NCERT Solutions for 11th Class Physics:Chapter 14- Oscillations



# NCERT Solutions for 11th Class Physics: Chapter 14-Oscillations

Class 11: Physics Chapter 14 solutions. Complete Class 11 Physics Chapter 14 Notes.

## NCERT Solutions for 11th Class Physics: Chapter 14-Oscillations

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**Exercises**

**14.1. Which of the following examples represent periodic motion?**

**(a) A swimmer completing one (return) trip from one bank of a river to the other and back.**

**(b) A freely suspended bar magnet displaced from its N-S direction and released.**

**(c) A hydrogen molecule rotating about its center of mass.**

**(d) An arrow released from a bow.**

**Answer**

(a) The swimmer's motion is not periodic. Though the motion of a swimmer is to and fro but will not have a definite period.

(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic because the magnet oscillates about its position with a definite period of time.

(c) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such a motion is periodic.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

**14.2. Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?**

**(a) the rotation of earth about its axis.**

**(b) motion of an oscillating mercury column in a U-tube.**

**(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.**

**(d) general vibrations of a polyatomic molecule about its equilibrium position.**

**Answer**

(a) It is priodic but not simple harmonic motion because it is not to and fro about a fixed point.

(b) It is a simple harmonic motion because the mercury moves to and fro on the same path, about the fixed position, with a certain period of time.

(c) It is simple harmonic motion because the ball moves to and fro about the lowermost point of the bowl when released. Also, the ball comes back to its initial position in the same period of time, again and again.

(d) A polyatomic molecule has many natural frequencies of oscillation. Its vibration is the superposition of individual simple harmonic motions of a number of different molecules. Hence, it is not simple harmonic, but periodic.

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**14.3. Figure 14.27 depicts four *x*-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?**

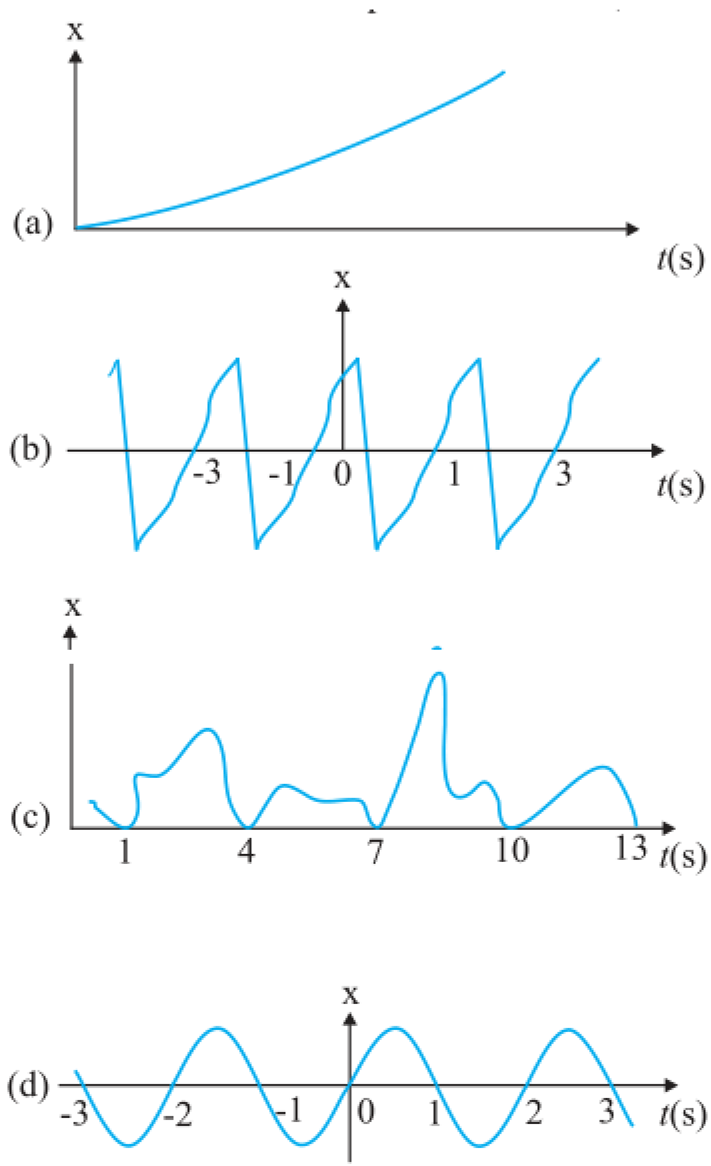


*Fig. 14.27*

**Answer**

(a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.

