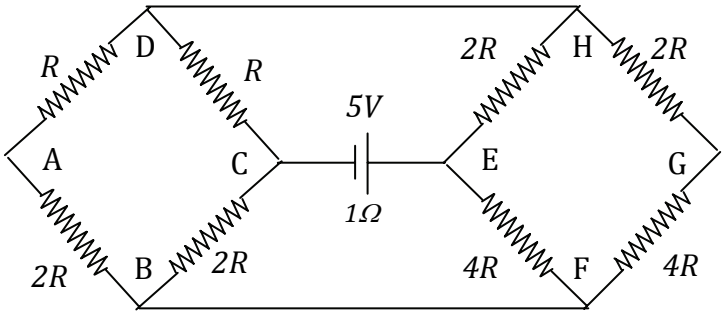
1. Which of the following statements is not true ?
(A) The resistance of intrinsic semiconductors decreases with increase of temperature.
(B) Doping pure Si with trivalent impurities give p-type semiconductors.
(C) The majority carriers in n-type semiconductors are holes.
(D) A p-n junction can act as a semiconductor diode.

2. Two resistive networks are connected through a battery of emf 5V and internal resistance 1 Ω, as shown below. The value of R for which the power delivered to the network by the battery will be maximum, is



(A) 1 Ω
(B) 0.5 Ω
(C) 1.2 Ω
(D) 2.4 Ω

3. A charged particle is released from rest in a region of constant and uniform electric and magnetic fields. The two fields are parallel to each other. The path of the particle's motion will be a
(A) straight line
(B) circle
(C) helix
(D) cycloid

4. A particle of charge e and mass m is executing a circular motion with a uniform angular speed ω. If the radius of the circular path be r, the angular momentum be L and magnetic moment due to circular loop be μ, then
(A) the current flowing in the circular path is proportional to the area of the loop
(B) L is proportional to the areal velocity
(C) the ratio of μ/L is in inversely proportional to the specific charge of the particle
(D) μ is proportional to m

Space for Rough Work

5. A moving coil galvanometer has a resistance of 100 Ω and shows full-scale deflection at a current of 100 μA. The galvanometer has to be used as an ammeter in the range of 0–100 mA so that 100 mA is the full-scale deflection current. A resistance R has to be connected in parallel. Then

(A) the value of R needed should be 1.0 Ω
(B) when this ammeter measures 100 mA, the current flowing in galvanometer is 40 μA
(C) for higher current measurement, value of R should be larger than the present value of R
(D) this new ammeter cannot measure −100 mA

6. An object is moving away from a vertical concave mirror of focal length 25 m. When the distance of the object is 100 m, the velocity of the object is 5 m/s and it accelerates at 2 m/s^2. The distance of the object from the image after 5 sec is

(A) 300 m
(B) 120 m
(C) 150 m
(D) 90 m

7. An electrostatic field $\vec{E} = \hat{i} + 2\hat{j} + 3\hat{k}$ passes through the surface $\vec{A} = 30\hat{j}$. The electric flux coming through the surface is

(A) 30 unit
(B) 90 unit
(C) 60 unit
(D) 120 unit

8. A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is

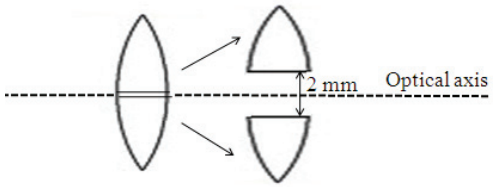
(A) 2P/3
(B) P
(C) 3P/2
(D) 2P

9. The current amplification factor of a transistor α is 0.9. The transistor is biased in a common-base configuration. In this connection, when the base current changes by 4 mA, the change in collector current is

(A) 4.44 mA
(B) 40 mA
(C) 36 mA
(D) 24 mA

Space for Rough Work

10. A bi-convex lens of focal length 10 cm is cut along the horizontal diameter and the two halves are kept 2 mm apart symmetrically about the optical axis as shown in the figure. A monochromatic point source of light is now placed at a distance 10 cm on the optical axis. Then which of the following statements is/are correct ?



(A) The rays emerging from each lens-half will be converging to a point on the optical axis at a distance of 5 cm from the lens-halves.

(B) The rays emerging from each lens-half will be parallel making an angle of 10^{-2} radian with the optical axis.

(C) Rays from each lens-half will be diverging as if the source is on the optic axis at a distance of 10/3 cm behind the lens-halves.

(D) The rays emerging from the upper lens-half will appear to come from a point 10/3 cm behind the lens-halves and 3 mm below the optical axis.

