

- Standard Form
- Types of Roots
- Relation Between Roots and Co-efficients
- Formation of a Quadratic Equation
- **Standard Form** : A polynomial $f(x) = ax^2 + bx + c$, $a, b, c \in \mathbb{R}$, $a \neq 0$ is called a quadratic polynomial.

If $f(\alpha) = 0$, then $x = \alpha$ is called a root of the quadratic equation.

- Generally we use three methods to find roots of a quadratic equation.

(1) Method of factorization (2) Method of perfect square (3) Method of formula.

- To find the roots by the method of formula, first find $D = b^2 - 4ac$ and then use $(\alpha, \beta) = \frac{-b \pm \sqrt{D}}{2a}$.

Note : (1) If $D = b^2 - 4ac > 0$, then the quadratic equation has two real and distinct roots.

(2) If $D = b^2 - 4ac = 0$, then the quadratic equation has two equal real roots.

(3) If $D = b^2 - 4ac < 0$, then the quadratic equation has two distinct complex roots. They are complex conjugates of each other. Here note that $i^2 = -1$.

- **Types of Real Roots** :

- (1) In a quadratic equation, if a and c have the same sign which is opposite to the sign of b , then both the roots are positive.
- (2) In a quadratic equation, if a and c are of opposite sign, then both the roots are of different signs.
- (3) If $b = 0$, then both the roots are additive inverses of each other.
- (4) If $c = a$, then both the roots are multiplicative inverses of each other.

- **Relationship Between Roots of a Quadratic Equation and Co-efficients** :

If α and β are roots of a given quadratic equation, $f(x) = ax^2 + bx + c = 0$, then

The sum of two roots $\alpha + \beta = \frac{-b}{a} = -\frac{\text{The coefficient of } x}{\text{The coefficient of } x^2}$ and

The product of two roots $\alpha\beta = \frac{c}{a} = \frac{\text{The constant term}}{\text{The coefficient of } x^2}$

- **Formation of a Quadratic Equation** :

If we know the roots of a quadratic equation, then the quadratic equation can be obtained by $x^2 - (\text{the sum of two roots})x + (\text{the product of two roots}) = 0$

- **Common Root** :

If two distinct quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$

(where $a_1, a_2 \neq 0$)

(1) have one root in common, then $(a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = (a_1c_2 - a_2c_1)^2$

(2) have both the roots common, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

● **Boundary Values :**

- (1) If $(x - \alpha)(x - \beta) < 0$, then $x \in (\alpha, \beta)$
- (2) If $(x - \alpha)(x - \beta) \leq 0$, then $x \in [\alpha, \beta]$
- (3) If $(x - \alpha)(x - \beta) > 0$, then $x \notin [\alpha, \beta]$ and
- (4) If $(x - \alpha)(x - \beta) \geq 0$, then $x \notin (\alpha, \beta)$

● **Maximum and Minimum Values of a Quadratic Polynomial :**

Let $p(x) = ax^2 + bx + c$; ($a \neq 0$) be a given polynomial. Let $D = b^2 - 4ac < 0$

- (i) If $a > 0$, then $p(x)$ has minimum value $\frac{4ac - b^2}{4a}$ at $x = \frac{-b}{2a}$ and
- (ii) If $a < 0$, then $p(x)$ has maximum value $\frac{4ac - b^2}{4a}$ at $x = \frac{-b}{2a}$.

● **Cubic Polynomial :**

If α, β, γ are the roots of $p(x) = ax^3 + bx^2 + cx + d$; ($a \neq 0$), then

- (1) The sum of the roots : $\alpha + \beta + \gamma = \frac{-b}{a}$
 - (2) The sum of the products of two roots taken at a time : $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$
 - (3) The product of the roots : $\alpha\beta\gamma = \frac{-d}{a}$
- The equation of a cubic polynomial with three roots $\alpha; \beta; \gamma$
- $$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$$

: Multiple Choice Questions :

- (1) If x_1 and x_2 are roots of the quadratic equation $x^2 - 3x + p = 0$ and x_3 and x_4 are roots of the quadratic equation $x^2 - 12x + q = 0$ and if all these four roots are in increasing G.P., then $p = \dots, q = \dots$
- (A) $p = 2; q = 32$ (B) $p = 4; q = 16$ (C) $p = 4; q = 32$ (D) $p = 2; q = 16$

Sol. : Here $x_1 + x_2 = 3$ and $x_1x_2 = p$. Similarly, $x_3 + x_4 = 12$ and $x_3x_4 = q$.

Now, x_1, x_2, x_3, x_4 are in G.P.

$\therefore$ We can take $x_2 = x_1r$; $x_3 = x_1r^2$; $x_4 = x_1r^3$

$\therefore x_1 + x_2 = x_1 + x_1r = 3$ and $x_3 + x_4 = 12$

$\therefore x_1(1+r) = 3$

$x_1r^2 + x_1r^3 = 12$

$\therefore x_1r^2(1+r) = 12$

$\therefore r^2(3) = 12$

$\therefore r^2 = 4$

$\therefore r = 2$

($r > 0$)

$\therefore x_1 = 1$ and $x_2 = 2$. So, $p = x_1x_2 = 1(2) = 2$

$x_3 = x_1r^2 = 4$, $x_4 = x_1r^3 = 8$

$\therefore q = x_3 \cdot x_4 = 4(8) = 32$

So we get $p = 2; q = 32$

Ans. : (A)

- (2) Real roots of the equation $|x^2 - 5x + 6| = x + 6$ are

(A) 0, 2,

(B) 0, 6

(C) 6, 8

(D) 4, 6

Sol. : $|x^2 - 5x + 6| = x + 6$

$\therefore |(x-3)(x-2)| = x + 6$

Case 1 $x \leq 2 \Rightarrow (x-2) \leq 0$ and $(x-3) < 0$

$\therefore |(x-3)(x-2)| = (x-3)(x-2)$

$\therefore x^2 - 5x + 6 = x + 6$

$\therefore x^2 - 6x = 0$

$\therefore x(x-6) = 0$

$\therefore x = 0$ or $x = 6$. But $x \leq 2$, so $x = 0$

Case 2 $x \geq 3$. So, $(x-3) \geq 0$ and $(x-2) \geq 1 > 0$

$\therefore (x-3)(x-2) \geq 0$. Hence $|(x-3)(x-2)| = (x-3)(x-2)$

$\therefore x = 0$ or 6

But $x \geq 3$. So, $x = 6$.

Case 3 $x > 2$ and $x < 3$. So $x \in (2, 3)$

$\therefore (x-2) > 0$ and $(x-3) < 0$

$\therefore (x-3)(x-2) < 0$

$\therefore |(x-3)(x-2)| = -(x^2 - 5x + 6)$

$\therefore -(x^2 - 5x + 6) = x + 6$

$\therefore -6 + 5x - x^2 = x + 6$

$\therefore x^2 - 4x + 12 = 0$

$\therefore$ This has no real roots.

$\therefore x = 0$ or 6

Ans. : (B)

(3) If the roots of the equation $3x^2 + 2x(k^2+1) + k^2 - 3k + 2 = 0$ have opposite signs, then $k \in$

(A) $(-1, 0)$

(B) $(0, 1)$

(C) $(1, 2)$

(D) $\left(\frac{1}{2}, \frac{3}{2}\right)$

Sol. : We know that if the roots of a quadratic equation have opposite signs, then the product of the roots is negative.

$\therefore \frac{c}{a} < 0$

$\therefore \frac{k^2 - 3k + 2}{3} < 0$

$\therefore k^2 - 3k + 2 < 0$

$\therefore (k-2)(k-1) < 0$

$\therefore k \in (1, 2)$

Ans. : (C)

(4) If $e^{\cot x} - e^{-\cot x} = 4$, then $\cot x =$

(A) $\log_{10}(2+\sqrt{5})$

(B) $\log_{10}(2-\sqrt{5})$

(C) $\log_e(2-\sqrt{5})$

(D) $\log_e(2+\sqrt{5})$

Sol. : $e^{\cot x} - \frac{1}{e^{\cot x}} = 4$

Let, $e^{\cot x} = m$

$$\therefore m - \frac{1}{m} = 4$$

$$\therefore m^2 - 4m - 1 = 0$$

$$\therefore m = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

But $m = e^{\cot x} > 0$. So, $m = 2 + \sqrt{5}$

$$\therefore e^{\cot x} = 2 + \sqrt{5}$$

$$\therefore \cot x = \log_e (2 + \sqrt{5})$$

Ans. : (D)

- (5) The quadratic equation whose roots are reciprocals of the roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0, c \neq 0$) is

(A) $bx^2 + cx - a = 0$ (B) $cx^2 + ax - b = 0$ (C) $cx^2 + bx + a = 0$ (D) $bx^2 - cx + a = 0$

Sol. : Suppose α and β are the roots of the equation $ax^2 + bx + c = 0$.

$$\therefore \alpha + \beta = \frac{-b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

Now, $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the required equation,

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-b}{c} \text{ and } \frac{1}{\alpha\beta} = \frac{a}{c}$$

$$\therefore \text{The required equation is } x^2 - \left(\frac{-b}{c}\right)x + \frac{a}{c} = 0$$

$$\therefore cx^2 + bx + a = 0$$

→ **Second method :**

$$\text{If we take } x = \frac{1}{y}, \frac{a}{y^2} + \frac{b}{y} + c = 0$$

$$\therefore cy^2 + by + a = 0$$

If α, β are roots of $ax^2 + bx + c = 0$, then $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $cx^2 + bx + a = 0$.

Ans. : (C)

- (6) If the equation $x^2 - 3kx + 2e^{2\log|k|} - 1 = 0$ has real roots, so that the product of the roots is 31, then $k = \dots$

(A) ± 1

(B) ± 2

(C) ± 3

(D) ± 4

Sol. : $x^2 - 3kx + 2e^{2\log|k|} - 1 = 0$

$$\therefore x^2 - 3kx + 2e^{\log k^2} - 1 = 0$$

$$\therefore x^2 - 3kx + (2k^2 - 1) = 0$$

$$\therefore \text{Now product of roots} = 31. \quad \text{So, } 2k^2 - 1 = 31$$

$$\therefore k^2 = 16$$

$$\therefore k = \pm 4$$

Ans. : (D)

- (7) If the difference between two roots of the quadratic equation $x^2 - bx + c = 0$ is 1, then which of the following is true ?

(A) $b^2 = 4c + 1$ (B) $b^2 = 4c - 1$ (C) $c^2 = 4b - 1$ (D) $a^2 = 4c + 1$

Sol. : Suppose α and β are the roots of $x^2 - bx + c = 0$ with $\beta = \alpha + 1$.

The sum of two roots $= \alpha + \beta = \alpha + \alpha + 1 = 2\alpha + 1 = b$ (1)

and the product of two roots $= c = \alpha(\alpha + 1) = \alpha^2 + \alpha$

$$\therefore c = \left(\frac{b-1}{2}\right)^2 + \left(\frac{b-1}{2}\right) \quad \text{(From (1))}$$

$$\begin{aligned} \therefore 4c &= (b-1)^2 + 2(b-1) \\ &= b^2 - 2b + 1 + 2b - 2 \end{aligned}$$

$$4c = b^2 - 1$$

$$b^2 = 4c + 1$$

Ans. : (A)

→ **Second method :**

$$(\alpha - \beta)^2 = 1 \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1$$

$$\Rightarrow b^2 - 4c = 1 \Rightarrow b^2 = 4c + 1$$

Ans. : (A)

- (8) If the two roots of the equation $\frac{a}{x-a} + \frac{b}{x-b} = 1$ are additive inverses of each other, then $2(a+b) = \dots\dots$

(A) -1 (B) 2 (C) 0 (D) $\frac{1}{2}$

Sol. : $\frac{a}{x-a} + \frac{b}{x-b} = 1$

$$\therefore ax - ab + bx - ab = (x-a)(x-b)$$

$$\therefore (a+b)x - 2ab = x^2 - (a+b)x + ab$$

$$\therefore x^2 - 2(a+b)x + 3ab = 0$$

Roots of this equation are additive inverses of each other.

$$\therefore \alpha + \beta = 0. \quad \text{So, coefficient of } x = 0$$

$$\therefore a + b = 0$$

$$\therefore 2(a+b) = 0$$

Ans. : (C)

- (9) If $\sin \alpha$ and $\cos \alpha$ are the roots of the quadratic equation $ax^2 + bx + c = 0$, then which of the following is true ?

