

	COMPLEX NUMBERS Class XI	
Q.1)	Convert the given complex number into polar form $z = \frac{1+7i}{(2-i)^2}$	
Sol.1)	$z = \frac{1+7i}{(2-i)^2}$ First, we have to convert in to standard form $z = \frac{1+7i}{4+i^2-4i}$ $z = \frac{1+7i}{4-1-4i} \dots\dots\dots \{i^2 = -1\}$ $z = \frac{1-7i}{3-4i} \text{ then rationalize}$ $z = \frac{1-7i}{3-4i} \times \frac{3+4i}{3+4i}$ $z = \frac{3-4i+21i-28}{3+4i+21i-28}$ $z = \frac{9+16i-25}{-25+25i}$ $z = \frac{-16i}{-25+25i}$ $z = -1 + i \text{ (standard form)}$ Here, $a = -1$ and $b = 1$ Now, $r z = \sqrt{a^2 + b^2}$ $r = \sqrt{1+1} = \sqrt{2}$ $\tan \alpha = \left \frac{b}{a} \right = \left \frac{1}{-1} \right = -1 = 1$ $\Rightarrow \tan \alpha = 1$ $\Rightarrow \alpha = \frac{\pi}{4}$ Now, z is in 2nd quadrant $\therefore \theta = \pi - \alpha$ $\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ Polar form: $z = r(\cos \theta + i \sin \theta)$ $z = \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \text{ ans.}$	
Q.2)	Convert the given complex number in to polar form $z = \frac{-16}{1+i\sqrt{3}}$	
Sol.2)	We have, $z = \frac{-16}{1+i\sqrt{3}}$ Rationalize $z = \frac{-16}{1+i\sqrt{3}} \times \frac{(1-i\sqrt{3})}{(1-i\sqrt{3})}$ $z = \frac{-16 + i16\sqrt{3}}{1 - 3i^2}$ $z = \frac{-16 + i16\sqrt{3}}{4}$ $z = -4 + i4\sqrt{3}$ Here, $a = -4$ and $b = 4\sqrt{3}$ Now, $r = \sqrt{a^2 + b^2} = \sqrt{16 + 48} = \sqrt{64} = 8$ $\therefore r = 8$ $\tan \alpha = \left \frac{b}{a} \right = \left \frac{4\sqrt{3}}{-4} \right = -\sqrt{3} = \sqrt{3}$	

	$\Rightarrow \tan \alpha = \sqrt{3}$ $\Rightarrow \alpha = \frac{\pi}{3}$ Now, z is in 2nd quadrant $\therefore \theta = \pi - \alpha$ $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ Polar form: $z = r(\cos \theta + i \sin \theta)$ $z = 8 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) \text{ ans.}$	
Q.3)	Convert the given complex number in to polar form $z = \frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$	
Sol.3)	We have, $z = \frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$ $\Rightarrow z = \frac{-1+i}{\frac{1}{2} + \frac{i\sqrt{3}}{2}}$ $\Rightarrow z = \frac{-2+2i}{1+i\sqrt{3}}$ Rationalize $z = \frac{(2+2i)}{(1+i\sqrt{3})} \times \frac{(1-i\sqrt{3})}{(1-i\sqrt{3})}$ $z = \frac{-2 + 2\sqrt{3}i + 2i - 2\sqrt{3}i^2}{1 - 3i^2}$ $z = \frac{-2 + 2\sqrt{3}i + 2i - 2\sqrt{3}}{1 + 3}$ $z = \frac{(2\sqrt{3} - 2)}{4} + \frac{i(2\sqrt{3} + 2)}{4}$ $z = \left(\frac{\sqrt{3} - 