(b) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.

(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

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**14.4. Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):**

**(a)** $\sin \omega t - \cos \omega t$

**(b)** $\sin^3 \omega t$

**(c)** $3 \cos (\pi/4 - 2\omega t)$

**(d)** $\cos \omega t + \cos 3\omega t + \cos 5\omega t$

**(e)** $\exp(-\omega^2 t^2)$

**Answer**

(a) SHM

The given function is:

$\sin \omega t - \cos \omega t$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right]$$
$$= \sqrt{2}\left[\sin\omega t \times \cos\frac{\pi}{4} - \cos\omega t \times \sin\frac{\pi}{4}\right]$$
$$= \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$$

This function represents SHM as it can be written in the form: *a sin (ωt + Φ)*

Its period is: 2π/ω

(b) Periodic but not SHM

The given function is:

$\sin^3 \omega t$ = 1/4 [3 sin *ωt* - sin3*ωt*]

The terms sin *ωt* and sin *ωt* individually represent simple harmonic motion (SHM). However, the superposition of two SHM is periodic and not simple harmonic.

Ites period is: 2π/ω

(c) SHM

The given function is:

$$3\cos\left[\frac{\pi}{4} - 2\,\omega t\right]$$
$$= 3\cos\left[2\,\omega t - \frac{\pi}{4}\right]$$

This function represents simple harmonic motion because it can be written in the form: *a cos (ωt + Φ)*Its period is: 2π/2ω = π/ω

(d) Periodic, but not SHM

The given function is $\cos\omega t + \cos 3\omega t + \cos 5\omega t$. Each individual cosine function represents SHM. However, the superposition of three simple harmonic motions is periodic, but not simple harmonic.

(e) Non-periodic motion

The given function $\exp(-\omega^2 t^2)$ is an exponential function. Exponential functions do not repeat themselves. Therefore, it is a non-periodic motion.

(f) The given function $1 + \omega t + \omega^2 t^2$ is non-periodic.

**14.5. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is**

**(a) at the end A,**

**(b) at the end B,**

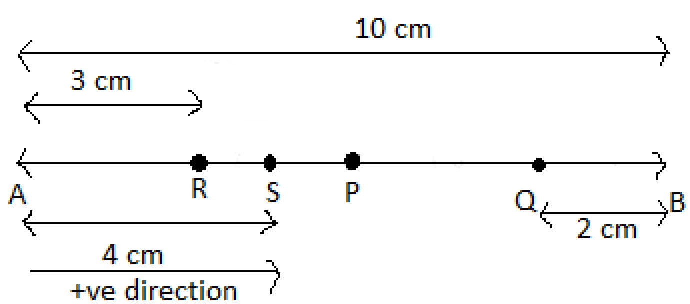
**(c) at the mid-point of AB going towards A,**

**(d) at 2 cm away from B going towards A,**

**(e) at 3 cm away from A going towards B, and**

**(f) at 4 cm away from B going towards A.**

**Answer**



From above figure, where A and B represent the two extreme positions of a SHM. FOr velocity, the direction from A to B is taken to b positive. The acceleration and the force, along AP are taken as positive and alsong Bp are taken as negative.

(a) At the end A, the particle executing SHM is momentarily at rest being its extreme position of motion. Therefore, its velocity is zero. Acceleration is positive becaue it is directed along AP, Force is also Positive since the force is directed along AP.

(b) At the end B, velocity is zero. Here, accleration and force are negative as they are directed along BP.

(c) At the mid point of AB going towards A, the particle is at its mean psoiton P, with a tendancy to move along PA. Hence, velocity is positive. Both accleration and force are zero.

(d) At 2 cm away from B going towards A, the particle is at Q, with a tendancy to move along QP, which is negative direction. Therefore, velocity, acceleration and force all are positive.

