CHAPTER – 1 : MEASUREMENT OF ANGLES

1.1 In Geometry angles are measured in terms of a right angle. This, however, is an inconvenient unit of measuremsent on account of its size.

1.2 In the **Sexagesimal** system of measurement a right angle is divided into 90 equal parts called Degrees. Each degree is divided into 60 equal parts called **Minutes,** each minute into 60 equal parts called **Seconds.**

The symbols 1°, $1'$, and $1''$ are used to denote a Degree, a Minute, and a Second respectively.

Thus 60 Seconds ($60''$) make One Minute ($1'$),

60 Minutes ($60'$) make One Degree (1°),

and 90 Degree (90°) make One Right Angle.

This system is well established and is always used in the practical application of Trigonometry. It is not however very convenient on account of the multipliers 60 and 90.

1.3 On this account another system of measurement called the **Centesimal,** or French, system has been proposed. In this system the right angle is divided into 100 equal parts, called **Grades;** each grade is subdivided into 100 **Minutes,** and each minute into 100 **Seconds.**

The symbols 1^g, $1^{`}$, and $1^{``}$ are used to denote a Grade, a Minute, and a Second respectively.

Thus 100 Seconds ($100'$) make One Minute ($1^{`}$),

100 Minutes ($100'$) make One Grade (1^g),

100 Grades (100^g) make One Right Angle.

1.4 This system would be much more convenient to use than the ordinary Sexagesimal system.

As a preliminary, however, to its practical adoption, a large number of tables would have to be recalculated. For this reason the system has in practice never been used.

1.5 To convert Sexagesimal into Centesimal Measure, and vice versa.

Since a right angle is equal to 90° and also to 100^g, we have

$$90^\circ = 100^g.$$

$$\therefore \qquad 1^\circ = \frac{10^g}{9}, \text{ and } 1^g = \frac{9^\circ}{10}.$$

Hence, to change degrees into grades, add on one-ninth; to change grades into degrees, subtract one-tenth.

EXAMPLE 1

$$36^\circ = \left(36 + \frac{1}{9} \times 36\right)^g = 40^g,$$

and

$$64^g = \left(64 - \frac{1}{10} \times 64\right)^\circ = (64 - 6{\cdot}4)^\circ = 57{\cdot}6^\circ.$$

If the angle does not contain an integral number of degrees, we may reduce it to a fraction of a degree and then change to grades.

In practice it is generally found more convenient to reduce any angle to a fraction of a right angle. The method will be seen in the following examples;

EXAMPLE 2

Reduce 63° 14′ 51″ to Centesimal Measure.

Solution : We have

$$51'' = \frac{17'}{20} = {\cdot}85',$$

and

$$14'\ 51'' = 14{\cdot}85' = \frac{14{\cdot}85^\circ}{60} = 0{\cdot}2475^\circ,$$

$$\therefore \qquad 63^\circ\ 14'\ 51'' = 63{\cdot}2475^\circ = \frac{63{\cdot}2475}{90} \text{ rt. angle}$$

$$= 0{\cdot}70275 \text{ rt. angle}$$

$$= 70{\cdot}275^g = 70^g\ 27{\cdot}5' = 70^g\ 27'\ 50''.$$

EXAMPLE 3

Reduce 94^g 23\` 87\`\` to Sexagesimal Measure.

$$94^g\ 23`\ 87`` = {\cdot}942387 \text{ right angle}$$

$$\frac{90}{84{\cdot}81483} \text{ degrees}$$

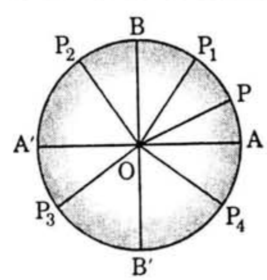
$$\frac{60}{48{\cdot}8898} \text{ minutes}$$

$$\frac{60}{53{\cdot}3880} \text{ seconds.}$$

$$\therefore \qquad 94^g\ 23'\ 87'' = 84^\circ\ 48'\ 53{\cdot}388''.$$

1.6 ANGLES OF ANY SIZE.

Suppose AOA′ and BOB′ to be two fixed lines meeting at right angles in O, and suppose a revolving line OP (turning about a fixed point at O) to start from OA and revolve in a direction opposite to that of the hands of a watch.



For any position of the revolving line between OA and OB, such as OP_1, it will have turned through an angle

AOP_1, which is less than a right angle.

For any position between OB and OA′, such as OP_2, and angle AOP_2, through which it has turned is greater than a right angle.

For any position OP_3, between OA′ and OB′, the angle traced out is AOP_3, *i.e.*

$AOB + BOA' + A'OP_3$,

i.e., 2 right angles + $A'OP_3$,

so that the angle described is greater than two right angles.

For any position OP_4, between OB′ and OA, the angle turned through is similarly greater than three right angles.

When the revolving line has made a complete revolution, so that it coincides once more with OA, the angle through which it has turned is 4 right angles.

If the line OP still continues to revolve, the angle through which it has turned, when it is for the second time in the position, OP_1, is not AOP_1 but 4 right angles + AOP_1.

Similarly, when the revolving line, having made two complete revolutions, is once more in the position OP_2, the angle it has traced out is 8 right angles + AOP_2.

1.7 If the revolving line OP be between OA and OB, it is said to be in the first quadrant; if it be between OB and OA′, it is in the second quadrant; if between OA′ and OB′, it is in the third quadrant; if it is between OB′ and OA, it is in the fourth quadrant.

1.8 EXAMPLE

What is the position of the revolving line when it has turned through (i) 225°, (ii) 480°, and (iii) 1050°?

(i) Since 225° = 180° + 45°, the revolving line has turned through 45° more than two right angles, and it is therefore in the third quadrant and halfway between OA′ and OB′.

(ii) Since 480° = 360° + 120°, the revolving line has turned through 120° more than one complete revolution, and is therefore in the second quadrant, *i.e.* between OB and OA′, and makes an angle of 30° with OB.

(iii) Since 1050° = 11 × 90° + 60°, the revolving line has turned through 60° more than eleven right angles, and is therefore in the fourth quadrant, *i.e.* between OB′ and OA, and makes 60° with OB′.

SOLVED EXAMPLES

Express in terms of a right angle the angles

1. 60°	***2. 75°15′.***	***3. 63°17′25″.***
4. 130°30′.	***5. 210°31′30″***	***6. 370°20′48″.***

Solutions.

1. $60° = \frac{60}{90}$ rt. angle $= \frac{2}{3}$ (right angle)

2. $75°\,15' = 75° + 15' = 75° + \frac{1°}{4} = \frac{301°}{4} = \mathbf{\frac{301}{360}}$ **(rt. angle)**

3. $63°\,17'\,25'' = 63°\,17'\,\frac{25'}{60} = 63\,\frac{209°}{12 \times 60}$

$$= \frac{45569}{720 \times 90} \text{ rt. angle} = \frac{45569}{64800} \textbf{ rt. angle}$$

4. $$130^\circ\ 30' = 130^\circ \frac{1^\circ}{2} = \frac{261^\circ}{2} = \frac{261}{2 \times 90} \textbf{ rt. angle}$$

$$= \frac{29}{20} \text{ right angle} = 1\frac{\mathbf{9}}{\mathbf{20}} \textbf{ rt. angle}$$

5. $$210^\circ\ 31'\ 30'' = 210^\circ\ 31' \frac{30'}{60} = 210^\circ \frac{63'}{2} = 210^\circ \frac{63^\circ}{120} = \frac{25251^\circ}{120}$$

$$= \frac{25251}{120 \times 90} \text{ rt. angle} = 2\frac{\mathbf{3661}}{\mathbf{10800}} \textbf{ rt. angle.}$$

6. $$370^\circ\ 20'\ 48'' = 370^\circ\ 20' \frac{48'}{60} = 370^\circ \frac{104'}{5}$$

$$= 370^\circ \frac{104^\circ}{5 \times 60} = 370^\circ \frac{26^\circ}{75}$$

$$= \frac{27776}{90 \times 75} \text{ rt. angle}$$

$$= \frac{27776}{3375} \text{ rt. angle}$$

$$= \mathbf{4}\frac{\mathbf{388}}{\mathbf{3375}} \textbf{ rt. angle.}$$

Express in grades, minutes, and seconds the angles.