11. An infinitely long straight conductor has a circular loop of radius R meter. If a current I ampere flows through the conductor, then the magnetic induction at the centre of the circular loop is

(A) $\frac{\mu_0}{4\pi}\frac{2I}{R(\pi+1)}$ Tesla

(B) $\frac{\mu_0}{4\pi}\frac{2I}{R}$ Tesla

(C) $\frac{\mu_0}{4\pi}\frac{2I}{R(\pi-1)}$ Tesla

(D) Zero

12. Energy liberated per fission is about 200 MeV of $_{92}U^{235}$ nuclei. A fission reactor of power 1 MW consumes a mass *x* of $_{92}U^{235}$ per day. Here *x* is equal to

(A) 1 gm

(B) 10 gm

(C) 1 kg

(D) 10 kg

Space for Rough Work

13. A free electron has a wave function $\psi(x, t) = \sin(kx - \omega t)$. Given that $h = 6.626 \times 10^{-34}$ J-s, when $k = 50\ nm^{-1}$, the momentum of the electron in kg-m/s is

(A) 5.26×10^{-22}

(B) 2.62×10^{-32}

(C) 1.26×10^{-12}

(D) 6.62×10^{-24}

14. A non-conducting solid sphere of radius R is uniformly charged. At a distance r from the centre of the sphere the electric field (magnitude) due to sphere

(A) $E \propto 1/r^2$ for $r < R$ (inside sphere)

(B) $E \propto 1/r^2$ for $0 < r < \infty$ (everywhere except centre)

(C) $E \propto 1/r^2$ for $R < r < \infty$ (outside sphere)

(D) $E = 0$ at $r = R$ (at the surface)

15. The ground state wave function associated with a particle in a potential box of width L (i.e., $x = 0$ to $x = L$) is given by

(A) $\sqrt{(2/L)} \sin[(x/L)]$

(B) $\sqrt{(2/L)} \cos[(x/2L)]$

(C) $\sqrt{(2/L)} \sin[2(x/L)]$

(D) $\sqrt{(2/L)} \cos[2(x/L)]$

16. A circular loop of copper wire has a radius r and mass m. The loop is at rest on flat table in the horizontal plane xy. The earth's magnetic field at this point is $\hat{i}\, B_x + \hat{k} B_z$. When a current I flows through the loop, the loop starts tilting. The minimum value of current is

(A) $\dfrac{mg}{\pi r B_x}$

(B) $\dfrac{mg}{\pi r \sqrt{B_x^2 + B_z^2}}$

(C) $\dfrac{mg}{\pi r \sqrt{B_x B_z}}$

(D) $\dfrac{mg}{\pi r B_z}$

Space for Rough Work

17. A hydrogen atom is in its n^{th} excited state. The magnetic moment due to the electron of this excited hydrogen atom is

(A) $\frac{ne\hbar}{2m}$

(B) $\frac{me\hbar}{2n}$

(C) $\frac{neh}{2\pi m}$

(D) $\frac{meh}{2\pi n}$

18. 5.0 kg of steam at 200° C is kept in a frictionless piston-cylinder based container at a pressure of 400 kPa. Heat is transferred to steam at constant pressure in a quasi-static process till the temperature reaches 250° C. Assume the specific volumes of steam at 200° C and 250° C as 0.53434 m^3/kg and 0.59520 m^3/kg respectively. The work done by the steam is then

(A) 121.7 kJ
(B) –55.3 kJ
(C) 30.4 kJ
(D) –53.5 kJ

19. A plane diffraction grating has 100 lines per mm. The grating is illuminated by sodium light of wavelength 5890Å. The number of orders that will be visible is

(A) 6
(B) 12
(C) 10
(D) 16

20. The electric fields of two electromagnetic waves in a certain region are $E_1 = E_0 e^{ik\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}y\right)}$ and $E_2 = E_0 e^{ik\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}y\right)}$ respectively. The angle between these electromagnetic fields is

(A) 60°
(B) 30°
(C) 45°
(D) 120°

Space for Rough Work

21. A charged particle is thrown in a uniform magnetic field of flux density 1.5 Wb/m^2 with a speed of 2×10^7 m/sec making an angle of 30° with the direction of field. The particle experiences a force equal to

(A) 1.2×10^{12} N

(B) 2.4×10^{12} N

(C) 12×10^{12} N

(D) 24×10^{12} N

22. The differential equation describing the oscillation of a particle is given by $2\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 12x = 0$. The oscillation of the particle is

(A) underdamped

(B) critically damped

(C) overdamped

(D) free SHM

23. The electric field of an electromagnetic wave is given by $= E_o\cos(kz - ct)\hat{x} + E_o \sin(kz - ct)\hat{y}$. The wave is

(A) elliptically polarized

(B) left circularly polarized

(C) linearly polarized

(D) right circularly polarized

24. A thermal neutron has a speed v at temperature T=300K and kinetic energy given by $\frac{m_n v^2}{2} = \frac{3kT}{2}$. Take mass of neutron as $m_n = 1.67 \times 10^{-27}$ kg and $k = 1.38 \times 10^{-23}$ $m^2kgs^{-2}K^{-1}$. The de-Broglie wavelength associated with this neutron is

(A) 32 pm

(B) 23.3 pm

(C) 14 pm

(D) 1.22 pm

25. In a Young's double-slit experiment using a monochromatic light of wavelength 488 nm, the separation between the slits is 0.320 mm and interference fringes are formed on the screen. How many interference fringes will be observed in the angular range of $-30° < \theta < +30°$? Here θ is measured from the direction of the central fringe.

