

## P.G. ENTRANCE EXAMINATION, APRIL 2022

## MATHEMATICS

Time : Two Hours

Maximum : 200 Marks

*Each question carries 4 marks.**1 mark will be deducted for each wrong answer.*1.  $x^2 - 4y^2 - 2x + 16y - 40 = 0$  represents :

- (a) A pair of straight lines.
- (b) An ellipse.
- (c) A hyperbola.
- (d) A parabola.

2. Suppose  $G$  is a cyclic group and  $\sigma, \tau \in G$  are such that  $\text{order}(\sigma) = 12$  and  $\text{order}(\tau) = 21$ . Then the order of the smallest group containing  $\sigma$  and  $\tau$  is :

- (a) 261.
- (b) 84.
- (c) 3.
- (d) 84.

3. The product of two negative real numbers is not negative. Is given by ?

- (a)  $\exists x \forall y ((x < 0) \wedge (y < 0) \rightarrow (xy > 0))$ .
- (b)  $\exists x \exists y ((x < 0) \wedge (y < 0) \wedge (xy > 0))$ .
- (c)  $\forall x \exists y ((x < 0) \wedge (y < 0) \wedge (xy > 0))$ .
- (d)  $\forall x \forall y ((x < 0) \wedge (y < 0) \rightarrow (xy > 0))$ .

Turn over

4. If  $z_1, z_2$  are any Complex numbers, then  $|z_1 + z_2|^2 + |z_1 - z_2|^2 =$

(a)  $2\{|z_1|^2 - |z_2|^2\}$ .

(b)  $2\{|z_1|^2 + |z_2|^2\}$ .

(c)  $\{|z_1|^2 + |z_2|^2\}$ .

(d) None of these.

5. If the amplitude of the Complex number  $z$  be  $\theta$ , then amplitude of  $iz$  is :

(a)  $-\theta$ .

(b)  $\theta + \pi/2$ .

(c)  $\theta + \pi$ .

(d) None of these.

6. The value of  $\int_C \frac{z^2 + 5z + 6}{(z - 2)} dz$ , where  $C$  is  $|z| = 3$  is :

(a)  $2\pi i$ .

(b) 1.

(c) 0.

(d)  $40\pi i$ .

7. Which of the following statements is correct ?

- (a) Every LP problem has at least one optimal solution.
- (b) Every LP problem has a unique solution.
- (c) If an LP problem has two optimal solutions, then it has infinitely many solutions.
- (d) If a feasible region is unbounded then LP problem has no solution.

8. Which one of the following is true ?

- (a) The number of groups of order 14 up to isomorphism is 2.
- (b) The number of groups of order 14 up to isomorphism is 3.
- (c) The number of groups of order 26 up to isomorphism is 1.
- (d) The number of groups of order 14 up to isomorphism is 1.

9. Laplace integral transform of  $\frac{\sin x}{x}$  :

(a) Does not exist.

(b) Is  $\tan^{-1} p$ .

(c) Is  $\frac{\pi}{2} - \tan^{-1} p$ .

(d) Is  $\frac{d}{dp} \left[ \frac{1}{p^2 + 1} \right]$ .

10. Which one of the following sets has no extreme point ?

(a)  $\{(x, y) : x \geq 0, y \geq 0\}$ .

(b)  $\{(x, y) : x^2 + y^2 \leq 1\}$ .

(c)  $\{(x, y) : x \geq \alpha, \alpha \in \mathbf{R}\}$ .

Turn over

11. Let  $\{a_n\}$  be a sequence of positive real numbers. The series  $\sum_{n=1}^{\infty} a_n$  converges if the series :

(a)  $\sum_{n=1}^{\infty} a_n^2$  converges.

(b)  $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$  converges.

(c)  $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$  converges.

(d)  $\sum_{n=1}^{\infty} \frac{a_n}{a_{n+1}}$  converges.

