

Statement : A sentence is called a statement, if it is either true or false but not both and its truth or falseness can be definitely decided upon without ambiguity.

Simple Statement : A statement which cannot be decomposed into two or more statements is called a simple statement.

Negation : If a statement is true, then its negation is false and if a given statement is false, then its negation is true.

The truth of simple statement and its negation is shown in the tabular form as below :

p	$\sim p$
T	F
F	T

The symbol for complementation is ‘ $\sim$ ’ and it is an operation equivalent to negation in mathematical logic.

$$\sim(\sim p) = p$$

Compound Statement : When two or more simple statements are joined using the logical connectives, the statement formed is called a compound statement.

Logical connectives : Sometimes two or more simple statements are joined by connectives ‘or’ or ‘and’. These connectives are called logical connectives.

The Conjunction of Statements : If two or more simple statements are joined by logical connective ‘and’, the statement obtained is called the conjunction of given statements.

Here, given statements are called **component statements**.

- The conjunction of two simple statements is denoted by $p \wedge q$. If the truthvalue of both the simple statements is T, then the truthvalue of $p \wedge q$ is T, otherwise it is F.

The above information can be summarised in the tabular form as follows :

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Intersection of sets A and B is symbolically represented by $A \cap B$ and it is an operation equivalent to conjunction in mathematical logic.

Disjunction : If two or more simple statements are joined by logical connective ‘or’, the statement obtained is called the disjunction of given statements.

The disjunction of two simple statements p and q is denoted by $p \vee q$.

If the truthvalue of both the simple statements is F, then the truthvalue of $p \vee q$ is F, otherwise it is T.

The above information can be written in the tabular form as follows :

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Union of sets A and B is symbolically represented by $A \cup B$ and it is an operation equivalent to disjunction in mathematical logic.

Negation of Compound Statements : Negation of the conjunction of two simple statements is the disjunction of negations of two given simple statements. In symbol, it is expressed as $\sim(p \wedge q) = (\sim p) \vee (\sim q)$.

If two simple statements p and q are true, then the negation of conjunction will be false, otherwise the negation of conjunction will be true.

The above information can be written in the tabular form as under :

p	q	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

The negation of the disjunction of two simple statements is the conjunction of the negations of two given simple statements. In symbol, it is expressed as $\sim(p \vee q) = (\sim p) \wedge (\sim q)$.

If two simple statements p and q are false, then the negation of disjunction will be true, otherwise the negation of disjunction will be false.

The above information can be written in the tabular form as under :

p	q	$p \vee q$	$\sim(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Results of the negation of a compound statement are equivalent to De'Morgan's laws of set operation.

Use of logical Connective 'or' in terms of inclusive/exclusive sense :

When two events do not exist simultaneously, at that time logical connectives can be used in exclusive sense, but when two events can exist simultaneously, then logical connectives can be used in inclusive sense.

Connectives and their negations :

'there exists', 'for all' or 'for each' phrases are also known as connectives.

The symbol for 'there exists' is $\exists$ which is called existential quantifier.

The symbol for the phrase 'for each' is $\forall$ which is called universal quantifier.

Their negations can be denoted in symbols obtained as follows :

$$\sim(\text{for some } p) = \text{for all } \sim p. \text{ i.e. } \sim(\exists p) = \forall(\sim p)$$

$$\sim(\text{for all } p) = \text{for some } \sim p. \text{ i.e. } \sim(\forall p) = \exists(\sim p)$$

Implication and double implication : If p and q are any two statements, the statement of the form, ‘if p , then q ’ is called an implication and is denoted by $p \Rightarrow q$. It is called a conditional statement.

Implication is expressed by any one of the following :

(i) If p then q (ii) q , if p , (iii) p only if q (iv) p is a sufficient condition for q . (v) q is a necessary condition for p .

p is called the hypothesis or the antecedent and q is called the conclusion or the consequent.

In implication, if antecedent is true and the consequent is false, then the implication is false, otherwise it will be true.

$$(p \Rightarrow q) = (\sim p) \vee q$$

The above information can be written in the tabular form as follows :

p	q	$p \Rightarrow q$	$\sim p$	$(\sim p) \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Double Implication : If a given statement is in the form, ‘ p if and only if q ’, then it is called a biconditional statement or double implication. It is denoted by $p \Leftrightarrow q$.

