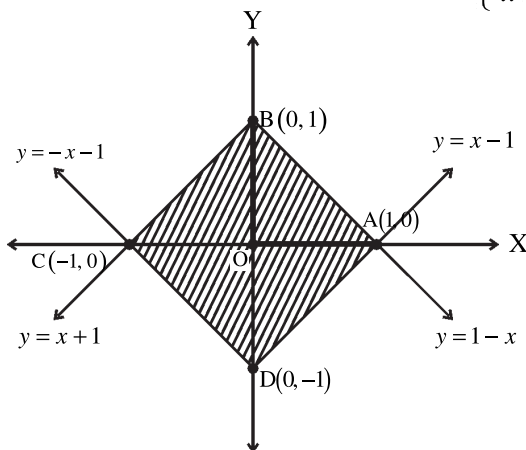
Ans. : (B)

(36) Area bounded by curves $y = |x| - 1$ and $y = -|x| + 1$ is

- (A) 1 (B) 2 (C) 3 (D) 4 (IIT : 2002)

Sol. : Curve $y = |x| - 1 = \begin{cases} x - 1, & x \geq 0 \\ -x - 1, & x \leq 0 \end{cases}$ and

$$y = -|x| + 1 = \begin{cases} -x + 1, & x \geq 0 \\ x + 1, & x \leq 0 \end{cases}$$



$$\begin{aligned}
 \text{Required area } \Delta &= \text{Area of ABCDA} \\
 &= 4 (\text{Area of } \triangle ABO) \\
 &= 4 \left(\frac{1}{2} \times OA \times OB \right) \\
 &= 4 \times \frac{1}{2} \\
 &= 2
 \end{aligned}$$

Ans. : (B)

Figure 18.42

Second method :

$$\Delta = 4 \int_0^1 (1-x) dx = 4 \left[x - \frac{x^2}{2} \right]_0^1 = 4 \left[1 - \frac{1}{2} \right] = 2$$

(37) The area bounded by the curve $y = 2x - x^2$ and the line $y = -x$ is

- (A) $\frac{35}{6}$ (B) $\frac{46}{6}$ (C) $\frac{9}{2}$ (D) $\frac{45}{2}$

(AIEEE : 2002)

Sol. : The curve $y = 2x - x^2 \Rightarrow y = -(x^2 - 2x + 1) + 1 \Rightarrow y - 1 = -(x - 1)^2$ is a parabola having vertex $P(1, 1)$.

Taking $y = 0$ in $y = 2x - x^2$, we get $2x - x^2 = 0 \Rightarrow x(2 - x) = 0$

$$\therefore x = 0 \text{ or } x = 2$$

$\therefore$ The curve intersects X-axis at $O(0, 0)$ and $B(2, 0)$.

Now, solving $y = 2x - x^2$ and $y = -x$, we get

$$-x = 2x - x^2$$

$$\therefore x^2 - 3x = 0$$

$$\therefore x(x - 3) = 0$$

$$\therefore x = 0 \text{ or } x = 3$$

$$\therefore x = 0 \Rightarrow y = 0 \text{ and } x = 3 \Rightarrow y = -3$$

$\therefore$ The curve and the line intersect at $O(0, 0)$ and $C(3, -3)$.

$$\begin{aligned} \text{Required area } A &= \int_0^3 (y_1 - y_2) dx \\ &= \int_0^3 (2x - x^2 + x) dx \\ &= \int_0^3 (3x - x^2) dx \\ &= \left[\frac{3x^2}{2} - \frac{x^3}{3} \right]_0^3 = \frac{27}{2} - \frac{27}{3} = \frac{27}{6} = \frac{9}{2} \end{aligned}$$

$$\begin{aligned} \text{Note : Infact, } A &= \int_0^2 (2x - x^2) dx - \left[\int_0^3 -x dx - \int_2^3 (2x - x^2) dx \right] \\ &= \int_0^2 (2x - x^2) dx + \int_2^3 (2x - x^2) dx - \int_0^3 -x dx \\ &= \int_0^3 (2x - x^2 + x) dx \end{aligned}$$

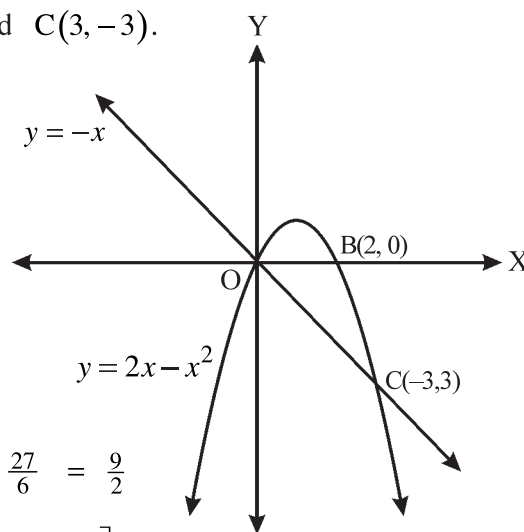


Figure 18.43

Ans. : (C)

- (38) The area bounded by the parabola $(y - 2)^2 = x - 1$, the X-axis and the tangent to the parabola at the point whose y-coordinate is 3 is

- (A) 3 (B) 6 (C) 9 (D) None of these

(AIEEE : 2009)

Sol. : Parabola $(y - 2)^2 = (x - 1)$ has vertex $(1, 2)$.

For a point on the parabola whose y-coordinate 3, $(3 - 2)^2 = x - 1 \Rightarrow x = 2$.