7. $\boldsymbol{30^\circ}$ ***8.*** $\boldsymbol{81^\circ}$ ***9.*** $\boldsymbol{138^\circ 30'}$.

10. $\boldsymbol{35^\circ 47' 15''}$ ***11.*** $\boldsymbol{235^\circ 12' 36''}$ ***12.*** $\boldsymbol{475^\circ 13' 48''}$.

Solutions.

7. $$30^\circ = 30 \times \frac{100^g}{90} = \frac{100^g}{3}\ \mathbf{33^g\ 33'\ 33.3''}$$

8. $$81^\circ = 81 \times \frac{10^g}{9} = \mathbf{90^g}.$$

9. $$138^\circ\ 30' = 138\frac{1}{4} \times \frac{10^g}{9} = \frac{277}{2} \times \frac{10^g}{9}$$

$$= \frac{2770}{18} = 153\frac{16''}{18} = \mathbf{153^g\ 88'\ 88.8''}$$

10. $$35^\circ\ 47'\ 15'' = 35^\circ\ 47\frac{1}{2}' = 35^\circ + \frac{180^\circ}{4 \times 60} = 35\frac{63^\circ}{80}$$

$$= \frac{2863}{80} \times \frac{10''}{9} = \frac{2863}{72} = \mathbf{39^g\ 76'\ 38.8''}$$

11. $$235^\circ\ 12'\ 36'' = 235^\circ\ 12\frac{3}{5}' = 235^\circ + \frac{63^\circ}{5 \times 60}$$

$$= 275\frac{21^\circ}{100} = \frac{23521}{100} \times \frac{10^g}{9} = \frac{23521}{90}$$

$$= \mathbf{261^g\ 34'\ 44.4''}$$

12. $475° \, 13' \, 48'' = 475° + 13\frac{4}{5}{}' = 475° + \frac{69°}{300}$

$= 475° + \frac{23°}{100} = \frac{47523°}{100} = \frac{47523 \times 10^g}{100 \times 9}$

$= \frac{47523^g}{90}$

$= \mathbf{528^g \; 3' \; 33.3''}$

Express in terms of right angles, and also in degrees, minutes, and seconds the angle

13. 120^g. ***14. $45^g \, 35' 24''$*** ***15. $39^g \, 45' 36''$.***

16. $255^g \, 8' 9''$ ***17. $75^g \, 0' 5''$.***

Solutions.

13. $120^g = \frac{120 \times 9°}{10} = 108°$

$= \frac{108}{90}$ rt. angle

$= \frac{6}{5}$ rt. angle = **1.2 rt. angle**

14. $45^g \, 35' \, 24'' = 45^g \, 35.24'$

$= 45.3524^g$

$= \frac{45 \cdot 3524 \times 9°}{10}$

$= 40° + .81716 \times 60'$

$= 40' \, 49' + .0296 \times 60''$

$= \mathbf{40° \, 49' \, 1.776''}$

Again, $45^g \, 35' \, 24'' = \frac{45 \cdot 3524}{100}$ rt. angle

= 0. 453524 rt. angle.

15. $39^g \, 45' \, 36'' = 39.4536^g = \frac{39 \cdot 4536 \times 9°}{10} = \mathbf{35° \, 30' \, 29.664''}$

Again, $39^g \, 45' \, 36'' = 39.4536^g = \frac{39 \cdot 4536}{100}$ rt. angle

= 0 .394536 rt. angle

16. $255^g \, 8' \, 9'' = \frac{255 \cdot 0809 \times 9°}{10} = 229.57281° = 299° \, 34' \, 22.116''$

Again, $255^g \, 8' \, 9'' = \frac{255 \cdot 0809}{100}$ rt. angle

= 2.550809 rt. angle.

17. $75^g \, 0' \, 5'' = \frac{759 \cdot 0005 \times 9°}{10} = 683.10045° = 683° \, 6' \, 1.62''$

Again $75^g \, 0' \, 5'' = \frac{759 \cdot 0005}{100}$ rt. angle = **7.590005 rt. angle.**

DETAILED SOLUTIONS TO MEASUREMENT OF ANGLES BY SL LONEY

Mark the position of the revolving line when it has traced out the following angles:

18. $\frac{4}{8}$ *right angle.* *19.* $3\frac{1}{2}$ *right angle.* *20.* $13\frac{1}{3}$ *right angles.*

21. 120°. *22.* 315° *23.* 745°

24. 1185°. *25.* 150^g. *26.* 420^g.

27. 875^g. *28.* 875^g

Solutions.

18. $$\frac{4}{3} \text{ rt. angle } = 1\frac{1}{3} \text{ rt. angle} = (90^\circ + 30^\circ)$$

Here, since the revolving line makes an angle more than one and less than two rt. angles with the original line, hence it will be in the second quadrant.

19. $$3\frac{1}{2} \text{ rt. angle } = 3 \text{ rt. angle} + 45^\circ$$

Here, since revolving line makes an angle greater than three and less than four rt. angles with the given line, hence it will lie in the fourth quadrant.

20. $$13\frac{1}{3} \text{ rt. angle } = 12 \text{ rt. angle} + 1\frac{1}{3} \text{ rt. angle}$$

Here, since the revolving line makes three complete revolutions and $1\frac{1}{3}$ rt. angle with the base, hence revolving line is in second quadrant.

21. $$120^\circ = 90^\circ + 30^\circ.$$

Here, since revolving line makes an angle greater than one rt. angle and less than 2 rt. angles, hence it will lie in the second fourth quadrant.

22. $$315^\circ = 270^\circ + 45^\circ = 3 \text{ rt. angle} + 45^\circ$$

Here, since the revolving line makes an angle greater than three and less than four rt. angles with the given straight line, hence it will lie in the fourth quadrant.

23. $$745^\circ = 2 \times 360^\circ + 25^\circ$$

Here, since revolving line makes two complete revolutions and an angle of 25° from its original position, hence in the final position revolving line will lie in the first quadrant.

24. $$1185^\circ = 3 \times 360^\circ + (90^\circ + 15^\circ)$$

Here since, revolving line makes three complete revolutions and an angle of 15° more than a right angle with its original position, hence it will lie in the second quadrant.

25. $$150^g = \frac{150}{100} \text{ rt. angles} = 1\frac{1}{2} \text{ rt. angles}$$

Here, since revolving line here makes an angle of $\frac{1}{2}$ rt. angle more than a rt. angle, hence revolving line will lie in the second quadrant.

26. $$420^g = \frac{420}{100} \text{ rt. angle } = 4\frac{1}{5} \text{ rt. angles}$$

Here, since revolving line makes one complete revolution and an angle of 18°, hence in the final position the revolving line will be found in the first quadrant.

27. $875^g = \frac{875}{100}$ r.t angles $= 8.75$ r.t angles.

Here, since revolving line makes two complete revolutions and an angle of 67.5°, hence the revolving line will lie in the first quadrant.

28. ***How many degrees, minutes and seconds are respectively passed over in 11 $\frac{1}{9}$ minutes by the hour and minute hands of a watch?***

Solution. Since, in 60 minutes, the minute hand turns through 360°.

$\therefore$ In $\frac{100}{9}$ minutes it turns through $\frac{360}{60} \times \frac{100°}{9}$

$$= \frac{200°}{3} = \mathbf{66° \, 40'}$$

Since, in 720 minutes, *i.e.* in 12 hrs. the hour had turns through 360°.