(A) $a^2 = b^2 - 2ac$ (B) $a^2 = b^2 - 4ac$ (C) $b^2 = c^2 - 4ab$ (D) $c^2 = a^2 - 4bc$

Sol. : Here, the sum of two roots is $\sin \alpha + \cos \alpha = \frac{-b}{a}$ and the product of the two roots is

$$\sin \alpha \cos \alpha = \frac{c}{a} \quad \text{(1)}$$

$$\therefore (\sin \alpha + \cos \alpha)^2 = \left(\frac{-b}{a}\right)^2 = \frac{b^2}{a^2}$$

$$\therefore 1 + 2\left(\frac{c}{a}\right) = \frac{b^2}{a^2} \quad \text{(From (1))}$$

$$\therefore a^2 + 2ac = b^2$$

$$\therefore a^2 = b^2 - 2ac$$

Ans. : (A)

- (10) If the ratio of the roots of the quadratic equation $x^2 + bx + c = 0$ is equal to the ratio of the roots of the equation $x^2 + px + q = 0$, then which of the following is true ?
 (A) $ap^2 = bq^2$ (B) $b^2q = cp^2$ (C) $a^2q = bp^2$ (D) $(aq)^2 = (bq)^2$

Sol. : Suppose α and β are the roots of the quadratic equation $x^2 + bx + c = 0$.

Then $\alpha + \beta = -b$, $\alpha\beta = c$.

Also, if m and n are the roots of the quadratic equation $x^2 + px + q = 0$.

Then $m + n = -p$, $mn = q$.

Now, $\frac{\alpha}{\beta} = \frac{m}{n}$ by hypothesis,

$$\therefore \frac{\alpha + \beta}{\alpha - \beta} = \frac{m + n}{m - n}$$

$$\therefore \frac{(\alpha + \beta)^2}{(\alpha - \beta)^2} = \frac{(m + n)^2}{(m - n)^2}$$

$$\therefore \frac{(\alpha + \beta)^2}{[(\alpha + \beta)^2 - 4\alpha\beta]} = \frac{(m + n)^2}{[(m + n)^2 - 4mn]}$$

$$\therefore \frac{(-b)^2}{[(-b)^2 - 4(c)]} = \frac{(-p)^2}{[(-p)^2 - 4q]}$$

$$\therefore \frac{b^2}{b^2 - 4(c)} = \frac{p^2}{p^2 - 4q}$$

$$\therefore b^2p^2 - b^2(4q) = b^2p^2 - (4cp^2)$$

$$\therefore b^2(4q) = (p^2 4c)$$

$$\therefore b^2q = cp^2$$

Ans. : (B)

→ **Second method :** Suppose $\alpha, \alpha\gamma$ and $\beta, \beta\gamma$ are the roots of the given equations.

$$\alpha(1 + \gamma) = -b, \beta(1 + \gamma) = -p, \alpha^2\gamma = c, \beta^2\gamma = q$$

$$\therefore \frac{\alpha}{\beta} = \frac{b}{p}, \frac{\alpha^2}{\beta^2} = \frac{c}{q}. \text{ So, } \left(\frac{b}{p}\right)^2 = \frac{c}{q}. \quad \text{So, } b^2q = p^2c \quad \text{Ans. : (B)}$$

- (11) If $x = \sqrt{20 + \sqrt{20 + \sqrt{20 + \dots \infty}}}$, then the value(s) of x is/are ($x > 0$)

(A) 5, -4

(B) 4

(C) 5

(D) 4, -5

Sol. : $x = \sqrt{20 + \sqrt{20 + \sqrt{20 + \dots \infty}}}$

$$\therefore x = \sqrt{20 + x}$$

$$\therefore x^2 = 20 + x$$

$$\therefore x^2 - x - 20 = 0$$

$$\therefore (x - 5)(x + 4) = 0$$

$$\therefore x = 5 \text{ or } x = -4$$

But $x > 0$, so $x \neq -4$

$$\therefore x = 5$$

Ans. : (C)

- (12) If the sum of the roots of a quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of squares of reciprocals of those roots, then which of the following is true ?

(A) ab^2, a^2c and bc^2 are in A.P.

(B) ab^2, a^2c and bc^2 are in G.P.

(C) $\frac{b}{c}, \frac{a}{b}$ and $\frac{c}{a}$ are in A.P.

(D) $\frac{b}{c}, \frac{a}{b}, \frac{c}{a}$ are in G.P.

Sol. : Suppose α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$.

$$\text{Now, } \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\therefore \alpha + \beta = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\text{Here } \alpha + \beta = \frac{-b}{a} \text{ and } \alpha\beta = \frac{c}{a}.$$

$$\therefore \frac{-b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\left(\frac{c}{a}\right)^2} = \frac{b^2 - 2ac}{c^2}$$

$$\therefore -bc^2 = ab^2 - 2a^2c$$

$$\therefore 2a^2c = ab^2 + bc^2$$

$$\therefore ab^2, a^2c \text{ and } bc^2 \text{ are in A.P.}$$

Ans. : (A)

(13) The roots of the equation $x^{\frac{1}{4}(\log_2 x) - \frac{3}{4}} = 2$ are

(A) $\frac{1}{2}, 6$

(B) $\frac{1}{6}, 2$

(C) $\frac{1}{2}, 16$

(D) $\frac{1}{16}, 2$

Sol. : $x^{\frac{1}{4}(\log_2 x) - \frac{3}{4}} = 2$

$$\therefore \frac{\log_2 x - 3}{4} = \log_x 2$$

$$\therefore (\log_2 x) - 3 = \frac{4 \log_2 2}{\log_2 x}$$

$$\therefore (\log_2 x)^2 - 3 \log_2 x - 4 = 0$$

$$\therefore (\log_2 x + 1)(\log_2 x - 4) = 0$$

$$\therefore \log_2 x = -1 \text{ or } \log_2 x = 4$$

$$\therefore x = 2^{-1} \text{ or } x = 2^4 \text{ i.e. } x = \frac{1}{2} \text{ or } x = 16$$

Ans. : (C)

(14) If $\frac{x^2 + 2x + 7}{2x + 3} < 6; x \in \mathbb{R}$, then

(A) $x > 11$ or $x < -1$

(B) $x < \frac{-3}{2}$ or $-1 < x < 11$

(C) $x > 11$ or $x < \frac{-3}{2}$

(D) $\frac{-3}{2} < x < -1$

Sol. : Given that $\frac{x^2 + 2x + 7}{2x + 3} < 6; x \in \mathbb{R}$.

$$\therefore \frac{x^2+2x+7}{2x+3} - 6 < 0$$

$$\therefore \frac{x^2-10x-11}{2x+3} < 0$$

Case 1 $x^2 - 10x - 11 < 0$ and $2x + 3 > 0$

$$\therefore (x - 11)(x + 1) < 0 \text{ and } x > \frac{-3}{2}$$

$$\therefore -1 < x < 11 \text{ and } x > \frac{-3}{2}$$

But $x > \frac{-3}{2}$. So $-1 < x < 11$ is possible.

Thus $x < \frac{-3}{2}$ or $-1 < x < 11$ is true.

Case 2 $x^2 - 10x - 11 > 0$ and $2x + 3 < 0$

$$(x - 11)(x + 1) > 0 \text{ and } x < \frac{-3}{2}$$

$$\therefore (x > 11 \text{ or } x < -1) \text{ and } x < \frac{-3}{2}$$

$$\therefore x < -\frac{3}{2}$$

Ans. : (B)

(15) If $\cos \alpha$ is a root of the quadratic equation $15x^2 + 8x - 12 = 0$; $0 < x < 1$, then $\tan^2 \frac{\alpha}{2} = \dots\dots$

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{1}{5}$

(D) 5

Sol. : Given that $15x^2 + 8x - 12 = 0$;

$$0 < x < 1$$

$$\therefore (3x - 2)(5x + 6) = 0$$

$$\therefore x = \frac{2}{3} \text{ or } x = \frac{-6}{5}$$

But $0 < x < 1$, so $x = \frac{2}{3} = \cos \alpha$

$$\therefore \tan^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha} = \frac{1 - \frac{2}{3}}{1 + \frac{2}{3}} = \frac{1}{5}$$

Ans. : (C)

(16) If $a, b, c \in \mathbb{R}$; $a + b + c = 0$, $c \neq 0$, then the quadratic equation $4ax^2 + 3bx + 2c = 0$ has
 a, c , have same sign.

(A) one positive and one negative root

(B) two complex roots

(C) two real roots

(D) both roots are zero

Sol. : Here $4ax^2 + 3bx + 2c = 0$ is given.

$$\therefore \Delta = 9b^2 - 32ac$$

$$= 9(-a - c)^2 - 32ac$$

$$(a + b + c = 0 \Rightarrow b = -a - c)$$

$$= 9(a^2 + c^2 + 2ac) - 32ac$$

$$= 9a^2 + 9c^2 - 14ac$$

$$= \frac{9a^2}{c^2} - \frac{14a}{c} + 9$$

$$= \left(\frac{3a}{c} - \frac{7}{3}\right)^2 + \frac{32}{9} \geq \frac{32}{9} > 0$$

So the quadratic equation has two real roots. (Why not (A) or (D) ?)

Ans. : (C)

(17) The equation $5^{2x^2-7x+7} = 25$ has roots.

- (A) 0 (B) more than two (C) exactly two. 1, and $\frac{2}{5}$ (D) exactly two. 1 and $\frac{5}{2}$

Sol. : Here $5^{2x^2-7x+7} = 5^2$ is given.

$$\therefore 2x^2 - 7x + 7 = 2$$

$$\therefore 2x^2 - 7x + 5 = 0$$

$$\therefore (2x - 5)(x - 1) = 0$$

$$\therefore x = 1 \text{ or } x = \frac{5}{2}$$

Ans. : (D)

(18) Maximum value of the function $\frac{5}{9x^2+6x+14}$ is

- (A) 5 (B) $\frac{13}{5}$ (C) $\frac{5}{13}$ (D) 13

Sol. : $\frac{5}{9x^2+6x+14}$ attains the maximum value only if the quadratic function $9x^2 + 6x + 14$ has the minimum value.

$$\text{Now,, } 9x^2 + 6x + 14 = 9x^2 + 6x + 1 + 13 = (3x + 1)^2 + 13 \geq 13$$

So, the lower limit of $9x^2 + 6x + 14$ is 13 and it is minimum at $x = -\frac{1}{3}$.

Hence, the maximum value of $\frac{5}{9x^2+6x+14}$ is $\frac{5}{13}$.

Ans. : (C)

Note : The minimum value of $9x^2 + 6x + 14 = \frac{4ac - b^2}{4a} = \frac{504 - 36}{36} = \frac{468}{36} = 13$.

(19) If x is a real number, then $\frac{x^2+14x+9}{x^2+2x+3} \in$

- (A) $[1, 3]$ (B) $[4, -5]$ (C) $[-5, 4]$ (D) $\left[\frac{1}{3}, 3\right]$

Sol. : Suppose, $y = \frac{x^2+14x+9}{x^2+2x+3}$

$$\therefore x^2y + 2xy + 3y = x^2 + 14x + 9$$

$$\therefore (y-1)x^2 + (y-7)2x + 3y-9 = 0$$

In order that $x \in \mathbb{R}$ exists, we must have $b^2 - 4ac \geq 0$.

$$\therefore 4(y-7)^2 - 4(y-1)(3y-9) \geq 0$$

$$\therefore (y-7)^2 - 3(y-1)(y-3) \geq 0$$

$$\therefore (y^2 - 14y + 49) - 3(y^2 - 4y + 3) \geq 0$$

$$\therefore -2y^2 - 2y + 40 \geq 0$$

$$\therefore y^2 + y - 20 \leq 0$$

$$\therefore (y+5)(y-4) \leq 0$$

$$\therefore -5 \leq y \leq 4$$

$$\therefore \frac{x^2+14x+9}{x^2+2x+3} \in [-5, 4]$$

Ans. : (C)

(20) If $x \in \mathbb{R}$, then the maximum and the minimum values of $\frac{x^2 - 2x + 4}{x^2 + 2x + 4}$ are respectively.