$p \Leftrightarrow q$ is true if both p and q are true or both are false.

The above information can be summarised in the tabular form as under :

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Contrapositive statement :

The contrapositive statement of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$. See $p \Rightarrow q = (\sim q) \Rightarrow (\sim p)$.

Table :

(a)			(b)		
p	q	$p \Rightarrow q$	$\sim q$	$\sim p$	$(\sim q) \Rightarrow (\sim p)$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Converse of a statement : For two statements p and q , the converse of, ‘if p then q ’ is ‘if q then p ’.

Converse of $p \Rightarrow q$ is $q \Rightarrow p$.

Double implication is a compound statement of implication and its converse.

$$\sim(p \Rightarrow q) = p \wedge (\sim q)$$

Validating argument : We can use the following two methods to verify the validity of argument :

(i) **Direct method :** To prove a statement with 'if p then q '; assuming p is true, prove q is true. This is called direct method of proof.

(ii) **Method of contradiction (Method of reductio ad absurdum) :** Here we assume that q is not true and come to a contradiction against hypothesis.

The truthvalue of tautology is T and the truthvalue of fallacy is F. Tautology is denoted by t and fallacy is denoted by c .

Important Results :

- | | |
|---|---|
| (1) $p \vee t = t$ | (2) $p \wedge t = p$ |
| (3) $p \vee (\sim p) = t$ | (4) $(p \wedge (\sim p)) = c$ |
| (5) $p \wedge c = c$ | (6) $p \vee c = p$ |
| (7) $[p \Rightarrow (p \vee q)] = t$ | (8) $[q \Rightarrow (p \vee q)] = t$ |
| (9) $[(p \wedge q) \Rightarrow p] = t$ | (10) $[(p \wedge q) \Rightarrow q] = t$ |
| (11) $[\{(p \vee q) \wedge (\sim p)\} \Rightarrow q] = t$ | (12) $[\{(p \vee q) \wedge (\sim q)\} \Rightarrow p] = t$ |
| (13) $[\{(p \Rightarrow q) \wedge (q \Rightarrow r)\} \Rightarrow (p \Rightarrow r)] = t$ | |

: Multiple Choice Questions :

- (1) The negation of $x \in A \cap B \Rightarrow (x \in A \text{ and } x \in B)$ is
- (A) $x \in A \cup B \Rightarrow (x \in A \text{ or } x \in B)$ (B) $x \in A \cap B \text{ or } (x \in A \text{ and } x \in B)$
 (C) $x \in A \cap B \text{ and } (x \notin A \text{ or } x \notin B)$ (D) $x \notin A \cap B \text{ and } (x \in A \text{ and } x \in B)$

Sol. : Here $p : x \in A \cap B$ and $q : x \in A \text{ and } x \in B$.

The given statement is of the form $p \Rightarrow q$. Its negation is

$$\sim(p \Rightarrow q) = p \wedge (\sim q)$$

$$= x \in A \cap B \text{ and } \sim(x \in A \text{ and } x \in B)$$

$$= x \in A \cap B \text{ and } (x \notin A \text{ or } x \notin B)$$

Ans. : (C)

- (2) The negation of ' $x \in \mathbb{R}$ if and only if $|x| \geq 0$ ' is

- (A) $\{x \in \mathbb{R}^+ \text{ and } |x| > 0\}$ and $\{|x| > 0 \text{ and } x \in \mathbb{R}\}$
 (B) $\{x \in \mathbb{R} \text{ and } |x| \not\geq 0\}$ and $\{|x| \geq 0 \text{ and } x \notin \mathbb{R}\}$
 (C) $\{x \in \mathbb{R} \text{ and } |x| \not\geq 0\}$ or $\{|x| \geq 0 \text{ and } x \notin \mathbb{R}\}$
 (D) $\{x \in \mathbb{R}^+ \text{ and } |x| \not\geq 0\}$ or $\{|x| < 0 \text{ and } x \notin \mathbb{R}\}$

Sol. : Here $p : x \in \mathbb{R}$ and $q : |x| \geq 0$ are given statements.