$\therefore$ In $\frac{100}{9}$ minutes the hour had turns through $\frac{360}{720} \times \frac{100°}{9}$

$$= \frac{50°}{9} = \mathbf{5° \, 33' \, 20''}$$

29. ***The number of degrees in one acute angle of a right angled triangle is equal to the number of grades in the other; express both the angles in degrees.***

Solution: Suppose the first acute angle is $x°$

Hence, other actual angle $= x^g$

But $x^g = \frac{9}{10} x°$

Now since the sum of both the acute angles is a right angles,

$\therefore$ $x + \frac{9}{10} x = 90°$,

or $\frac{19}{10} x = 90°$

or $x = \frac{90 \times 10}{19} = \frac{900}{19} = 47 \frac{7°}{19}$

$\therefore$ First acute angle $= 47 \frac{7°}{19}$

and, second angle $= 42 \frac{12°}{19}$.

30. ***Prove that the number of Sexagesimal minutes in any angle is to the number of Centesimal minutes in the same angle as 27: 50.***

Solution. Let any angle is $x°$.

$\therefore$ $x° = 60 \, x'$

Again, $x° = \frac{10}{9} x^g = \frac{10x \times 100}{9} = \frac{1000}{9} x^g$

$\therefore$ Required ratio $= \frac{60x}{\frac{1000x}{9}} = \frac{27}{50} = \mathbf{27 : 50}$

31. ***Divide 44°8′ into two parts such that the number of Sexagesimal seconds in one part may be equal to the number of Centesimal seconds in the other part.***

Solution. Suppose one part is $x°$

Hence other part is $(44° 8' - x°)$

$\therefore \quad x° = 60 \times 60x'' = 3600x''$

or $\quad (44° 8' - x°) = \left(\frac{662 - 15x}{15}\right) \times \left(\frac{10}{9}\right)^g = \frac{(662 - 15x)100000''}{135}$

$\therefore \quad \frac{(662 - 15x)100000''}{135} = 3600x$

or $\quad \left(\frac{662}{135} - \frac{15x}{135}\right) 1000 = 36x$

or $\quad \left(\frac{132 \cdot 4}{27} - \frac{3x}{27}\right) 1000 = 36x$

or $\quad 27 \times 36x = (1324 - 30x)\, 100$

or $\quad 972x = 132400 - 3000x$

or $\quad 3972x = 132400$

or $\quad x = \frac{132400}{3972} = 33\frac{1}{3}$

Hence, one part is $33\frac{1}{3} = 33° 20°$

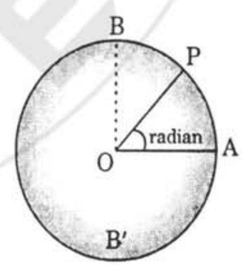
and other part is $(44° 8' - 33° 20') = 10° 48'$

CIRCULAR MEASURE

1.9 A third system of measurement of angles has been devised, and it is this system which is used in all the higher branches of Mathematics.

The unit used is obtained thus;

Take *any* circle APBB′, whose centre is O, and from any point A measure off an arc AP whose length is equal to the radius of the circle. Join OA and OP.



The angle AOP is the angle which is taken as the unit of circular measurement, *i.e.* it is the angle in terms of which in this system we measure all others.

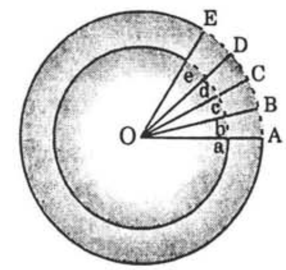
This angle is called **A Radian** and is often denoted by 1°.

1.10 It is clearly essential to the proper choice of a unit that it should be a *constant* quantity; hence we must shew that the Radian is a constant angle. This we shall do in the following articles.

1.11 THEOREM

The length of the circumference of a circle always bears a constant radio to its diameter.

Take any two circles whose common centre is O. In the larger circle inscribe a regular polygon of n sides, ABCD....



Let OA, OB, OC,... meet the smaller circle in the points a, b, c, d, ... and join ab, bc, cd, ...

Then, by Geometry, $abcd$.... is a regular polygon of n sides inscribed in the smaller circle.

Since Oa = Ob, and OA = OB, the lines ab and AB must be parallel, and hence

$$\frac{AB}{ab} = \frac{OA}{Oa}.$$

Also the outer polygon ABCD... being regular, its perimeter, *i.e.* the sum of its sides, is equal to n.AB. Similarly for the inner polygon.

$$\frac{\text{Perimeter of the outer polygon}}{\text{Perimeter of the inner polygon}} = \frac{n.AB}{n.ab} = \frac{AB}{ab} = \frac{OA}{Oa} \qquad ....(i)$$

This relation exists whatever be the number of sides in the polygons.

Let the number of sides be indefinitely increased (*i.e.* let n become inconceivably great) so that finally the perimeter of the outer polygon will be the same as the circumference of the outer circle, and the perimeter of the inner polygon the same as the circumference of the inner circle.

The relation (i) will then become

$$\frac{\text{Circumference of outer circle}}{\text{Circumference of inner circle}} = \frac{OA}{Oa}$$

$$= \frac{\text{Radius of outer circle}}{\text{Radius of inner circle}}$$

Hence,
$$\frac{\text{Circumference of outer circle}}{\text{Radius of outer circle}}$$

$$= \frac{\text{Circumference of inner circle}}{\text{Radius of inner circle}}$$

Since there was no restriction whatever as to the sizes of the two circles, it follows that the quantity

$$\frac{\text{Circumference of any circle}}{\text{Radius of that circle}}$$

is **the same for all circles**.

Hence the ratio of the circumference of a circle to its radius, and therefore also to its diameter, is a constant quantity.

1.12 In the previous article we have shown that the ratio $\frac{\text{Circumference}}{\text{Diameter}}$ is the same for all circles. The value of this constant ratio is always denoted by the Greek letter π [pronounced "Pi"], so that π is a number.

Hence, $\dfrac{\text{Circumference}}{\text{Diameter}}$ = the constant number π.

We have therefore the following theorem;

The circumference of a circle is always equal to π times its diameter or 2π times its radius.

1.13 Unfortunately the value of π is not a whole number, nor can it be expressed in the form of a vulgar fraction, and hence not in the form of decimal fraction, terminating or recurring.

The number π is an incommensurable magnitude, *i.e.* a magnitude whose value cannot be exactly expressed as the ratio of two whole numbers.

Its value, correct to 8 places of decimals, is

$$3{\cdot}14159265.....$$

The fraction $\dfrac{22}{7}$ gives the value of π correctly for the first two decimal places; for

$$\frac{22}{7} = 3{\cdot}14285.....$$

The fraction $\dfrac{355}{113}$ is a more accurate value of π, being correct to 6 places of decimals; for $\dfrac{355}{113} = 3{\cdot}14159203.....$

[**N.B.** The fraction $\dfrac{355}{113}$ may be remembered thus; write down the first three numbers repeating each twice, thus 113355; divide the number thus obtained into two parts and let the first part be divided into the second, thus 113)355(.

The quotient is the value of π to 6 places of decimals.]

To sum up. **An approximate value of π, correct to 2 places of decimals, is the fraction $\dfrac{22}{7}$; a more accurate value is $3{\cdot}14159.....$**

By division, we can show that

$$\frac{1}{\pi} = {\cdot}3183098862.....$$

1.14 EXAMPLE 1

The diameter of a wheel is 28 cm.; through what distance does its centre move during one revolution of the wheel along the ground?

Solution. The radius r is here 14 cm.

$\therefore$ circumference $= 2\pi r.\ 2\pi.14 = 28\pi$ cm.

If we take $\pi = \dfrac{22}{7}$, then

circumference $= 28 \times \dfrac{22}{7}$ cm. $= 88$ cm. approximately.

If we give π the more accurate value $3{\cdot}14159265...$,
circumference $= 28 \times 3{\cdot}14159265...$ cm. $= 87{\cdot}96459$ cm.

EXAMPLE 2

What must be the radius of a circular running path, round which on athlete must run 5 times in order to describe 1760 metres?