12. If  $\int_0^1 \int_{2y}^2 e^{x^2} dx dy = k(e^4 - 1)$ , then  $k$  equals :

(a)  $\frac{1}{8}$ .

(b)  $\frac{1}{2}$ .

(c)  $\frac{1}{3}$ .

(d)  $\frac{1}{4}$ .

13. Which among the following is not a characteristic of gradient of a scalar field ?

(a) Line integral of a gradient is path-independent.

(b) Closed line integral of a gradient is zero.

(c) Gradient is a measure of the maximum rate of change in the field.

(d) Gradient is a scalar quantity.

14. Order of the permutation  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 6 & 4 \end{pmatrix}$  is:

(a) 2.

(b) 3.

(c) 6.

(d) 4.

15. Consider the following subset of  $Z$  :

$$H = \{30x + 42y + 70z \mid x, y, z \in Z\}$$

Which among the following is/are wrong ?

- (a)  $H$  is a subgroup of  $Z$ .
- (b) 2 generator for  $H$ .
- (c) 3 generator for  $H$ .
- (d) All of the above are wrong.

16. Which among the following is correct ?

(a)  $\beta(m, n) = \frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)}$ .

(b)  $\frac{\beta(m, n)}{\Gamma(m)} = \Gamma(m+n)$ .

(c)  $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma((m+n))}$ .

(d)  $\beta(m, n) = \frac{\Gamma(m)\Gamma(m+n)}{\Gamma(n)}$ .

17. How many elements in the Power set of set  $A = \{\{\Phi\}, \{\Phi, \{\Phi\}\}\}$  ?

- (a) 4.
- (b) 2.
- (c) 8.
- (d) 6.

18. The interior angles of polygon are in arithmetic progression. The smallest angle is 120 and the common difference is 5. Then the number of sides of the polygon is :
- (a) 9 or 16.
  - (b) 9.
  - (c) 16.
  - (d) 13.
19. Choose the incorrect option for  $n > 2$  :
- (a)  ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$ .
  - (b)  ${}^n C_r = 1$  for  $r = 0$  and  $r = n$ .
  - (c)  ${}^n C_r = {}^{n-1} C_{r-1} + {}^{n-1} C_r$ .
  - (d) None of the above.
20. A value of  $c$  for which the conclusion of mean value theorem holds for the function  $f(x) = \log_e x$  on the interval  $[1, 3]$  is :
- (a)  $2 \log_3 e$ .
  - (b)  $\frac{1}{2} \log_3 e$ .
  - (c)  $\text{Log}_3 e$ .
  - (d)  $\text{Log}_e 3$ .
21. The matrix  $A = \begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}$  is :
- (a) Positive definite.
  - (b) Positive semi definite.
  - (c) Negative semi definite.
  - (d) Negative definite.

22. If  $u = (x^2 + y^2)^{-\frac{1}{2}}$  then  $x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} =$

(a)  $2(x^2 + y^2)^{-\frac{1}{2}}$ .

(b)  $(x^2 + y^2)^{-\frac{1}{2}}$ .

(c)  $-(x^2 + y^2)^{-\frac{1}{2}}$ .

(d)  $\frac{1}{2}(x^2 + y^2)^{-\frac{1}{2}}$ .

23. The nature of the PDE  $u_{xx} - xu_{yy} = 0$  is:

(a) Parabolic.

(b) Hyperbolic.

(c) Elliptic.

(d) None of the above.

24. The error bound for the 5<sup>th</sup> approximation to the solution of the non-linear equation  $f(x) = 0$  in  $[1.5, 2]$  using bisection method is :

(a)  $\frac{1}{32}$ .

(b)  $\frac{1}{64}$ .

(c)  $\frac{1}{16}$ .

(d)  $\frac{1}{128}$ .

Turn over

25. Jacobian of the transformation

$$x = v/u, \text{ and } y = v.$$

(a)  $-\frac{v}{u^2}$ .