(e) At 3 sm away from A going towards B, the particle is at R, with a tendancy to move along RP, which is positive direction. Here, velocity, acceleration all are positive.

(f) At 4 cm away from A going towards A, the particles is at S, with a tendancy to move along SA, which is negative direction. Therefore, velocity is negative but acclereation is directed towards mean position, along SP. Hence it is positive and also force is positive similarly.

**14.6. Which of the following relationships between the acceleration *a* and the displacement *x* of a particle involve simple harmonic motion?**

**(a) $a = 0.7x$**

**(b) $a = -200x^2$**

**(c) $a = -10x$**

**(d) $a = 100x^3$**

**Answer**

In SHM, acceleration $a$ is related to displacement by the relation of the form $a = -kx$, which is for relation (c).

**14.7. The motion of a particle executing simple harmonic motion is described by the displacement function,**

**$x(t) = A\cos(\omega t + \varphi)$.**

**If the initial ($t = 0$) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is $\pi\ s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM: $x = B\sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.**

**Answer**

Intially, at t = 0;

Displacement, $x = 1$ cm

Intial velocity, $v = \omega$ cm/ sec.

Angular frequency, $\omega = \pi$ rad/$s^{-1}$

It is given that,

$x(t) = A\cos(\omega t + \Phi)$

$1 = A\cos(\omega \times 0 + \Phi) = A\cos\Phi$

$A\cos\Phi = 1$ **...(i)**

Velocity, $v = dx/dt$

$\omega = -A\,\omega\sin(\omega t + \Phi)$

$1 = -A\sin(\omega \times 0 + \Phi) = -A\sin\Phi$

$A\sin\Phi = -1$ **...(ii)**

Squaring and adding equations **(i)** and **(ii)**, we get:

$A^2(\sin^2\Phi + \cos^2\Phi) = 1 + 1$

$A^2 = 2$

$\therefore A = \sqrt{2}$ cm

Dividing equation **(ii)** by equation **(i)**, we get:

$\tan\Phi = -1$

$\therefore \Phi = 3\pi/4, 7\pi/4, \ldots\ldots$

SHM is given as:

$x = B\sin(\omega t + \alpha)$

Putting the given values in this equation, we get:

$1 = B\sin[\omega \times 0 + \alpha] = 1 + 1$

$B\sin\alpha = 1$ **...(iii)**

Velocity, $v = \omega B\cos(\omega t + \alpha)$

Substituting the given values, we get:

$\pi = \pi B\sin\alpha$

$B\sin\alpha = 1$ **...(iv)**

Squaring and adding equations **(iii)** and **(iv)**, we get:

$B^2 [\sin^2\alpha + \cos^2\alpha] = 1 + 1$

$B^2 = 2$

$\therefore B = \sqrt{2}$ cm

Dividing equation (iii) by equation (iv), we get:

$B\sin\alpha / B\cos\alpha = 1/1$

$\tan\alpha = 1 = \tan\pi/4$

$\therefore \alpha = \pi/4, 5\pi/4, \ldots\ldots$

**14.8. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?**

**Answer**

Maximum mass that the scale can read, $M = 50$ kg

Maximum displacement of the spring = Length of the scale, $l = 20$ cm = 0.2 m

Time period, $T = 0.6$ s

Maximum force exerted on the spring, $F = Mg$

where,

$g$ = acceleration due to gravity = 9.8 m/s²

$F = 50 \times 9.8 = 490$

∴ Spring constant, $k = F/l = 490/0.2 = 2450\ N\ m^{-1}$.

Mass $m$, is suspended from the balance.

Time Period, $t = 2\pi\sqrt{\frac{m}{k}}$

$\therefore m = \left(\frac{T}{2\pi}\right)^2 \times k = \left(\frac{0.6}{2 \times 3.14}\right)^2 \times 2450 = 22.36\ kg$

∴ Weight of the body = $mg = 22.36 \times 9.8 = 219.167$ N

Hence, the weight of the body is about 219 N.