Solution: The circumference must be $\frac{1}{5} \times 1760$, *i.e.* 352 metres.

Hence, if r be the radius of the path in metres, we have

$$2\pi r = 352,$$

i.e. $$r = \frac{176}{\pi} \text{ metres.}$$

Taking $$\pi = \frac{22}{7}, \text{ we have}$$

$$r = \frac{176 \times 7}{22} = 56 \text{ metres.}$$

Taking the more accurate value $\frac{1}{\pi} = \cdot 31831$, we have

$$\pi = 176 \times \cdot 31831 = \mathbf{56{\cdot}02256 \text{ metres.}}$$

SOLVED EXAMPLES

1. ***If the radius of the earth be 6400 km., what is the length of its circumference?***

Solution. Radius of the earth = 6400 kms.

So, its circumference $= 6400 \times 2\pi$ kms.

$= 6400 \times 2 \times 3.14159265$ kms.

= **40212 kms.** (approx)

2. ***The wheel of a railway carriage is 90 cm. in diameter and makes 3 revolutions in second; how fast is the train going?***

Solution. Circumference of the wheel $= 90 \times \frac{22}{7}$ cms. $= \frac{1980}{7}$ cms.

In 1 sec. the wheel runs $= 3 \times \frac{1980}{7}$ cms.

$= \frac{5940}{7}$ cms. = 848.2 cms

$\therefore$ Speed of the train = 848.2 cms./ sec.

3. ***A mill sail whose length is 540 cm. makes 10 revolutions per minute. What distance does its end travel in an hour?***

Solution. Circumference of the wheel of the mill $= 2 \times \frac{22}{7} \times 540$ cms.

$\therefore$ In one minute the wheel runs $= \frac{2 \times 22 \times 540 \times 10}{7}$ cms.

In one hour the wheel will run through $= \frac{2 \times 22 \times 540 \times 10 \times 60}{7 \times 100 \times 1000}$ kms.

= **20.36 kms.**

4. ***Assuming that the earth describes in one year a circle, of 149,700,000 km. radius, whose centre is the sun, how many miles des the earth travel in a year?***

Solution. Distance travelled by the earth

$= $ circumference of this circle

$= 2.149700000\pi$ kms.

$= 299400000\,\pi$ kms.

$= 299400000 \times \frac{22}{7}$ kms.

$= 94060000$ kms. (approx)

5. ***The radius of a carriage wheel is 50 cm., and in $\frac{1}{9}$ the of a second it turns through 80° about its centre, which is fixed; how many km, does a point on the rim of the wheel travel in one hour?***

Solution. Since, in $\frac{1}{9}$ sec. radius rotates by an angle of 80°

Hence, in $\frac{1}{9}$ sec. the end point move $= 2 \times \frac{22}{7} \times 50 \times \frac{80}{360}$ cms.

So in one hour, the end point will move

$$= \frac{2 \times 22 \times 50 \times 9 \times 80 \times 3600}{7 \times 360 \times 3 \times 10000} \text{ kms.}$$

$= 23$ kms. (approx)

1.15 THEOREM

The radian is a constant angle.

Take the figure of Art. 9. Let the arc AB be a quadrant of the circle, *i.e.* one-quarter of the circumference.

By Art. 12, the length of AB is therefore $\frac{\pi r}{2}$, where r is the radius of the circle.

By Geometry, we know that angles at the centre of any circle are to one another as the arcs on which they stand.

Hence $$\frac{\angle AOP}{\angle AOB} = \frac{\text{arc AP}}{\text{arc AB}}$$

$$= \frac{r}{\frac{\pi r}{2}} = \frac{2}{\pi},$$

i.e. $$\angle AOP = \frac{2}{\pi}.\angle AOB.$$

But we defined the angle AOP to be a Radian.

Hence $$\text{a Radian} = \frac{2}{\pi}.\angle AOB$$

$$= \frac{2}{\pi} \text{ of a right angle.}$$

Since a right angle is a constant angle, and since we have shown (Art. 12) that π is a constant quantity, it follows that a Radian is a constant angle, and is therefore the same whatever be the circle from which it is derived.

1.16 MAGNITUDE OF A RADIAN

By the previous article,

$$\text{a Radian} = \frac{2}{\pi} \times \text{a right angle} = \frac{180^\circ}{\pi}$$

$$= 180^\circ \times \cdot 3183098862\ldots = 57\cdot 2957795^\circ$$

$$= 57^\circ\ 17'\ 44\cdot 8'' \text{ nearly.}$$

1.17 Since $\qquad \text{a Radian} = \dfrac{2}{\pi}$ of a right angle,

therefore a right angle $\qquad = \dfrac{\pi}{2}$ radians,

so that $\qquad 180^\circ = 2$ right angles $= \pi$ radians,

and $\qquad 360^\circ = 4$ right angles $= 2\pi$ radians.

Hence, when the revolving line (Art. 6) has made a complete revolution, it has described an angle equal to 2π radians; when it has made three complete revolutions, it has described an angle of 6π radians; when it has made n revolutions; it has described an angle of $2n\pi$ radians.

1.18 In practice the symbol "c" is generally omitted, and instead of "an angle r^{c}" we find written "an angle π."

The student must notice this point carefully. If the unit, in terms of which the angle is measured, be not mentioned, he must mentally supply the word "radians." Otherwise he will easily fall into the mistake of supposing that π stands for 180°. It is true that π radians (π^c) is the same as 180°, but π itself is a number, and a number only.

1.19 To convert Circular Measure into Sexagesimal Measure or Centesimal Measure and vice versa.

The student should remember the relations

$$\text{Two right angles} = 180^\circ = 200^g = \pi \text{ radians.}$$

The conversion is then merely Arithmetic.

EXAMPLE

(i) $\qquad 0\cdot 45\pi^c = \cdot 45 \times 180^\circ = 81^\circ = 90^g.$

(ii) $\qquad 3^c = \dfrac{3}{\pi} \times \pi^c = \dfrac{3}{\pi} \times 180^\circ = \dfrac{3}{\pi} \times 200^g.$

(iii) $\qquad 40^\circ\ 15'\ 36'' = 40^\circ\ 15\tfrac{3}{5}' = 40\cdot 26^\circ$

$$= 40\cdot 26 \times \frac{\pi^c}{180} = 2236\pi \text{ radians.}$$

(iv) $\qquad 40^g\ 15'\ 36'' = 40.1536g = 40.1536 \times \dfrac{\pi}{200}$ radians

$$= \cdot 200768\pi \text{ radians.}$$

1.20 EXAMPLE 1

The angles of a triangle are in A.P. and the number of grades in the least is to the number of radians in the greatest as 40 : π; find the angle is degrees.

Solution. Let the angles be $(x-y)^\circ$, x°, and $(x+y)^\circ$.

Since the sum of the three angles of a triangle is 180°, we have

$$180 = x - y + x + y = 3x,$$

$$\therefore \quad x = 60.$$

Hence, required angles are

$$(60-y)^\circ,\ 60^\circ, \text{ and } (60+y)^\circ,$$

Now $$(60-y)^\circ = \frac{10}{9}\times(60-y)^g,$$

and $$(60+y)^\circ = \frac{\pi}{180}\times(60+y) \text{ radians}.$$

Hence $$\frac{10}{9}(60-y) : \frac{\pi}{180}(60+y) :: 40 : \pi,$$

$$\therefore \quad \frac{200}{\pi}\,\frac{60-y}{60+y} = \frac{40}{\pi},$$

i.e. $$5(60-y) = 60+y,$$

i.e. $$y = 40.$$

The angles are therefore 20°, 60°, and 100°.

EXAMPLE 2

Express in the three systems of angular measurement the magnitude of the angle of a regular decagon.

By Geometry, we know that all the interior angles of any rectilinear figure together with four right angles are equal to twice as many right angles as the figure has sides.

Let the angle of a regular decagon contain x right angles, so that all the angles are together equal to $10x$ right angles.