$\therefore$ We find the equation of the tangent at $(2, 3)$.

$$(y - 2)^2 = x - 1$$

$$\therefore 2(y-2) \frac{dy}{dx} = 1$$

$$\therefore \frac{dy}{dx} = \frac{1}{2(y-2)}$$

$$\therefore \left(\frac{dy}{dx} \right)_{(2,3)} = \frac{1}{2(3-2)} = \frac{1}{2}$$

$\therefore$ The slope of the tangent is $\frac{1}{2}$ and it passes through (2, 3).

$\therefore$ The equation of the tangent is $(y-3) = \frac{1}{2}(x-2)$

$$\therefore 2y-6 = x-2$$

$$\therefore x = 2y-4$$

Taking $y = 0$ in the equation of parabola $(y-2)^2 = x-1$ we get $x = 5$.

$\therefore$ The parabola intersects X-axis at (5, 0).

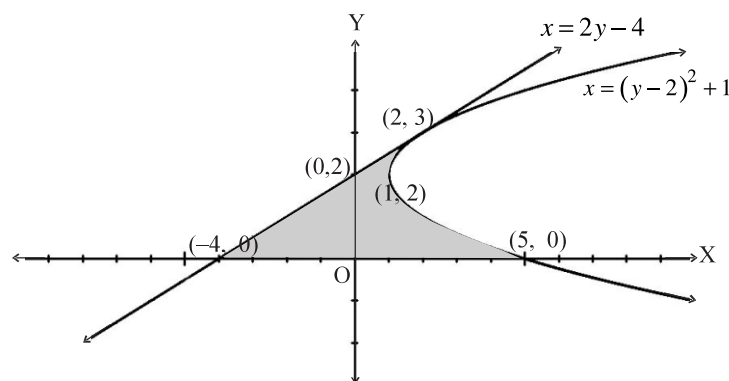


Figure 18.44

$$\text{Required area } A = \int_0^3 (x_1 - x_2) dy \quad (\text{Explain !})$$

$$= \int_0^3 \left[\left((y-2)^2 + 1 \right) - (2y-4) \right] dy$$

$$= \left[\frac{(y-2)^3}{3} + y \right]_0^3 - \left[y^2 - 4y \right]_0^3$$

$$= \left(\frac{1}{3} + 3 \right) - \left(\frac{-8}{3} - 0 \right) - (9 - 12) = 3 + 3 + 3 = 9 \quad \text{Ans. : (C)}$$

(39) The area of the region bounded by the curve $x = -4y^2$ and $x-1 = -5y^2$ is

(A) $\frac{2}{3}$ (B) $\frac{3}{4}$ (C) $\frac{3}{2}$ (D) $\frac{4}{3}$ (IIT : 2006)

Sol. : Given curve $x = -4y^2 \Rightarrow y^2 = -\frac{1}{4}x$ is a parabola having centre $O(0, 0)$

and curve $x-1 = -5y^2 \Rightarrow y^2 = -\frac{1}{5}(x-1)$ is a parabola having vertex $A(1, 0)$.

Solving $y^2 = -\frac{1}{4}x$ and $y^2 = -\frac{1}{5}(x-1)$,

$$-\frac{1}{4}x = -\frac{1}{5}(x-1)$$

$$\Rightarrow -5x = -4x + 4 \Rightarrow x = -4$$

Taking $x = -4$, we get $y = \pm 1$.

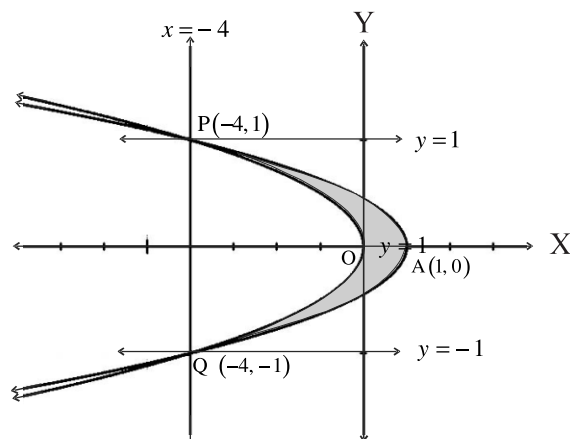


Figure 18.45

Given curves intersect at $P(-4,1)$ and $Q(-4,-1)$.

Required area = Area of region OPAQO = 2 Area of region OAPO (Explain !)

$$= 2 \int_0^1 (x_1 - x_2) dy$$

$$= 2 \int_0^1 \left[(1-5y^2) - (-4y^2) \right] dy$$

$$= 2 \int_0^1 (1-y^2) dy$$

$$= 2 \left[y - \frac{y^3}{3} \right]_0^1 = 2 \left(1 - \frac{1}{3} \right) = 2 \left(\frac{2}{3} \right) = \frac{4}{3}$$

Ans. : (D)

(40) For which value of b , the area bounded by $y = x - bx^2$ and $y = \frac{1}{b}x^2$ becomes maximum ? $b > 0$.