- (A) $3, \frac{1}{3}$ (B) $\frac{1}{3}, 3$ (C) $\frac{1}{3}, \frac{1}{3}$ (D) $3, 3$

Sol. : Suppose $y = \frac{x^2 - 2x + 4}{x^2 + 2x + 4}$

$$\therefore x^2y + 2xy + 4y = x^2 - 2x + 4$$

$$\therefore (y-1)x^2 + 2(y+1)x + 4(y-1) = 0$$

In order that $x \in \mathbb{R}$ exists, we must have $b^2 - 4ac \geq 0$.

$$\therefore 4(y+1)^2 - 4(y-1)4(y-1) \geq 0$$

$$\therefore (y+1)^2 - 4(y-1)^2 \geq 0$$

$$\therefore [y+1-2(y-1)](y+1+2(y-1)) \geq 0$$

$$\therefore (3-y)(3y-1) \geq 0 \text{ i.e. } (y-3)(3y-1) \leq 0$$

$$\therefore \frac{1}{3} \leq y \leq 3$$

$$\therefore \frac{x^2 - 2x + 4}{x^2 + 2x + 4} \text{ has the minimum value } \frac{1}{3} \text{ and the maximum value } 3.$$

Note : They are attained at $x = 2$ and $x = -2$ respectively.

Ans. : (A)

(21) If $\alpha \neq \beta$ and $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$, then the quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

- (A) $3x^2 - 19x + 3 = 0$ (B) $3x^2 - 19x - 3 = 0$ (C) $3x^2 - 16x + 1 = 0$ (D) $x^2 + 19x - 3 = 0$

Sol. : Here $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$

$$\therefore \alpha^2 - 5\alpha + 3 = 0 \text{ and } \beta^2 - 5\beta + 3 = 0. \text{ So, } \alpha, \beta \text{ are roots of the equation } x^2 - 5x + 3 = 0.$$

$$\therefore \alpha + \beta = 5, \alpha\beta = 3$$

$$\therefore \text{If } \frac{\alpha}{\beta} \text{ and } \frac{\beta}{\alpha} \text{ are roots of the required equation, then}$$

$$\text{The sum of roots} = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{25 - 6}{3} = \frac{19}{3}$$

$$\text{The product of roots} = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$$

$$\therefore \text{The required quadratic equation is } x^2 - \left(\frac{19}{3}\right)x + 1 = 0.$$

$$\therefore 3x^2 - 19x + 3 = 0$$

Ans. : (A)

(22) The equation $\frac{\log 3 + \log(x^2 + 2)}{\log(x-2)} = 2$ has real root(s).

- (A) 0 (B) 1 (C) 2 (D) 3

Sol. : $\frac{\log 3 + \log(x^2 + 2)}{\log(x-2)} = 2$

$$\therefore \log [3 (x^2 + 2)] = 2 \log (x - 2) = \log (x - 2)^2$$

$$\therefore 3 (x^2 + 2) = (x - 2)^2$$

$$\therefore 3x^2 + 6 = x^2 - 4x + 4$$

$$\therefore 2x^2 + 4x + 2 = 0$$

$$\therefore x^2 + 2x + 1 = 0$$

$$\therefore (x + 1)^2 = 0$$

$$\therefore x = -1 \quad \text{But for } x = -1, \log (x - 2) \text{ is not possible.}$$

$$\therefore x \neq -1$$

$$\therefore \text{No solution is possible.}$$

Ans. : (A)

(23) The real roots of the equation $\sqrt{x-2} (x^2 - 4x + 3) = 0$ are

(A) 1, 2

(B) 1, 3

(C) 2, 3

(D) 1, 2, 3

Sol. : $\sqrt{x-2} (x^2 - 4x + 3) = 0$

$$\therefore \sqrt{x-2} (x - 1)(x - 3) = 0$$

$$\therefore x = 1 \text{ or } 2 \text{ or } 3$$

$$\text{But } x - 2 \geq 0$$

$$\therefore x = 2, 3$$

Ans. : (C)

(24) If x, y, z are distinct and real, then $x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$ is always

(A) non-negative

(B) negative

(C) 0

(D) complex

Sol. : $x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$

$$= (x)^2 + (2y)^2 + (3z)^2 - (2y)(3z) - x(3z) - x(2y)$$

$$= \frac{1}{2} [(x - 2y)^2 + (2y - 3z)^2 + (x - 3z)^2] \geq 0$$

$$\text{So } x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy \text{ is always non-negative.}$$

Ans. : (A)

(25) If $\log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$ and $x \neq y$, then $x + y = \dots$

(A) 8

(B) $\frac{1}{2^3}$

(C) $8 - 2^{\frac{1}{3}}$

(D) $8 + 2^{\frac{1}{3}}$

Sol. : Here given that $\log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$.

$$\therefore \text{We have } \log_2 x + \frac{1}{\log_2 x} = \frac{10}{3}. \text{ So, } \frac{(\log_2 x)^2 + 1}{\log_2 x} = \frac{10}{3}$$

Let $\log_2 x = m$, so $3(m^2 + 1) = 10m$

$$\therefore 3m^2 - 10m + 3 = 0$$

$$\therefore (3m - 1)(m - 3) = 0$$

$$\therefore m = 3 \text{ or } m = \frac{1}{3}$$

$$\therefore \log_2 x = 3 \text{ or } \log_2 x = \frac{1}{3}$$

Similarly, we get $\log_2 y = 3$ or $\log_2 y = \frac{1}{3}$.

Now, since $x \neq y$ by taking, $\log_2 x = 3$ we have, $\log_2 y \neq 3$. So, $\log_2 y = \frac{1}{3}$.

or by taking $\log_2 x = \frac{1}{3}$, we have $\log_2 y = 3$.

$$\therefore x = 2^3 \text{ and } y = 2^{\frac{1}{3}} \text{ or } x = 2^{\frac{1}{3}} \text{ and } y = 2^3.$$

$$\therefore x + y = 2^3 + 2^{\frac{1}{3}} = 8 + 2^{\frac{1}{3}}$$

Ans. : (D)

(26) If $x = \sqrt{12 - \sqrt{140}}$, then $x + \frac{1}{x} = \dots\dots$

(A) $\sqrt{7} + \sqrt{5}$

(B) $\frac{\sqrt{7} - \sqrt{5}}{2}$

(C) $\frac{3\sqrt{7} - \sqrt{5}}{2}$

(D) $\frac{\sqrt{7} - 3\sqrt{5}}{2}$

Sol. : $x = \sqrt{12 - \sqrt{140}} = \sqrt{12 - 2\sqrt{35}} = \sqrt{7} - \sqrt{5}$

(square root of a surd)

$$\therefore \frac{1}{x} = \frac{1}{\sqrt{7} - \sqrt{5}} \times \frac{\sqrt{7} + \sqrt{5}}{\sqrt{7} + \sqrt{5}} = \frac{\sqrt{7} + \sqrt{5}}{2}$$

$$\therefore x + \frac{1}{x} = \sqrt{7} - \sqrt{5} + \frac{\sqrt{7} + \sqrt{5}}{2}$$

$$= \frac{3\sqrt{7} - \sqrt{5}}{2}$$

Ans. : (C)

(27) If 3, 3 are the roots of the quadratic equation $x^2 + ax + \beta = 0$ and -8, 2 are the roots of the equation $x^2 + \alpha x + b = 0$ then the roots of the equation $x^2 + ax + b = 0$ are

(A) 2, 8

(B) -2, -8

(C) 8, -2

(D) -8, 2

Sol. : Since 3, 3 are roots of the equation $x^2 + ax + \beta = 0$, the sum of the roots $3 + 3 = -a$. So we get $a = -6$.

Also, -8 and 2 are roots of $x^2 + \alpha x + b = 0$. So product of roots $= -8(2) = b$.

So, $b = -16$.

So we get a new quadratic equation $x^2 - 6x - 16 = 0$.

$$\therefore (x - 8)(x + 2) = 0$$

$$\therefore x = 8 \text{ or } x = -2.$$

Ans. : (C)

- (28) Out of two students wrote a wrong constant term of a quadratic equation at the time of solving it and obtained the sum of correct roots as 3. Another student wrote correct coefficient of x^2 and constant term of that equation which are 1 and -18 respectively. The correct roots of the equation are

(A) 3, 6 (B) $-3, -6$ (C) $-6, 3$ (D) 6, -3

Sol. : Here, the sum of the roots = 3

$$\text{and the product of the roots} = \frac{-18}{1} = -18$$

$$\therefore \text{The quadratic equation is } x^2 - 3x - 18 = 0$$

$$\therefore (x - 6)(x + 3) = 0$$

$$\therefore x = 6 \text{ or } x = -3$$

Ans. : (D)

- (29) One of the roots of the equation $(5+2\sqrt{6})^{x-3} + (5-2\sqrt{6})^{x-3} = 10$ is

(A) 4 (B) 3 (C) 8 (D) 2

Sol. : Here $5-2\sqrt{6} = \frac{1}{5+2\sqrt{6}}$

$$\text{Now, } (5+2\sqrt{6})^{x-3} + (5-2\sqrt{6})^{x-3} = 10$$

$$\therefore (5+2\sqrt{6})^{x-3} + \left(\frac{1}{5+2\sqrt{6}}\right)^{x-3} = 10$$

$$\text{Let } (5+2\sqrt{6})^{x-3} = m$$

$$\therefore m + \frac{1}{m} = 10$$

$$\therefore m^2 - 10m + 1 = 0$$

$$\therefore m = \frac{10 \pm \sqrt{100-4}}{2}$$

$$\therefore m = 5 \pm 2\sqrt{6}$$

$$\therefore m = (5+2\sqrt{6})^{x-3} = (5+2\sqrt{6})^1 \text{ or } (5+2\sqrt{6})^{-1}$$

$$\therefore x - 3 = \pm 1$$

$$\therefore x = 4 \text{ or } 2$$

Ans. : (A),(D)

- (30) If the quadratic equation $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ has equal roots, then a, b, c are in

(A) Arithmetic progression

(B) Geometric progression

(C) Harmonic progression

(D) Arithmetico-Geometric progression

Sol. : $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ has equal roots.

$$\therefore \Delta = 0$$

$$\therefore 4b^2(a + c)^2 - 4(a^2 + b^2)(b^2 + c^2) = 0$$

$$\therefore 4b^2(a^2 + 2ac + c^2) - 4(a^2b^2 + a^2c^2 + b^4 + b^2c^2) = 0$$

$$\therefore 8ab^2c - 4a^2c^2 - 4b^4 = 0$$

$$\therefore b^4 + a^2c^2 - 2ab^2c = 0$$

$$\therefore (b^2 - ac)^2 = 0$$

$$\therefore b^2 = ac$$

$$\therefore a, b, c \text{ are in G.P.}$$

Ans. : (B)

(31) Roots of $\log_2(x^2 + 5x + 10) = 2$ are

(A) 2, 3

(B) -2, 3

(C) -3, 2

(D) -2, -3

Sol. : $\log_2(x^2 + 5x + 10) = 2$

$$\therefore x^2 + 5x + 10 = 2^2 = 4$$

$$\therefore x^2 + 5x + 6 = 0$$

$$\therefore (x + 3)(x + 2) = 0$$

$$\therefore x = -2 \text{ or } x = -3.$$

Ans. : (D)

(32) If α and β are roots of $ax^2 + bx + c = 0$, then $\frac{1}{a\alpha+b} + \frac{1}{a\beta+b} = \dots\dots$

(A) $\frac{c}{ab}$

(B) $\frac{a}{bc}$

(C) $\frac{b}{ac}$

(D) $\frac{1}{abc}$

Sol. : $\frac{1}{a\alpha+b} + \frac{1}{a\beta+b} = \frac{a\beta+b+a\alpha+b}{(a\alpha+b)(a\beta+b)}$

$$= \frac{a(\alpha+\beta)+2b}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$$

$$= \frac{a\left(\frac{-b}{a}\right)+2b}{a^2\left(\frac{c}{a}\right)+ab\left(\frac{-b}{a}\right)+b^2} = \frac{-b+2b}{ac-b^2+b^2} = \frac{b}{ac}$$

Ans. : (C)

(33) If $x \in \mathbb{R}$ and $k = \frac{x^2+x+1}{x^2-x+1}$, then $k \in \dots\dots$

(A) $\left(\frac{1}{3}, 3\right)$

(B) $\left[\frac{1}{3}, 3\right]$

(C) $k \leq \frac{1}{3}$

(D) $k \geq 3$

Sol. : Given that $k = \frac{x^2+x+1}{x^2-x+1}$.