**14.9. A spring having with a spring constant 1200 N $m^{-1}$ is mounted on a horizontal table as shown in Fig. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.**

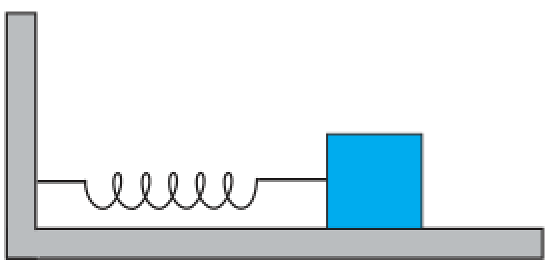


Fig. 14.28

**Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.**

**Answer**

Spring constant, $k$ = 1200 N $m^{-1}$

Mass, $m$ = 3 kg

Displacement, $A$ = 2.0 cm = 0.02 cm

(i) Frequency of oscillation $v$, is given by the relation:

$$v = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where, $T$ is time period

$$\therefore v = \frac{1}{2 \times 3.14}\sqrt{\frac{1200}{3}} = 3.18\, m/s$$

Hence, the frequency of oscillations is 3.18 cycles per second.

(ii) Maximum acceleration ($a$) is given by the relation:

$a = \omega^2 A$

where,

$$\omega = Angular\ frequency = \sqrt{\frac{k}{m}}$$

$A$ = maximum displacement

$$\therefore a = \frac{k}{m}A = \frac{1200 \times 0.02}{3} = 8\, ms^{-2}$$

Hence, the maximum acceleration of the mass is 8.0 $m/s^2$.

(iii) Maximum velocity, $v_{max} = A\omega$

$$= A\sqrt{\frac{k}{m}} = 0.02 \times \sqrt{\frac{1200}{3}} = 0.4\, m/s$$

Hence, the maximum velocity of the mass is 0.4 m/s.

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**14.10. In Exercise 14.9, let us take the position of mass when the spring is unstreched as *x* = 0, and the direction from left to right as the positive direction of *x*-axis. Give *x* as a function of time t for the oscillating mass if at the moment we start the stopwatch (*t* = 0), the mass is**

**(a) at the mean position,**

**(b) at the maximum stretched position, and**

**(c) at the maximum compressed position.**

**In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?**

**Answer**

Distance travelled by the mass sideways, $a$ = 2.0 cm

Angular frequency of oscillation:

$$\omega = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{1200}{3}} = \sqrt{400} = 20\, rad\, s^{-1}$$

(a) As time is noted from the mean position, hence using

$x = a \sin \omega t$, we have $x = 2 \sin 20 t$

(b) At maximum stretched position, the body is at the extreme right position, with an intial phase of π/2 rad. Then,

$$x = a \sin\left(\omega t + \frac{\pi}{2}\right) = a \cos \omega\, t = 2 \cos 20\, t$$

(c) At maximum compressed position, the body is at left position, with an intial phase of 3 π/2 rad. Then,

$$x = a \sin\left(\omega t + \frac{3\pi}{2}\right) = -a \cos \omega\, t = -2 \cos 20\, t$$

The functions neither differ in amplitude nor in frequency. They differ in intial phase.

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**14.11. Figures 14.29 correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure.**

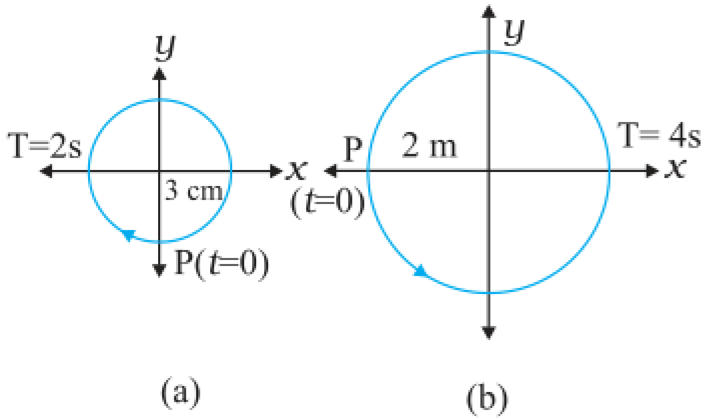


*Fig. 14.29*

**Obtain the corresponding simple harmonic motions of the *x*-projection of the radius vector of the revolving particle P, in each case.**