The corollary therefore states that

$$10x + 4 = 20,$$

so that $$x = \frac{8}{5} \text{ right angles}.$$

But, $$\text{one right angle} = 90^\circ = 100^g = \frac{\pi}{2} \text{ radians}.$$

Hence, $$\text{the required angle} = 144^\circ$$

$$= 160^g = \frac{4\pi}{5} \text{ radians}.$$

DETAILED SOLUTIONS TO MEASUREMENT OF ANGLES BY SL LONEY

SOLVED EXAMPLES

1. *Express in degrees, minute and seconds.* $\frac{\pi^c}{3}$.

Solution. $\frac{\pi^c}{3} = \frac{180°}{3} = \mathbf{60°}$.

2. *Express in degrees* $\frac{4\pi^c}{3}$.

Solution. $\frac{4\pi^c}{3} = \frac{4 \times 180°}{3} = \mathbf{240°}$

3. *Express in degrees* $10\pi^c$.

Solution. $10\ \pi^c = 10 \times 180° = \mathbf{1800°}$

4. *Express in degrees* 1^c.

Solution. $1° = \frac{180°}{\pi} = 180 \times .31831 = \mathbf{57°\ 17'\ 44.8''}$

5. *Express in degrees* 8^c.

Solution. $8° = 8° \times \frac{180°}{8} = \mathbf{458°\ 21'\ 58.4''}$

6. *Express in grades, minutes and seconds* $\frac{4\pi^c}{5}$.

Solution. $\frac{4\pi^c}{5} = \left(\frac{4 \times 200}{5}\right)^g = \mathbf{160g}$

7. *Express in grades* $\frac{7\pi^c}{6}$

Solution. $\frac{7\pi^c}{6} = \frac{7 \times 200^g}{6} = 233g\ 33'\ 33.3''$

8. *Express in grades* $10\pi^c$.

Solution. $10\pi^c = 10 \times 200^g = 2000^g$

9. *Express in radians* $60°$

Solution. $60° = 60 \times \frac{\pi^c}{180} = \frac{\pi^c}{3}$

10. *Express in radians* $110° 30'$.

Solution. $110°\ 30' = \frac{1105}{10} \times \frac{\pi^c}{180} = \frac{221\pi^c}{360}$

11. ***Express in radians 175° 45′.s***

Solution. $175° \; 45' = 175.75° = \frac{17575}{1000} \times \frac{\pi^c}{180}$

$= \frac{703}{720}$ pc.

12. ***Express in radians 47° 25′ 36″.***

Solution. $47°25°36' = \frac{3557°}{75} = \frac{3557\pi^c}{75 \times 180} = \frac{3557}{13500}\pi^c$

13. ***Express in radians 395°.***

Solution. $395° = 395 \times \frac{\pi^c}{180} = \frac{79\pi^c}{36}$

14. ***Express in radians 60^g.***

Solution. $60^g = \frac{60 \times \pi^c}{200} = \frac{3\pi^c}{100}$

15. ***Express in radians 110^g 30′.***

Solution. $110g.\; 30' = 110.3^g = \frac{1103}{2000}\pi^c$

16. ***Express in radians 345^g 25′ 36″.***

Solution. $345^g \; 25' \; 36'' = 345.\,2536^g$

$= \frac{3452536}{20000}\pi^c = 1.726268\,\pi^c$

17. ***The difference between the acute angles of a right–angled triangle is $\frac{2}{5}\pi$ radians; express the angle in degrees.***

Solution. Suppose the actue angles of right angled triangle are $x°$ and $y°$;

We know $\frac{2}{5}\pi^c = \frac{2 \times 180°}{5} = 72°$

$\therefore \quad x + y = 90°$ ...(*i*)

and $\quad x - y = 72°$ ...(*ii*)

Solving equations (*i*) and (*ii*), we get

$x = 81°$ or $y = 9°$

Hence, required acute angles are **81° and 9°**

18. ***One angle of a triangle is $\frac{2}{3}$ x grades another is $\frac{3}{2}$ x degrees, whilst the third is $\frac{\pi x}{75}$ radians; express them all in degrees.***

Solution. $\frac{2}{3}x^g = \frac{2}{3}x \times \frac{9°}{10} = \frac{3}{5}x°$

and $$\frac{\pi x^c}{75} = \frac{x}{75} \times 180° = \frac{12x°}{5}$$

But $$\frac{3}{5}x° + \frac{3}{2}x° + \frac{12}{5}x° = 180°$$

$\therefore$ $$6x + 15x + 24x = 180 \times 10$$

or $$45x = 1800$$

or $$x = 40°$$

Hence, three angles of the triangle are **24°, 60° and 96°.**

19. ***The circular measure of two angles of a triangle are respectively $\frac{1}{2}$ and $\frac{1}{3}$; what is the number of degrees in the third angle?***

Solution. One angle of the triangle $= \frac{1^c}{2} = 28.64789°$

Second angle of the triangle $= \frac{1^c}{3} = 19.09859°$.

Hence, third angle of triangle $= 180° - (28.64789° + 19.09859°)$

$= 0132.25352° =$ **132° 15′ 12.6″**

20. ***The angle of a triangle are in A.P. and the number of degrees in the least is to the number of radians in the greatest as 60 to π; find the angles in degrees.***

Solution. The three angles are in A.P.; if y is common difference, let these angle be $(x + y)°$, $x°$ and $(x - y)°$

$\therefore$ $$x + y + x + x - y = 180°$$

or $$x = 60°$$

According to the Question,

$$(x - y)' : (x + y) \times \frac{\pi^c}{180} = 60 : \pi$$

or $$\pi(\pi - y) = (x + y)\frac{\pi}{180} \times 60$$

or $$3(x - y) = (x + y)$$

or $$4y = 2x$$

or $$y = x/2 \qquad \therefore \qquad y = 30°$$

Hence, three angles are **30°, 60° and 90°**.

21. ***The angles of a triangle are in A.P. and the number of radians in the least angle is to the number of degrees in the mean angle as 1: 120. Find the angles in radians.***

Solution. The three angles are in (A.P) and y is common difference, let these angles be $(x + y)°$, $x°$ and $(x - y)°$

$\therefore$ $$x + y + x + x - y = 180°$$

or $$x = 60°$$

$$\text{Smallest angle} = \left(\frac{\pi}{3} - y\right)^c = \frac{(x/3) - y}{60} = \frac{1}{120}$$

or $$y = \frac{2\pi - 3}{6} = \left(\frac{\pi}{3} - \frac{1}{2}\right)^c$$

DETAILED SOLUTIONS TO MEASUREMENT OF ANGLES BY SL LONEY

Hence smallest angle $= \frac{\pi}{3} - \left(\frac{\pi}{3} - \frac{1}{2}\right)^c = \frac{1}{2}^c$

and biggest angle $= \left(\frac{\pi}{3} + \frac{\pi}{3} - \frac{1}{2}\right)^c = \left(\frac{4\pi - 3}{6}\right)^c$.

22. ***Find the magnitude, in radians and degrees, of the interior angle of***
(i) a regular pentagon
(ii) a regular heptagon
(iii) a regular octagon
(iv) a regular duodecagon, and (v) a regular polygon of 27 sides.

Solution.

(i) Angle of regular pentagon $= \frac{1}{5}$ ($2\times 5 - 4$) rt. angle

$= 180° = \frac{3\pi^c}{5}$.

(ii) Angle of regular heptagon $= \frac{1}{7}$ ($2\times 7 - 4$) rt. angle

$= \frac{10 \times 90}{7} = 128\ \frac{4}{7}^{\circ} = \frac{900 \times \pi^c}{7 \times 180} = \frac{5\pi^c}{4}$

(iii) Angle of the regular octagon $= \frac{1}{8}$ ($8\times 2 - 4$) rt. angle

$= 135° = \frac{3\pi^c}{4}$

(iv) Internal angle of the 12 sided regular polygon

$= \frac{1}{12}$ ($12\times 2 - 4$) rt. angle

$= 150° = \frac{5\pi^c}{6}$

(v) Internal angle of the 17 sided regular polygon

$= \frac{1}{17}$($17\times 2 - 4$) rt. angle $= 158\frac{14}{17}^{\circ}$

$= \frac{2700 \times \pi^c}{17 \times 80} = \frac{15\pi^c}{17}$.