$$\therefore x^2 + x + 1 = kx^2 - kx + k$$

$$\therefore (k-1)x^2 - (k+1)x + (k-1) = 0$$

Now in order that $x \in \mathbb{R}$ satisfies this equation, $\Delta = b^2 - 4ac \geq 0$

$$\therefore (k+1)^2 - 4(k-1)(k-1) \geq 0$$

$$\therefore (k+1)^2 - [2(k-1)]^2 \geq 0$$

$$\therefore (k+1-2k+2)(k+1+2k-2) \geq 0$$

$$\therefore (3-k)(3k-1) \geq 0$$

$$\therefore (k-3)(3k-1) \leq 0$$

$$\therefore \frac{1}{3} \leq k \leq 3$$

$$\therefore k \in \left[\frac{1}{3}, 3\right]$$

Ans. : (B)

$$(34) \quad \text{If } \alpha \text{ and } \beta \text{ are roots of } x^2 - m(x+1) - n = 0, \text{ then } \frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + n} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + n} = \dots \dots$$

(A) 0

(B) 1

(C) 2

(D) 4

Sol. : $x^2 - m(x+1) - n = 0$

$$\therefore x^2 - mx - (m+n) = 0 \text{ has roots } \alpha \text{ and } \beta. \quad (1)$$

$$\therefore \alpha + \beta = m; \alpha\beta = -(m+n)$$

$$\therefore n = -\alpha\beta - m$$

$$= -\alpha\beta - (\alpha + \beta)$$

$$\therefore \frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha + n} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta + n} = \frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + 2\alpha - \alpha\beta - \alpha - \beta} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + 2\beta - \alpha\beta - \alpha - \beta}$$

$$= \frac{\alpha^2 + 2\alpha + 1}{\alpha^2 + \alpha - \alpha\beta - \beta} + \frac{\beta^2 + 2\beta + 1}{\beta^2 + \beta - \alpha\beta - \alpha}$$

$$= \frac{(\alpha+1)^2}{(\alpha-\beta)(\alpha+1)} + \frac{(\beta+1)^2}{(\beta+1)(\beta-\alpha)}$$

$$= \frac{\alpha+1}{\alpha-\beta} - \frac{\beta+1}{\alpha-\beta}$$

$$= \frac{\alpha-\beta}{\alpha-\beta}$$

$$= 1$$

Ans. : (B)

(35) If the roots of the equation $x^2 + x - 1 = 0$ are in ratio $m : n$, then which of the following is true ?

(A) $2m+n(3+\sqrt{5})=0$

(B) $2m = 3n$

(C) $3m - 2n(\sqrt{5}+1) = 0$

(D) $3m = 2n$

Sol. : Suppose α and β are roots of the equation $x^2 + x - 1 = 0$.

Here $\frac{\alpha}{\beta} = \frac{m}{n}$. Also, $\alpha + \beta = -1$; $\alpha\beta = -1$

$$\therefore \frac{\alpha+\beta}{\alpha-\beta} = \frac{m+n}{m-n} \text{ (Componendo-Dividendo)}$$

$$\therefore \frac{(\alpha+\beta)^2}{(\alpha-\beta)^2} = \frac{(m+n)^2}{(m-n)^2}$$

$$\therefore \frac{(\alpha+\beta)^2}{(\alpha+\beta)^2 - 4\alpha\beta} = \frac{(m+n)^2}{(m-n)^2}$$

$$\therefore \frac{1}{1+4} = \frac{(m+n)^2}{(m-n)^2}$$

$$\therefore \frac{1}{5} = \left(\frac{m+n}{m-n} \right)^2$$

$$\therefore \frac{m+n}{m-n} = \frac{1}{\sqrt{5}}$$

$$\therefore \frac{m+n+m-n}{m+n-m+n} = \frac{1+\sqrt{5}}{1-\sqrt{5}} \quad (\text{Componendo-Dividendo})$$

$$\therefore \frac{m}{n} = \frac{\sqrt{5}+1}{1-\sqrt{5}} \times \frac{(1+\sqrt{5})}{(1+\sqrt{5})} = \frac{6+2\sqrt{5}}{-4}$$

$$\therefore \frac{m}{n} = \frac{-3}{2} - \frac{\sqrt{5}}{2} = \frac{-1}{2} (3+\sqrt{5})$$

$$\therefore 2m + n (3+\sqrt{5}) = 0$$

Ans. : (A)

Note : If $\frac{m+n}{m-n} = \frac{-1}{\sqrt{5}}$, then we get $2m + n (3-\sqrt{5}) = 0$ which is not given in options.

Second method : Let $\alpha = mk$, $\beta = nk$

$$\therefore \alpha + \beta = (m+n)k = -1, \quad \alpha\beta = mnk^2 = -1.$$

$$\therefore k = \frac{-1}{m+n}, \quad k^2 = \frac{-1}{mn}. \quad \text{So, } \frac{1}{(m+n)^2} = \frac{-1}{mn}. \quad \text{So, } m^2 + 3mn + n^2 = 0$$

$$\therefore \frac{m}{n} = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2}$$

$$\therefore 2m = (-3 \pm \sqrt{5})n$$

$$\therefore 2m + (3+\sqrt{5})n = 0 \quad (\text{As given option})$$

(36) The real roots of the equation $(x+1)(x+2)(x+3)(x+4) = 120$ are

(A) 6, 1

(B) -1, 6

(C) -6, -1

(D) -6, 1

Sol. : $(x+1)(x+2)(x+3)(x+4) = 120$

$$\therefore [(x+1)(x+4)][(x+2)(x+3)] = 120$$

$$\therefore (x^2 + 5x + 4)(x^2 + 5x + 6) = 0$$

$$\text{Let } x^2 + 5x = m$$

$$\therefore (m+4)(m+6) = 120$$

$$\therefore m^2 + 10m - 96 = 0$$

$$\therefore (m+16)(m-6) = 0$$

$$\therefore m = -16 \text{ or } m = 6$$

$$\therefore x^2 + 5x = -16 \text{ or } x^2 + 5x = 6$$

$$(x^2 + 5x = m)$$

$$\therefore x^2 + 5x + 16 = 0 \text{ or } x^2 + 5x - 6 = 0$$

$$\text{Taking } x^2 + 5x + 16 = 0, \quad \Delta = (5)^2 - 4(1)(16) = 25 - 64 = -39 < 0$$

$\therefore$ The equation has no real solution.

$$\text{Taking, } x^2 + 5x - 6 = 0, \quad (x+6)(x-1) = 0.$$

$$\therefore x = -6 \text{ or } x = 1$$

Ans. : (D)

Note : $(x+1)(x+2)(x+3)(x+4) = 2.3.4.5 = (-5)(-4)(-3)(-2)$ gives two real roots $x = 1$ and $x = -6$.

- (37) A quadratic equation whose roots are rational, has the sum of the squares of the roots 40 and the sum of the cubes of the roots 208. It is

(A) $x^2 - 4x - 12 = 0$ (B) $x^2 - 4x + 12 = 0$ (C) $x^2 + 4x - 12 = 0$ (D) $x^2 + 4x + 12 = 0$

Sol. : Suppose α and β are the roots of the required quadratic equation.

$$\alpha^2 + \beta^2 = 40 \text{ and } \alpha^3 + \beta^3 = 208$$

$$\therefore (\alpha + \beta)^2 - 2\alpha\beta = 40 \text{ and } (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = 208$$

$$\therefore (\alpha + \beta)(40 - \alpha\beta) = 208$$

$$\text{Let } \alpha + \beta = m \text{ and } \alpha\beta = n.$$

$$\therefore m^2 - 2n = 40 \text{ and } m(40 - n) = 208.$$

$$\therefore \frac{m^2 - 40}{2} = n. \text{ Substitute this value of } n \text{ in the equation } m(40 - n) = 208.$$

$$\therefore m \left[40 - \left(\frac{m^2 - 40}{2} \right) \right] = 208$$

$$\therefore m[80 - m^2 + 40] = 416$$

$$\therefore 120m - m^3 = 416$$

$$\therefore m^3 - 120m + 416 = 0$$

$$\therefore (m - 4)(m^2 + 4m - 104) = 0$$

$$\therefore m = 4 \text{ or } m^2 + 4m - 104 = 0$$

$$\text{Now, for } m^2 + 4m - 104 = 0, \Delta = 16 - 4(1)(-104) = 432$$

$$\therefore \sqrt{\Delta} = 12\sqrt{3} \text{ which is irrational. But } \alpha, \beta \text{ are rational.}$$

$$\therefore m = 4. \text{ So, } n = \frac{16 - 40}{2} = -12$$

$$\therefore \alpha + \beta = 4; \alpha\beta = -12$$

$$\therefore x^2 - 4x - 12 = 0 \text{ is the required quadratic equation.}$$

Ans. : (A)

Note : Option (B) and (D) have complex roots. Now, we have two options (A) and (C). The roots of (A) are $\alpha = 6$ and $\beta = -2$. They satisfies the given condition. So, answer is (A).

- (38) The solution set of the equation $3 \left(x^2 + \frac{1}{x^2} \right) + 16 \left(x + \frac{1}{x} \right) + 26 = 0$ is

(A) $\left\{ 1, 3, \frac{1}{3} \right\}$ (B) $\left\{ -1, -3, \frac{-1}{3} \right\}$ (C) $\left\{ -1, 3, \frac{-1}{3} \right\}$ (D) $\left\{ 1, 3, \frac{-1}{3} \right\}$

Sol. : We have $3 \left(x^2 + \frac{1}{x^2} \right) + 16 \left(x + \frac{1}{x} \right) + 26 = 0.$

$$\text{Let } \left(x + \frac{1}{x} \right) = m. \text{ Then } \left(x^2 + \frac{1}{x^2} \right) = m^2 - 2$$

$$\therefore 3(m^2 - 2) + 16m + 26 = 0$$

$$\therefore 3m^2 + 16m + 20 = 0$$

$$\therefore (3m + 10)(m + 2) = 0$$

$$\therefore m = \frac{-10}{3} \text{ or } m = -2$$

$$\therefore \text{Let } m = \frac{-10}{3}. \text{ So, } x + \frac{1}{x} = \frac{-10}{3}.$$

$$\therefore 3x^2 + 10x + 3 = 0. \text{ So, } (3x + 1)(x + 3) = 0$$

$$\therefore \text{We get } x = \frac{-1}{3} \text{ or } x = -3.$$

$$\text{If } m = -2, x + \frac{1}{x} = -2.$$

$$\therefore x^2 + 2x + 1 = 0$$

$$\therefore (x + 1)^2 = 0. \text{ So, } x = -1$$

$$\therefore \text{So the set of possible values of } x \text{ is } \left\{-3, -1, \frac{-1}{3}\right\}.$$

Ans. : (B)

(39) The number of real roots of the equation $27^{\frac{1}{x}} + 12^{\frac{1}{x}} = 2 \cdot 8^{\frac{1}{x}}$ is

(A) 0

(B) 1

(C) 2

(D) 3

Sol. : $27^{\frac{1}{x}} + 12^{\frac{1}{x}} = 2 \cdot 8^{\frac{1}{x}}$

$$\therefore \left(\frac{27}{8}\right)^{\frac{1}{x}} + \left(\frac{12}{8}\right)^{\frac{1}{x}} = 2$$

$$\therefore \left(\frac{3}{2}\right)^{\frac{3}{x}} + \left(\frac{3}{2}\right)^{\frac{1}{x}} = 2$$

$$\text{Let } \left(\frac{3}{2}\right)^{\frac{1}{x}} = m$$

$$\therefore m^3 + m = 2$$

$$\therefore m^3 + m - 2 = 0$$

$$\therefore (m^2 + m + 2)(m - 1) = 0$$

$$\therefore m = 1 \text{ or } m^2 + m + 2 = 0$$

Now, for $m^2 + m + 2 = 0$, $\Delta = 1 - 4(1)(2) = -7 < 0$. So there is no real root.

$$\therefore m = 1$$

$$\therefore \left(\frac{3}{2}\right)^{\frac{1}{x}} = 1 = \left(\frac{3}{2}\right)^0$$

$$\therefore \frac{1}{x} = 0 \text{ which is not possible.}$$

$\therefore$ The number of real roots is 0.

Ans. : (A)

- (40) If one of the roots of the equation $ax^2 + bx + c = 0$ is equal to the n th power of its other root then $(ac^n)^{\frac{1}{n+1}} + (a^n c)^{\frac{1}{n+1}} = \dots$
- (A) $-a$ (B) $-b$ (C) $-c$ (D) $-ab$

Sol. : Suppose α and β are the roots of the equation $ax^2 + bx + c = 0$.

Now, we know $\alpha = \beta^n$.