**Answer**

(a) Time period, t = 2 s

Amplitude, A = 3 cm

At time, t = 0, the radius vector OP makes an angle π/2 with the positive *x*-axis,.e., phase angle $\Phi = +\pi/2$

Therefore, the equation of simple harmonic motion for the *x*-projection of OP, at the time *t*, is given by the displacement equation:

$$x = A\cos\left[\frac{2\pi t}{T} + \emptyset\right]$$

$$= 3\cos\left(\frac{2\pi t}{2} + \frac{\pi}{2}\right) = -3\sin\left(\frac{2\pi t}{2}\right)$$

$$\therefore x = -3\sin \pi\, t\ cm$$

(b) Time Period, t = 4 s

Amplitude, a = 2 m

At time t = 0, OP makes an angle π with the x-axis, in the anticlockwise direction, Hence, phase angle *Φ* = +π

Therefore, the equation of simple harmonic motion for the *x*-projection of OP, at the time *t*, is given as:

$$x = a\cos\left[\frac{2\pi t}{T} + \emptyset\right]$$
$$= 2\cos\left(\frac{2\pi t}{4} + \pi\right)$$
$$\therefore x = -2\cos\left(\frac{\pi}{2}t\right)\ m$$

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**14.12. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (*t* = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (*x* is in cm and *t* is in *s*).**

**(a) $x = -2\sin(3t + \pi/3)$**

**(b) $x = \cos(\pi/6 - t)$**

**(c) $x = 3\sin(2\pi t + \pi/4)$**

**(d) $x = 2\cos\pi t$**

**Answer**

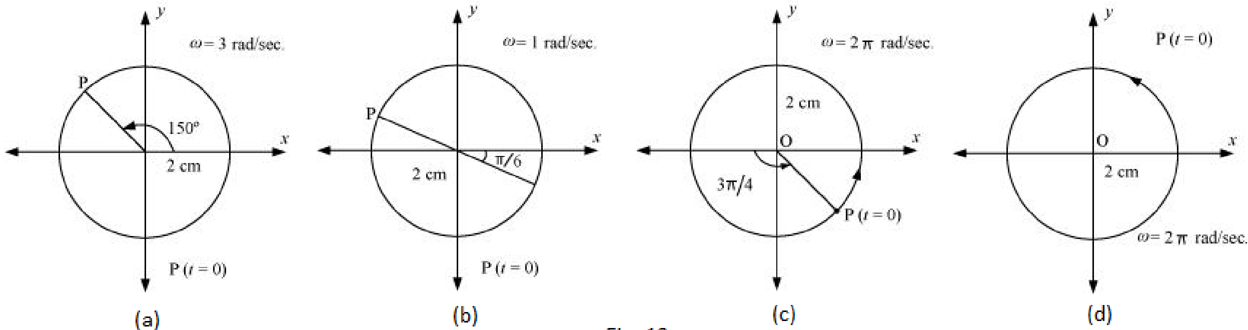


*Fig. 10*

(a)

$$x = -2\sin\left(3t + \frac{\pi}{3}\right) = +2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$$
$$= 2\cos\left(3t + \frac{5\pi}{6}\right)$$

If this equation is compared with the standard SHM equation

$x = A\cos\left(\frac{2\pi}{T}t + \Phi\right)$, then we get:

Amplitude, $A$ = 2 cm

Phase angle, $\Phi$ = 5π/6 = 150°.

Angular velocity = ω = 2π/$T$ = 3rad/sec.

The motion of the particle can be plotted as shown in fig. 10(a).

(b)

$$x = \cos\left(\frac{\pi}{6} - t\right)$$
$$= \cos\left(t - \frac{\pi}{6}\right)$$

If this equation is compared with the standard SHM equation

$x = A\cos\left(\frac{2\pi}{T}t + \Phi\right)$, then we get:

Amplitude, A = 1

Phase angle, $\Phi$ = -π/6 = -30°.

Angular velocity, ω = 2π/$T$ = 1 rad/s.