(A) $\frac{1}{2}$

(B) 1

(C) 2

(D) None of these

(IIT : 1997)

Sol. : Curves $y = x - bx^2$ and $y = \frac{1}{b}x^2$ are given.

$$\therefore \text{ Solving, } \frac{1}{b}x^2 = x - bx^2 \Rightarrow (1+b^2)x^2 - bx = 0$$

$$\Rightarrow x \left[(1+b^2)x - b \right] = 0$$

$$\Rightarrow x = 0 \text{ or } x = \frac{b}{1+b^2}$$

$$\begin{aligned}
\therefore \text{ Required area} &= \left| \int_0^{\frac{b}{1+b^2}} (y_1 - y_2) dx \right| \\
&= \left| \int_0^{\frac{b}{1+b^2}} \left(x - bx^2 - \frac{x^2}{b} \right) dx \right| \\
&= \left| \left[\frac{x^2}{2} - \frac{bx^3}{3} - \frac{x^3}{3b} \right]_0^{\frac{b}{1+b^2}} \right| \\
&= \left| \left[\frac{x^2}{2} - \left(\frac{b^2+1}{b} \right) \frac{x^3}{3} \right]_0^{\frac{b}{1+b^2}} \right| \\
&= \left| \frac{1}{6} \left(\frac{b}{b^2+1} \right)^2 \right| = \frac{1}{6} \frac{b^2}{(b^2+1)^2}
\end{aligned}$$

$$\text{Let } f(b) = \frac{1}{6} \left(\frac{b}{b^2+1} \right)^2$$

$$\log f(b) = 2 \log b - 2 \log(b^2+1) - \log 6$$

$$\frac{f'(b)}{f(b)} = \frac{2}{b} - 2 \frac{2b}{b^2+1} \quad (\text{differentiating w.r.t. } b)$$

$$\frac{dA}{db} = - \frac{2(b-1)(b+1)}{b(b^2+1)} \cdot A$$

For a maximum or a minimum value of A we have, $\frac{dA}{db} = 0$,

$$\frac{dA}{db} = 0 \Rightarrow b = 1 \quad (b > 0)$$

Also, $\frac{dA}{db} > 0$ where $0 < b < 1$ and $\frac{dA}{db} < 0$ where $b > 1$.

$\therefore$ The area becomes maximum for $b = 1$.

Ans. : (B)

(41) The area bounded by $y^2 = 9x$ and $x^2 = 9y$ is

(A) 9

(B) 27

(C) 54

(D) 18

Sol. : Both the curves $y^2 = 9x$ and $x^2 = 9y$ are parabola having vertex origin.

Clearly one point of intersection is $O(0, 0)$

Solving both equations,

$$x^2 = 9y \Rightarrow x^4 = 81y^2$$

$$\Rightarrow x^4 = 81(9x)$$

$$\Rightarrow x^3 = 9^3$$

$$\Rightarrow x = 9 \text{ and } y = 9$$

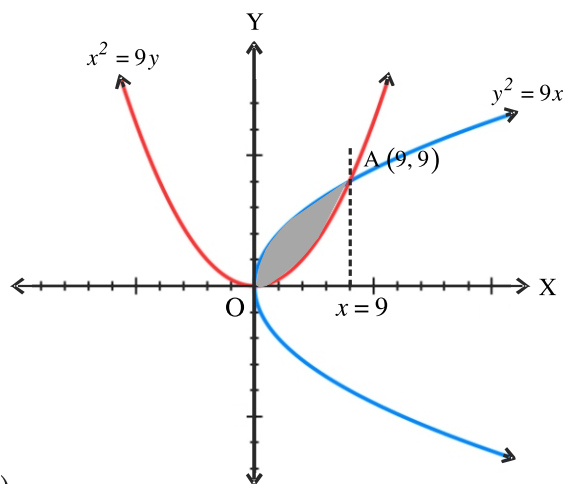


Figure 18.46

$\therefore$ The other point of intersection is $A(9, 9)$.

$\therefore$ The points of intersection are $O(0, 0)$ and $A(9, 9)$.

The curves are $y_1 = 3\sqrt{x}$ and $y_2 = \frac{x^2}{9}$

$$\therefore \text{ Required area} = \int_0^9 (y_1 - y_2) dx$$

$$= \int_0^9 \left(3\sqrt{x} - \frac{x^2}{9} \right) dx$$

$$= \left[3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{27} \right]_0^9$$

$$= \left[2x^{\frac{3}{2}} - \frac{x^3}{27} \right]_0^9 = (54 - 27) = 27 \quad \text{Ans. : (B)}$$

(42) The area bounded by the curve $4y = 3x^2$ and the line $3x - 2y + 12 = 0$ is

(A) 27

(B) 108

(C) 54

(D) 13

Sol. : Here parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$ are given.

Solving $3x^2 = 4y$ and $3x + 12 = 2y$

$$\therefore 3x^2 = 2(3x + 12)$$

$$\therefore x^2 - 2x - 8 = 0$$

$$\therefore (x - 4)(x + 2) = 0$$

$$x = 4 \text{ or } x = -2$$

If $x = 4$, then $y = 12$ and if $x = -2$, then $y = 3$

The points of intersection are $A(4, 12)$ and $B(-2, 3)$.

Now, $y_1 = \frac{3x^2}{4}$ and $y_2 = \frac{3x+12}{2}$

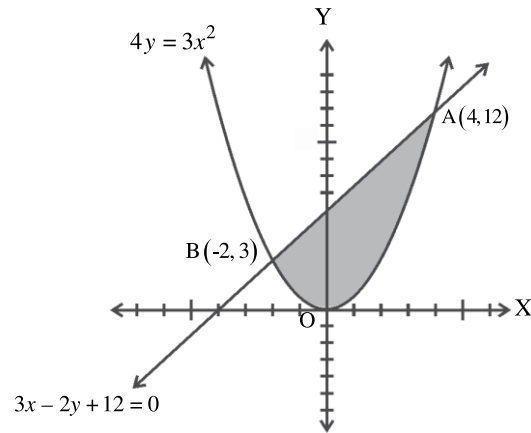


Figure 18.47

$$\begin{aligned} \text{Required area} &= \int_{-2}^4 \left(\frac{3x+12}{2} - \frac{3x^2}{4} \right) dx \\ &= \frac{1}{4} \int_{-2}^4 (6x + 24 - 3x^2) dx \\ &= \frac{1}{4} \left[3x^2 + 24x - x^3 \right]_{-2}^4 \\ &= \frac{1}{4} [(48 + 96 - 64) - (12 - 48 + 8)] \\ &= \frac{1}{4} (108) = 27 \end{aligned}$$