(b)  $\frac{v}{u^2}$ .

(c)  $-\frac{v^2}{u}$ .

(d)  $-\frac{2v}{u^2}$ .

26. Which among the following is False :

(a)  $\frac{1}{x}$  is not uniformly continuous on  $(0, 1]$ .

(b)  $x^2$  is not uniformly continuous on  $[0, \infty)$

(c)  $\frac{1}{x}$  is uniformly continuous on  $[1, \infty)$

(d)  $x^2$  is not uniformly continuous on  $[1, 2]$ .

27. Choose the wrong statement :

(a) Every differentiable function is continuous.

(b) Every uniformly continuous function is continuous.

(c) Every continuous function is differentiable.

(d) There exists continuous functions that are nowhere differentiable.

28. The number of possible solutions of the equation  $f(x) = 0$ , given that  $f(x)$  continuous in  $[a, b]$  and  $f(a)f(b) < 0$  is :

- (a) 0.
- (b) 2.
- (c) 7.
- (d) 10.

29. The  $n^{\text{th}}$  derivative of  $g$  at  $x = 0$  is given by  $g^{(n)}(0) = \frac{\sqrt{n+7}}{n^3}$  for  $n \geq 1$ . Then the coefficient for the term containing  $x^2$  in the Maclaurin series of  $g$  is :

- (a)  $\frac{3}{8}$ .
- (b)  $\frac{3}{16}$ .
- (c)  $\frac{3}{12}$ .
- (d)  $\frac{3}{8}$ .

30. The inverse of the matrix  $M = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$  is :

- (a)  $M - I$ .
- (b)  $M^2 - I$ .
- (c)  $I - M^2$ .
- (d)  $I - M$ .

where  $I$  is the identity matrix.

31. Radius of convergence of the power series  $\sum_{n=1}^{\infty} \frac{x^n}{n \cdot 5^n}$  is :
- (a) 1.  
(b) 2.  
(c) 0.  
(d) 5.
32. System of equations  $x + y = 1; y + z = 1; x + z = 1$  is :
- (a) Inconsistent.  
(b) Consistent with a single non-trivial solution.  
(c) Consistent with infinitely many solutions.  
(d) Only trivial solution exists.
33. How many words of 3 consonants and 2 vowels can be formed from a given set of 7 consonants and 4 vowels ?
- (a) 25200.  
(b) 21000.  
(c) 12000.  
(d) 22500.
34. Let  $x = \frac{5!}{4 \times 3!}$ . Which of the following expressions is not equal to  $x$  :
- (a)  $5 \times \frac{1}{0!}$ .  
(b)  $5 \times \frac{1}{0}$ .  
(c)  $5 \times \frac{6}{3! + (3 \times 2) + (3 \times 2 \times 1)}$ .  
(d)  $5 \times \frac{18}{3! + (3 \times 2) + (3 \times 2 \times 1)}$ .

35. The value of  $r$  such that the ratio of  ${}^3P_r$  and  ${}^4P_{r-1}$  will be  $\frac{1}{2}$  is :
- (a) 1.  
(b) 2.  
(c) 3.  
(d) 4.
36. If  $(p + q)^{\text{th}}$  term of a G.P. is  $m$  and  $(p - q)^{\text{th}}$  term is  $n$ , then the  $p^{\text{th}}$  term will be :
- (a)  $m/n$ .  
(b)  $\sqrt{mn}$ .  
(c)  $mn$ .  
(d) 0.
37. If the HCF of 65 and 117 can be expressed in the form  $65m - 117$ , then the value of  $m$  is :
- (a) 4.  
(b) 2.  
(c) 1.  
(d) 3.
38. Which of the following Diophantine equation cannot be solved :
- (a)  $6x + 51y = 22$ .  
(b)  $33x + 14y = 115$ .  
(c)  $14x + 35y = 93$   
(d)  $11x + 13y = 21$ .

