

The background is a vibrant blue with various white and light blue mathematical sketches. At the top left, there's a triangle with vertices labeled 'a', 'b', and 'c', and an angle labeled 'C'. To its right is a circular diagram with a radius 'r' and an angle '2'. Below these are a ruler with markings from 1 to 5, a compass, and a pencil. At the bottom, there's a 3D cube with dashed lines for hidden edges, a 30-degree angle, and a pen. The overall theme is mathematics and geometry.

CLASS X MATHEMATICS NOTES

POLYNOMIALS

Key Notes and Important Questions with
Answers

Key Points

- Polynomial :** If x is a variable, n is a natural number and $a_0, a_1, a_2, a_3, \dots, a_n$ are real numbers, then $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, ($a_n \neq 0$) is called a polynomial in x .
- Polynomials of degree 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
- A quadratic polynomial is an algebraic expression of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
- Zeros of a polynomial $p(x)$ are precisely the x – coordinates of the points where the graph of $y = p(x)$ intersects the x –axis, *i.e.*, $x = a$ is a zero of polynomial $p(x)$ if $p(a) = 0$
- A polynomial can have at most the same number of zeroes as the degree of the polynomial.
- (i) If one zero of a quadratic polynomial $p(x)$ is negative of the other, then coefficient of x is 0.
(ii) If zeroes of a quadratic polynomial $p(x)$ are reciprocal of each other, then coefficient of $x^2 =$ constant term.
- Relationship between zeroes and coefficients of a polynomial
If α And β Are zeroes of $p(x) ax^2 + bx + c$ ($a \neq 0$), then
Sum of zeroes $= \alpha + \beta = -\frac{b}{a}$
Product of zeroes $= \alpha\beta = \frac{c}{a}$
- If α, β are zeroes of a quadratic polynomial $p(x)$, then
 $p(x) = k [x^2 - (\text{sum of zeroes}) x + \text{product of zeroes}]$
 $\Rightarrow p(x) = k [x^2 - (\alpha + \beta)x + \alpha\beta]$; where k is any non-zero real number.
- Graph of linear polynomial $p(x) = ax + b$ is a straight line.
- Division Algorithm states that given any polynomials $p(x)$ and $g(x)$, there exist polynomial $q(x)$ and $r(x)$ such that: $p(x) = g(x) \cdot q(x) + r(x)$; $g(x) \nmid r(x) \Rightarrow$

[where either $r(x) = 0$ or degree $r(x) <$ degree $g(x)$]

Important
Questions
with
Solutions

VERY SHORT ANSWER TYPE QUESTIONS

1. What will be the number of zeroes of a linear polynomial $p(x)$ if its graph (i) passes through the origin. (ii) doesn't intersect or touch x -axis at any point?
2. Find the quadratic polynomial whose zeroes are $(5 + 2\sqrt{3})$ and $(5 - 2\sqrt{3})$
3. If one zero of $p(x) = 4x^2 - (8k^2 - 40k)x - 9$ is negative of the other, find values of k .
4. What number should be added to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of polynomial so obtained.
5. How many (i) maximum (ii) minimum number of zeroes can a quadratic polynomial have?
6. What will be the number of real zeroes of the polynomial $x^2 + 1$?
7. If α and β are zeroes of polynomial $6x^2 - 7x - 3$, then form a quadratic polynomial where zeroes are 2α and 2β
8. If α and $\frac{1}{\alpha}$ are zeroes of $4x^2 - 17x + k - 4$, find value of k .
9. What will be the number of zeroes of the polynomials whose graphs are parallel to (i) y -axis (ii) x -axis
10. What will be number of zeroes of the polynomials whose graphs are either touching or intersecting the axis only at the points:
(i) $(-3, 0)$, $(0, 2)$ & $(3, 0)$ (ii) $(0, 4)$, $(0, 0)$ and $(0, -4)$

SHORT ANSWER TYPE (I) QUESTIONS

11. If -3 is one of the zeroes of the polynomial $(k - 1)x^2 + kx + 1$, find the value of k .
12. If the product of zeroes of $ax^2 - 6x - 6$ is 4, find the value of a . Hence find the sum of its zeroes.
13. If α and β are zeroes of the polynomial $x^2 - a(x + 1) - b$ such that $(\alpha + 1)(\beta + 1) = 0$, find the value of b .
14. If zeroes of $x^2 - kx + 6$ are in the ratio 3 : 2, find k .