Also, $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$. Substitute, $\alpha = \beta^n$. We get, $\beta^n \cdot \beta = \frac{c}{a}$

$$\therefore \beta^{n+1} = \frac{c}{a}. \text{ So, } \beta = \left(\frac{c}{a}\right)^{\frac{1}{n+1}}. \quad \text{So, } \alpha = \left(\frac{c}{a}\right)^{\frac{n}{n+1}}.$$

$$\therefore \text{From } \alpha + \beta = \frac{-b}{a}, \left(\frac{c}{a}\right)^{\frac{n}{n+1}} + \left(\frac{c}{a}\right)^{\frac{1}{n+1}} = \frac{-b}{a}$$

$$\therefore \frac{\frac{c^{\frac{n}{n+1}}}{a^{\frac{n}{n+1}}}}{\frac{c^{\frac{1}{n+1}}}{a^{\frac{1}{n+1}}}} + \frac{c^{\frac{1}{n+1}}}{a^{\frac{1}{n+1}}} = \frac{-b}{a} \quad \text{So, } \frac{\frac{c^{\frac{n}{n+1}}}{a^{\frac{n}{n+1}}} \cdot \frac{1}{a^{\frac{1}{n+1}}} + \frac{c^{\frac{1}{n+1}}}{a^{\frac{1}{n+1}} \cdot c^{\frac{1}{n+1}}}}{\frac{c^{\frac{n}{n+1}}}{a^{\frac{n}{n+1}}} \cdot \frac{1}{a^{\frac{1}{n+1}}}} = \frac{-b}{a}$$

$$\therefore \frac{(c^n \cdot a)^{\frac{1}{n+1}} + (a^n \cdot c)^{\frac{1}{n+1}}}{a} = \frac{-b}{a}. \quad \text{So, } (c^n \cdot a)^{\frac{1}{n+1}} + (a^n \cdot c)^{\frac{1}{n+1}} = -b \quad \text{Ans. : (B)}$$

- (41) If $ax + by = 1$ and $px^2 + qy^2 = 1$ has only one root, then which of the following is true ?
- (A) $\frac{a^2}{p} + \frac{b^2}{q} = 1$ (B) $x = \frac{-a}{p}$ (C) $x = \frac{b}{q}$ (D) $a^2b^2 = pq$

Sol. : $ax + by = 1$. So, $y = \frac{1-ax}{b}$

Substitute the value of y in the equation $px^2 + qy^2 = 1$. We get $px^2 + q \left(\frac{1-ax}{b}\right)^2 = 1$

$$\therefore b^2px^2 + q(1-ax)^2 = b^2$$

$$\therefore pb^2x^2 + q - 2aqx + qa^2x^2 = b^2$$

$$\therefore (qa^2 + pb^2)x^2 - 2aqx + q - b^2 = 0. \text{ Since it has only one root, } \Delta = 0$$

$$\therefore 4a^2q^2 - 4(qa^2 + pb^2)(q - b^2) = 0$$

$$\therefore 4a^2q^2 - 4(a^2q^2 - a^2b^2q + b^2pq - b^4p) = 0$$

$$\therefore a^2q - pq + b^2p = 0 \quad (b \neq 0)$$

$$\therefore a^2q + b^2p = pq$$

$$\therefore \frac{a^2}{p} + \frac{b^2}{q} = 1 \quad \text{Ans. : (A)}$$

- (42) If $\log_{10} a + \log_{10} \sqrt{a} + \log_{10} \sqrt[4]{a} + \dots = b > 0$ and $\frac{\sum_{n=1}^b (2n-1)}{\sum_{n=1}^b (3n+1)} = \frac{20}{7 \log_{10} a}$, then $a = \dots$

- (A) 100 (B) 1000 (C) 10000 (D) 100000

Sol. : We have $\log_{10} a + \log_{10} \sqrt{a} + \log_{10} \sqrt[4]{a} + \dots = b$.

$$\therefore \log_{10} a + \log_{10} a^{\frac{1}{2}} + \log_{10} a^{\frac{1}{4}} + \dots = b$$

$$\therefore \log_{10} (a \cdot a^{\frac{1}{2}} \cdot a^{\frac{1}{4}} \dots) = b$$

$$\therefore \log_{10} a^{1+\frac{1}{2}+\frac{1}{4}+\dots} = b$$

$$\therefore \log_{10} a^{\frac{1}{1-\frac{1}{2}}} = b$$

$$\therefore \log_{10} a^2 = b$$

$$\therefore 2 \log_{10} a = b$$

$$\frac{\sum_{n=1}^b (2n-1)}{\sum_{n=1}^b (3n+1)} = \frac{20}{7 \log_{10} a}$$

$$\therefore \frac{2 \cdot \frac{b(b+1)}{2} - b}{3 \cdot \frac{b(b+1)}{2} + b} = \frac{20}{7 \cdot \frac{b}{2}}$$

$$\therefore \frac{b^2}{3b^2 + 5b} = \frac{20}{7b} \cdot \text{So, } \frac{b^2}{3b+5} = \frac{20}{7}$$

$$\therefore 7b^2 - 60b - 100 = 0$$

$$\therefore (7b + 10)(b - 10) = 0$$

$$\therefore b = \frac{-10}{7} \text{ or } b = 10$$

But $b \neq \frac{-10}{7}$, because $b \in \mathbb{N}$. So, $b = 10$

$$\therefore \log_{10} a = 5. \quad \text{So, } a = 10^5 = 100000$$

Ans. : (D)

(43) If for $x \in \mathbb{R}$, $3^{72} \left(\frac{1}{3}\right)^x \left(\frac{1}{3}\right)^{\sqrt{x}} > 1$, then $x \in \dots$

(A) $[0, 64]$

(B) $(0, 64]$

(C) $[0, 64]$

(D) $(0, 64)$

Sol. : $3^{72} \left(\frac{1}{3}\right)^x \left(\frac{1}{3}\right)^{\sqrt{x}} > 1$

$$\therefore 3^{72} (3^{-1})^x (3^{-1})^{\sqrt{x}} > 1$$

$$\therefore 3^{72-x-\sqrt{x}} > 3^0$$

$$\therefore 72 - x - \sqrt{x} > 0$$

$$\therefore x + \sqrt{x} - 72 < 0$$

$$\therefore (\sqrt{x} - 8)(\sqrt{x} + 9) < 0. \quad \text{So, } -9 < \sqrt{x} < 8 \quad \text{But } \sqrt{x} \geq 0$$

$$\therefore 0 \leq \sqrt{x} < 8. \quad \text{So, } 0 \leq x < 64$$

$$\therefore x \in [0, 64)$$

Ans. : (A)

- (44) The roots of the equation $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$ are always
($a \neq b$)

(A) real, equal (B) complex (C) real unequal (D) purely imaginary

Sol. : $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$

$$\therefore 3x^2 - 2(a + b + c)x + (ab + bc + ca) = 0$$

$$\text{Now, } D = 4(a + b + c)^2 - 4(3)(ab + bc + ca)$$

$$= 4[a^2 + b^2 + c^2 - ab - bc - ca]$$

$$= 2[(a - b)^2 + (b - c)^2 + (c - a)^2] > 0$$

Since $D > 0$, the roots are always real and unequal.

Ans. : (C)

- (45) For $a, b, c \in \mathbb{R}$; $a \neq 0$, if α is a root of the equation $a^2x^2 + bx + c = 0$ and β is a root of the equation $a^2x^2 - bx - c = 0$ where, $0 < \alpha < \beta$, then a root γ of the equation $a^2x^2 + 2bx + 2c = 0$ always satisfies
(IIT : 1989)

(A) $\gamma = \alpha + \frac{\beta}{2}$ (B) $\gamma = \frac{\alpha + \beta}{2}$ (C) $\gamma = \frac{\alpha}{2} + \beta$ (D) $\alpha < \gamma < \beta$

Sol. : Since α is a root of the equation $a^2x^2 + bx + c = 0$, $a^2\alpha^2 + b\alpha + c = 0$

$$\therefore b\alpha + c = -a^2\alpha^2 \quad (1)$$

Also β is a root of the equation $a^2x^2 - bx - c = 0$. So, $a^2\beta^2 - b\beta - c = 0$

$$\therefore b\beta + c = a^2\beta^2 \quad (2)$$

Now, if γ is a zero of $f(x) = a^2x^2 + 2bx + 2c$, then $f(\gamma) = a^2\gamma^2 + 2b\gamma + 2c = 0$

$$\therefore \text{Taking } x = \alpha \text{ in } f(x) = a^2x^2 + 2bx + 2c,$$

$$\therefore f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c$$

$$= a^2\alpha^2 + 2(b\alpha + c) = a^2\alpha^2 - 2a^2\alpha^2 = -a^2\alpha^2 < 0 \quad (\text{From (1)})$$

Also, by taking $x = \beta$ in $f(x) = a^2x^2 + 2bx + 2c$, we get

$$\therefore f(\beta) = a^2\beta^2 + 2b\beta + 2c$$

$$= a^2\beta^2 + 2(b\beta + c) = a^2\beta^2 + 2a^2\beta^2 = 3a^2\beta^2 > 0 \quad (\text{From (2)})$$

Since $f(\alpha) < 0$ and $f(\beta) > 0$. There exists $\gamma \in (\alpha, \beta)$ such that $f(\gamma) = 0$.

$$\therefore \alpha < \gamma < \beta \text{ is true.}$$

Ans. : (D)

- (46) If α, β are the real roots of the equation $x^2 + px + q = 0$ and α^4, β^4 are the roots of the equation $x^2 - rx + s = 0$, then the roots of the equation $x^2 - 4qx + 2q^2 - r = 0$ are always (IIT : 1989)

(A) real and distinct (B) real and equal
(C) complex (D) one real and one pure imaginary

Sol. : α and β are the roots of the equation $x^2 + px + q = 0$. So, $\alpha + \beta = -p$ and $\alpha\beta = q$.

Also, α^4 and β^4 are the roots of the equation $x^2 - rx + s = 0$.

$\therefore \alpha^4 + \beta^4 = r$ and $\alpha^4 \beta^4 = s$. So, $(\alpha\beta)^4 = s$.

$\therefore$ For the equation $x^2 - 4qx + 2q^2 - r = 0$,

$$\begin{aligned} D = b^2 - 4ac &= 16q^2 - 4(2q^2 - r) \\ &= 16q^2 - 8q^2 + 4r \\ &= 8q^2 + 4r \\ &= 4(2q^2 + r) \\ &= 4[\alpha^4 + \beta^4 + 2\alpha^2\beta^2] = 4(\alpha^2 + \beta^2)^2 = [2(\alpha^2 + \beta^2)]^2 > 0 \end{aligned}$$

$\therefore D > 0$. The roots are real and distinct.

Ans. : (A)

(47) If the quadratic equation in variable x , $(\cos p - 1)x^2 + x \cos p + \sin p = 0$ has real roots, then,
 $p \in \dots\dots$ (IIT : 1990)

- (A) $(0, \pi]$ (B) $(-\pi, 0)$ (C) $(0, 2\pi)$ (D) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

Sol. : Since the equation $(\cos p - 1)x^2 + x \cos p + \sin p = 0$ has real roots,

$$\begin{aligned} D = b^2 - 4ac &\geq 0 \\ \therefore \cos^2 p - 4(\cos p - 1)\sin p &\geq 0 \\ \therefore \cos^2 p + 4\sin p(1 - \cos p) &\geq 0 \\ \therefore \cos^2 p + 4\sin p(2\sin^2 \frac{p}{2}) &\geq 0 \end{aligned}$$

$$\text{Now, } \cos^2 p \geq 0 \text{ and } \sin^2 \frac{p}{2} \geq 0$$

$\therefore$ If $\sin p \geq 0$, then we will have $D \geq 0$.

If $p \in (0, \pi]$, then we have $\sin p \geq 0$.

$$\therefore p \in (0, \pi]$$

Ans. : (A)

(48) $\forall x \in \mathbb{R}$, if quadratic polynomial $f(x) = ax^2 + bx + c > 0$, then $g(x) = f(x) + f'(x) + f''(x)$ satisfies
 $\dots\dots (x \in \mathbb{R})$ (IIT 1990)

- (A) $g(x) < 0$ (B) $g(x) > 0$ (C) $g(x) = 0$ (D) $g(x) \geq 0$

Sol. : [Note : If $a > 0$ and $D = b^2 - 4ac < 0$, then $f(x) > 0$.]