(A) 321

(B) 231

(C) 655

(D) 565

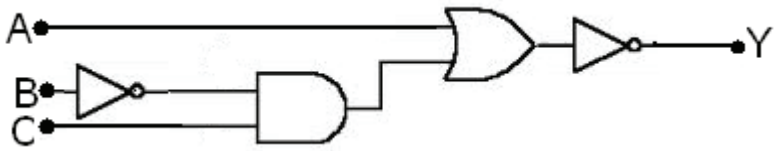
Space for Rough Work

26. The radii of two soap bubbles are r and 2r respectively. The excess pressures inside the bubbles are in the ratio
(A) 2 : 1
(B) 1 : 4
(C) 4 : 1
(D) 1 : 2

27. The dimension of a quantity is given by $ML^2T^{-2}I^{-2}$, where M is mass, L is length, T is time, and I is electric current. Which of the following quantities has this dimension ?
(A) Capacitor
(B) Inductance
(C) Magnetic Flux
(D) Electric Flux

28. In a hydrogen atom, radius of the first Bohr orbit of electron is 5.3×10^{-11} m. The de Broglie wavelength associated with this electron is then
(A) 3.3×10^{-11} m
(B) 33×10^{-11} m
(C) 6.62×10^{-13} m
(D) 66.2×10^{-13} m

29. The output of the logic gate is



(A) $\overline{A} + \overline{B \cdot C}$
(B) $\overline{A + B \cdot C}$
(C) $A + \overline{B} \cdot C$
(D) $\overline{A + \overline{B} \cdot C}$

30. What is the minimum thickness of a soap bubble needed for constructive interference in reflected light, if the light incident on the film is 750 nm ? Assume that the refractive index of the film is n = 4/3.
(A) 140.6 nm
(B) 281.2 nm
(C) 70.3 nm
(D) 210.9 nm

Space for Rough Work

31. For an ideal gas the Joule-Thomson coefficient will be

(A) > 0

(B) 0

(C) < 0

(D) cannot be predicted.

32. Entropy is a measure of

(A) randomness

(B) orderliness

(C) reactivity

(D) feasibility

33. The efficiency of a Carnot engine would be unity when

(A) the sink temperature is 0 °C

(B) the source temperature is 1000 °C

(C) the sink temperature is 0 K

(D) the source temperature is 100000 K

34. Free energy change, $\Delta G=0$ when

(A) catalyst is added

(B) the system is under equilibrium

(C) reactants are completely consumed

(D) reactants are initially mixed thoroughly.

35. Given the following notation for an electrochemical cell :

$Pt(s) \mid H_2(g) \mid H^+(aq) \parallel Ag^+(aq) \mid Ag(s)$

Which of the following represents the overall balanced (net) cell reaction ?