Ans. : (A)

(43) The area bounded by $y = \cos x$ and $y = \sin x$; $0 \leq x \leq \frac{3\pi}{2}$ is (AIEEE : 2010)

- (A) $4\sqrt{2}-2$ (B) $4\sqrt{2}+2$ (C) $4\sqrt{2}-1$ (D) $4\sqrt{2}+1$

Sol. :

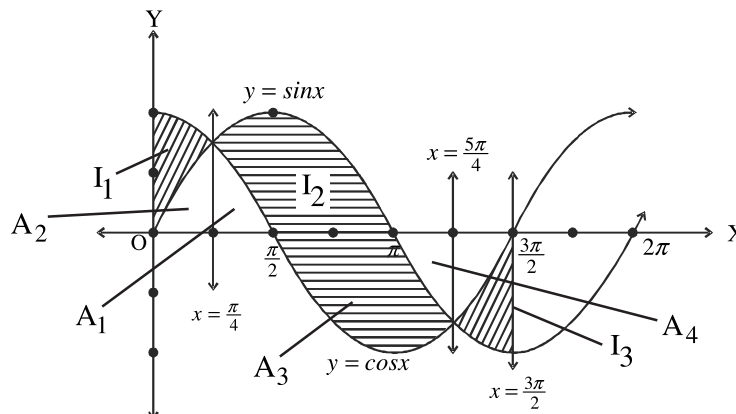


Figure 18.48

$$I_1 = \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx$$

$$= [\sin x + \cos x]_0^{\frac{\pi}{4}}$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (1)$$

$$= \sqrt{2} - 1.$$

$$I_2 = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x) dx$$

$$= [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$$

$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right)$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

$$|I_3| = \int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}} (\cos x - \sin x) dx$$

$$= [\sin x + \cos x]_{\frac{5\pi}{4}}^{\frac{3\pi}{2}}$$

$$= (0 - 1) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right)$$

$$= \sqrt{2} - 1$$

$$\text{Required area} = |I_1| + |I_2| + |I_3|$$

$$= \sqrt{2} - 1 + 2\sqrt{2} + \sqrt{2} - 1$$

$$= (4\sqrt{2} - 2)$$

Note : Let $I_2' = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x) dx + \int_{\frac{\pi}{2}}^{\pi} \sin x dx$, $I_2'' = \int_{\frac{\pi}{2}}^{\pi} \cos x dx + \int_{\pi}^{\frac{5\pi}{4}} (\cos x - \sin x) dx$

Then, $I_2' - I_2'' = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x dx + \int_{\frac{\pi}{2}}^{\pi} \sin x dx - \int_{\frac{\pi}{2}}^{\pi} \cos x dx - \int_{\pi}^{\frac{5\pi}{4}} (\cos x - \sin x) dx$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x dx - \int_{\frac{\pi}{2}}^{\pi} \cos x dx - \int_{\pi}^{\frac{5\pi}{4}} (\sin x - \cos x) dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x dx - \int_{\frac{\pi}{2}}^{\pi} (\cos x - \sin x) dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x) dx + \int_{\pi}^{\frac{5\pi}{4}} (\sin x - \cos x) dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x) dx = I_2$$

Ans. : (A)

(44) The area bounded by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and positive X-axis is

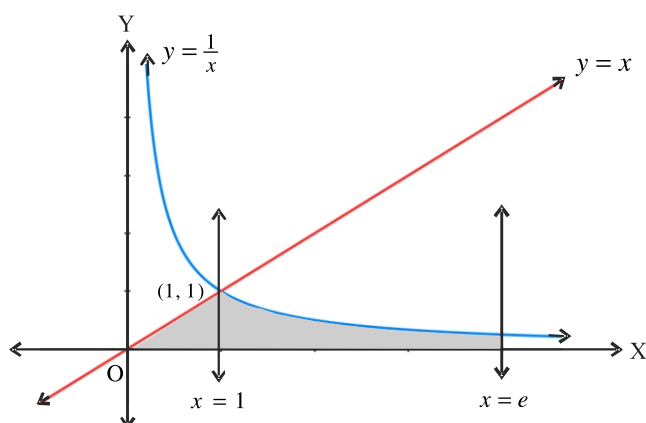
(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) $\frac{5}{2}$

Sol. :



(AIEEE : 2011)

Figure 18.49

For the points of intersection of $y = \frac{1}{x}$ and $y = x$, we have $\frac{1}{x} = x$.

We get $x^2 = 1$. So, $x = \pm 1$

$\therefore$ In the first quadrant we have (1, 1).

$$\text{Required area} = \int_0^1 x \, dx + \int_1^e \left(\frac{1}{x}\right) dx$$

$$= \left[\frac{x^2}{2} \right]_0^1 + [\log |x|]_1^e$$

$$= \frac{1}{2} + \log_e e - \log 1 = \frac{1}{2} + 1 = \frac{3}{2}$$

Ans. : (C)

(45) The area bounded by parabola $x^2 = \frac{y}{4}$, $x^2 = 9y$ and the line $y = 2$ is

(A) $20\sqrt{2}$

(B) $\frac{10\sqrt{2}}{3}$

(C) $\frac{20\sqrt{2}}{3}$

(D) $10\sqrt{2}$

Sol. : Taking $y = 2$ in the equation of the parabola $x^2 = \frac{y}{4}$ we get $x^2 = \frac{1}{2} \Rightarrow x = \pm \frac{1}{\sqrt{2}}$

$\therefore$ The parabola $x^2 = \frac{y}{4}$ intersects the line $y = 2$ at $\left(\frac{1}{\sqrt{2}}, 2\right)$ and $\left(-\frac{1}{\sqrt{2}}, 2\right)$.