The motion of the particle can be plotted as shown in fig. 10(b).

(c)

$$x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$$

$$= -3\cos\left[\left(2\pi t + \frac{\pi}{4}\right) + \frac{\pi}{2}\right] = -3\cos\left(2\pi t + \frac{3\pi 4}{4}\right)$$

If this equation is compared with the standard SHM equation

$x = A\cos\left(\frac{2\pi}{T}t + \emptyset\right)$,then we get:

Amplitude, $A$ = 3 cm

Phase angle, $\Phi$ = 3π/4 = 135°

Angular velocity, ω = 2π/$T$ = 2 rad/s.

The motion of the particle can be plotted as shown in fig. 10(c).

(d)

$x = 2\cos\pi t$

If this equation is compared with the standard SHM equation

$x = A\cos\left(\frac{2\pi}{T}t + \emptyset\right)$,then we get:

Amplitude, $A$ = 2 cm

Phase angle, $\Phi$ = 0

Angular velocity, ω = π rad/s.

The motion of the particle can be plotted as shown in fig. 10(d).

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**14.13. Figure 14.30 (a) shows a spring of force constant *k* clamped rigidly at one end and a mass *m* attached to its free end. A force *F* applied at the free end stretches the spring. Figure 14.30 (b) shows the same spring with both ends free and attached to a mass *m* at either end. Each end of the spring in Fig. 14.30(b) is stretched by the same force *F*.**

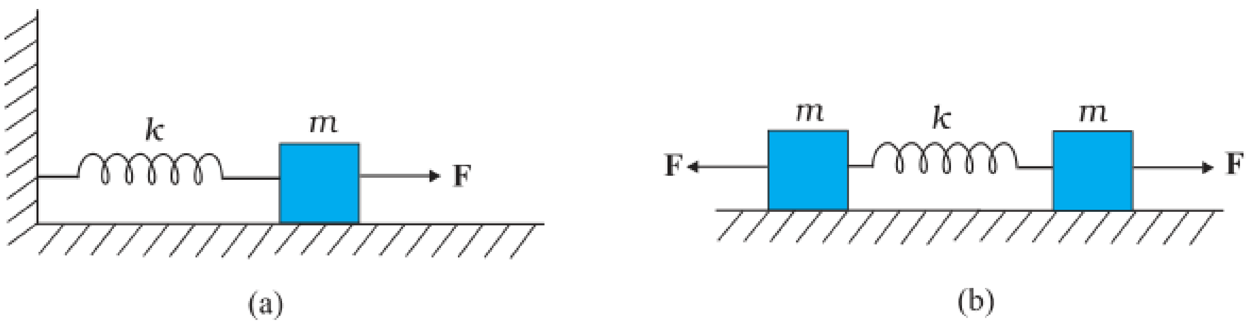


*Fig. 14.30*

**(a) What is the maximum extension of the spring in the two cases?**

**(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?**

**Answer**

(a) The maximum extension of the spring in both cases will = $F/k$, where $k$ is the spring constant of the springs used.

(b) In Fig.14.30(a) if $x$ is the extension in the spring, when mass m is returning to its mean position after being released free, then restoring force on the mass iss $F = -kx$, i.e., $F \propto x$

As, this $F$ is directed towards mean position of the mass, hence the mass attached to the spring will execute SHM.

Spring factor = spring constant = $k$

inertia factor = mass of the given mass = $m$

As time period,

$$T = 2\pi\sqrt{\frac{inertia\ factor}{spring\ factor}}$$

$$\therefore T = 2\pi\sqrt{\frac{m}{k}}$$

In Fig.14.30(b), we have a two body system of spring constant k and reduced mass, μ = $m \times m$ / $m + m = m/2$.

Inertia factor = $m/2$

Spring factor = $k$

$$\therefore time\ period, T = 2\pi\sqrt{\frac{m/2}{k}} = 2\pi\sqrt{\frac{m}{2k}}$$

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**14.14. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what is its maximum speed?**

**Answer**

Angular frequency of the piston, ω = 200 rad/ min.