(A) $H_2(g)+Ag^+(aq) \rightarrow 2H^+(aq)+Ag(s)$

(B) $H_2(g)+Ag(s) \rightarrow H^+(aq) +Ag^+(aq)$

(C) $Ag(s) +H^+(aq) \rightarrow Ag^+(aq) +H_2(g)$

(D) None of these

Space for Rough Work

36. The structure of heteropolyacid ammonium phosphomolybdate contains

(A) one tetrahedral and twelve octahedral units

(B) twelve tetrahedral and one octahedral units

(C) four tetrahedral and seven octahedral units

(D) seven tetrahedral and six octahedral units

37. The atoms in BF_4^- ion supply

(A) 32 valence electrons

(B) 31 valence electrons

(C) 41 valence electrons

(D) 36 valence electrons

38. The ground state configuration of CO is

(A) $1\sigma^2 1\pi^2 2\pi^4 2\sigma^2$

(B) $1\sigma^2 2\sigma^2 3\sigma^4 1\pi^2$

(C) $1\sigma^2 2\sigma^2 1\pi^4 3\sigma^2$

(D) $1\sigma^2 2\sigma^2 1\pi^2 3\sigma^4$

39. The reaction of BCl_3 with excess NaCl in acidic aqueous solution gives

(A) BCl_4^- and Na^+

(B) $BOCl_2$ and HCl

(C) H_3BO_3 and NaOCl

(D) H_3BO_3 and HCl

40. Amongst the following VO, V_2O_3, VO_2, V_2O_5, Cr_2O_3 and CrO_3 the pair of amphoteric oxides is

(A) VO, Cr_2O_3

(B) V_2O_3, Cr_2O_3

(C) VO_2, Cr_2O_3

(D) V_2O_5, CrO_3

Space for Rough Work

41. The ground state electronic configuration of the Ti^{3+} is

(A) $[Ar]3d^2$

(B) $[Ar]3d^1$

(C) $[Ne]3d^9$

(D) $[Ar]4s^23d^1$

42. Reaction of fluorapetite with conc. H_2SO_4 gives

(A) P_4, $CaSiO_3$ and HF

(B) H_3PO_4, $CaSO_4$ and CaF_2

(C) H_3PO_4, $CaSO_4$ and HF

(D) $H_4P_2O_7$, $CaSO_4$ and HF

43. The average N-O bond length and O-N-O angle in NO_2^- are, respectively

(A) 1.15 Å, 180º

(B) 1.24 Å, 120º

(C) 1.12 Å, 115º

(D) 1.24 Å, 115º

44. Conc. HNO_3 is yellow in colour due to the presence of

(A) NO_2

(B) NO

(C) N_2O

(D) N_2O_5

45. At 250 ºC molten NH_4NO_3 gives

(A) NO and H_2O

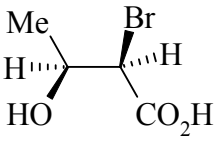
(B) N_2O and H_2O

(C) N_2O and H_2O_2

(D) NO_2 and H_2O

Space for Rough Work

46. Predict the absolute configuration for the following compound



(A) 2S, 3R

(B) 2S, 3S

(C) 2R, 3R

(D) 2R, 3S

47. The following compound cannot be resolved mainly due to the fact that

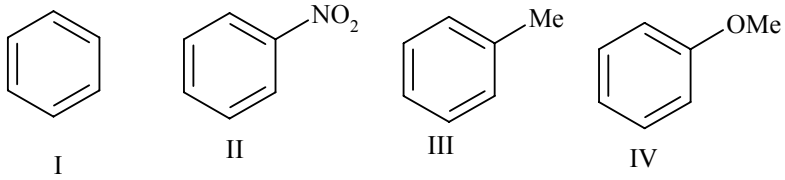
(A) The compound is not chiral

(B) Rapid interconversion between the enantiomers take place

(C) The compound is liquid in nature

(D) None of the above

48. Predict the correct order of affinity towards electrophilic substitution reaction of the following substrates :



(A) IV > III > I >II

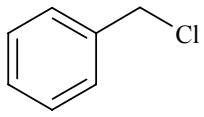
(B) III > IV > I >II

(C) IV > I >III >II

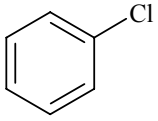
(D) I > II > III >IV

Space for Rough Work

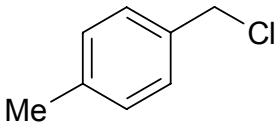
49. Predict the correct order of affinity towards nucleophilic substitution (S_N1) reaction of the following substrates :



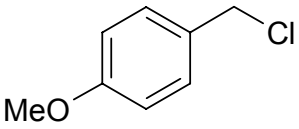
I



II



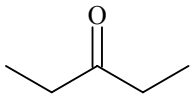
III

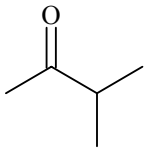


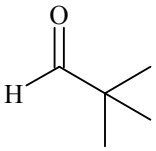
IV

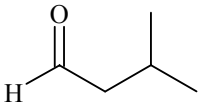
(A) II > I > IV > III
(B) III >IV > II > I
(C) IV > III > I > II
(D) I > III >IV >II

50. An aliphatic carbonyl compound (Mw = 86) gives a pair of oximes (with NH_2OH) which could be reduced to an amine that is resolvable, what is the correct structure of the parent carbonyl compound ?

(A) 

(B) 

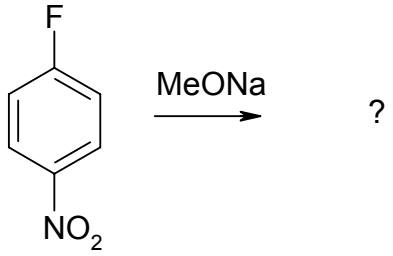
(C) 

(D) 

51. Reimer-Tiemann reaction of phenol involves the formation of which of the following intermediate ?
(A) Dichlorocarbene
(B) Trichlorocarbene
(C) Trichloromethane
(D) Nitrene

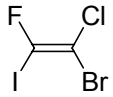
Space for Rough Work

52. The following reaction will yield which of the following product ?



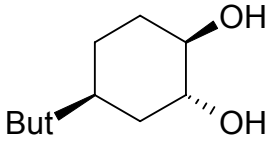
(A) o-Nitroanisole
(B) m-Nitroanisole
(C) p-Nitroanisole
(D) None of the above

53. What would be the double bond geometry in the following compound ?



(A) Z
(B) E
(C) E, Z
(D) None of the above

54. The following diol cannot be cleaved by periodic acid mainly due to



(A) Both the OH groups are in equatorial position
(B) Both the OH groups are in axial position
(C) One of the OH group is in equatorial position and another is in axial position
(D) None of the above

55. The number of gauche-butane interaction present in *cis*-1,2-dimethyl cyclohexane is
(A) 2
(B) 3
(C) 4
(D) None of the above

Space for Rough Work

56. What happens when a catalyst is added to a system at equilibrium ?

(A) The heat of reaction decreases.