Taking $y = 2$ in the equation of the parabola $x^2 = 9y$ we get $x^2 = 18 \Rightarrow x = \pm 3\sqrt{2}$

$\therefore$ The parabola $x^2 = 9y$ intersects the line $y = 2$ at $(3\sqrt{2}, 2)$ and $(-3\sqrt{2}, 2)$.

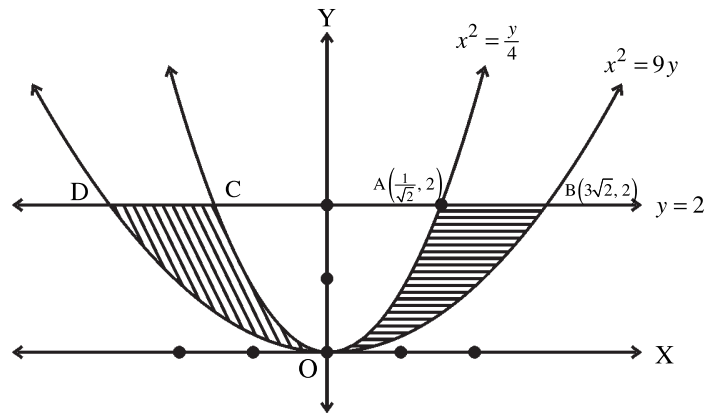


Figure 18.50

$$\begin{aligned}\text{Required area} &= (\text{Area of region } AOB) + (\text{Area of region } COD) \\ &= 2(\text{Area of region } AOB)\end{aligned}$$

$$= 2 \int_0^2 (x_1 - x_2) dy$$

$$= 2 \int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2} \right) dy$$

$$= 2 \int_0^2 \frac{5}{2} \sqrt{y} dy$$

$$= 5 \left[\frac{y^{3/2}}{3/2} \right]_0^2 = \frac{10}{3} \left[2^{3/2} \right] = \frac{10 \times 2 \times \sqrt{2}}{3} = \frac{20\sqrt{2}}{3} \quad \text{Ans. : (C)}$$

(46) The area bounded by the curve $y = \sqrt{x}$, $2y - x + 3 = 0$ and X-axis in the first quadrant is

(A) 36 (B) 18 (C) $\frac{27}{4}$ (D) 9 (JEE : 2013)

Sol. : Given curve $y = \sqrt{x}$ is a subset of $y^2 = x$ and line $x = 2y + 3$.

Solving we get $y^2 = 2y + 3$

$$\therefore y^2 - 2y - 3 = 0$$

$$\therefore (y-3)(y+1) = 0$$

$$\therefore y = 3 \text{ or } y = -1$$

If $y = 3$, then $x = 9$ and if $y = -1$, then $x = 1$

$\therefore$ The curve $y^2 = x$ and the line intersect at

$A(9, 3)$ and $B(1, -1)$.

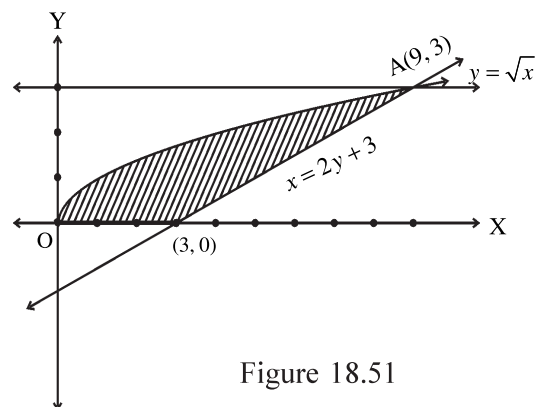


Figure 18.51

$$\begin{aligned}
 \therefore \text{ Required area} &= \int_0^3 (x_2 - x_1) dy \\
 &= \int_0^3 (2y + 3 - y^2) dy = \left[y^2 + 3y - \frac{y^3}{3} \right]_0^3 = 9 + 9 - 9 = 9 \quad \text{Ans. : (D)}
 \end{aligned}$$

(47) The area bounded by $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ in $\left[0, \frac{\pi}{2}\right]$ is

- (A) $4(\sqrt{2}-1)$ (B) $2(\sqrt{2}+1)$ (C) $2\sqrt{2}(\sqrt{2}-1)$ (D) $2\sqrt{2}(\sqrt{2}+1)$

(JEE : 2014 Adv)

Sol. : Here curve $f(x) = y = \sin x + \cos x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$, $0 \leq x \leq \frac{\pi}{2}$ and

$$g(x) = y = |\cos x - \sin x| = \begin{cases} \cos x - \sin x, & 0 \leq x \leq \frac{\pi}{4} \\ \sin x - \cos x, & \frac{\pi}{4} \leq x \leq \frac{\pi}{2} \end{cases}$$