Here $p \Leftrightarrow q$ is a biconditional statement. Its negation is,

$$\sim(p \Leftrightarrow q) = \sim[(p \Rightarrow q) \wedge (q \Rightarrow p)]$$

$$= \sim(p \Rightarrow q) \vee \sim(q \Rightarrow p)$$

$$= (p \wedge (\sim q)) \vee (q \wedge (\sim p))$$

Negation of given statement = $\{x \in \mathbb{R} \text{ and } |x| \not\geq 0\}$ or $\{|x| \geq 0 \text{ and } x \notin \mathbb{R}\}$ **Ans. : (C)**

- (3) For two statements p and q , $p \vee \sim(p \Rightarrow \sim q) = \dots\dots$

- (A) $\sim p \vee q$ (B) $p \wedge (\sim q)$ (C) p (D) q

Sol. : $p \vee \sim(p \Rightarrow \sim q) = p \vee (p \wedge \sim(\sim q)) = p \vee (p \wedge q) = p$

Ans. : (C)

- (4) If the truthvalues of simple statements p, q, r and s are respectively T, F, F, T, then the truthvalue of $((\sim p \wedge q) \vee r) \Rightarrow \sim s$ is

(A) F

(B) T

(C) F, if truthvalue of p and q is T

(D) T, if truthvalue of s is F

Sol. :

p	q	r	s	$\sim p$	$\sim s$	$\sim p \wedge q$	$(\sim p \wedge q) \vee r$	$((\sim p \wedge q) \vee r) \Rightarrow \sim s$
T	F	F	T	F	F	F	F	T

Ans. : (B)

- (5) $\sim[\sim p \wedge (p \Leftrightarrow q)] = \dots$

(A) $q \wedge p$

(B) $p \vee q$

(C) $\sim p$

(D) $\sim q$

Sol. : $\sim[\sim p \wedge (p \Leftrightarrow q)] = \sim(\sim p) \vee \sim(p \Leftrightarrow q)$
 $= p \vee \sim[(p \Rightarrow q) \wedge (q \Rightarrow p)]$
 $= [p \vee (p \wedge \sim q)] \vee (q \wedge \sim p)$
 $= p \vee (q \wedge \sim p)$
 $= (p \vee q) \wedge (p \vee \sim p)$
 $= p \vee q$

Ans. : (B)

- (6) The contrapositive statement of 'If $x^2 = 25$, then $(x = 5 \text{ or } x = -5)$ ' is

(A) If $x^2 \neq 25$, then $x \neq 5$ and $x \neq -5$

(B) If $x \neq 5$ or $x \neq -5$, then $x^2 = 25$

(C) If $x \neq 5$ and $x \neq -5$, then $x^2 \neq 25$

(D) If $x \neq 5$ and $x \neq -5$, then $x^2 = 25$

Sol. : Here, $p : x^2 = 25$; $q : x = 5$, $r : x = -5$

The given statement is of the form $p \Rightarrow (q \vee r)$. Its contrapositive statement is $\sim(q \vee r) \Rightarrow \sim p$.

$\therefore$ Contrapositive statement of given statement :

If $(x \neq 5 \text{ and } x \neq -5)$, then $x^2 \neq 25$

Ans. : (C)

- (7) The contrapositive statement and the converse statement of 'If the points P, Q and R are collinear, then ΔPQR is not possible' are respectively and

(A) **Contrapositive statement** : If ΔPQR is possible, then the points P, Q and R are collinear.

Converse : If ΔPQR is not possible, then the points P, Q and R are collinear.

(B) **Contrapositive statement** : If ΔPQR is not possible, then the points P, Q and R are not collinear.

Converse : If ΔPQR is possible, then the points P, Q and R are not collinear.

(C) **Contrapositive statement** : If ΔPQR is not possible, then the points P, Q and R are collinear.

Converse : If ΔPQR is not possible, then the points P, Q and R are not collinear.

(D) **Contrapositive statement** : If ΔPQR is possible, then the points P, Q and R are not collinear.

Converse : If ΔPQR is not possible, then the points P, Q and R are collinear.

Sol. : Here p : The points P, Q and R are collinear.

q : ΔPQR is not possible.

The given statement is of the form $p \Rightarrow q$. Its contrapositive statement is $\sim q \Rightarrow \sim p$.

$\therefore$ **Contrapositive statement** : If ΔPQR is possible, then the points P, Q and R are not collinear.

Also, converse statement of $p \Rightarrow q$ is $q \Rightarrow p$.