23. ***The angle in one regular polygon is to that in another as 3 : 2; also the number of sides in the first is twice that in the second; how many sides have the polygons?***

Solution . Suppose the second regular polygon has x side

Thus, the first regular polygon has $2x$ sides

$\therefore$ Each angle of the first polygon $= \frac{(2x \times 2 - 4)}{2x}$ rt. angle

and each angle of the second polygon

$= \left(\frac{2.x - 4}{x}\right)$ rt. angle

Since, $\frac{4x - 4}{2x} : \frac{2x - 4}{x} = 3 : 2$

$$\therefore \qquad \frac{4x-4}{x} = \frac{6x-12}{x}$$

or $\qquad 4x - 4 = 6x - 12$

or $\qquad 2x = 8$

$\therefore \qquad x = 4$

Hence, the number of sides in the first and second polygons is respectively 8 and 4.

24. ***The number of sides in two regular polygons are as 5 : 4, and the difference between their angles is 9°; find the number of sides in the polygons.***

Solution. Let the number of sides in first regular polygon = $5x$.

$\therefore$ Number of sides in second regular polygon = $4x$.

One internal angle of $5x$ sided polygon = $\frac{1}{5x}$ $(2 \times 5x - 4)$ rt. angle.

One internal angle of $4x$ sided polygon = $\frac{1}{4x}$ $(2 \times 4x - 4)$ rt. angle

$\therefore$ Difference between two internal angles

$$\therefore \qquad = \left(\frac{10x-4}{5x} - \frac{2x-1}{x}\right) \times 90 = 9$$

$$\therefore \qquad \left(\frac{10x-4-10x+5)}{5x}\right) 90 = 9$$

or $\qquad \frac{1}{5x} \times 90 = 9$

or $\qquad x = 2$

Hence, number of sides are 10 and 8 respectively.

25. ***Find two regular polygons such that the number of their sides may be as 3 to 4 and the number of degrees in an angle of the first to the number of grades in an angle of the second as 4 to 5.***

Solution. Suppose the two polygons have respectively $3x$ and $4x$ sides.

$\therefore$ One angle of each $\frac{6x-4}{3x}$ rt. angles. and $\frac{8x-4}{4x}$ rt. angles.

But according to the question

$$\frac{6x-4}{3x} \times 90 = \frac{8x-4}{4x} \times 100 = 4 : 5$$

$$\therefore \qquad \frac{30x-20}{3x} \times 90 = \frac{32x-16}{4x} \times 100$$

or $\qquad 900x - 600 = 800x - 400$

or $\qquad 100x = 200$

or $\qquad x = 2$

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26. ***The angles of a quadrilateral are in A.P. and the greatest is double the least; express the least angle in radians.***

Solution. Let the smallest angle of quadrilateral is $(x-3y)^c$.

So angles of quadrilateral are $(x-3y)^c$, $(x-y)^c$, $(x+y)^c$ and $(x+3y)^c$.

$\therefore \quad x - 3y + x \;\; y + x + y + x + 2y = 2\pi,$

or $$x = \frac{\pi}{2}.$$

Hence, smallest and biggest anlges of quadrilateral are

$$\left(\frac{\pi}{2} - 3y\right)^c \text{ and } \left(\frac{\pi}{2} + 3y\right)^c$$

But $$2\left(\frac{\pi}{2} - 3y\right) = \left(\frac{\pi}{2} + 3y\right),$$

or $$y = \frac{\pi^c}{18}$$

Hence, $$\text{smallest angle} = \frac{\pi}{2} - \frac{3\pi}{18} = \frac{\pi^c}{18} = \frac{\pi^c}{3}$$

27. ***Find in radians, degrees, and grades the angle between the hour hand and the minute hand of a clock at***

(i) half past three

(ii) twenty minutes to six

(iii) a quarter past eleven.

Solution. Since, the circumference of the watch is divided into 60 divisions.

$\therefore$ each division = $6°$.

(i) At 3.30, the minutes hand will be at 30 minutes and hour hand will in the middle of 15 minutes and 20 minutes

$\therefore$ difference between two hands $= (30 - 17\frac{1}{2}) = 12\frac{1}{2}$ minutes.

Hence, $$\text{angle between the hands} = 6 \times \frac{25}{2} = 75°$$

$$= 75 \cdot \frac{\pi^c}{180} = \frac{5\pi^c}{12}$$

$$= 75° \times \frac{10^g}{9} = \frac{250^g}{3} = \mathbf{83\frac{1}{3}^g}.$$

(ii) At twenty minutes to six, hour needle will remain in advance of minute needle by

$$(10 + \frac{5}{3})\text{ mts.} = \frac{35}{3} \times 6°$$
$$= 70°$$

$$= \frac{70 \times 10^g}{9} = 77\frac{7^g}{9} = \frac{70\pi^c}{180} = \mathbf{\frac{7p^c}{18}}.$$

(iii) At 11-15 minutes, needle will remain in advance of hour needle by $15 + (5 - \frac{5}{4})$ minutes.

$$= \frac{75}{4} \times 6° = 112\frac{1}{2}^{\circ} = \frac{225 \times 10^{g}}{2 \times 9} = 125^{g} = \frac{\mathbf{225}}{\mathbf{2}} \times \frac{\pi^{c}}{\mathbf{0180}}\ \frac{\mathbf{5p}^{c}}{\mathbf{8}}$$

28. ***Find the times***

(i) between four and five O' clock when the angle between the minute-hand and the hour-hand is 78°,

(ii) between seven and eight O' clock when this angle is 54°.

Solution. *(i)* Since, at 4 O' clock minute hand remains behind the hour hand by 120°

So that the difference between two needles remains 78°, minutes needles has to move $120° - 78° = 42°$.

or $(120 + 78°) = 198°$ more than the hour hand

$\therefore$ Minute-hand moves 42° more in

$$\frac{60 \times 42}{330} = 7\frac{7}{11} \text{ minutes.}$$

Minute-hand moves 198° 330 more in

$$\frac{60 \times 198}{330} = 36 \text{ minutes.}$$

Required time at $\frac{7}{11}$ past 4 O' clock and 36 minutes past 4.

(ii) Since, at 7 O' clock needle remains behind the hour needle by 210°.

So that difference between the two hands remains 54°, minute hand has to move $210° - = 54° = 156°$,

or $120° + 54° = 264°$ more than the hour needle.

But minute hand moves 330° more in 60 mts.

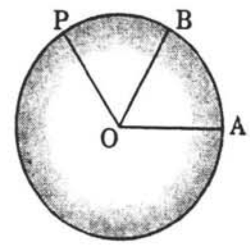
$\therefore$ Minute hand move 156° more in $= \frac{60 \times 156}{330}$ mts $= 28\frac{4}{11}$ mts.

$\therefore$ Minute hand move 156° 264° more in $= \frac{60 \times 264}{330}$ 48 mts.

Hence, required time is $28\frac{4}{11}$ mts. past 7. and 48 mts. past 7.

1.21 Theorem:

The number of radians in any angle whatever is equal to a fraction, whose numerator is the arc which the angle subtends at the centre of any circle, and whose denominator is the radius of that circle.



Let AOP be the angle which has been described by a line starting from OA and revolving into the position OP.

Where centre O and any radius, describe a circle cutting OA and OP in the points A and P.

Let the angle AOB be a radian, so that the arc AB is equal to the radius OA.

