2017
Test Code: QMA

1. A multiple choice test has 10 questions. Each question has 4 choices : A, B, C, D. In how many ways can the test be answered?
   (a) $10 + 4$
   (b) 10×4
   (c) 10^4
   (d) 4^{10}

2. $\int \ln(e^{2x}) dx$ is equal to
   (a) $x + c$
   (b) $x^2 + c$
   (c) $\ln(x) + c$
   (d) $e^x + c$

3. For any real number x let $g(x)$ denote the determinant of the matrix

$$\begin{bmatrix} 2 & x & 0 & 0 \\ x & 5 & 0 & 0 \\ 0 & 0 & x & 5 \\ 0 & 0 & 5 & 2 \end{bmatrix}$$

   The solutions of $g(x) = 0$ are
   (a) $(\frac{25}{2}, -10, 10)$
   (b) $(25, \sqrt{10}, -\sqrt{10})$
   (c) $(2, \sqrt{10}, -\sqrt{10})$
   (d) $(\frac{25}{2}, \sqrt{10}, -\sqrt{10})$

4. I sold 2 books for Rs. 120 each. My profit on one was 20 percent and the loss on the other was 20 percent. Then, on the whole, I
   (a) lost Rs. 5
   (b) lost Rs. 10
   (c) gained Rs. 5
   (d) neither gained nor lost

5. The function $f(x) = x^2 e^{-2x}$, for $x > 0$. Then the maximum value of $f(x)$ is

(a) $\frac{1}{e}$

(b) $\frac{1}{2e}$

(c) $\frac{1}{e^2}$

(d) none of these

6. Let f and g be differentiable functions such that $f'(x) = 2g(x)$ and $g'(x) = -f(x)$ and let $T(x) = [f(x)]^2 - [g(x)]^2$. Then $T'(x)$ is equal to

(a) $T(x)$

(b) 0

(c) $2f(x)g(x)$

(d) $6f(x)g(x)$

7. The value of $\int_0^{\pi} \frac{e^{\cos x}}{e^{-\cos x} + e^{\cos x}}\, dx$ is equal to

(a) π

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$

(d) 1

8. The velocity, v, of a body moving on a straight track varies according to the following rule,

$$v = \begin{cases} 2t + 13, & 0 \le t \le 5 \\ 3t + 8, & 5 < t \le 7 \\ 4t + 1, & t > 7 \end{cases}$$

The distances are measured in metres and time, t, in seconds. The distance in metres moved by the body in 10 seconds is equal to

(a) 127

(b) 247

(c) 186

(d) 323

9. In an election 10 percent of the voters on the voters list did not cast vote and 60 voters cast their ballot papers blank. There were only two candidates. The winner was supported by 47 percent of all voters in the list and he got 308 votes more than his rival. the number of voters in the list was

   (a) 3600
   (b) 9200
   (c) 4575
   (d) None of the above

10. Let $F(x) = e^x$ and $G(x) = e^{-x}$ and $H(x) = G(F(x))$ where x is a real number. Then $\frac{dH}{dx}$ at $x = 0$ is equal to

   (a) 1
   (b) -1
   (c) $-\frac{1}{e}$
   (d) $-e$

11. If $f(2) = 5$ and $f'(2) = 1$ then the value of $\lim_{x \to 2} \frac{xf(2)-2f(x)}{x-2}$ is equal to

   (a) 0
   (b) 1
   (c) 2
   (d) 3

12. The value of $\int_0^2 |x^2 - 2| dx$ is

   (a) $\frac{4}{3}(2\sqrt{2} - 1)$
   (b) $\frac{4}{3}$
   (c) $\frac{8\sqrt{2}}{3}$
   (d) $\frac{4}{3}(2 - \sqrt{2})$

13. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A^n is equal to

   (a) $2^{n-1}A - (n-1)I$
   (b) $nA - (n-1)I$
   (c) $2^{n-1}A + (n-1)I$
   (d) $nA + (n-1)I$

14. The value of $\int_{e^{-1}}^{e^2} |\frac{\log_e x}{x}| dx$ is

(a) $\frac{3}{2}$
(b) $\frac{5}{2}$
(c) 3
(d) 5

15. f is a quadratic function whose graph is a parabola opening upward and has a vertex on the x-axis. The graph of the new function g defined by $g(x) = 2 - f(x-5)$ has a range defined by

(a) $[-5, \infty)$
(b) $[2, \infty)$
(c) $(-\infty, 2]$
(d) $(-\infty, 0]$

16. In how many different ways can the letters A, A, B, B, B, C, D, E be arranged so that the position of letter C is always to the right of the letter D?