$\therefore$ **Converse** : If ΔPQR is not possible, then points P, Q and R are collinear. **Ans. : (D)**

(8) The converse of $(p \wedge (\sim q)) \Rightarrow r$ is

(A) $\sim r \Rightarrow (\sim p \vee q)$

(B) $r \Rightarrow (\sim p \wedge \sim q)$

(C) $(\sim p \vee q) \Rightarrow \sim r$

(D) All of these (A), (B) and (C)

Sol. : The converse statement of $(p \wedge (\sim q)) \Rightarrow r$ is $r \Rightarrow (p \wedge (\sim q))$.

$$\therefore r \Rightarrow (p \wedge (\sim q))$$

$$= \sim r \vee (p \wedge \sim q)$$

$$(p \Rightarrow q = \sim p \vee q)$$

$$= (p \wedge \sim q) \vee (\sim r)$$

$$= [\sim(\sim p \vee q)] \vee (\sim r)$$

$$(p \wedge \sim q = \sim(\sim p \vee q))$$

$$= (\sim p \vee q) \Rightarrow (\sim r)$$

$$\therefore \text{Converse of } (p \wedge (\sim q)) \Rightarrow r \text{ is } (\sim p \vee q) \Rightarrow (\sim r).$$

Ans. : (C)

(9) p : It rains today.

q : I am going to school.

r : I will meet my friend.

s : I will go to watch a movie.

If it doesn't rain today or if I won't go to school, then I will meet my friend and I will go to watch a movie. This statement can be represented as in logical form.

(A) $\sim(p \wedge q) \Rightarrow (r \wedge s)$ (B) $(\sim p \wedge \sim q) \Rightarrow (r \wedge s)$ (C) $\sim(p \vee q) \Rightarrow (r \vee s)$ (D) $(p \wedge q) \Rightarrow (r \vee s)$

Sol. : Here, if it doesn't rain today is negation of p and I won't go to school is negation of q . Also, the statement is of 'if, then' form.

$$\therefore \text{Given statement is of form } (\sim p \vee \sim q) \Rightarrow (r \wedge s).$$

$$\therefore (\sim p \vee \sim q) \Rightarrow (r \wedge s)$$

$$= \sim(p \wedge q) \Rightarrow (r \wedge s)$$

Ans. : (A)

(10) $\sim[p \Rightarrow (\sim p \vee q)] = \dots\dots$

(A) $p \vee (p \vee \sim r)$

(B) $p \Rightarrow \sim(p \vee q)$

(C) $p \Leftrightarrow q$

(D) $p \wedge \sim q$

Sol. : $p \Rightarrow (\sim p \vee q) = \sim p \vee (\sim p \vee q) = ((\sim p) \vee (\sim p)) \vee q$

$$(p \Rightarrow q = \sim p \vee q)$$

$$= \sim p \vee q$$

$$\therefore \sim[p \Rightarrow (\sim p \vee q)] = \sim[\sim p \vee q] = p \wedge \sim q$$

Ans. : (D)

(11) $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to

[Online : 2014]

(A) p

(B) q

(C) $\sim p$

(D) $\sim q$

Sol. : $\sim(p \vee q) \vee (\sim p \wedge q) = (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$$= \sim p \wedge (\sim q \vee q)$$

$$= \sim p \wedge t$$

$$(q \vee \sim q = t)$$

$$= \sim p$$

Ans. : (C)

(12) $\sim[p \wedge (q \Rightarrow \sim r)] = \dots\dots$

(A) $p \vee (q \wedge r)$

(B) $\sim p \vee (q \wedge r)$

(C) $p \vee (q \vee r)$

(D) $\sim p \wedge (q \wedge r)$

Sol. : $\sim[p \wedge (q \Rightarrow \sim r)] = (\sim p) \vee \sim(q \Rightarrow \sim r)$

$$= \sim p \vee (q \wedge \sim(\sim r))$$

$$= \sim p \vee (q \wedge r)$$

Ans. : (B)

(13) p : 10 is an even natural number q : 12 is divisible by 8

r : l.c.m. of 6 and 8 is 24. Then, which of the following is true ?

(A) $(p \wedge r) \vee q$

(B) $(p \Rightarrow q) \vee r$

(C) only (A)

(D) (A) and (B) both

Sol. : Here, statement p and statement r are true and statement q is false.

