SAMPLE QUESTIONS (MCQ -TYPE)

1. The highest power of 7 that divides $100!$ is

   (A) 14 (B) 15 (C) 16 (D) 18.

2. The number of 6-digit positive integers whose sum of the digits is at least 52 is

   (A) 21 (B) 22 (C) 27 (D) 28.

3. The sum of all 3-digit numbers that leave a remainder of 2 when divided by 3 is

   (A) 189700 (B) 164850 (C) 164750 (D) 149700.

4. Suppose that 6-digit numbers are formed using each of the digits $1, 2, 3, 7, 8, 9$ exactly once. The number of such 6-digit numbers that are divisible by 6 but not divisible by 9 is equal to

   (A) 120 (B) 180 (C) 240 (D) 360.

5. The solution of the differential equation

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

   is

   (A) $x^2 + y^2 = cy$, where c is a constant
   (B) $x^2 + y^2 = cx$, where c is a constant
   (C) $x^2 - y^2 = cy$, where c is a constant
   (D) $x^2 - y^2 = cx$, where c is a constant.

6. If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$ and $g'(a) = 2$, then the value of

$$\lim_{x \to a} \frac{g(x)f(a) - f(x)g(a)}{x - a}$$

   is

   (A) -5 (B) -3 (C) 3 (D) 5.

7. $(\cos 100^\circ + i \sin 100^\circ)(\cos 110^\circ + i \sin 110^\circ)$ is equal to

(A) $\frac{1}{2}(\sqrt{3} - i)$ (B) $\frac{1}{2}(-\sqrt{3} - i)$ (C) $\frac{1}{2}(-\sqrt{3} + i)$ (D) $\frac{1}{2}(\sqrt{3} + i)$.

8. The value of $\frac{1}{2 \sin 10^\circ} - 2 \sin 70^\circ$ is

(A) $-\frac{1}{2}$ (B) -1 (C) $\frac{1}{2}$ (D) 1.

9. For $0 \leq x < 2\pi$, the number of solutions of the equation

$$\sin^2 x + 2 \cos^2 x + 3 \sin x \cos x = 0$$

is

(A) 1 (B) 2 (C) 3 (D) 4.

10. The chance of a student getting admitted to colleges A and B are 60% and 40%, respectively. Assume that the colleges admit students independently. If the student is told that he has been admitted to at least one of these colleges, what is the probability that he has got admitted to college A?

(A) 3/5 (B) 5/7 (C) 10/13 (D) 15/19.

11. Given a positive integer m, we define $f(m)$ as the highest power of 2 that divides m. If n is a prime number greater than 3, then

(A) $f(n^3 - 1) = f(n - 1)$
(B) $f(n^3 - 1) = f(n - 1) + 1$
(C) $f(n^3 - 1) = 2f(n - 1)$
(D) none of the above is necessarily true.

12. How many triplets of real numbers (x, y, z) are simultaneous solutions of the equations $x + y = 2$ and $xy - z^2 = 1$?

(A) 0 (B) 1 (C) 2 (D) infinitely many.

13. Let V be the vector space of all 4×4 matrices such that the sum of the elements in any row or any column is the same. Then the dimension of V is

(A) 8 (B) 10 (C) 12 (D) 14.

14. If the system of equations

$$\begin{array}{rcrcrcl} ax & + & y & + & z & = & 0 \\ x & + & by & + & z & = & 0 \\ x & + & y & + & cz & = & 0 \end{array}$$

with $a, b, c \neq 1$ has a non-trivial solution, the value of

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$$

is

(A) 1 (B) -1 (C) 3 (D) -3.

15. The rank of the matrix $\begin{bmatrix} 0 & 1 & t \\ 2 & t & -1 \\ 2 & 2 & 0 \end{bmatrix}$ equals

(A) 3 for any real number t

(B) 2 for any real number t

(C) 2 or 3 depending on the value of t

(D) 1, 2 or 3 depending on the value of t.

16. If S and S' are the foci of the ellipse $3x^2 + 4y^2 = 12$ and P is a point on the ellipse, then the perimeter of the triangle PSS' is

(A) 4 (B) 6 (C) 8 (D) dependent on the coordinates of P.

17. The reflection of the the point $(1, 2)$ with respect to the line $x+2y = 15$ is

(A) (3, 6) (B) (6, 3) (C) (5, 10) (D) (10, 5).

18. For the differential equation

$$\frac{dy}{dx} + xe^{-y} + 2x = 0,$$

it is given that $y = 0$ when $x = 0$. When $x = 1$, y is given by

(A) $\ln\left(\frac{3}{2e} - \frac{1}{2}\right)$ (B) $\ln\left(\frac{3e}{2} - \frac{1}{4}\right)$

(C) $\ln\left(\frac{3}{e} - \frac{1}{2}\right)$ (D) $\ln\left(\frac{3}{2e} - \frac{1}{4}\right)$.