(B) The reaction follows an alternative pathway of lower activation energy.

(C) The potential energy of the reactants decreases.

(D) The potential energy of the products decreases.

57. Which of the following is false ?

(A) Phase diagram provides information on the transformation rates.

(B) Phase diagram indicates the relative amounts of different phases that can be found under given equilibrium conditions.

(C) Phase diagram indicates the temperature at which different phases start to melt.

(D) Solid solubility limits are depicted by the phase diagram.

58. In a condensed system having a single-component, if the degree of freedom is zero, maximum number of co-existing phases will be

(A) 0

(B) 1

(C) 2

(D) 3

59. Which of the following has maximum entropy of vapourisation ?

(A) water (*l*)

(B) toluene (*l*)

(C) diethyl ether (*l*)

(D) acetone (*l*)

60. The internal energy of one mole of an ideal gas is

(A) 4RT

(B) (5/2)RT

(C) (7/2)RT

(D) (3/2)RT

Space for Rough Work

61. Shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is

(A) $3\sqrt{10}$
(B) $3\sqrt{30}$
(C) $10\sqrt{3}$
(D) $3\sqrt{3}$

62. Condition that the plane $x + y + z = 1$ should touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2uz + v$ is

(A) $(2u + v + 1)^2 = 2v^2 + u^2 - u$
(B) $(u + 2v + 1)^2 = v^2 + 2u^2 - u$
(C) $(u + 2v + 1)^2 = u^2 + 2v^2 - v$
(D) $(2u + v + 1)^2 = 2u^2 + v^2 - v$

63. Locus of the points from which three mutually perpendicular lines can be drawn to intersect the conic $z = 0, ax^2 + by^2 = 1$ is

(A) $2(ax^2 + by^2) + z^2(a - b) = 1$
(B) $ax^2 + by^2 + z^2(a - b) = 1$
(C) $ax^2 + by^2 + z^2(a + b) = 1$
(D) $2(ax^2 + by^2) + z^2(a + b) = 1$

64. Laplace transform of $\int_t^\infty \frac{e^{-\theta}}{\theta} d\theta =$

(A) $\frac{1}{s} \ln (s + 1)$
(B) $s \ln(s + 1)$
(C) $\frac{1}{s} \ln\left(\frac{1}{s} + 1\right)$
(D) $s \ln\left(\frac{1}{s} + 1\right)$

65. If Laplace transform of F(t) is f(s), then Laplace transform of $\frac{d^3F}{dt^3}$ is

(A) $s^2f(s) + s^2F(0) + sF'(0) + F''(0)$
(B) $s^2f(s) - s^2F(0) + sF'(0) - F''(0)$
(C) $s^2f(s) - s^2F(0) - sF'(0) - F''(0)$
(D) $s^2f(s) + s^2F(0) + sF'(0) - F''(0)$

Space for Rough Work

66. $J_n(x)$ denotes the Bessel's function of order n. Solution of the differential equation

$9x^2y'' + 9xy' + (36x^4 - 16)y = 0$ is

(A) $c_1 J_{\frac{1}{3}}(x^2) + c_2 J_{-\frac{2}{3}}(x^2)$

(B) $c_1 J_{\frac{1}{3}}(x^2) + c_2 J_{-\frac{1}{3}}(x^2)$

(C) $c_1 J_{\frac{2}{3}}(x^2) + c_2 J_{-\frac{2}{3}}(x^2)$

(D) $c_1 J_{\frac{2}{3}}(x^2) + c_2 J_{-\frac{1}{3}}(x^2)$

where c_1 and c_2 are arbitrary constants.

67. If $\{x_n\}$ is the sequence of iterates used for Secant method to find root of an equation $f(x) = 0$, then for $n \geq 1$, $x_{n+1} - x_n =$

(A) $\dfrac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$

(B) $\dfrac{x_{n-1} - x_n}{f(x_n) - f(x_{n-1})} f(x_n)$

(C) $\dfrac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_{n-1})$

(D) $\dfrac{x_{n-1} - x_n}{f(x_n) - f(x_{n-1})} f(x_{n-1})$

68. Let $f(x) = x^n$, $n \geq 0$ be an integer and $x_0, x_1, \ldots, x_n$ are n + 1 distinct numbers. Then the divided difference $f[x_0, x_1, \ldots, x_n] =$

(A) 0

(B) 1

(C) (n + 1)!

(D) n!