$\therefore (p \wedge r)$ is true and q is false. Their disjunction becomes true.

$\therefore$ Truthvalue of $(p \wedge r) \vee q$ is T.

Also, in $p \Rightarrow q$, p being true and q being false, $p \Rightarrow q$ is false.

r being true, $(p \Rightarrow q) \vee r$ is true.

Thus, (A) and (B), both the options are true.

Short method : r being true, (B) is true.

$p \wedge r$ being true, (A) is true.

Ans. : (A), (B), (D)

(14) If $[(p \Rightarrow q) \wedge (p \Rightarrow r)] \Rightarrow (p \Rightarrow r)$ is logically true, then this statement is

(A) p

(B) q

(C) a tautology

(D) a contradiction

Sol. :

p	q	r	$p \Rightarrow q$	$p \Rightarrow r$	$(p \Rightarrow q) \wedge (p \Rightarrow r)$	$[(p \Rightarrow q) \wedge (p \Rightarrow r)] \Rightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	F	T
T	F	F	F	F	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

From the last column, we can say that the given statement is a tautology.

Ans. : (C)

(15) The equivalent statement of $p \Rightarrow (q \Rightarrow p)$ is

(Online 2008 : 2013)

(A) $p \Rightarrow (p \Leftrightarrow q)$

(B) $p \Rightarrow (p \Rightarrow q)$

(C) $p \Rightarrow (p \vee q)$

(D) $p \Rightarrow (p \wedge q)$

Sol. : $p \Rightarrow (q \Rightarrow p) = \sim p \vee (\sim q \vee p)$

$$= \sim p \vee (p \vee \sim q)$$

(Commutative law)

$$= (\sim p \vee p) \vee (\sim q)$$

(Associative law)

$$= t \vee (\sim q)$$

$$= t$$

(1)

$$\text{Now, } p \Rightarrow (p \Leftrightarrow q) = (\sim p) \vee [(p \Rightarrow q) \wedge (q \Rightarrow p)]$$

$$= \sim p \vee [(\sim p) \vee q] \wedge (\sim q \vee p)$$

$$= [(\sim p) \vee (\sim p \vee q)] \wedge [(\sim p) \vee (\sim q \vee p)]$$

$$= [(\sim p \vee \sim p) \vee q] \wedge [(\sim p) \vee (p \vee \sim q)]$$

$$= [\sim p \vee q] \wedge [(\sim p \vee p) \vee \sim q]$$

$$= (\sim p \vee q) \wedge (t \vee \sim q)$$

$$= (\sim p \vee q) \wedge t$$

$$= p \Rightarrow q \neq t$$

$$p \Rightarrow (p \Rightarrow q) = (\sim p) \vee (\sim p \vee q) = ((\sim p) \vee q) = (p \Rightarrow q) \neq t$$

$$\text{and } p \Rightarrow (p \vee q) = \sim p \vee (p \vee q)$$

$$= (\sim p \vee p) \vee q = t \vee q = t$$

(Associative law) (2)

From (1) and (2), it is clear that the statement equivalent to the given statement is

$$p \Rightarrow (p \vee q).$$

$$p \Rightarrow (p \wedge q) = (\sim p) \vee (p \wedge q) = ((\sim p) \vee p) \wedge (\sim p \vee q) = (\sim p) \vee q \neq t$$

Ans. : (C)

- (16) For $S \subset \mathbb{R}$, where $S \neq \emptyset$, which of the following is the negation of the statement p given below ?

Statement p : There is a rational number $x \in S$, such that $x > 0$

- (A) There is no rational number $x \in S$, such that $x \leq 0$
 (B) For every rational number $x \in S$, $x \leq 0$.
 (C) $x \in S$ and $x \leq 0 \Rightarrow x$ is an irrational number.
 (D) $\exists$ a rational number $x \in S$, such that $x \leq 0$

Sol. : Here statement $p : \exists$ a rational number $x \in S$, such that $x > 0$ has negation.

$\sim p : \forall$ rational numbers $x \in S$, $x \leq 0$

$\therefore \sim p : \text{for every rational number } x \in S, x \leq 0.$

Ans. : (B)

- (17) The negation of the statement 'if I become a teacher, then I will own a school' is
 [AIEEE : 2012]

- (A) I will become a teacher and I will not own a school.
 (B) Either I will not become a teacher or I will not own a school.
 (C) Neither I will become a teacher nor I will own a school.
 (D) I will not become a teacher or I will own a school.