By Geometry, we have

$$\frac{\angle AOP}{\text{A Radian}} = \frac{\angle AOP}{\angle AOB} = \frac{\text{arc AP}}{\text{arc AB}} = \frac{\text{arc AP}}{\text{Radius}}$$

so that
$$\angle AOP = \frac{\text{arc AP}}{\text{Radius}} \text{ of a Radian.}$$

Hence the theorem is proved.

1.22 **EXAMPLE 1**

Find the angle subtended at the centre of a circle of radius 3 cm. by an arc of length 1 cm.

Solution: The number of radians in the angle $= \frac{\text{arc}}{\text{radius}} = \frac{1}{3}$.

Hence,

$$\text{the angle } = \frac{1}{3} = \text{radian}$$
$$= \frac{1}{3}\cdot\frac{2}{\pi} \text{ right angle}$$
$$= \frac{2}{3\pi} \times 90^\circ = \frac{60^\circ}{\pi}$$
$$= 19\frac{1}{11}^\circ, \text{ on taking } \pi \text{ equal to } \frac{22}{7}.$$

EXAMPLE 2

In a circle of 5 cm. radius, what is the length of the arc which subtends an angle of 33°15′ at the centre?

If x cm. be the required lengh, we have

Solution:
$$\frac{x}{5} = \text{number of radians in } 33^\circ\ 15'$$
$$= \frac{33\frac{1}{4}}{180}\pi \text{ (Art. 19)}$$
$$= \frac{133}{720}\pi.$$

$$\therefore \quad x = \frac{133}{144}\pi \text{ cm.} = \frac{133}{144} \times \frac{22}{7} \text{ cm. nearly.}$$
$$= 2\frac{65}{72} \text{ cm. nearly.}$$

EXAMPLE 3

Assuming the average distance of the earth from the sun to be 149,700,000 km., and the angle subtended by the sun at the eye of a person on the earth to be 32′, find the sun's diameter.

Let D be the diameter of the sun in km.

Solution:

The angle subtended by the sun being very small, its diameter is very approximately equal to a small arc of a circle whose centre is the eye of the observer. Also the sun subtends an angle of 32′ at the centre of this circle.

Hence, by Art. 21, we have

$$\frac{D}{149{,}700{,}000} = \text{number of radians in } 32'$$

$$= \text{number of radians in } \frac{8^\circ}{15}$$

$$= \frac{8}{15} \times \frac{\pi}{180} = \frac{2\pi}{675}.$$

$$\therefore \quad D = \frac{299{,}400{,}000}{675}\pi \text{ km.}$$

$$= \frac{299{,}400{,}000}{675} \times \frac{22}{7} \text{ km. (approximately)}$$

$$= 1{,}390{,}000 \text{ km. (approx)}$$

EXAMPLE 4

Assuming that a person of normal sight can read print at such a distance that the letters subtend an angle of 5′ at his eye, find what is the height of the letters that he can read at a distance

(i) of 12 metres, and

(ii) 1320 metres.

Let x be the required height in metres.

Solution:

(i) In this case, x is very nearly equal to the arc of a circle, of radius 12m., which subtends an angle of 5′ at its centre.

Hence
$$\frac{x}{12} = \text{number of radians in } 5'$$

$$= \frac{1}{12} \times \frac{\pi}{180}.$$

$$\therefore \quad x = \frac{\pi}{180} \text{ m.} = \frac{1}{180} \times \frac{22}{7} \text{ metres (approx)} = 1{\cdot}7 \text{ cm.}$$

(ii) In this case, the height y is given by

$$\frac{y}{1320} = \text{number of radians in } 5'$$

$$= \frac{1}{12} \times \frac{\pi}{180},$$

$$\therefore \quad y = \frac{11}{18}\pi = \frac{11}{18} \times \frac{22}{7} \text{ metres} = \mathbf{1{\cdot}9 \text{ metres.}}$$

SOLVED EXAMPLES

$$\left[Assume\,\pi = 3.14159...and\,\frac{1}{\pi} = \cdot 31831.\right]$$

1. ***Find the number of degrees subtended at the centre of a circle by an arc whose length is 0.37 times the radius.***

Solution. Number of radians in the angle $= \dfrac{\text{arc}}{\text{radius}} = 0.357$

Hence $357 \text{ radians} = 0.357 \times \dfrac{2}{\pi}$ right angle

$= 0.357 \times 2 \times .31831 \times 90°$

$=$ **20.454°.**

2. ***Express in radians and degrees the angle subtended at the centre of a circle by an arc whose length is 15 cm., the radius of the circle being 25 cm.***

Solution. Angle in radians $= \dfrac{\text{arc}}{\text{radius}}$

$= \dfrac{15}{25} = \dfrac{3}{5}$ radians

$\therefore$ $\dfrac{3}{5}$ radians $= \dfrac{3}{5} \times \dfrac{2}{\pi}$ rt. angle

$= \dfrac{3}{5} \times 2 \times 90 \times .31831°$

$=$ **34° 22′ 38.9″**

3. ***The value of the divisions on outer rim of graduated circle is 5′ and between successive graduation is 0.1 cm. Find the radius of the circle.***

Solution. $\text{Angle} = \dfrac{5°}{60} = \dfrac{\text{Arc}}{\text{Radius}} = \dfrac{\cdot 1^c}{\text{Radius}} = \dfrac{\cdot 1}{\text{Radius}} \times \dfrac{180°}{\pi}$

$\therefore$ $\text{Radius} = \dfrac{0.1 \times 180 \times 60}{\pi \times 5} = 68.75496$ cms.

$=$ **68.75 cms.** nearly.

4. ***The diameter of a graduated cirlce is 72 cm. and the graduations on its rim are 5′ apart; find the distance from one graduation to another.***

Solution. $\text{Angle} = \dfrac{5°}{60} = \dfrac{\text{Arc}}{72/2} \times \dfrac{180'}{\pi}$

$\therefore$ $\text{Arc} = \dfrac{5 \times 36 \times \pi}{60 \times 180} = \dfrac{\pi}{60} = \dfrac{3 \cdot 14159}{60} =$ **0.5236 cms.**

5. ***Find the radius of a globe which is such that the distance between two places on the same meridian whose latitude differs by 1°10′ may be 0.5 cm.***

Solution. $$\text{Angle } 1^\circ\, 10' = \frac{7^\circ}{6} = \frac{}{2 \times \text{radius}} \times \frac{180^\circ}{\pi}$$

$\therefore$ $$\text{Radius} = \frac{180^\circ \times 6}{2 \times \pi \times 7} = \mathbf{24.555\ cms.}$$

6. ***Taking the radius of the earth as 6400 km., find the difference in latitude of two places, one of which is 100 km. north of the other.***

Solution. $$\text{Required difference} = \frac{\text{Arc}}{\text{Radius}} = \frac{100}{6400} \times \frac{180^\circ}{\pi}$$

$$= \frac{1}{64} \times \frac{180}{22} \times 7$$
$$= \mathbf{53'\, 43''}$$

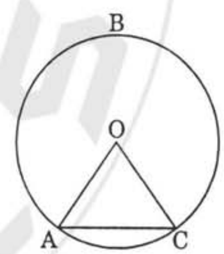
7. ***Assuming the earth to be a sphere and the distance between two parallels of latitude, which subtends an angle of 1° at the earth's centre to be $69\frac{1}{9}$ km., find the radius of the earth.***

Solution. $$\text{Angle } 1^\circ = \frac{622}{9 \times \text{Radius}} \times \frac{180^\circ}{\pi}$$

or $$\text{Radius} = \frac{622 \times 180}{9 \times} = 3959.7764 \text{ kms.}$$
$$= \mathbf{3959.8\ kms.} \text{ (approx)}$$

8. ***The radius of a certain circle is 30 cm., find approximately the length of an arc of this circle; if the length of the chord of the are be 30 cm. also.***

Solution. Let ABC be the circle whose centre is O and AC is chord.