19. Let $G = \{a_1, a_2, \ldots, a_{12}\}$ be an Abelian group of order 12. Then the order of the element $\left(\prod_{i=1}^{12} a_i\right)$ is

(A) 1 (B) 2 (C) 6 (D) 12.

20. A function $f : \mathbb{R}^2 \to \mathbb{R}$ is called degenerate on x_i if $f(x_1, x_2)$ remains constant when x_i varies ($i = 1, 2$). Define

$$f(x_1, x_2) = \left|2^{\pi i/x_1}\right|^{x_2} \quad \text{for } x_1 \neq 0,$$

where $i = \sqrt{-1}$. Then which of the following statements is true?

(A) f is degenerate on both x_1 and x_2

(B) f is degenerate on x_1 but not on and x_2

(C) f is degenerate on x_2 but not on and x_1

(D) f is neither degenerate on x_1 nor on x_2.

21. Suppose that the number plate of a vehicle contains two vowels followed by four digits. However to avoid confusion, the letter 'O' and the digit '0' are not used in the same number plate. How many such number plates can be formed?

(A) 164025 (B) 190951 (C) 194976 (D) 219049.

22. A coin, with probability p $(0 < p < 1)$ of getting head, is tossed until a head appears for the first time. If the probability that the number of tosses required is even is $2/5$, then the value of p is

(A) $2/7$ (B) $1/3$ (C) $5/7$ (D) $2/3$.

23. Let A be a 2×2 matrix with real entries. Now consider the function $f_A(x) = Ax$. If the image of every circle under f_A is a circle of the same radius, then

(A) A must be an orthogonal matrix
(B) A must be a symmetric matrix
(C) A must be a skew-symmetric matrix
(D) none of the above must necessarily hold.

24. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $\lim_{n\to\infty} f^n(x)$ exists for every $x \in \mathbb{R}$, where $f^n(x) = f \circ f^{n-1}(x)$ for $n \geq 2$. Define

$$S = \left\{ \lim_{n\to\infty} f^n(x) : x \in \mathbb{R} \right\} \quad \text{and} \quad T = \{x \in \mathbb{R} : f(x) = x\}.$$

Then which of the following is necessarily true?

(A) $S \subset T$ (B) $T \subset S$

(C) $S = T$ (D) None of the above.

25. Let a, b, c be non-zero real numbers such that

$$\int_0^1 (1 + \cos^8 x)(ax^2 + bx + c)dx = \int_0^2 (1 + \cos^8 x)(ax^2 + bx + c)dx = 0.$$

Then the quadratic equation $ax^2 + bx + c = 0$ has

(A) no roots in (0, 2)
(B) one root in (0, 2) and one root outside this interval
(C) one repeated root in (0, 2)
(D) two distinct real roots in (0, 2).

26. If $t = \binom{200}{100}/4^{100}$, then

(A) $t < \frac{1}{3}$ (B) $\frac{1}{3} < t < \frac{1}{2}$ (C) $\frac{1}{2} < t < \frac{2}{3}$ (D) $\frac{2}{3} < t < 1$.

27. A general election is to be scheduled on 5 days in May such that it is not scheduled on two consecutive days. In how many ways can the 5 days be chosen to hold the election?

(A) $\binom{26}{5}$ (B) $\binom{27}{5}$ (C) $\binom{30}{5}$ (D) $\binom{31}{5}$.

28. Consider the functions $f, g : [0,1] \to [0,1]$ given by

$$f(x) = \frac{1}{2}x(x+1) \quad \text{and} \quad g(x) = \frac{1}{2}x^2(x+1).$$

Then the area enclosed between the graphs of f^{-1} and g^{-1} is

(A) 1/4 (B) 1/6 (C) 1/8 (D) 1/24.

29. Let $\psi : \mathbb{R} \to \mathbb{R}$ be a continuous function with $\psi(y) = 0$ for all $y \notin [0,1]$ and $\int_0^1 \psi(y)dy = 1$. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Then the value of

$$\lim_{n\to\infty} n \int_0^{100} f(x)\psi(nx)dx$$

is

(A) $f(0)$ (B) $f'(0)$ (C) $f''(0)$ (D) $f(100)$.

30. Consider the function h defined on $\{0, 1, \ldots, 10\}$ with $h(0) = 0$, $h(10) = 10$ and

$$2[h(i) - h(i-1)] = h(i+1) - h(i) \quad \text{for } i = 1, 2, \ldots, 9.$$

Then the value of $h(1)$ is

(A) $\frac{1}{2^9 - 1}$ (B) $\frac{10}{2^9 + 1}$ (C) $\frac{10}{2^{10} - 1}$ (D) $\frac{1}{2^{10} + 1}$.