1}{2} \right) + i \left(\frac{\sqrt{3} + 1}{2} \right)$ Here, $a = \frac{\sqrt{3}-1}{2}$ and $b = \frac{\sqrt{3}+1}{2}$ Now, $r = \sqrt{a^2 + b^2}$ $= \sqrt{\left(\frac{\sqrt{3}-1}{2} \right)^2 + \left(\frac{\sqrt{3}+1}{2} \right)^2}$ $\Rightarrow r = \sqrt{\frac{3+1-2\sqrt{3}}{4} + \frac{3+1+2\sqrt{3}}{4}}$ $\Rightarrow r = \sqrt{\frac{4+4}{4}} = \sqrt{\frac{8}{4}} = \sqrt{2}$ $\Rightarrow r = \sqrt{2}$ $\tan \alpha = \left \frac{b}{a} \right = \left \frac{\frac{\sqrt{3}+1}{2}}{\frac{\sqrt{3}-1}{2}} \right $ $\Rightarrow \tan \alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ Divide by $\sqrt{3}$ $\Rightarrow \tan \alpha = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$ $\Rightarrow \tan \alpha = \frac{\tan\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{6}\right)}{1 - \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)} \dots \dots \dots \{ = \tan(A + B) \}$	

	$\Rightarrow \tan \alpha = \tan \left(\frac{\pi}{4} + \frac{\pi}{6} \right)$ $\Rightarrow \tan \alpha = \tan \left(\frac{5\pi}{12} \right)$ $\Rightarrow \alpha = \frac{5\pi}{12}$ since, z is in 1st quadrant $\therefore \theta = \alpha$ $\theta = \frac{5\pi}{12}$ Polar form: $z = r(\cos \theta + i \sin \theta)$ $z = \sqrt{2} \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right) \text{ ans.}$	
	Finding the value of a POYNOMIAL:-	
Q.4)	If $x = -5 + 2\sqrt{-4}$. Find the value of $x^4 + 9x^3 + 35x^2 - x + 4$	
Sol.4)	We have, $x = -5 + 2\sqrt{-4}$ $\Rightarrow x = -5 + 4i \dots\dots\dots \{\sqrt{-4} = 2i\}$ $\Rightarrow x + 5 = 4i$ Squaring $\Rightarrow x^2 + 25 + 10x = 16i^2$ $\Rightarrow x^2 + 25 + 10x = 16$ $\Rightarrow x^2 + 10x + 41 = 0$ $ \begin{array}{r} x^2 + 10x + 41 \overline{) x^4 + 9x^3 + 35x^2 - x + 4} \\ \underline{-(x^4 + 10x^3 + 41x^2)} \\ -x^3 - 6x^2 - x + 4 \\ \underline{-(-x^3 - 10x^2 - 41x)} \\ 4x^2 + 40x + 4 \\ \underline{-(4x^2 + 40x + 164)} \\ -160 \end{array} $ Now $x^4 + 9x^3 + 35x^2 - x + 4 = (x^2 + 10x + 41)(x^2 - x + 4) - 16 \dots\dots\{dividend = (division)(quotient) + remainder\}$ $\Rightarrow x^4 + 9x^3 + 35x^2 - x + 4 = 0(x^2 - x + 4) - 160 \dots\dots\dots \text{from eq.(i)}$ $= -160 \text{ ans.}$	
Q.5)	Find the value of $x^3 + 7x^2 - x + 16$ when $x = 1 + 2i$	