Sol. : p : I will become a teacher.

q : I will own a school.

Here, given statement is of the form $p \Rightarrow q$.

The negation of $p \Rightarrow q$ is $\sim(p \Rightarrow q) = \sim(\sim p \vee q) = p \wedge \sim q$.

$\therefore p \wedge \sim q$: I will become a teacher and I will not own a school.

Ans. : (A)

- (18) For integers $m, n > 1$, the following three statements are given :

$p : n$ is divisible by m .

$q : n^2$ is divisible by m .

$r : m$ is a prime. Then, which of following is symbolic presentation ?

[Online : 2013]

- (A) $(q \wedge r) \Rightarrow p$ (B) $(p \wedge q) \Rightarrow r$ (C) $q \Rightarrow r$ (D) $q \Rightarrow p$

If m is a prime and n^2 is divisible by m , then n is also divisible by m .

Sol. : $(q \wedge r) \Rightarrow p$ is the symbolic presentation.

Ans. : (A)

- (19) If the truthvalue of r is F, where $r : p \Rightarrow (\sim p \vee q)$, then the truthvalues of p and q are respectively and
 [Online : 2013]

- (A) F, F (B) T, T (C) T, F (D) F, T

Sol. : Truthvalue of $p \Rightarrow (\sim p) \vee q$ is F.

$\therefore$ Truthvalue of p is T and truthvalue of $(\sim p) \vee q$ is F.

$\therefore$ Truthvalue of q is F.

Ans. : (C)

- (20) Equivalent statement of $\sim(p \Leftrightarrow \sim q)$ is
 [JEE : 2014]

- (A) $p \Leftrightarrow q$ (B) $\sim p \Leftrightarrow \sim q$ (C) t (D) c

Sol. :

p	q	$\sim p$	$\sim q$	$p \Leftrightarrow \sim q$	$\sim(p \Leftrightarrow \sim q)$	$p \Leftrightarrow q$	$(\sim p) \Leftrightarrow (\sim q)$
T	T	F	F	F	T	T	T
T	F	F	T	T	F	F	F
F	T	T	F	T	F	F	F
F	F	T	T	F	T	T	T

It is clear from the last two columns that $\sim(p \Leftrightarrow \sim q) = p \Leftrightarrow q = (\sim p) \Leftrightarrow (\sim q)$ **Ans. : (A), (B)**

- (21) The contrapositive of the statement 'I will go to school if it does not rain' is [Online : 2014]
- (A) If it rains, I will not go to school. (B) If I will not go to school, it rains.
(C) If it rains, I will go to school. (D) If I will go to school, it rains.

Sol. : Here p : It does not rain and q : I will go to school.

Here, I will go to school if it does not rain is a statement of the form $p \Rightarrow q$. Its contrapositive statement is $\sim q \Rightarrow \sim p$.

Thus, contrapositive statement : If I will not go to school, it rains.

Ans. : (B)

- (22) For three simple statements p, q and r , the logically equivalent statement of $p \Rightarrow (q \vee r)$ is [Online : 2014]

- (A) $(p \vee q) \Rightarrow r$ (B) $(p \Rightarrow q) \vee (p \Rightarrow r)$
(C) $(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$ (D) $(p \Rightarrow q) \wedge (p \Rightarrow \sim r)$

Sol. : $p \Rightarrow (q \vee r)$

$$= \sim p \vee (q \vee r)$$

$$= (\sim p \vee q) \vee (\sim p \vee r)$$

$$= (p \Rightarrow q) \vee (p \Rightarrow r)$$

OR

							(a)						(b)		
p	q	r	$\sim q$	$\sim r$	$p \vee q$	$q \vee r$	$p \Rightarrow (q \vee r)$	$p \Rightarrow q$	$p \Rightarrow r$	$p \Rightarrow \sim q$	$p \Rightarrow \sim r$	$(p \vee q) \Rightarrow r$	$(p \Rightarrow q) \vee (p \Rightarrow r)$	$(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$	$(p \Rightarrow q) \wedge (p \Rightarrow \sim r)$
T	T	T	F	F	T	T	T	T	T	F	F	T	T	F	F
T	T	F	F	T	T	T	T	T	F	F	T	F	T	F	T
T	F	T	T	F	T	T	T	F	T	T	F	T	T	T	F
T	F	F	T	T	T	F	F	F	F	T	T	F	F	F	F
F	T	T	F	F	T	T	T	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T	T	T	T	F	T	T	T
F	F	T	T	F	F	T	T	T	T	T	T	T	T	T	T
F	F	F	T	T	F	F	T	T	T	T	T	T	T	T	T