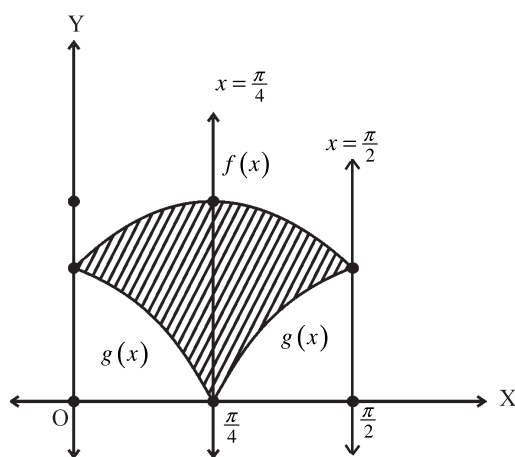


Figure 18.52

$$\begin{aligned}
 \text{Required area } A &= \int_0^{\frac{\pi}{4}} \{(\sin x + \cos x) - (\cos x - \sin x)\} dx \\
 &\quad + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \{(\sin x + \cos x) - (\sin x - \cos x)\} dx \\
 &= \int_0^{\frac{\pi}{4}} 2\sin x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\cos x \, dx \\
 &= -2[\cos x]_0^{\frac{\pi}{4}} + 2[\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}} \\
 &= -2\left[\frac{1}{\sqrt{2}} - 1\right] + 2\left[1 - \frac{1}{\sqrt{2}}\right] \\
 &= -\sqrt{2} + 2 + 2 - \sqrt{2} = 4 - 2\sqrt{2} = 2\sqrt{2}(\sqrt{2}-1).
 \end{aligned}$$

Ans. : (C)

- (48) Find the area of the region formed by $A = \{(x, y) \mid x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$.

(JEE : 2014 Main)

- (A) $\frac{\pi}{2} + \frac{4}{3}$ (B) $\frac{\pi}{2} - \frac{4}{3}$ (C) $\frac{\pi}{2} - \frac{2}{3}$ (D) $\frac{\pi}{2} + \frac{2}{3}$

Sol. : Here curve $x^2 + y^2 = 1$ is a circle having centre $O(0, 0)$ and radius 1, while $y^2 = 1 - x$ is a parabola having vertex at $(1, 0)$.

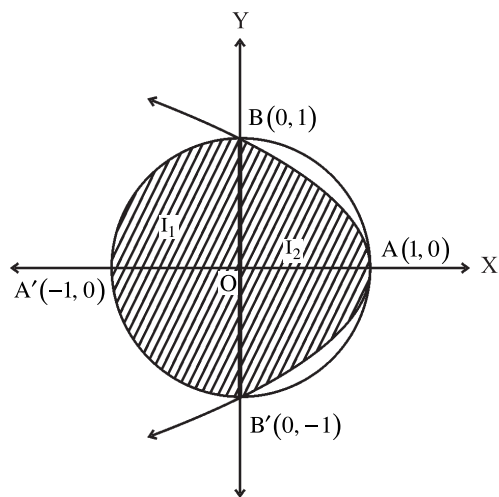


Figure 18.53

$$\begin{aligned}
 \text{Required area } A &= I_1 + I_2 \\
 &= \frac{1}{2}\pi r^2 + 2 \text{ (Area of region } OABO) \\
 &= \frac{1}{2}\pi(1)^2 + 2 \int_0^1 x \, dy \\
 &= \frac{1}{2}\pi(1)^2 + 2 \int_0^1 (1 - y^2) \, dy \\
 &= \frac{\pi}{2} + 2 \left[y - \frac{y^3}{3} \right]_0^1 = \frac{\pi}{2} + 2 \left(1 - \frac{1}{3} \right) = \frac{\pi}{2} + \frac{4}{3} \quad \text{Ans. : (A)}
 \end{aligned}$$

- (49) Common tangents of the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at points P, Q and the parabola at points R, S. Find the area of quadrilateral PQRS.

- (A) 3 (B) 6 (C) 9 (D) 15

(JEE : 2014 Adv)

Sol. : If a tangent of the parabola $y = mx + \frac{2}{m}$ touches the circle $x^2 + y^2 = 2$, then perpendicular distance from centre to this tangent is radius.

$$\therefore \left| \frac{2}{m\sqrt{1+m^2}} \right| = \sqrt{2}$$

$$\therefore m^4 + m^2 = 2$$

$$\therefore m^4 + m^2 - 2 = 0$$

$$\therefore (m^2 - 1)(m^2 + 2) = 0$$

$$\therefore m = \pm 1 \text{ or } m^2 = -2 \text{ which is not possible.}$$

$$\therefore m = \pm 1$$

The equations of the tangents are $y = x + 2$ and $y = -x - 2$.

The tangents intersect at $A(-2, 0)$.

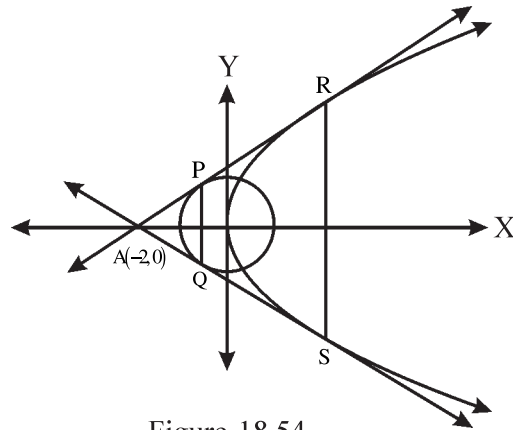


Figure 18.54

From $y = \pm(x + 2)$ and $x^2 + y^2 = 2$,

$$x^2 + (x + 2)^2 = 2. \quad \text{So, } 2x^2 + 4x + 2 = 0$$

$$x = -1 \text{ and } y = \pm 1$$

$$\therefore P(-1, 1) \text{ and } Q(-1, -1)$$

From $y = \pm(x + 2)$ and $y^2 = 8x$, we have

$$(x + 2)^2 = 8x$$

$$\therefore x^2 - 4x + 4 = 0$$

$$x = 2 \text{ and } y = \pm 4$$

$$\therefore \text{Coordinates of P, Q, R, S}$$

$$P(-1, 1), Q(-1, -1), R(2, 4), S(2, -4)$$

The area of the region PQSR = $\frac{1}{2}((\text{Sum of the lengths of parallel sides})$