Space for Rough Work

69. Value of $\int_1^2 \frac{dx}{x}$ using Simpson's $\frac{1}{3}$ rule, with h = 0.25, upto 6 decimal places is

(A) 0.693254
(B) 0.731212
(C) 0.681154
(D) 0.729318

70. For a given function y = f(x), approximate value of f'(1 + 0.02) =

(A) $\frac{1}{.02[f(1) - 4f(1 + 0.01) + 3f(1 + 0.02)]}$

(B) $\frac{1}{.02[f(1) - 3f(1 + 0.01) + 4f(1 + 0.02)]}$

(C) $\frac{1}{.02[f(1) - 2f(1 + 0.01) + f(1 + 0.02)]}$

(D) $\frac{1}{.02[f(1) + 2f(1 + 0.01) - f(1 + 0.02)]}$

71. Legendre polynomial of degree 4 is

(A) $\frac{1}{4}(30x^4 - 35x^2 + 3)$

(B) $\frac{1}{4(30x^4 + 35x^2 - 3)}$

(C) $\frac{1}{8}(35x^4 - 30x^2 + 3)$

(D) $\frac{1}{8}(35x^4 - 30x^2 - 3)$

72. Laplace transform of $5e^{2t} \sinh 2t$ is

(A) $\frac{10}{(s + 2)^2 + 4}$

(B) $\frac{5}{(s + 2)^2 + 4}$

(C) $\frac{5}{(s - 2)^2 - 4}$

(D) $\frac{10}{(s - 2)^2 - 4}$

Space for Rough Work

73. Let f be a function from the set of positive integers to itself. If f(n) is defined as the maximum of n and 50, then f is

(A) one to one but not onto
(B) onto but not one to one
(C) both one to one and onto
(D) neither one to one nor onto

74. Let S be the set of all functions from the set of integers to itself. Then the relation R = {(f, g) : f(0) = g(1) and f(1) = g(0)} is

(A) only reflexive
(B) only symmetric
(C) only transitive
(D) an equivalence relation

75. Consider the equivalence relation R = {(x, y) : x – y is an integer} on the set of all real numbers. Then the equivalence class of 1 with respect to R is

(A) {x + 1 : x is a real number}
(B) set of all real numbers
(C) set of all integers
(D) set of all positive integers

76. Which of the following is a Hermitian matrix ?

(A) $\begin{pmatrix} -4 & 0 & -3i \\ 0 & 5 & 0 \\ -3i & 0 & -4 \end{pmatrix}$

(B) $\begin{pmatrix} 4 & 0 & 3i \\ 0 & 5 & 0 \\ -3i & 0 & -4 \end{pmatrix}$

(C) $\begin{pmatrix} 4 & 0 & 3i \\ 0 & 5 & 0 \\ 3i & 0 & -4 \end{pmatrix}$

(D) $\begin{pmatrix} 4 & 0 & -3i \\ 0 & 5 & 0 \\ -3i & 0 & -4 \end{pmatrix}$

Space for Rough Work

77. The inverse of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ is

(A) $\begin{pmatrix} 2 & 0 & 0 \\ 5 & 1 & 0 \\ 4 & 1 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

(D) None of these

78. The rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 3 \\ 2 & 2 & 2 & 6 \\ -1 & -1 & -1 & -3 \end{pmatrix}$ is

(A) 1
(B) 2
(C) 3
(D) 4

79. Maximum number of linearly independent columns in $\begin{pmatrix} 1 & 2 & 1 & 3 \\ 2 & 3 & -1 & -6 \\ 3 & -2 & -4 & -2 \end{pmatrix}$ is

(A) 1
(B) 2
(C) 3
(D) 4

Space for Rough Work

80. Which of the following is an eigenvector of $\begin{pmatrix} 1 & 0 & 3 \\ 1 & 2 & 1 \\ 3 & 0 & 1 \end{pmatrix}$?

(A) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

(B) $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

(D) $\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$

81. $x = 2k$, $y = -k$, $z = 0$ is a solution of the system

$x + 2y + 2z = 0$

$2x + 4y + 2z = 0$

$-3x - 6y - 4z = 1$ for

(A) $k = 0$

(B) $k = 1$

(C) $k = 2$

(D) No value of k

82. Under matrix multiplication the set of all 2×2 matrices with real entries and determinant equal to one is

(A) not a group

(B) a non commutative group

(C) a commutative group

(D) a cyclic group

Space for Rough Work

83. The function $f(x) = |x|$ from the group G of non-zero real numbers under multiplication to the group G' of positive real numbers under multiplication is

(A) a homomorphism

(B) not a homomorphism

(C) not a onto homomorphism

(D) an isomorphism

84. Let G be the group of integers modulo 8 (under addition modulo 8) and $H = \{0, 4\}$ be a subgroup of G. Then total number of left cosets of H in G is

(A) 1

(B) 2

(C) 3

(D) 4

85. If G is a finite group with fewer than 100 elements and G has subgroups of orders 10 and 25, then what is the order of G ?