Sol.5)	We have, $x = 1 + 2i$ $\Rightarrow x - 1 = 2i$ Squaring $\Rightarrow x^2 + 1 - 2x = 4i^2$ $\Rightarrow x^2 + 1 - 2x = -4$ $\Rightarrow x^2 - 2x + 5 = 0 \dots\dots\dots \text{eq.(i)}$ Now, $ \begin{array}{r} x^2 - 2x + 5 \overline{) x^3 + 7x^2 - x + 16} \\ \underline{-(x^3 + 2x^2 + 5x)} \\ 9x^2 - 6x + 16 \\ \underline{-(9x^2 - 18x + 35)} \\ 12x - 29 \end{array} $ $\{dividend = (division)(quotient) + remainder\}$ $\therefore x^3 + 7x^2 - x + 16 = (x^2 - 2x + 5)(x + 9) + 12x - 29$ $= 0(x + 9) + 12x - 29 \dots\dots\dots \because x^2 - 2x + 5 = 0 \text{ from eq. (i)}$ $= 12x - 29$ $= 12(1 + 2i) - 29$	

	$= 12 + 24i - 29$ $= -17 + 24i \text{ ans.}$	
	Square Root Of A Complex Numbers:-	
Q.6)	Find $\sqrt{-15 - 8i}$	
Sol.6)	Let $x + iy = \sqrt{-15 - 8i}$ Squaring $x^2 + i^2 y^2 + 2ixy = 15 - 8i$ $\Rightarrow (x^2 + y^2) + 2ixy = -15 - 8i$ Equating real & imaginary parts $\Rightarrow x^2 - y^2 = -15 \text{ (i)}$ And $2xy = -8$ Now $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2 \text{ (to be remembered)}$ $(x^2 + y^2)^2 = (-15)^2 + (-8)^2$ $(x^2 + y^2)^2 = 289$ $\Rightarrow x^2 + y^2 = 17 \text{ (ii)}$ Adding eq. (i) & (ii) $\Rightarrow (x^2 - y^2) + (x^2 + y^2) = -15 + 17$ $\Rightarrow 2x^2 = 2$ $\Rightarrow x^2 = 1$ $\Rightarrow x = \pm 1$ Put $x^2 = 1$ in eq. (ii) $\Rightarrow 1 + y^2 = 17$ $\Rightarrow y^2 = 16$ $\Rightarrow y = \pm 4$ Now, $\sqrt{-15 - 8i} = \pm(1 - 4i) \text{ ans.}$	
Q.7)	Find the value of square of $1 - i$	
Sol.7)	Let $x + iy = \sqrt{1 - i}$ Squaring $x^2 + i^2 y^2 + 2ixy = 1 - i$ $\Rightarrow (x^2 + y^2) + 2ixy = 1 - i$ Equating real & imaginary parts $\Rightarrow x^2 - y^2 = 1 \text{ (i)}$ And $2xy = -1$ Now $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2 \text{ (to be remembered)}$ $\Rightarrow (x^2 + y^2)^2 = 1 + 1$ $\Rightarrow (x^2 + y^2)^2 = 2$ $\Rightarrow x^2 + y^2 = \sqrt{2} \text{ (ii)}$ Adding eq. (i) & (ii) $\Rightarrow (x^2 - y^2) + (x^2 + y^2) = \sqrt{2} + 1$ $\Rightarrow 2x^2 = \sqrt{2} + 1$ $\Rightarrow x^2 = \frac{\sqrt{2} + 1}{2}$ $\Rightarrow x = \pm \sqrt{\frac{\sqrt{2} + 1}{2}}$ Put $x^2 = \frac{\sqrt{2} + 1}{2}$ in eq. (ii) $\Rightarrow \frac{\sqrt{2} + 1}{2} + y^2 = \sqrt{2}$	

	$\Rightarrow y^2 = \sqrt{2} - \left(\frac{\sqrt{2}+1}{2}\right)$ $\Rightarrow y^2 = \frac{2\sqrt{2}-\sqrt{2}-1}{2}$ $\Rightarrow y = \sqrt{\frac{\sqrt{2}-1}{2}}$ Now, $1 - i = \pm \left(\sqrt{\frac{\sqrt{2}+1}{2}} - \sqrt{\frac{\sqrt{2}-1}{2}} i \right)$ ans.									
	On Equality:-									
Q.8)	Find real value of $x + y$ for which the complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate of each other.									