(a) 1680
(b) 2160
(c) 2520
(d) 3240

17. If $A(\alpha, \beta) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & e^{\beta} \end{bmatrix}$, then $[A(\alpha, \beta)]^{-1}$ is equal to

(a) $A(\alpha, -\beta)$
(b) $A(-\alpha, \beta)$
(c) $A(-\alpha, -\beta)$
(d) $A(\alpha, \beta)$

18. If $y = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$, where $0 < y < 1$, then the value of x is

(a) $\frac{1}{2} \log_2 \frac{y}{1-y}$
(b) $\frac{1}{2} \log_2 \frac{1+y}{1-y}$
(c) $\frac{1}{2} \log_2 \frac{1+y}{y}$
(d) $\frac{1}{2} \log_2 \frac{2+y}{2-y}$

19. Two particles are moving with uniform velocities u and v respectively along x and y axes, each directed towards origin. If the particles are at distance, a, and ,b, from the origin respectively, the time at which they will be nearest to each other will be equal to

   (a) $\frac{au}{u^2+v^2}$
   (b) $\frac{bu}{u^2+v^2}$
   (c) $\frac{au+bv}{u^2+v^2}$
   (d) $\frac{au}{bv}$

20. Let the function $f(x) = x^2 + x + \sin x - \cos x + \log(1 + |x|)$ be defined in the interval $[-1, 1]$. The function $g(x)$ defined on $[-1, 1]$ satisfying $g(-x) = -f(x)$ is

   (a) $x^2 + x + \sin x + \cos x - log(1 + |x|)$
   (b) $-x^2 + x + \sin x + \cos x - log(1 + |x|)$
   (c) $-x^2 + x + \sin x - \cos x + log(1 + |x|)$
   (d) none of these

21. Let $x, y, z \in (0, \frac{\pi}{2})$. If $u = \ln(\tan x + \tan y + \tan z)$, then

$$\sin(2x)\frac{du}{dx} + \sin(2y)\frac{du}{dy} + \sin(2z)\frac{du}{dz}$$

   is equal to

   (a) 1
   (b) 2
   (c) u
   (d) $2u$

22. If the sum of the real roots of the quadratic equation $ax^2 + bx + c = 0$ (where a, b and c are non zero real numbers) is equal to the sum of squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in

   (a) Geometric Progression
   (b) Harmonic Progression
   (c) Arithmetic Progression
   (d) Arithmetico-Geometric Progression

23. The function $f(x) = \exp(-|x|) + \sin(x) + \cos^3 x$ is:

(a) differentiable at $x = 0$

(b) continuous but not differentiable at $x = 0$

(c) a bounded function which is not continuous at $x = 0$

(d) an unbounded function which is continuous at $x = 0$

24. For $-1 \leq x \leq 1$ and $x \neq 0$ consider the function $f(x) = \frac{1}{\sin x} - \frac{1}{e^x - 1}$. Then $\lim_{x \to 0} f(x)$ is

(a) not defined

(b) 0

(c) 1

(d) $\frac{1}{2}$

25. A five digit number divisible by 3 is to be formed using the numerals $0, 1, 2, 3, 4$ and 5 without repetition. The total number of ways in which this can be done is equal to

(a) 64

(b) 720

(c) 256

(d) 216

26. If $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x dx$, then $I_4 + 12 I_2$ is equal to

(a) 4π

(b) $3(\frac{\pi}{2})^3$

(c) $(\frac{\pi}{2})^2$

(d) $4(\frac{\pi}{2})^3$

27. If α, β, γ are the roots of the equation $x^3 - x^2 - 1 = 0$, with $\alpha \neq 1, \beta \neq 1$ and $\gamma \neq 1$, then the value of $\frac{1+\alpha}{1-\alpha} + \frac{1+\beta}{1-\beta} + \frac{1+\gamma}{1-\gamma}$ is equal to

(a) 5

(b) 6

(c) -6

(d) -5

28. If $S_r = \alpha^r + \beta^r + \gamma^r$, then the value of $\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix} =$

(a) 0

(b) $(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$

(c) $(\alpha + \beta + \gamma)^6$

(d) $(\alpha - \beta)^2(\beta - \gamma)^2(\gamma - \alpha)^2$

29. The domain (set of real values of x for which $f(x)$ is well defined) of the function $y = f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ is

(a) $[-2, 1)$

(b) $[-3, -2]$ excluding -2.5

(c) $[0, 1]$ excluding 0

(d) None of these

30. The sum of the series $\frac{x-1}{x+1} + \frac{1}{2}\frac{x^2-1}{(x+1)^2} + \frac{1}{3}\frac{x^3-1}{(x+1)^3} + \frac{1}{4}\frac{x^4-1}{(x+1)^4} + \ldots$ to ∞, $(x > 0)$ is

(a) $\log_e x$

(b) $2\log_e x$

(c) $\log_e(\frac{1}{x+1})$

(d) None of these