From columns (a) and (b) of the above table, it is clear that,

$$[p \Rightarrow (q \vee r)] = [(p \Rightarrow q) \vee (p \Rightarrow r)]$$

Ans. : (B)

- (23) The contrapositive of the statement 'if I do not feel well, then I will go to see a doctor' is [Online : 2014]

- (A) If I feel well, then I will not go to see a doctor.
(B) If I go to see a doctor, then I will feel well.
(C) If I do not go to see a doctor, then I will feel well.
(D) If I go to see a doctor, then I will not feel well.

Sol. : Here p : I do not feel well. q : I will go to see a doctor.

The given statement is of the form $p \Rightarrow q$. Its contrapositive statement is $\sim q \Rightarrow \sim p$.

Thus, the contrapositive statement of 'If I do not feel well, then 'I will go to see a doctor' is 'If I do not go to see a doctor, then I will feel well.'

Ans. : (C)

: Question with More Than One Correct Answers :

(24) Which of the following statements is a tautology ?

- (A) $(p \wedge q) \Rightarrow p$ (B) $[(p \vee q) \wedge (\sim q)] \Rightarrow p$ (C) $p \wedge t$ (D) $p \vee (\sim p)$

Sol. : Consider the statements p , q and t . We can find out which of the above statements is a tautology using the table given below :

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
p	q	t	$\sim p$	$\sim q$	$p \vee (\sim p)$	$p \wedge t$	$p \wedge q$	$(p \wedge q) \Rightarrow p$	$p \vee q$	$(p \vee q) \wedge (\sim q)$	$[(p \vee q) \wedge (\sim q)] \Rightarrow p$
T	T	T	F	F	T	T	T	T	T	F	T
T	F	T	F	T	T	T	F	T	T	T	T
F	T	T	T	F	T	F	F	T	T	F	T
F	F	T	T	T	T	F	F	T	F	F	T

It is clear from columns (6), (9) and (12) that,

$p \vee (\sim p)$, $(p \wedge q) \Rightarrow p$ and $[(p \vee q) \wedge (\sim q)] \Rightarrow p$ are tautologies.

Ans. : (A), (B), (D)

Second method :

$$\begin{aligned}(p \wedge q) \Rightarrow p &= (\sim p) \vee (\sim q) \vee p \\ &= [(\sim p) \vee (p)] \vee (\sim q) \\ &= t\end{aligned}$$

$$[(p \vee q) \wedge (\sim q)] \Rightarrow p = [(\sim p) \wedge (\sim q)] \vee q \vee p = \sim(p \vee q) \vee (p \vee q) = t$$

$$p \wedge t = p$$

$$p \vee (\sim p) \Rightarrow t$$

Ans. : (A), (B), (D)

(25) Which of the following statements are fallacies ?

- (A) $(p \wedge \sim q)$ (B) $(p \wedge \sim q) \wedge (\sim p \wedge q)$ (C) $(p \Rightarrow q) \wedge (q \Rightarrow r)$ (D) $p \wedge c$

Sol. : For statements p , q , r and c ,

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
p	q	r	c	$\sim q$	$p \wedge \sim q$	$\sim p$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \wedge c$
T	T	T	F	F	F	F	F	F	T	T	T	F
T	T	F	F	F	F	F	F	F	T	F	F	F
T	F	T	F	T	T	F	F	F	F	T	F	F
T	F	F	F	T	T	F	F	F	F	T	F	F
F	T	T	F	F	F	T	T	F	T	T	T	F
F	T	F	F	F	F	T	T	F	T	F	F	F
F	F	T	F	T	F	T	F	F	T	T	T	F
F	F	F	F	T	F	T	F	F	T	T	T	F

It is clear from the above columns (9) and (13) that,

$(p \wedge \sim q) \wedge (\sim p \wedge q)$ and $p \wedge c$ are fallacies.