39. The value of  $\sqrt{6 + (\sqrt{6 + (\sqrt{6 + \dots}})}$  is:

(a) 6.

(b) 3.

(c) 36.

(d) 2.

40. The last two digits of decimal expansion of  $17^{256}$  are :

(a) 18.

(b) 38.

(c) 68.

(d) 81.

41. Which among the following is a critical point of  $f(x, y) = x^2 + xy - 2y - 2x + 1$ .

(a) (2, -2).

(b) (-2, 2).

(c) (2, 2).

(d) (-2, 2).

42. The value of the triple integral  $\iiint_V (x^2 y + 1) dx dy dz$ , where V is the region given by

$x^2 + y^2 \leq 1, 0 \leq z \leq 2$  is :

(a)  $\pi$

(b)  $2\pi$ .

(c)  $3\pi$ .

(d)  $4\pi$ .

43. The curl of vector field  $\vec{F} = x^2 y\vec{i} + xyz\vec{j} + z^2 y\vec{k}$  at the point (0, 1, 2) is :

(a)  $4\vec{i} - 2\vec{j} + 2\vec{k}$ .

(b)  $4\vec{i} + 2\vec{j} + 2\vec{k}$ .

(c)  $4\vec{i} + 2\vec{k}$ .

(d)  $2\vec{i} + 4\vec{k}$ .

44. If  $\vec{u} = x^2 y\vec{i} + y^2 x^3 \vec{j} - 3x^2 z^2 \vec{k}$  and  $\phi = x^2 yz$ , then  $(\vec{u} \cdot \nabla) \phi$  at the point (1, 2, 1) is :

(a) 1.

(b) 3.

(c) 8.

(d) 6.

45. The  $n^{\text{th}}$  derivative of  $y = e^x (\sin x + \cos x)$  is :

(a)  $(\sqrt{2})^n e^x \left[ \sin \left( x + \frac{n\pi}{4} \right) + \cos \left( x + \frac{n\pi}{4} \right) \right]$ .

(b)  $(\sqrt{2})^n e^x \left[ \sin \left( x + \frac{n\pi}{2} \right) + \cos \left( x + \frac{n\pi}{2} \right) \right]$ .

(c)  $e^x \left[ \sin \left( x + \frac{n\pi}{4} \right) + \cos \left( x + \frac{n\pi}{4} \right) \right]$ .

(d)  $(-1)^n n! e^x \left[ \sin^n x + \cos^n x \right]$ .

46. Which of the following is not a valid expression of the volume of the solid

$$\left\{ (x, y, z) \in \mathbb{R}^3 \mid 1 \leq x \leq 2, 0 \leq y \leq \frac{2}{x}, 0 \leq z \leq x \right\}.$$

(a)  $\int_1^2 \int_0^{2/x} \int_0^x dz dy dx.$

(b)  $\int_1^2 \int_0^x \int_0^{2/x} dy dz dx.$

(c)  $\int_0^2 \int_1^x \int_0^{2/x} dy dx dz.$

(d)  $\int_0^2 \int_{\max\{z, 1\}}^2 \int_0^{2/x} dy dx dz.$

47. Solution of the LP problem

Minimize  $z = -3x + 2y$  subject to  $0 \leq x \leq 4, 1 \leq y \leq 6, x + y \leq 5$  is :

(a) - 10.

(b) 0.

(c) 2.

(d) 10.

48. If  $\phi$  is Euler phi function then  $\phi(\phi(1001))$  is :

(a) 144.

(b) 192.

(c) 298.

(d) 96.

49. If eigen values of the matrix  $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$  are 4 and 8, then :

(a)  $x = 4, y = 0.$

(b)  $x = 5, y = 8.$

(c)  $x = -3, y = 9.$

(d)  $x = -4, y = 10.$

50. Which of the following is not a Fermat prime ?

(a) 5.

(b) 17.

(c) 31.

(d) 257.