Here $f(x) = ax^2 + bx + c > 0$ is given.

$$\therefore f'(x) = 2ax + b \text{ and } f''(x) = 2a$$

$$\text{Also, } g(x) = f(x) + f'(x) + f''(x)$$

$$\begin{aligned} &= ax^2 + bx + c + 2ax + b + 2a \\ &= ax^2 + (2a + b)x + (b + c + 2a) \end{aligned}$$

For the polynomial $g(x)$, $D = (2a + b)^2 - 4a(b + c + 2a)$

$$= 4a^2 + 4ab + b^2 - 4ab - 4ac - 8a^2 = b^2 - 4a^2 - 4ac = (b^2 - 4ac) - 4a^2 < 0$$

Since, $a > 0$ and $D < 0$, we have $g(x) > 0$. $\forall x \in \mathbb{R}$

Ans. : (B)

- (49) If c and d are the roots of the quadratic equation $(x - a)(x - b) - k = 0$, then the quadratic equation with roots a and b is (IIT : 1992)

- (A) $(x - c)(x - d) + k = 0$ (B) $(x + c)(x + d) - k = 0$
(C) $(x - c)(x - d) - k = 0$ (D) $(x + c)(x + d) + k = 0$

Sol. : Given equation is $(x - a)(x - b) - k = 0$.

$$\therefore x^2 - ax - bx + ab - k = 0$$

$$\therefore x^2 - (a + b)x + ab - k = 0 \text{ has roots } c \text{ and } d.$$

$$\therefore \text{The sum of the roots} = c + d = (a + b) \text{ and the product of the roots} = cd = (ab - k)$$

So, now for the quadratic equation with roots a and b , the sum of the roots $a + b = c + d$ and $ab = cd + k$.

The required quadratic equation with roots a and b is $x^2 - (c + d)x + cd + k = 0$

$$\therefore (x - c)(x - d) + k = 0$$

Ans. : (A)

- (50) If one of the roots for the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is common, then the value of $a + b$ is ($a \neq b$) (IIT : 1986)

- (A) 0 (B) -1 (C) 1 (D) 2

Sol. : Suppose the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ have a common root α .

$$\therefore \alpha^2 + a\alpha + b = 0 \text{ and } \alpha^2 + b\alpha + a = 0 \quad (1)$$

$$\therefore \alpha^2 + a\alpha + b = \alpha^2 + b\alpha + a$$

$$\therefore a\alpha - b\alpha = a - b$$

$$\therefore \alpha(a - b) = (a - b)$$

$$\therefore \alpha = 1$$

$$(a \neq b)$$

Substitute $\alpha = 1$ in equation (1), we get, $1 + b + a = 0$

$$\therefore a + b = -1.$$

Ans. : (B)

- (51) If one of the roots of the equation $x^2 + px + q = 0$; $p, q \in \mathbb{R}$ is $2 + \sqrt{3}i$, then, (IIT : 1982)

- (A) $p = -4, q = 7$ (B) $p = 4, q = 7$ (C) $p = 4, q = -7$ (D) $p = -4, q = -7$

Sol. : $\alpha = 2 + \sqrt{3}i$ is one of the roots of the equation $x^2 + px + q = 0$. So its second root will be $\beta = 2 - \sqrt{3}i$.

The sum of the roots $= \alpha + \beta = -p = 2 + \sqrt{3}i + 2 - \sqrt{3}i$. So, $p = -4$

and the product of the roots $= q = \alpha\beta = (2 + \sqrt{3}i)(2 - \sqrt{3}i)$

$$= 4 - 3i^2 = 4 + 3 = 7 \quad (i^2 = -1)$$

$$\therefore p = -4 \text{ and } q = 7$$

Ans. : (A)

- (52) If a, b, c are non-zero real numbers such that

$$\int_0^1 (1 + \cos^8 x)(ax^2 + bx + c) dx = \int_0^2 (1 + \cos^8 x)(ax^2 + bx + c) dx,$$

then the quadratic equation $ax^2 + bx + c = 0$ has (IIT : 1982)

- (A) no roots in $(0, 2)$ (B) at least one root in $(1, 2)$
(C) two roots in $(0, 2)$ (D) two complex roots.

Sol. : Suppose, $f(t) = \int_0^t (1 + \cos^8 x)(ax^2 + bx + c) dx$, then f is continuous on $[1, 2]$ and differentiable on $(1, 2)$.

Also, $f(1) = f(2)$

So from Roll's theorem there exist $k \in (1, 2)$ such that $f'(k) = 0$.

Now, $f(t) = \int_0^t (1 + \cos^8 x)(ax^2 + bx + c) dx$

$$\therefore f'(x) = (1 + \cos^8 x)(ax^2 + bx + c)$$

$$\therefore f'(k) = (1 + \cos^8 k)(ak^2 + bk + c) = 0$$

$$\therefore ak^2 + bk + c = 0$$

$$\therefore k \text{ is a root of } ax^2 + bx + c = 0$$

But since $k \in (1, 2)$, the quadratic equation $ax^2 + bx + c = 0$ has atleast one root in $(1, 2)$.

Ans. : (B)

- (53) If the roots of the quadratic equation $3x^2 + 2x + p(p-1) = 0$ are of opposite sign, then the set of values of p is (IIT : 1992)

(A) $(-\infty, 0)$

(B) $(0, 1)$

(C) $(1, \infty)$

(D) $(0, \infty)$

Sol. : If the roots of the quadratic equation are of opposite sign, then the product of the roots must be negative.

$$\therefore \alpha \beta < 0$$

$$\therefore \frac{p(p-1)}{3} < 0. \quad \text{So, } 0 < p < 1 \text{ i.e. } p \in (0, 1)$$

Ans. : (B)

- (54) If α and β are the roots of the quadratic equation $x^2 + x + 1 = 0$, then find the quadratic equation whose roots are α^{19} and β^7 (IIT : 1994)

(A) $x^2 - x - 1 = 0$

(B) $x^2 - x + 1 = 0$

(C) $x^2 + x - 1 = 0$

(D) $x^2 + x + 1 = 0$

Sol. : $x^2 + x + 1 = 0$ has roots α and β .

$$\therefore \alpha, \beta = \frac{-1 \pm \sqrt{1-4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{3}i}{2} = \omega \text{ and } \omega^2$$

(If $\omega^3 - 1 = 0$, then $(\omega - 1)(\omega^2 + \omega + 1) = 0$)

Let $\alpha = \omega$ and $\beta = \omega^2$ then,

$$\alpha^{19} + \beta^7 = \omega^{19} + (\omega^2)^7 = \omega^{19} + \omega^{14}$$

$$= (\omega^3)^6 \cdot \omega + (\omega^3)^4 \cdot \omega^2$$

$$= \omega + \omega^2$$

$$(\omega^3 = 1)$$

$$= -1$$

$$(1 + \omega + \omega^2 = 0)$$

The product of the roots $\alpha^{19} \cdot \beta^7 = \omega^{19} \cdot \omega^{14} = \omega^{33} = (\omega^3)^{11} = 1$

$\therefore$ The required equation is $x^2 - (-1)x + 1 = 0$

$$\therefore x^2 + x + 1 = 0$$

Ans. : (D)

- (55) If p, q, r are positive and in A.P., then the quadratic equation $px^2 + qx + r = 0$ has real roots if (IIT : 1995)

(A) $\left| \frac{r}{p} - 7 \right| \geq 4\sqrt{3}$ (B) $\left| \frac{p}{r} - 7 \right| < 4\sqrt{3}$ (C) $\left| \frac{p}{r} + 7 \right| < 4\sqrt{3}$ (D) $\frac{p^2}{r} = 7 + 4\sqrt{3}$

Sol. : Since p, q, r are in A.P., $2q = p + r$ (1)

The equation $px^2 + qx + r = 0$ has real roots if $D = b^2 - 4ac \geq 0$.

i.e. $q^2 - 4pr \geq 0$

i.e. $\left(\frac{p+r}{2} \right)^2 - 4pr \geq 0$ (From (1))

i.e. $(p+r)^2 - 16pr \geq 0$

i.e. $p^2 + r^2 - 14pr \geq 0$

i.e. $1 + \left(\frac{r}{p} \right)^2 - \frac{14r}{p} \geq 0$ ($p^2 \neq 0$) (2)

Now, for $\left(\frac{r}{p} \right)^2 - 14 \left(\frac{r}{p} \right) + 1 = 0$, $\frac{r}{p} = \frac{14 \pm \sqrt{196 - 4(1)(1)}}{2} = 7 \pm \frac{\sqrt{192}}{2} = 7 \pm \frac{8\sqrt{3}}{2}$

$\therefore \frac{r}{p} = 7 \pm 4\sqrt{3}$

$\therefore$ From (2), it is necessary that $\frac{r}{p} \leq 7 - 4\sqrt{3}$ or $\frac{r}{p} \geq 7 + 4\sqrt{3}$

i.e. $\frac{r}{p} - 7 \leq -4\sqrt{3}$ or $\frac{r}{p} - 7 \geq 4\sqrt{3}$

i.e. $\left| \frac{r}{p} - 7 \right| \geq 4\sqrt{3}$ **Ans. : (A)**

- (56) If the two quadratic equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ have a common root, then $(a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1) = \dots\dots\dots$. (IIT 1992)

(A) $-(a_1c_2 - a_2c_1)^2$ (B) $(a_1a_2 - c_1c_2)^2$ (C) $(a_1c_1 - a_2c_2)^2$ (D) $(a_1c_2 - a_2c_1)^2$

Sol. : Suppose the equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ have a common root α .

$\therefore a_1\alpha^2 + b_1\alpha + c_1 = 0$ and $a_2\alpha^2 + b_2\alpha + c_2 = 0$

$\therefore \begin{bmatrix} \alpha^2 & \alpha & 1 \\ b_1 & c_1 \\ b_2 & c_2 \end{bmatrix} = \begin{bmatrix} -\alpha & 1 \\ a_1 & c_1 \\ a_2 & c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$

$\therefore \frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$

$$\therefore \alpha^2 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}; \quad \alpha = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

$$\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} = \left(\frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \right)^2$$

$$\therefore (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1) = (a_1c_2 - a_2c_1)^2 \quad \text{Ans. : (D)}$$

- (57) If the difference between the roots of the quadratic equation $x^2 + px + 8 = 0$ is 2, then $p = \dots\dots$.
(IIT 1992)

(A) ± 2

(B) ± 4

(C) ± 6

(D) ± 8

Sol. : Suppose the quadratic equation $x^2 + px + 8 = 0$ has two roots α and β .

$$\therefore \alpha + \beta = -p \text{ and } \alpha\beta = 8. \text{ Also, } \alpha - \beta = 2 \text{ is given.}$$

$$\therefore 2\alpha = 2 - p. \text{ From } \alpha - 2 = \beta, \quad \frac{2-p}{2} - 2 = \beta.$$

$$\text{So, } \frac{-p-2}{2} = \beta$$

Now, substituting values of α and β in $\alpha\beta = 8$,

$$\text{we get, } \left(\frac{2-p}{2} \right) \left(\frac{-p-2}{2} \right) = 8$$

$$\therefore (p+2)(p-2) = 32$$

$$\therefore p^2 - 4 = 32. \quad \text{So, } p^2 = 36.$$

$$\text{Hence, } p = \pm 6$$

$$\text{Ans. : (C)}$$

- (58) If the two roots of the quadratic equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal, then a, b, c are in sequence.
(IIT : 1993)

(A) Arithmetic

(B) Geometric

(C) Harmonic

(D) Arithmetico-Geometric

Sol. : Since $\alpha = 1$ satisfies the given equation, $\beta = 1$.

$$\text{So, } \alpha\beta = 1$$

$$\therefore \frac{c(a-b)}{a(b-c)} = 1$$

$$\therefore ca - bc = ab - ac$$

$$\therefore 2ac = b(a+c)$$

$$\therefore b = \frac{2ac}{a+c}$$

$\therefore a, b, c$ are in Harmonic progression.

$$\text{Ans. : (C)}$$

- (59) The roots of the quadratic equation $kx^2 + 1 = kx + 3x - 11x^2$ are real and equal for the values of k .
(IIT : 1993)

(A) -11, -3

(B) 5, 7

(C) 5, -7

(D) 7, -5

Sol. : $kx^2 + 1 = kx + 3x - 11x^2$.

$$\therefore (11+k)x^2 - (k+3)x + 1 = 0 \text{ has real and equal roots.}$$

$$\therefore D = (k+3)^2 - 4(k+11) = 0$$

$$\therefore k^2 + 2k - 35 = 0$$

$$\therefore (k+7)(k-5) = 0$$

$$\therefore k = -7 \text{ or } 5$$

$$\text{Ans. : (C)}$$

- (60) If the roots of the quadratic equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = \dots$ (AIEEE : 2005)

(A) -2 (B) 2 (C) 3 (D) 1

Sol. : Suppose the equation $x^2 - bx + c = 0$ has roots α and β which are two consecutive integers.

$$\therefore \beta = \alpha + 1, \quad \alpha \in \mathbb{Z}$$

$$\text{Also, } \alpha + \beta = b \quad \alpha\beta = c$$

$$\therefore \alpha + \alpha + 1 = b \quad \text{and} \quad \alpha(\alpha + 1) = c$$

$$\therefore 2\alpha + 1 = b$$

$$\therefore b^2 - 4c = (2\alpha + 1)^2 - 4\alpha(\alpha + 1) = 4\alpha^2 + 4\alpha + 1 - 4\alpha^2 - 4\alpha = 1 \quad \text{Ans. : (D)}$$

$$\begin{aligned} \text{Second method : } |\alpha - \beta| = 1 &\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1 \\ &\Rightarrow b^2 - 4c = 1 \end{aligned}$$

- (61) If the sum of squares of the roots of the quadratic equation $x^2 - (a - 2)x - a - 1 = 0$ attains minimum value, then $a = \dots$ (AIEEE : 2005)

(A) 0 (B) 1 (C) 2 (D) 3

Sol. : Suppose the quadratic equation $x^2 - (a - 2)x - a - 1 = 0$ has roots α and β .