$\times (\text{The perpendicular distance between them})$

$$= \frac{(2+8)(3)}{2} = \frac{30}{2} = 15$$

Ans. : (D)

: Question with More Than One Correct Answer :

(50) The area bounded by the curve $y = \log x$, $y = 0$ and $x = 3$ is

- (A) $(\log 9 - 2)$ (B) $(\log 27 - 2)$ (C) $\log \frac{27}{e^2}$ (D) less than three

Sol. :

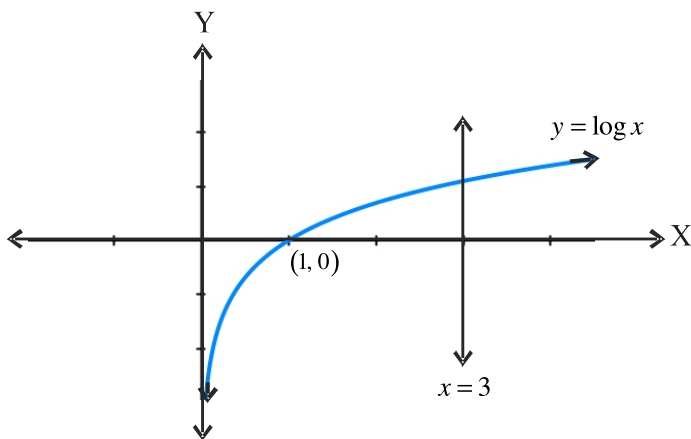


Figure 18.55

$$\begin{aligned} \text{Required area } A &= \int_1^3 y \, dx \\ &= \int_1^3 \log x \, dx \\ &= [x \log x - x]_1^3 \\ &= 3 \log 3 - 2 \end{aligned}$$

$$A = \log 27 - 2 \quad (\text{B})$$

$$\begin{aligned} &= \log 27 - \log_e e^2 \\ &= \log \left(\frac{27}{e^2} \right) \quad (\text{C}) \end{aligned}$$

$$= \log \left(\frac{27}{e^2} \right) < 3 \quad (\text{D})$$

Ans. : (B), (C), (D)

: Assertion and Reason Type Questions :

Explanation : Each of these questions contains statement 1 (Assertion) and statement 2 (Reason). Each question has four choices, option (A), (B), (C) and (D), which are as follows. You have to select the correct choice.

- (A) Statement 1 is true, statement 2 is true. Statement 2 is the correct explanation for statement 1.
 (B) Statement 1 is true, statement 2 is true. Statement 2 is not a correct explanation for statement 1.
 (C) Statement 1 is true, statement 2 is false.
 (D) Statement 1 is false, statement 2 is true.

(51) **Statement 1 :** The area bounded by the curve $y = \sin^2 x$ in $[0, \pi]$ is bigger than the area bounded by $y = \sin x$ in $[0, \pi]$.

Statement 2 : If $0 < t < 1$, then $t^2 < t$.

Sol. : If $0 < t < 1$, then $t^2 < t$

$$\therefore \sin^2 x < \sin x$$

$$\therefore \int_0^\pi \sin^2 x \, dx < \int_0^\pi \sin x \, dx$$

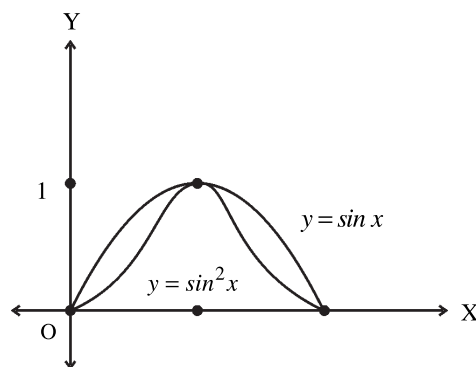


Figure 18.56

Statement 1 is false and statement 2 is true.

Ans. : (D)

(52) **Statement 1** : If $f(x) = (x-1)(x-2)(x-3)$, then the area bounded by $f(x)$ and lines $x = 2.2$, $x = 2.8$ and X-axis is given by $\int_{2.2}^{2.8} (x-1)(x-2)(x-3) dx$.

Statement 2 : $(x-1)(x-2)(x-3) \leq 0$, where $x \in [2.2, 2.8]$

Sol. : It is clear from the figure that if, $x \in [2.2, 2.8]$, then $(x-1)(x-2)(x-3) < 0$.

If $2.2 < x < 2.8$, then $(x-1) > 0, (x-2) > 0, (x-3) < 0$

$\therefore (x-1)(x-2)(x-3) < 0$

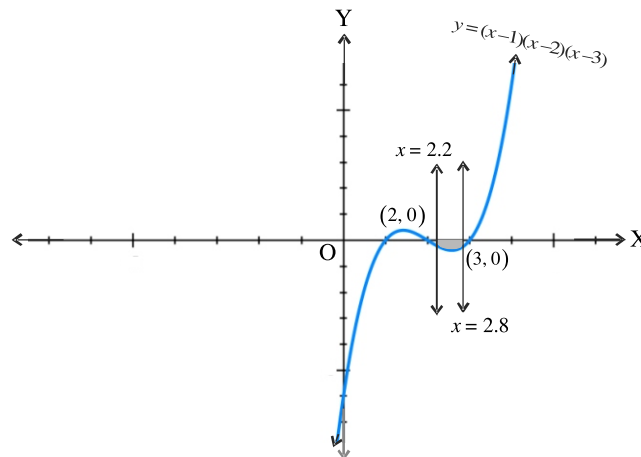


Figure 18.57

$$\therefore \text{Required area} = \left| \int_{2.2}^{2.8} f(x) dx \right| = - \int_{2.2}^{2.8} (x-1)(x-2)(x-3) dx$$

$\therefore$ Thus statement 1 is false and statement 2 is true.