(A) 25

(B) 50

(C) 75

(D) 100

86. Which of the following cannot be order of a field ?

(A) 2

(B) 4

(C) 6

(D) 9

87. A ring that is cyclic under addition is

(A) not necessarily a commutative ring

(B) a commutative ring

(C) an integral domain

(D) a field

Space for Rough Work

88. The value of $\lim_{n \to \infty} \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + ... + \frac{1}{\sqrt{n^2+n}} \right)$ is

(A) 0
(B) 1
(C) ∞
(D) does not exist

89. Let $f(x) = \begin{cases} x, & \text{for } 0 < x < 1 \\ 2 - x, & \text{for } 1 \leq x \leq 2 \\ x - \frac{x^2}{2}, & \text{for } x > 2, \end{cases}$

then $f(x)$ is

(A) not differentiable at $x = 2$
(B) differentiable at $x = 1$
(C) differentiable at both $x = 1$ and $x = 2$
(D) not differentiable at $x = 1$ but differentiable at $x = 2$

90. For $|x| \leq 1$, Maclaurin series expansion of $f(x) = \cot^{-1}x$ is

(A) $\frac{\pi}{2} + \left(x - \frac{x^3}{3} + \frac{x^5}{5} - ... \right)$

(B) $\frac{\pi}{2} - x - \frac{x^3}{3} - \frac{x^5}{5} - \ldots$

(C) $\frac{\pi}{2} - \left(x - \frac{x^3}{3} + \frac{x^5}{5} - ... \right)$

(D) None of these

91. The radius of curvature of $x^2 + y^2 + 6x + 8y = 0$ at (0,0) is

(A) 2
(B) 3
(C) 4
(D) 5

Space for Rough Work

92. The curve $y - 3 = 6(x - 2)^5$ has a point of inflexion at

(A) $x = 2$

(B) $x = 3$

(C) $x = -2$

(D) $x = -3$

93. Value of $\int \sqrt{1 + \sin x}\, dx$ is

(A) $2 \cos \frac{x}{2} + 2 \sin \frac{x}{2} + c$

(B) $-2 \cos \frac{x}{2} + 2 \sin \frac{x}{2} + c$

(C) $2 \cos \frac{x}{2} - 2 \sin \frac{x}{2} + c$

(D) $-2 \cos \frac{x}{2} - 2 \sin \frac{x}{2} + c$

94. The value of $\int \frac{dx}{\sqrt{(2x - x^2)^3}}$ is

(A) $\frac{x - 1}{\sqrt{2x - x^2}} + c$

(B) $\frac{x + 1}{\sqrt{2x - x^2}} + c$

(C) $\frac{x^2 - 1}{\sqrt{2x - x^2}} + c$

(D) $\frac{x^2 + 1}{\sqrt{2x - x^2}} + c$

95. The area of the region bounded by the curve $y = x(x - 1)(x - 2)$ and the x-axis is

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

Space for Rough Work

96. General solution of $y\,dx - 4(x + y^6)dy = 0$ is

(A) $x = 2y^6 + cy^4$

(B) $x = 2y^5 + cy^3$

(C) $x = cy^6 + y^4$

(D) $x = cy^5 + y^3$

97. Which of the following is NOT an exact equation ?

(A) $(2x - 1)dx + (3y + 7)dy = 0$

(B) $(5x + 4y)dx + (4x - 8y^3)dy = 0$

(C) $(x^2 - y^2)dx + (x^2 - 2xy)dy = 0$

(D) $(2xy^2 - 3)dx + (2x^2y + 4)dy = 0$

98. General solution of $12y'' - 5y' - 2y = 0$ is

(A) $y = C_1e^{2x} + C_2e^{-\frac{x}{4}}$

(B) $y = C_1e^{\frac{2x}{3}} + C_2e^{-\frac{x}{4}}$

(C) $y = C_1e^{-\frac{2x}{3}} + C_2e^{\frac{x}{4}}$

(D) $y = C_1e^{-\frac{2x}{3}} + C_2e^{-\frac{x}{4}}$

99. Which of the following is the general solution of $y'' + y = \sin x$?

(A) $y = C_1 \cos x + C_2 \sin x - \frac{1}{2}x \cos x$

(B) $y = C_1 \cos x + C_2 \sin x + \frac{1}{2}$

(C) $y = C_1 \cos x + C_2 \sin x + \frac{1}{2} - \frac{1}{6}\cos 2x$

(D) $y = C_1 \cos x + C_2 \sin x + \frac{1}{6}\cos 2x$

Space for Rough Work

100. The directional derivative of $f(x, y, z) = \sqrt{x^2 y + 2y^2 z}$ at (–2, 2, 1) in the direction of the origin is

(A) –1
(B) 1
(C) 2
(D) –2

101. The divergence of $f(x, y, z) = 3x^2 y\vec{i} + 2xz^3\vec{j} + y^4\vec{k}$ is

(A) $2xy$
(B) $3xy$
(C) $5xy$
(D) $6xy$

102. Area of the triangle determined by the points $P_1(1, 2, 4)$, $P_2(1, -1, 3)$, $P_3(-1, -1, 2)$ in square units is

(A) $\frac{5}{2}$
(B) $\frac{7}{2}$
(C) $\frac{9}{2}$
(D) $\frac{11}{2}$

103. Which of the following is a linearly independent set of vectors ?

(A) {(2, 5, 7), (1, 3, 4), (0, 1, 1)}
(B) {(1, 2, 3), (2, 5, 7), (1, 3, 4)}
(C) {(1, 2, 3), (1, 3, 5), (2, 5, 7)}
(D) {(2, 5, 7), (1, 3, 5), (1, 2, 2)}