$\alpha^2 + \beta^2$ attains the minimum value.

$$\text{Here } \alpha + \beta = a - 2 \text{ and } \alpha\beta = -(a + 1).$$

$$\begin{aligned} \therefore \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= (a - 2)^2 + 2(a + 1) \\ &= a^2 - 2a + 6 = (a - 1)^2 + 5 \geq 5 \end{aligned}$$

$\therefore$ The minimum value of $\alpha^2 + \beta^2$ is 5 and it occurs when $a - 1 = 0$.

$$\therefore a = 1 \quad \text{Ans. : (B)}$$

- (62) If $m\angle R = \frac{\pi}{2}$ in ΔPQR , and the roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$) are $\tan\left(\frac{P}{2}\right)$ and

$\tan\left(\frac{Q}{2}\right)$, then which of the following is true? (AIEEE : 2005)

(A) $a = b + c$ (B) $b = c$ (C) $b = a + c$ (D) $c = a + b$

Sol. : Here $ax^2 + bx + c = 0$ has roots $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$.

$$\therefore \tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = \frac{-b}{a}; \text{ and } \left(\tan\left(\frac{P}{2}\right) \cdot \tan\left(\frac{Q}{2}\right)\right) = \frac{c}{a}$$

$$\therefore \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = \frac{\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right)}{1 - \tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)} = \frac{\frac{-b}{a}}{1 - \frac{c}{a}} = \frac{-b}{a - c} \quad (1)$$

Now in ΔPQR , $m\angle P + m\angle Q + m\angle R = \pi$

$$\therefore m\angle P + m\angle Q = \pi - \frac{\pi}{2} \quad (m\angle R = \frac{\pi}{2})$$

$$\therefore m\angle P + m\angle Q = \frac{\pi}{2} \quad (2)$$

$$\therefore \text{From (1) and (2), } \tan \frac{P+Q}{2} = \tan \frac{\pi}{4} = \frac{-b}{a-c}$$

$$\therefore 1 = \frac{-b}{a-c}$$

$$\therefore a - c = -b$$

$$\therefore c = a + b$$

Ans. : (D)

(63) If x is a real number, then the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is (AIEEE : 2006)

(A) $\frac{1}{4}$

(B) 1

(C) 41

(D) $\frac{17}{7}$

Sol. : Let $\frac{3x^2+9x+17}{3x^2+9x+7} = y$

$$\therefore 3x^2 + 9x + 17 = 3x^2y + 9xy + 7y$$

$$\therefore 3x^2(y-1) + 9x(y-1) + 7y - 17 = 0$$

Now, in order that x is real, it is necessary that,

$$D = b^2 - 4ac \geq 0.$$

$$\therefore 81(y-1)^2 - 12(y-1)(7y-17) \geq 0$$

$$\therefore 81(y^2 - 2y + 1) - 12(7y^2 - 24y + 17) \geq 0$$

$$\therefore -y^2 + 42y - 41 \geq 0$$

$$\therefore y^2 - 42y + 41 \leq 0$$

$$\therefore (y-1)(y-41) \leq 0$$

$$\therefore 1 \leq y \leq 41$$

$$\therefore \text{The maximum value of } \frac{3x^2+9x+17}{3x^2+9x+7} \text{ is 41.}$$

Ans. : (C)

(64) For $m \in \dots$, both the roots of the quadratic equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4 . (AIEEE : 2006)

(A) $(-2, 0)$

(B) $(3, \infty)$

(C) $(-1, 3)$

(D) $(1, 4)$

Sol. : $x^2 - 2mx + m^2 - 1 = 0$

$$(x-m)^2 = 1. \quad \text{So, } x-m = \pm 1$$

$$x = m+1 \text{ or } x = m-1$$

$$\text{Now, } -2 < x < 4. \quad \text{So, } -2 < m-1 < m+1 < 4$$

$$\therefore m > -1 \text{ and } m < 3$$

$$\therefore m \in (-1, 3)$$

Ans. : (C)

(65) If the two roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$ respectively, then $2 + q - p = \dots$. (AIEEE : 2006)

(A) 0

(B) 1

(C) 2

(D) 3

Sol. : Here the roots of $x^2 + px + q = 0$ are $\tan 30^\circ$ and $\tan 15^\circ$ respectively.

$$\therefore \tan 30^\circ + \tan 15^\circ = -p \quad \text{and} \quad (\tan 30^\circ)(\tan 15^\circ) = q$$

$$\text{Also, } \tan 45^\circ = \tan (30^\circ + 15^\circ) = \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ}$$

$$\therefore 1 = \frac{-p}{1-q}$$

$$\therefore 1 - q = -p. \quad \text{So, } q - p = 1. \quad \text{So, } 2 + q - p = 3 \quad \text{Ans. : (D)}$$

(66) If the roots of the quadratic equation $bx^2 + cx + a = 0$ are complex, then the polynomial $3b^2x^2 + 6bcx + 2c^2$

(AIEEE 2009)

$$(A) > 4ab \quad (B) < 4ab \quad (C) > -4ab \quad (D) < -4ab$$

Sol. : The quadratic equation $bx^2 + cx + a = 0$ has complex roots.

$$\therefore D = c^2 - 4ab < 0. \quad \text{So, } c^2 < 4ab$$

$$\therefore -c^2 > -4ab$$

$$\text{Now for, } 3b^2x^2 + 6bcx + 2c^2, \quad 3b^2 > 0,$$

$$\begin{aligned} \text{The minimum value of the polynomial is } \frac{4(3b^2)(2c^2) - 36b^2c^2}{4(3b^2)} &= \frac{24b^2c^2 - 36b^2c^2}{12b^2} \\ &= \frac{-12b^2c^2}{12b^2} = -c^2 > -4ab \end{aligned}$$

Ans. : (C)

(67) The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has

(AIEEE : 2012)

- (A) exactly two real roots (B) no real root
(C) exactly one real root (D) exactly four real roots.

Sol. : Given that $e^{\sin x} - e^{-\sin x} = 4$.

$$\text{Let } e^{\sin x} = m$$

$$\therefore m - \frac{1}{m} = 4. \quad \text{So, } m^2 - 4m - 1 = 0$$

$$\therefore m = \frac{4 \pm \sqrt{16 - 4(-1)}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$$\therefore e^{\sin x} = 2 \pm \sqrt{5}$$

But maximum value of $e^{\sin x}$ is e which is less than $2 + \sqrt{5}$,

$$\text{Also, } e^{\sin x} > 0 > 2 - \sqrt{5}$$

$\therefore$ No real root exists.

Ans. : (B)

(68) If the equation $2x^3 + 3x + k = 0$ has two distinct roots in $[0, 1]$, then the value of k is

(JEE-main : 2013)

- (A) between 1 and 2 (B) between 2 and 3 (C) between -1 and 0 (D) does not exist

Sol. : Here given that $f(x) = 2x^3 + 3x + k = 0$.

$$f'(x) = 6x^2 + 3$$

Now, if the equation has two or more real roots then by Roll's theorem, for some x between them $f'(x) = 0$.

But $6x^2 + 3 > 0$

∴ It is not possible that the equation has two or more distinct real roots for any value of k .

Ans. : (D)

- (69) If both the roots of the equation $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$ are common, then $a : b : c = \dots\dots$ (JEE-main : 2013)

- (A) $1 : 2 : 3$ (B) $3 : 2 : 1$ (C) $1 : 3 : 2$ (D) $3 : 1 : 2$

Sol. : If the roots of both the equations are common, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \frac{a}{1} = \frac{b}{2} = \frac{c}{3}$$

$$\therefore a : b : c = 1 : 2 : 3$$

Ans. : (A)

- (70) If $a \in \mathbb{R}$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ has no integer solution then the possible values of a are in interval. (JEE-main : 2014)

- (A) $(-1, 0) \cup (0, 1)$ (B) $(1, 2)$ (C) $(-2, -1)$ (D) $(-\infty, -2) \cup (2, \infty)$

Sol. : $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$

$$\therefore -3\{x\}^2 + 2\{x\} + a^2 = 0$$

$$\therefore \{x\} = \frac{-2 \pm \sqrt{4 - 4(-3)a^2}}{2(-3)} = \frac{-2 \pm \sqrt{4 + 12a^2}}{-6} = \frac{1 \pm \sqrt{1 + 3a^2}}{3} \neq 0 \text{ because } x \text{ is not integer.}$$

So, $a \neq 0$

$$\text{Now, since } 0 < \{x\} < 1, 0 < \frac{1 \pm \sqrt{1 + 3a^2}}{3} < 1$$

$$\therefore 0 < 1 + \sqrt{1 + 3a^2} < 3 \quad (\text{as } 1 - \sqrt{1 + 3a^2} < 0)$$

$$\therefore -1 < \sqrt{1 + 3a^2} < 2 \quad \text{i.e. } 0 \leq \sqrt{1 + 3a^2} < 2.$$

$$\therefore 1 + 3a^2 < 4$$

$$\therefore a^2 < 1$$

$$\therefore a \in (-1, 1). \text{ Also, } a \neq 0$$

$$\therefore a \in (-1, 0) \cup (0, 1)$$

Ans. : (A)

- (71) If α and β are the roots of the equation $px^2 + qx + r = 0$ (where $p \neq 0$) and p, q, r are in A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then $|\alpha - \beta| = \dots\dots$ (JEE-main : 2014)

- (A) $\frac{\sqrt{61}}{9}$ (B) $\frac{2\sqrt{17}}{9}$ (C) $\frac{\sqrt{34}}{9}$ (D) $\frac{2\sqrt{13}}{9}$

Sol. : Here α and β are the roots of the quadratic equation $px^2 + qx + r = 0$.

$$\therefore \alpha + \beta = \frac{-q}{p} \text{ and } \alpha\beta = \frac{r}{p}$$

Also, p, q, r are in A.P. So, $2q = p + r$ (1)

Now,, $\frac{1}{\alpha} + \frac{1}{\beta} = 4$. So, $\alpha + \beta = 4\alpha\beta$

$$\therefore \frac{-q}{p} = \frac{4r}{p}$$

$$\therefore q = -4r. \quad (2)$$

$$\therefore |\alpha - \beta| = \sqrt{(\alpha - \beta)^2} = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \sqrt{\frac{q^2}{p^2} - \frac{4r}{p}} \quad (3)$$

Now, from (1) and (2), $\frac{p+r}{2} = -4r$. So, $p + r = -8r$. So, $9r = -p$

$$\therefore \frac{r}{p} = \frac{-1}{9} \quad (4)$$

From (3), $|\alpha - \beta| = \sqrt{\frac{16r^2}{p^2} - \frac{4r}{p}} = \sqrt{\frac{16+36}{81}} = \frac{2\sqrt{13}}{9}$ **Ans. : (D)**

(72) If the quadratic equation $p(x) = 0$ with real coefficients has only pure imaginary roots, then which of the following is true for the equation $p(p(x)) = 0$? (JEE advanced : 2014)

- (A) It has only one real root (B) All roots are real
(C) It has two real and two imaginary roots. (D) It has neither real nor imaginary roots

Sol. : Let $p(x) = ax^2 + b$, where a and b have same sign.

$$\therefore p(p(x)) = a(ax^2 + b)^2 + b$$

If $x \in \mathbb{R}$ or $ix \in \mathbb{R}$, then $x^2 \in \mathbb{R}$

$$\therefore p(p(x)) \neq 0$$

$\therefore$ No real or pure imaginary number satisfies $p(p(x)) = 0$.