Ans. : (D)

(53) **Statement 1** : The area bounded by the curve $2x^2 + 3y^2 = 6$ is smaller than the area bounded by the curve $2|x| + 3|y| \leq 6$.

Statement 2 : The length of the major axis of the ellipse $2x^2 + 3y^2 = 6$ is greater than the maximum of the distance between any two points on X-axis if $2|x| + 3|y| \leq 6$.

Sol. : The given curve is $2x^2 + 3y^2 = 6$. So, $\frac{x^2}{3} + \frac{y^2}{2} = 1$ represents an ellipse.

$$a = \sqrt{3} \text{ and } b = \sqrt{2}.$$

$$2|x| + 3|y| \leq 6 \Rightarrow 2x + 3y \leq 6, -2x + 3y \leq 6,$$

$$2x - 3y \leq 6 \text{ and } -2x - 3y \leq 6 \text{ indicate subsets of four lines.}$$

$$\text{Solving } 2x + 3y = 6 \text{ and } 2x^2 + 3y^2 = 6$$

$$2x^2 + 3 \frac{(6-2x)^2}{9} = 6$$

$$6x^2 + 36 - 24x + 4x^2 = 18$$

$$\therefore 10x^2 - 24x + 18 = 0$$

$$\therefore 5x^2 - 12x + 9 = 0$$

$$\therefore \Delta = 144 - 180 < 0$$

$\therefore$ Similarly, other lines too do not intersect the ellipse.

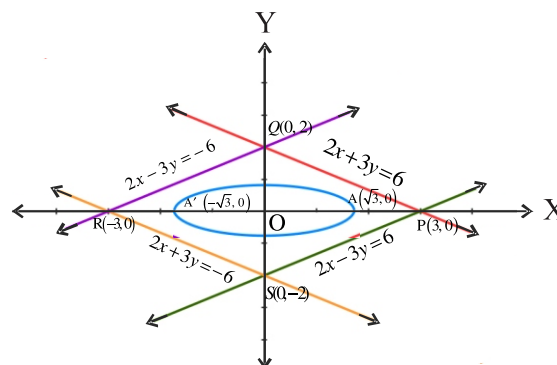


Figure 18.58

The area of an ellipse $= \pi ab = \sqrt{6} \pi$

and area bounded by $2|x| + 3|y| \leq 6$ is $= 4 \left(\frac{1}{2} \times b \times h \right) = 4 \times \frac{1}{2} \times 3 \times 2 = 12$

Since $\sqrt{6} \pi < 12 \Leftrightarrow 6\pi^2 < 144 \Leftrightarrow \pi^2 < 24$ statement 1 is true.

The length of the major axis of the ellipse $= AA' = 2\sqrt{3}$

The distance between any two points of $2|x| + 3|y| \leq 6$ on axes is 6.

Since $2\sqrt{3} < 6 \Leftrightarrow \sqrt{3} < 3$, statement 2 is false.

Ans. : (C)

(54) **Statement 1 :** The area of the region bounded by $|x| + |y| \leq 2$ is 8.

Statement 2 : $|x| + |y| = 2$ represents a square having sides of length $\sqrt{8}$.

Sol. : The region bounded by $|x| + |y| \leq 2$ is the region bounded by four sets satisfying $x + y \leq 2, -x + y \leq 2, x - y \leq 2$ and $-x - y \leq 2$.

The lines $x + y = 2, x + y = -2$ are parallel. $x - y = \pm 2$ are parallel.

Product of the slopes of $x + y = 2, x - y = 2$ is -1 .

Diagonal $AC = BD = 4$

$\therefore ABCD$ is a square.

$\triangle OAB$ is a right angled triangle.

$$\therefore OA^2 + OB^2 = AB^2$$

$$\therefore AB^2 = 4 + 4$$

$$\therefore AB = \sqrt{8}$$

$ABCD$ is a square having sides of length $\sqrt{8}$.

Its area $= l \times b = \sqrt{8} \times \sqrt{8} = 8$

Statement 1 is true.

Also, statement 2 is true and it is the correct explanation of statement 1.

Ans. : (A)

Second method : Area $= 4 \int_0^2 (2-x) dx = 4 \left[2x - \frac{x^2}{2} \right]_0^2 = 4(4-2) = 8$

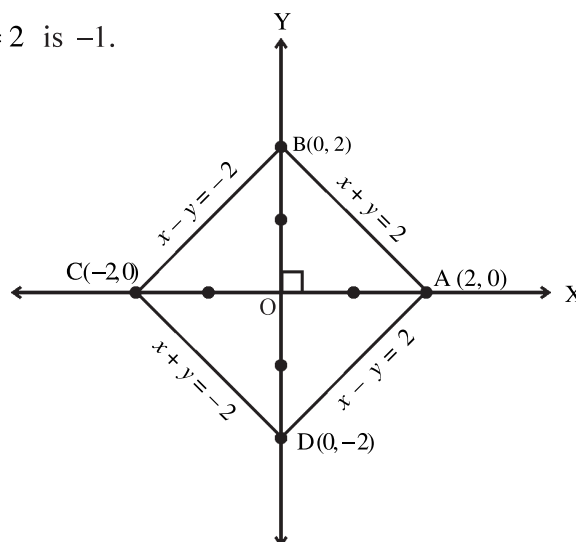


Figure 18.59