In Δ AOC, $$AO = OC = AC = 30 \text{ cm.}$$

$\therefore$ $$\angle AOC = 60^\circ = \frac{\pi}{3}$$

Hence, $$\text{arc } AC = \text{radius} \times \frac{\pi}{3} = 30 \times \frac{\pi}{3}$$
$$= 10\pi = \mathbf{31.4159\ cms.}$$

9. ***What is the ratio of the radil of two circles at the centre of which two arcs of the same length subtend angles of 60° and 75°?***

Solution. Suppose, length of the arc = x'

$\therefore$ $$\text{Radius of the circle} = \frac{x' \times 180}{\pi \times 60}$$

and, $$\text{radius of the second circle} = \frac{x \times 180}{\pi \times 75}$$

$\therefore$ $$\text{Required ratio} = \frac{x \times 180}{\pi \times 60} : \frac{x \times 189}{\pi \times 75} = 75 : 60 = \mathbf{5 : 4}$$

10. *If an arc, of length 10 cm., on a cirlce of 8 cm. diameter subtend at the centre of an angle 143°14′22″, find the value of π to 4 places of decimals.*

Solution. Angle at centre 143° 14′ 22′ $= \dfrac{257831}{1800}$

$$= \frac{10}{8/2} \times \frac{180}{\pi}$$

$$\therefore \quad \pi = \frac{10 \times 180 \times 1800}{257831 \times 4} = \mathbf{3.1416}$$

11. *If the circumference of a circle be divided into 5 parts which are in A.P., and if the greatest part be 6 times the least, find in radians the magnitudes of the angle that the parts subtend at the centre of the circle.*

Solution. Let 5 parts of the circumference of circle be

$$(x-2y),\quad (x-y),\ x,\ (x+y) \text{ and } (x+2y)$$

$$\therefore \quad 6(x-2y) = x+2$$

or $$5x = 14y$$

or $$y = \frac{5}{14}x \qquad \ldots(i)$$

If r is the radius of the circle, then its circumference $= 2\pi r$.

$$\therefore \quad (x-2y) + (x-y) + x + (x+y) + (x+2y) = 2\pi r.$$

$$x = \frac{2pr}{5}$$

Substituting the value of x in equation (i), we get

$$y = \frac{5}{14} \cdot \frac{2\pi}{5} = \frac{\pi r}{7}$$

Hence, angle subtended at the centre by the Ist part

$$= \frac{\frac{2\pi r}{5} - \frac{2}{7}\pi r}{r} = \frac{4\pi}{35}$$

Angle subtended at the centre by the 2nd part

$$= \frac{\frac{2\pi r}{5} - \frac{9}{7}\pi r}{r} = -\frac{4\pi}{35}$$

Angle subtended at the centre by the third part

$$= \frac{2\pi r}{r} \times \frac{1}{5} = \frac{14\pi}{35}$$

Angle subtended at the centre by the 4th part

$$= \frac{\pi r}{r}\left(\frac{2}{5} + \frac{1}{7}\right) = \frac{19\pi}{35}$$

Angle subtended at the centre by the 5th part

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$$= \frac{\pi r}{r}\left(\frac{2}{5}+\frac{2}{7}\right) = \frac{24\pi}{35}$$

Hence, the five parts of the circle subtended at the centre angles

$$\frac{4\pi}{35}, \frac{9\pi}{35}, \frac{13\pi}{35}, \frac{19\pi}{35} \text{ and } \frac{24\pi}{35}$$

12. ***The perimeter of a certain sector of a circle is equal to the length of the arc of a semicircle having the same radius; express the angle of the sector in degrees, minutes, and seconds.***

Solution. Suppose radius of circle be r.

Hence, arc of the semi-circle will be πr.

Circumference of sector = $2r$ + Arc

or $\pi r = 2r + \text{Arc}$

or $\text{Arc} = r(\pi - 2)$

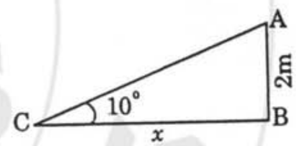
$\therefore$ Angle of sector $= \dfrac{r(\pi-2)^c}{r} = (\pi - 2) \times \dfrac{180°}{\pi}$

$$= \left(180 - \frac{360}{\pi}\right)^c = \mathbf{65° \; 24' \; 30.24''}$$

13. ***At what distance does a man, whose height is 2 m., subtended an angle of 10'?***

Solution. In the diagram AB is the man and C is the required point at a distance of x meteres from B.

Let required distance be x meters.



$\therefore \quad \dfrac{10}{60} = \dfrac{2}{x} \times \dfrac{180}{\pi}$

or $\quad x = \dfrac{2 \times 6 \times 180}{\pi} = \mathbf{687.5\ m.}$

14. ***Find the length which at a distance of 5280 m. will subtend an angle of 1' at the eye.***

Solution. In the above figure, BC = 5280 m. Let AB = x m. and $\angle$ ABC = 1' ; then

$$\frac{1}{60} \times \frac{\pi}{180} = \frac{x}{5280}$$

or $\quad x = \dfrac{1760 \times 3 \times \pi}{60 \times 180} = \dfrac{22 \times 3 \cdot 14159}{45}$ m.

$x = \mathbf{1.5359\ m.}$

15. ***Find approximately the distance at which a globe, $5\frac{1}{2}$ cm. in diameter, will subtend an angle of 6'.***

Solution. Let required distance be x cms.

$$6' = \frac{6}{60} = \frac{11}{2 \times x} \times \frac{180}{\pi}$$

or $\quad x = \dfrac{11 \times 180 \times 60}{6 \times 2 \times \pi} = 3151.2$ cms.

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16. ***Find approximately the distance of a tower whose height is 51 metres and which subtends at the eye an angle of 5 $\frac{5}{11}$.***

Solution. Suppose required length be x m.

$$\therefore \qquad \frac{60}{11\times 60} = \frac{51}{x}\times\frac{180}{\pi}$$

$$\text{or} \qquad x = \frac{51\times 180\times 11}{\pi}\text{ m.} = \mathbf{32142.9\ m.}$$

17. ***A church spire, whose height is known to be 10 metres, subtends an angle of 9′ at the eye: find approximately its distance.***

Solution. Suppose required length be x m.

$$\text{So} \qquad 9' = \frac{9}{60} = \frac{100}{x}\times\frac{180}{\pi}$$

$$\text{or} \qquad x = \frac{100\times 180\times 60}{9\times\pi}\text{ m.}$$

$$= \mathbf{3820\ m.}\text{ (approx.)}$$

18. ***Find approximately in minutes the inclination to the horizon of an incline which rises 1 $\frac{1}{6}$ metres in 210 metres.***

Solution. Here $\frac{7}{6}$ is the length of the arc and 210 m. is the radius.

$$\therefore \qquad \text{angle} = \frac{7}{2\times 210\times 3}\text{ radians} = \frac{7}{2\times 210\times 3}\text{ radians.}$$

$$= \frac{1}{180}\times\frac{2}{\pi}\times 90^\circ$$

$$= \frac{1^c}{\pi} = 0.31831$$

$$= \mathbf{19.099'}\text{ (approx.)}$$

19. ***The radius of the earth being taken to 6400 km., and the distance of the moon from the earth being 60 times the radius of the earth, find approximately the radius of the moon which subtends at the earth an angle 16′.***

Solution. Let Radius of moon be x km.

$$\therefore \qquad \frac{16}{60} = \frac{2x}{60\times 6400}\times\frac{180}{\pi}$$

$$\text{or} \qquad x = \frac{6400\times 16\times\pi}{2\times 180} = \mathbf{1788\ kms.}$$

20. ***When the moon is setting at any given place, the angle that is subtended at its centre by the radius of the earth passing through the given place is 57′. If the earth's radius be 6400 km., find approximately the distance of the moon.***

Solution. Let the required lenght x kms.

$$\therefore \qquad \frac{57}{60} = \frac{6400}{x}\times\frac{180^\circ}{\pi}$$

$$\text{or} \qquad x = \frac{6400\times 180\times 60}{57\times\pi} = \mathbf{38600\ kms.}$$