15. If one zero of the quadratic polynomial $(k^2 + k)x^2 + 68x + 6k$ is reciprocal of the other, find k .
16. If α and β are the zeroes of the polynomial $x^2 - 5x + m$ such that $\alpha - \beta = 1$, find m .
17. If the sum of squares of zeroes of the polynomial $x^2 - 8x + k$ is 40, find the value of k .
18. If α and β are zeroes of the polynomial $t^2 - t - 4$, form a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

SHORT ANSWER TYPE (II) QUESTIONS

19. If $(k + y)$ is a factor of each of the polynomials $y^2 + 2y - 15$ and $y^3 + a$, find values of k and a .
20. Obtain zeroes of $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ and verify relation between its zeroes and coefficients.
21. If $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by $(x^2 + 5)$, remainder comes out to be $(px + q)$, find values of p and q .
22. -5 is one of the zeroes of $2x^2 + px - 15$, zeroes of $p(x^2 + x) + k$ are equal to each other. Find the value of k .
23. Find the value of k such that $3x^2 + 2kx + x - k - 5$ has the sum of zeroes as half of their product.
24. If α and β are zeroes of $y^2 + 5y + m$, find the value of m such that $(\alpha + \beta)^2 - \alpha\beta = 24$.
25. If α and β are zeroes of $x^2 - x - 2$, find a polynomial whose zeroes are $(2\alpha + 1)$ and $(2\beta + 1)$.
26. Find values of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.
27. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$?
28. What must be added to $4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $x^2 - 2x - 3$?

LONG ANSWER TYPE QUESTIONS

29. Find all zeroes of the polynomial $2x^3 + x^2 - 6x - 3$ if two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.
30. If $\sqrt{2}$ is a zero of $(6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2})$, find its other zeroes.
31. If two zeroes of $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $(2 \pm \sqrt{3})$, find other zeroes.
32. On dividing the polynomial $x^3 - 5x^2 + 6x - 4$ by a polynomial $g(x)$, quotient and remainder are $(x - 3)$ and $(-3x + 5)$ respectively. Find $g(x)$.
33. If sum and product of two zeroes of the polynomial $x^3 + x^2 - 3x - 3$ are 0 and 3 respectively, find all zeroes of the polynomial.
34. If $-\frac{1}{2}$ is a zero of the polynomial $2x^3 + x^2 - 6x - 3$, find the sum and product of its other two zeroes.
35. Obtain all zeroes of the polynomial $2x^4 - 2x^3 - 7x^2 + 3x + 6$ if two factors of this polynomial are $\left(x \pm \sqrt{\frac{3}{2}}\right)$.
36. Sum and product of two zeroes of $x^4 - 4x^3 - 8x^2 + 36x - 9$ are 0 and -9 respectively. Find the sum and product of its other two zeroes.
37. A person distributes k books to some needy students. If k is a zero of the polynomial $x^2 - 100x - 20000$, then
(i) Find the number of books distributed
38. One zero of $x^3 - 12x^2 + 47x - 60$ is 3 and the remaining two zeroes are the number of trees planted by two students.
(i) Find the total number of trees planted by both students.

ANSWERS

1. (i) 1 (ii) 0
3. $k = 0, 5$
5. (i) 2 (ii) 0
7. $3x^2 - 7x - 6$
9. (i) 1 (ii) 0
11. $\frac{4}{3}$
13. 1
15. 5
17. 12
19. $k = 3, -5$ and $a = 27, -125$
21. $p = 2, q = 3$
23. 1
25. $x^2 - 4x - 5$
27. $14x - 10$
29. $\sqrt{3}, -\sqrt{3}, -\frac{1}{2}$
31. $-5, 7$
33. $\sqrt{3}, -\sqrt{3}, -1$
35. $2, -1, \pm\sqrt{\frac{3}{2}}$
37. (i) 200
38. (i) 9
2. $x^2 - 10x + 13$
4. 2
6. 0
8. $k = 8$
10. (i) 2 (ii) 1
12. $a = -\frac{3}{2}$, sum of zeroes = -4
14. $-5, 5$
16. 6
18. $4t^2 + t - 1$
20. $-\frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$
22. $\frac{7}{4}$
24. 1
26. $a = 1, b = 7$
28. $61x - 65$
30. $-\frac{\sqrt{2}}{2}, -\frac{2\sqrt{2}}{3}$
32. $x^2 - 2x + 3$
34. 0, 3
36. 4, 1

Practice-Test

Polynomials

Time : 50 Minutes

M.M. : 20

SECTION-A

1. If α and β are zeroes of a quadratic polynomial $p(x)$, then factorize $p(x)$. 1
2. If α and β are zeroes of $x^2 - x - 1$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$. 1

SECTION-B

3. If α and β are zeroes of $x^2 - (k + 6)x + 2(2k - 1)$. find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$. 2
4. Find a quadratic polynomial one of whose zeroes is $(3 + \sqrt{2})$ and the sum of its zeroes is 6. 2

SECTION-C

5. Find values of a and b if $(x^2 + 1)$ is a factor of the polynomial $x^4 + x^3 + 8x^2 + ax + b$. 3
6. If truth and lie are zeroes of the polynomial $px^2 + qx + r$, ($p \neq 0$) and zeroes are reciprocal to each other, 3
(i) Find the relation between p and r .

SECTION-D

7. On dividing the polynomial $x^3 + 2x^2 + kx + 7$ by $(x - 3)$, remainder comes out to be 25. Find quotient and the value of k . Also find the sum and product of zeroes of the quotient so obtained. 4
8. If β and $\frac{1}{\beta}$ are zeroes of the polynomial $(\alpha^2 + \alpha)x^2 + 61x + 6\alpha$, find values of β and α . 4