Ans. : (B), (D)

Second method :

$$(p \wedge (\sim q)) \wedge ((\sim p) \wedge q) = (p \wedge \sim p) \wedge (q \wedge (\sim q)) = c$$

$$p \wedge c = c$$

The remaining two options are clearly not fallacies.

Ans. : (B), (D)

: Assertion and Reason Type Questions :

Explanation : Each of these questions contains statement 1 (Assertion) and statement 2 (Reason). Each question has four choices, option (A), (B), (C) and (D), which are as follows. You have to select the correct choice.

- (A) Statement 1 is true, statement 2 is true. Statement 2 is the correct explanation for statement 1.
 (B) Statement 1 is true, statement 2 is true. Statement 2 is not a correct explanation for statement 1.
 (C) Statement 1 is true, statement 2 is false.
 (D) Statement 1 is false, statement 2 is true.

(26) **Statement 1:** $\sim(p \Leftrightarrow \sim q)$ and $p \Leftrightarrow q$ are logically equivalent. [IIT : 2009]

Statement 2: $\sim(p \Leftrightarrow \sim q)$ is a tautology.

Sol. : $\sim(p \Leftrightarrow \sim q) = \sim(\sim(p \Leftrightarrow q)) = p \Leftrightarrow q$

(1) p	(2) q	(3) $\sim q$	(4) $p \Leftrightarrow q$	(5) $p \Leftrightarrow (\sim q)$	(6) $\sim(p \Leftrightarrow (\sim q))$
T	T	F	T	F	T
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	F	T

$\therefore$ Statement 1 is true.

(Columns (4) and (6) are same.)

$$\begin{aligned}\sim[p \Leftrightarrow \sim q] &= \sim[[p \Rightarrow (\sim q)] \wedge [(\sim q) \Rightarrow p]] \\ &= \sim[(\sim p) \vee (\sim q)] \wedge [q \vee p] \\ &= (p \wedge q) \vee (\sim p \wedge \sim q)\end{aligned}$$

If p is true, q is true or both are false then only this statement becomes true..

Statement 2 is false.

Ans. : (C)

(27) **Statement 1:** $(p \wedge (\sim q)) \wedge ((\sim p) \wedge q)$ is a fallacy.

Statement 2: $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is a tautology.

[JEE : 2013]

Sol. : $(p \wedge (\sim q)) \wedge ((\sim p) \wedge q) = (p \wedge (\sim p)) \wedge ((\sim q) \wedge q) = c \wedge c = c$

$\therefore$ Statement 1 is true.

Also, contrapositive of $p \Rightarrow q$ is $\sim q \Rightarrow \sim p$.

$$\therefore (p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p) = t.$$

$\therefore$ Statement 2 is true but it is not a correct explanation for statement 1.

Ans. : (B)

(28) **Statement 1:** $A \Rightarrow (B \Rightarrow A)$ and $A \Rightarrow (A \vee B)$ are logically equivalent.

Statement 2: $\sim[(A \wedge B) \Rightarrow ((\sim A) \vee B)]$ is a tautology.

[Online : 2013]

Sol. : $A \Rightarrow (B \Rightarrow A) = \sim A \vee (\sim B \vee A)$

$$= (\sim A \vee A) \vee (\sim B)$$

(Associative law)

$$= t \vee (\sim B) = t$$

Now, $A \Rightarrow (A \vee B) = \sim A \vee (A \vee B)$

$$= (\sim A \vee A) \vee B$$

(Associative law)

$$= t \vee B = t$$

$\therefore A \Rightarrow (B \Rightarrow A)$ and $A \Rightarrow (A \vee B)$ are logically equivalent.

$\therefore$ Statement 1 is true.

Also, $\sim[(A \wedge B) \Rightarrow ((\sim A) \vee B)]$

$$= \sim[\sim(A \wedge B) \vee (\sim A \vee B)]$$

$$= (A \wedge B) \wedge (A \wedge \sim B)$$

$$= A \wedge (B \wedge (\sim B)) = A \wedge c = c$$

$\therefore$ Statement 2 is a fallacy.

Thus, statement 2 is false.

$\therefore$ Statement 1 is true and statement 2 is false.

Ans. : (C)