$\therefore$ No real or pure imaginary roots exist. **Ans. : (D)**

(73) α and β are the roots of the equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$; $n \geq 1$, then $\frac{a_{10} - 2a_8}{2a_9} = \dots\dots$

(JEE-main : 2014)

- (A) 6 (B) -6 (C) 3 (D) -3

Sol. : $x^2 - 6x - 2 = 0$

$$\therefore x^{10} - 6x^9 - 2x^8 = 0$$

$$\alpha \text{ and } \beta \text{ are roots of the equation, then } \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0 \quad (1)$$

$$\text{and } \beta^{10} - 6\beta^9 - 2\beta^8 = 0 \quad (2)$$

Subtracting (2) from (1), we get

$$\alpha^{10} - \beta^{10} - 6(\alpha^9 - \beta^9) - 2(\alpha^8 - \beta^8) = 0$$

$$\therefore a_{10} - 6a_9 - 2a_8 = 0$$

$$\therefore a_{10} - 2a_8 = 6a_9$$

$$\therefore \frac{a_{10} - 2a_8}{2a_9} = 3$$

Ans. : (C)

: Questions with More Than One Correct Answers :

- (74) If the two equations $x^2 - ax + b = 0$ and $x^2 + bx - a = 0$ has a common root, then which of the following is true ? (IIT : 1986)

(A) $a = b$ (B) $a = -b$ (C) $a + b = 1$ (D) $a - b = 1$

Sol. : Suppose $x^2 - ax + b = 0$ and $x^2 + bx - a = 0$ have a common root α .

$$\therefore \alpha^2 - a\alpha + b = 0 \text{ and } \alpha^2 + b\alpha - a = 0$$

$$\therefore -a\alpha + b = b\alpha - a$$

$$\therefore a + b = (a\alpha + b\alpha)$$

$$\therefore (a + b) = \alpha (a + b)$$

$$\therefore (a + b) (1 - \alpha) = 0$$

$$\therefore a + b = 0 \text{ or } \alpha = 1$$

$$\therefore a = -b \text{ or } 1 - a + b = 0$$

$$\therefore a - b = 1 \text{ or } a = -b$$

Ans. : (B), (D)

- (75) If p and q are the roots of the equation $x^2 + px + q = 0$, then the possible values of p are (IIT : 1995)

(A) 1 (B) 0 (C) $\frac{-1}{2}$ (D) -2

Sol. : p and q are the root of the equation $x^2 + px + q = 0$.

$$\therefore p^2 + p^2 + q = 0 \text{ and } q^2 + pq + q = 0$$

$$\therefore 2p^2 + q = 0 \text{ and } q(p + q + 1) = 0$$

$$\therefore 2p^2 + q = 0 \text{ and } (q = 0 \text{ or } q = -p - 1)$$

If $q = 0$, then from $2p^2 + q = 0$, we get, $p = 0$.

If $q = -p - 1$, then $2p^2 - p - 1 = 0$

$$\therefore (2p + 1)(p - 1) = 0$$

$$\therefore p = 0, 1 \text{ or } \frac{-1}{2} \text{ is possible.}$$

Ans. : (A), (B), (C)

- (76) If $2^{\cos 2x} = 3 \cdot 2^{\cos^2 x} - 4$, then $x = \dots\dots$ where $x \in [0, \pi]$

(A) 0 (B) π (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Sol. : We have $2^{\cos 2x} = 3 \cdot 2^{\cos^2 x} - 4$.

$$\therefore 2^{2\cos^2 x - 1} = 3 \cdot 2^{\cos^2 x} - 4$$

$$\therefore \frac{2^{2m}}{2} = 3 \cdot 2^m - 4 \quad (\text{assuming } \cos^2 x = m)$$

$$\therefore (2^m)^2 = 3 \cdot 2 \cdot 2^m - 8$$

$$\therefore (2^m)^2 - 6 \cdot 2^m + 8 = 0$$

$$\therefore (2^m - 2)(2^m - 4) = 0$$

$$\therefore 2^m = 2 \text{ or } 2^m = 4$$

$$\therefore m = 1 \text{ or } m = 2$$

$$\therefore \cos^2 x = 1 \text{ or } \cos^2 x = 2. \quad \text{But } |\cos^2 x| \leq 1$$

$$\therefore \cos^2 x = 1$$

$$\therefore \cos x = \pm 1$$

$$\therefore x = 0 \text{ or } \pi.$$

Ans. : (A), (B)

(77) If $7^{\log_7(x^2 - 4x + 5)} = x - 1$, then possible value of x are (IIT : 1990)

(A) 2

(B) 3

(C) 7

(D) -3, -2

Sol. : $7^{\log_7(x^2 - 4x + 5)} = x - 1$

$$\therefore x^2 - 4x + 5 = x - 1$$

$$\therefore x^2 - 5x + 6 = 0$$

$$\therefore (x - 3)(x - 2) = 0$$

$$\therefore x = 2 \text{ or } x = 3$$

Ans. : (A), (B)

(78) If for $0 \leq x \leq \pi$, $16^{\sin^2 x} + 16^{\cos^2 x} = 10$, then $x = \dots\dots$

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{3\pi}{4}$

Sol. : $16^{\sin^2 x} + 16^{\cos^2 x} = 10$

$$\therefore 16^{\sin^2 x} + 16 \cdot 16^{-\sin^2 x} = 10$$

$$\therefore \left(16^{\sin^2 x}\right)^2 - 10 \cdot 16^{\sin^2 x} + 16 = 0$$

Let $16^{\sin^2 x} = m$

$$\therefore m^2 - 10m + 16 = 0$$

$$\therefore (m - 8)(m - 2) = 0$$

$$\therefore m = 8 \text{ or } m = 2$$

$$\therefore 16^{\sin^2 x} = 8 \text{ or } 16^{\sin^2 x} = 2$$

$$\therefore 2^{4\sin^2 x} = 2^3 \text{ or } 2^{4\sin^2 x} = 2$$

$$\therefore 4 \sin^2 x = 3 \text{ or } 4 \sin^2 x = 1.$$

$$\text{So, } \sin x = \pm \frac{\sqrt{3}}{2} \text{ or } \sin x = \pm \frac{1}{2}$$

But $\sin x > 0$ as $x \in [0, \pi]$

$$\therefore \sin x = \frac{\sqrt{3}}{2} \text{ or } \sin x = \frac{1}{2}$$

$$\therefore x = \frac{\pi}{3} \text{ or } x = \frac{\pi}{6}$$

Ans. : (B), (C)

(79) If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca \in \dots\dots\dots$ (IIT : 1984)

- (A) $\left[-\frac{1}{2}, 2\right]$ (B) $[-1, 2]$ (C) $\left[-\frac{1}{2}, 1\right]$ (D) $\left[1, \frac{1}{2}\right]$

Sol. : $a^2 + b^2 + c^2 = 1$

$$\therefore (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca \geq 0$$

$$\therefore 0 \leq 1 + 2ab + 2bc + 2ca$$

$$\therefore \frac{-1}{2} \leq (ab + bc + ca)$$

$$\therefore ab + bc + ca \geq \frac{-1}{2}$$

$$\therefore ab + bc + ca \in \left[-\frac{1}{2}, 1\right] \text{ or } \left[-\frac{1}{2}, 2\right].$$

Ans. : (A), (C)

Section B

: Multiple Choice Questions with Integer Answers :

(80) If $x = 2 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$, then the value of $x^3 - 6x^2 + 6x$ is $\dots\dots\dots$

- (A) 1 (B) 2 (C) $\frac{2}{3}$ (D) 3

Sol. : $x = 2 + 2^{\frac{1}{3}} + 2^{\frac{2}{3}}$.

$$\therefore x - 2 = 2^{\frac{1}{3}} + 2^{\frac{2}{3}} \quad (1)$$

$$\therefore (x - 2)^3 = \left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)^3$$

$$\therefore x^3 - 8 - 6x(x - 2) = 2 + 2^2 + 3 \cdot 2^{\frac{1}{3}} \cdot 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)$$

$$\therefore x^3 - 6x^2 + 12x - 8 = 6 + 3 \cdot 2(x - 2) \quad (\text{From (1)})$$

$$\therefore x^3 - 6x^2 + 12x - 8 = 6 + 6x - 12$$

$$\therefore x^3 - 6x^2 + 6x = 2 \quad \text{Ans. : (B)}$$

(81) If $x + 1$ is one of the factors of the equation $x^4 + (p - 3)x^3 - (3p - 5)x^2 + (2p - 9)x + 6 = 0$, then $p = \dots\dots\dots$

- (A) 0 (B) 2 (C) 4 (D) -4

Sol. : $x + 1$ is one of the factor of the given equation,

$$1 - (3p - 5) + 6 = (p - 3) + (2p - 9)$$

$$\therefore 6p = 24$$

$$\therefore p = 4 \quad \text{Ans. : (C)}$$

(82) If both the roots of equations $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are common, then $2r - p = \dots\dots\dots$

- (A) 0 (B) 1 (C) -1 (D) 2

Sol. : Both the roots of equations $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$. i.e. $(6k + 2)x^2 + rx + 3k - 1 = 0$ and $(12k + 4)x^2 + px + 6k - 2 = 0$ are common.

$$\therefore \frac{6k+2}{12k+4} = \frac{r}{p} = \frac{3k-1}{6k-2} \quad \text{So, } \frac{r}{p} = \frac{1}{2} \quad \text{So, } 2r - p = 0$$

Ans. : (A)

Section : C

: Assertion and Reason Type Questions :

Explanation : Each of these questions contains statement 1 (Assertion) and statement 2 (Reason). Each question has four choices, option (A), (B), (C) and (D), which are as follows. You have to select the correct choice.

- (A) Statement 1 is true, statement 2 is true. Statement 2 is the correct explanation for statement 1.
- (B) Statement 1 is true, statement 2 is true. Statement 2 is not a correct explanation for statement 1.
- (C) Statement 1 is true, statement 2 is false.
- (D) Statement 1 is false, statement 2 is true.

(83) **Statement : 1** The squares of roots of the equation $ax^2 + bx + c = 0$ are roots of the quadratic equation $a^2x^2 + b^2x + c^2 = 0$.

Statement : 2 If $b^2 = ac$ and $a, b, c \in \mathbb{R}^+$, then a, b, c are in G.P.

Sol. : If α and β are the roots of the equation $ax^2 + bx + c = 0$, then $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$.

Now, α^2 and β^2 are roots of the equation $a^2x^2 + b^2x + c^2 = 0$.

$$\text{So, } \alpha^2 + \beta^2 = \frac{-b^2}{a^2} \text{ and } \alpha^2\beta^2 = \frac{c^2}{a^2}.$$

$$\therefore (\alpha + \beta)^2 - 2\alpha\beta = \frac{-b^2}{a^2} \quad \text{So, } \frac{b^2}{a^2} - \frac{2c}{a} = \frac{-b^2}{a^2} \quad (\alpha + \beta = \frac{-b}{a}; \alpha\beta = \frac{c}{a})$$

$$\therefore \frac{2b^2}{a^2} = \frac{2ac}{a^2}$$

$$\therefore b^2 = ac$$

$$\therefore a, b, c \text{ are in G.P.}$$

$\therefore$ Statement 1 and statement 2 are true and statement 2 is a correct explanation of statement 1.

Ans. : (A)

(84) **Statement : 1** The roots α, β of $x^2 - 5x + 6 = 0$ satisfy $|\alpha - \beta| = 1$.

Statement : 2 The roots α, β of $x^2 - px + q = 0$ satisfy $|\alpha - \beta| = 1$ if and only if $p^2 + 4q^2 = (2q + 1)^2$.

Sol. : $(\alpha - \beta) = 1$

$$\Leftrightarrow |\alpha - \beta|^2 = 1$$

$$\Leftrightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1$$

$$\Leftrightarrow p^2 - 4q = 1$$

$$\Leftrightarrow p^2 = 4q + 1$$

$$\Leftrightarrow p^2 + 4q^2 = 4q^2 + 4q + 1 = (2q + 1)^2$$

$\therefore$ Statement 2 is true.

For, $p = 5$ and $q = 6$

$$\therefore p^2 + 4q^2 = 25 + 144 = 169 = 13^2 = (2q + 1)^2$$

$\therefore$ Roots of $x^2 - 5x + 6 = 0$ satisfy $|\alpha - \beta| = 1$

$\therefore$ Statement 1 is true and statement 2 gives correct explanation of statement 1. **Ans. :** (A)