104. Which of the following is a subspace of R^3 ?

(A) $\{(x, y, z) \in R^3 \mid x \geq 0\}$
(B) $\{(x, y, z) \in R^3 \mid x^2 + y^2 + z^2 \leq 1\}$
(C) $\{(x, y, z) \in R^3 \mid x + y + z = 1\}$
(D) $\{(x, y, z) \in R^3 \mid 2x - y + z = 0\}$

Space for Rough Work

105. Rank of the system $x + y + 2z = 0$, $2x + 3y + 3z = 0$, $x + 3y + 5z = 0$ is

(A) 0

(B) 1

(C) 2

(D) 3

106. Basis of the image of the linear transformation $T : R^3 \to R^2$ defined by $T(x, y, z) = (x + y + z, 2x + 2y + 2z)$ is

(A) $\{(1, 2)\}$

(B) $\{(1, 1), (2, 0)\}$

(C) $\{(2, 3)\}$

(D) $\{(0, 1), (1, 0)\}$

107. Value of the $\int_0^1 \int_{x^2}^{x} (1 - 2xy)\, dy\, dx$ is

(A) $\frac{1}{6}$

(B) $\frac{1}{12}$

(C) 3

(D) 8

108. Volume of the region bounded by $z = x^2 + y^2$ and $z = 2x$ is

(A) $\frac{2\pi}{3}$

(B) $\frac{\pi}{4}$

(C) π

(D) $\frac{\pi}{2}$

Space for Rough Work

109. $\int_1^2 \left[\int_1^{x^2} f(x, y)\, dy \right] dx =$

(A) $\int_1^4 \left[\int_{\sqrt{y}}^1 f(x, y)\, dx \right] dy$

(B) $\int_1^4 \left[\int_{\sqrt{y}}^2 f(x, y)\, dx \right] dy$

(C) $\int_1^2 \left[\int_1^{\sqrt{y}} f(x, y)\, dx \right] dy$

(D) $\int_1^2 \left[\int_{\sqrt{y}}^2 f(x, y)\, dx \right] dy$

110. Γ stands for Gamma function. $\Gamma\left(-\frac{5}{2}\right) =$

(A) $\frac{8\sqrt{\pi}}{15}$

(B) $-\frac{8\sqrt{\pi}}{15}$

(C) $\frac{2\sqrt{\pi}}{15}$

(D) $-\frac{2\sqrt{\pi}}{15}$

111. Limit of the sequence $\{u_n\}$, where $u_1 = 1$ and $u_{n+1}^2 = 3u_n$ for $n \geq 1$, is

(A) 1

(B) 3

(C) $\sqrt{3}$

(D) does not exist

Space for Rough Work

112. What must be the value of f(1) so that the function $f(x) = \frac{x^2 - 1}{x^3 - 1}$ is continuous at $x = 1$?

(A) $\frac{2}{3}$

(B) 2

(C) $\frac{1}{3}$

(D) 1

113. Coefficient of third non-zero term in the expansion of $x^{80} - x^{40} + x^{20}$ in power of $(x - 1)$ is

(A) 60

(B) 3680

(C) 2570

(D) 90

114. $\lim_{(x, y) \to (0, 0)} (1 + x^2y^2)^{\frac{1}{x^2 + y^2}} =$

(A) 0

(B) – 1

(C) 1

(D) ∞

115. If $u = x^{\frac{y}{z}}$ then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ at (1, 1, 1) is

(A) 1

(B) 0

(C) – 1

(D) 3

116. Coefficient of $(x - 1)(y - 1)$ in the Taylor's expansion of x^y in power of $(x - 1)$ and $(y - 1)$ is

(A) 0

(B) 1

(C) 2

(D) $\frac{1}{2}$

Space for Rough Work

117. $f(x, y) = x^3 + y^3 - 3xy$ has minimum at

(A) (0, 0)

(B) (1, 1)

(C) (1, –1)

(D) (–1, –1)

118. Length of one of the sides of a right angled triangle having least perimeter and surface area 4 units is

(A) 2

(B) $2\sqrt{2}$

(C) $\sqrt{2}$

(D) None of these

119. The series $\sum_{n=0}^{\infty}(-1)^n (x+1)^n$ is convergent for

(A) $x \in [-2, 0]$

(B) $x \in [-2, 0)$

(C) $x \in (-2, 0)$

(D) $x \in (-2, 0]$

120. Inverse Laplace transform of $\frac{1}{\sqrt{2s+3}}$ is

(A) $\frac{te^{-\frac{3t}{2}}}{\sqrt{2\pi}}$

(B) $\frac{te^{\frac{3t}{2}}}{\sqrt{2\pi}}$

(C) $\frac{e^{-\frac{3t}{2}}}{\sqrt{2\pi t}}$

(D) $\frac{e^{\frac{3t}{2}}\sqrt{t}}{\sqrt{2\pi}}$

Space for Rough Work

Space For Rough Work