Sol.8)	$-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate of each other $\Rightarrow -3 + ix^2y = \overline{(x^2 + y) + 4i}$ $\Rightarrow -3 + ix^2y = (x^2 + y) - 4i$ Equating real & imaginary parts $\Rightarrow x^2 + y = -3 \text{ And } x^2y = -4$ $\therefore (-3 - y)y = -4$ $\Rightarrow -3 + y^2 = -4$ $\Rightarrow y^2 + 3y - 4 = 0$ $\Rightarrow (y + 4)(y - 1) = 0$ $\Rightarrow y = -4 \text{ and } y = 1$ Put $x^2y = -4$ <table><tr><td>when, $y = -4$</td><td>when $y = 1$</td></tr><tr><td>$x^2(-4) = -4$</td><td>$x^2(1) = -4$</td></tr><tr><td>$x^2 = 1$</td><td>$x^2 = -4$</td></tr><tr><td>$x = \pm 1$</td><td>$x = \pm 2i$</td></tr></table> $\therefore x = \pm 1 \text{ and } y = -4$ ans.	when, $y = -4$	when $y = 1$	$x^2(-4) = -4$	$x^2(1) = -4$	$x^2 = 1$	$x^2 = -4$	$x = \pm 1$	$x = \pm 2i$	
when, $y = -4$	when $y = 1$									
$x^2(-4) = -4$	$x^2(1) = -4$									
$x^2 = 1$	$x^2 = -4$									
$x = \pm 1$	$x = \pm 2i$									
Q.9)	If $(x + iy)^{\frac{1}{3}} = a + ib$, show that $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$									
Sol.9)	We have, $(x + iy)^{\frac{1}{3}} = a + ib$ cubing both sides $x + iy = (a + ib)^3$ $\Rightarrow x + iy = a^3 + i^3b^3 + 3a^2(ib) + 3a(i^2b^2)$ $\Rightarrow x + iy = a^3 - i^3b^3 + 3a^2b - 3ab^2$ $\Rightarrow x + iy = (a^3 - 3ab^2) + i(-b^3 + 3a^2b)$ Equating real & imaginary parts $x = a^3 - 3ab^2 \text{ and } y = -b^3 + 3a^2b$ Taking L.H.S. $\frac{x}{a} + \frac{y}{b}$ Putting the values of x and y $= \frac{(a^3 - 3ab^2)}{a} + \frac{(-b^3 + 3a^2b)}{b}$ $= \frac{a(a^2 - 3b^2)}{a} + \frac{b(-b^3 + 3a^2)}{b}$ $= a^2 - 3b^2 - b^3 + 3a^2$ $= 4a^2 - 3b^2$ $= 4(a^2 - b^2) \text{ RHS (proved)}$									
Q.10)	If $a + ib = \frac{c+i}{c-i}$ show that $\frac{b}{a} = \frac{2c}{c^2-1}$ and hence show $a^2 + b^2 = 1$									

Sol.10)	We have, $a + ib = \frac{c+i}{c-i}$ $a + ib = \frac{c+i}{c-i} \times \frac{c+i}{c+i}$ $\Rightarrow a + ib = \frac{(c+i)^2}{c^2-i^2}$ $\Rightarrow a + ib = \frac{c^2+i^2+2ic}{c^2+1}$ $\Rightarrow a + ib = \frac{(c^2-1)}{c^2+1} + \frac{2ic}{c^2+1}$ Equating real & imaginary parts $a = \frac{c^2-1}{c^2+1} \text{ and } b = \frac{2c}{c^2+1}$ Now $\frac{b}{a} = \frac{\frac{2c}{c^2+1}}{\frac{c^2-1}{c^2+1}}$ $\Rightarrow \frac{b}{a} = \frac{2c}{c^2-1} \text{ (proved)}$ Now $a^2 + b^2 = \left(\frac{c^2-1}{c^2+1}\right)^2 + \left(\frac{2c}{c^2+1}\right)^2$ $= \frac{c^4+1-2c^2}{(c^2+1)^2} + \frac{4c^2}{(c^2+1)^2}$ $= \frac{c^4+2c^2+1}{(c^2+1)^2}$ $= \frac{(c^2+1)^2}{(c^2+1)^2}$ $a^2 + b^2 = 1 \text{ (proved)}$
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