Stroke = 1.0 m

Amplitude, $A$ = 1.0/2 = 0.5 m

The maximum speed ($v_{max}$) of piston is given by the relation:

$v_{max}$ = Aω = 200 × 0.5 = 100m/min.

**14.15. The acceleration due to gravity on the surface of moon is 1.7 ms$^{-2}$. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? ($g$ on the surface of earth is 9.8 ms$^{-2}$)**

**Answer**

Acceleration due to gravity on the surface of moon,$g'$ = 1.7 m s$^{-2}$

Acceleration due to gravity on the surface of earth, $g$ = 9.8 m s$^{-2}$

Time period of a simple pendulum on earth, $T$ = 3.5 s

Hence, the time period of the simple pendulum on the surface of moon is 8.4 s.

**14. 16.**

*Answer the following questions:*

(a) *Time period of a particle in SHM depends on the force constant k and mass m of the particle:*

$$T = 2\pi\sqrt{\frac{m}{k}}$$

*A simple pendulum executes SHM approximately.*
*Why then is the time period of a pendulum independent of the mass of the pendulum?*

(b) *The motion of a simple pendulum is approximately simple harmonic for small angle oscillations.*

*For larger angles of oscillation, a more involved analysis shows that T is greater than* $2\pi\sqrt{\frac{l}{g}}$.

*Think of a qualitative argument to appreciate this result.*

(c) *A man with a wrist watch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?*

(d) *What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?*

**Answer:**

(a) In case of a spring, $k$ does not depend upon $m$. However, in case of a simple pendulum, $k$ is directly proportional to $m$ and hence the ratio $\frac{m}{k}$ is a constant quantity.

(b) The restoring force for the bob of the pendulum is given by

$$F = -mg \sin\theta$$

If $\theta$ is small, then $\sin\theta = \theta = \frac{y}{l}$ $\quad\therefore\quad F = -\frac{mg}{l}y$

*i.e.*, the motion is simple harmonic and time period is $T = 2\pi\sqrt{\frac{l}{g}}$.

Cleraly, the above formula is obtained only if we apply the approximation $\sin\theta \approx \theta$.

For large angles, this approximation is not valid and $T$ is greater than $2\pi\sqrt{\frac{l}{g}}$.

(c) The wristwatch uses an electronic system or spring system to give the time, which does not change with acceleration due to gravity. Therefore, watch gives the correct time.

(d) During free fall of the cabin, the acceleration due to gravity is zero. Therefore, the frequency of oscillations is also zero i.e., the pendulum will not vibrate at all.

**14. 17. A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?**

**Answer**

In this case, the bob of the pendulum is under the action of two accelerations.

(i) Acceleration due to gravity '$g$' acting vertically downwards.

(ii) Centripetal acceleration $a_c = \frac{v^2}{R}$ acting along the horizontal direction.

$\therefore$ Effective acceleration, $g' = \sqrt{g^2 + a_c^2}$

or $$g' = \sqrt{g^2 + \frac{v^4}{R^2}}$$

Now time period, $T' = 2\pi\sqrt{\frac{1}{g}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$.

**14. 18. A cylindrical piece of cork of density of base area A and height h floats in a liquid of density $\rho_1$. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period**

$$T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$$

**where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).**

**Answer**

Say, initially in equilibrium, $y$ height of cylinder is inside the liquid. Then,
Weight of the cylinder = upthrust due to liquid displaced

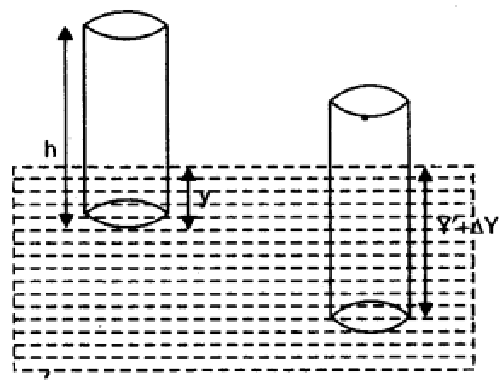
$\therefore$ $$Ah\rho g = Ay\rho_l g$$

When the cork cylinder is depressed slightly by $\Delta y$ and released, a restoring force, equal to additional upthrust, acts on it. The restoring force is

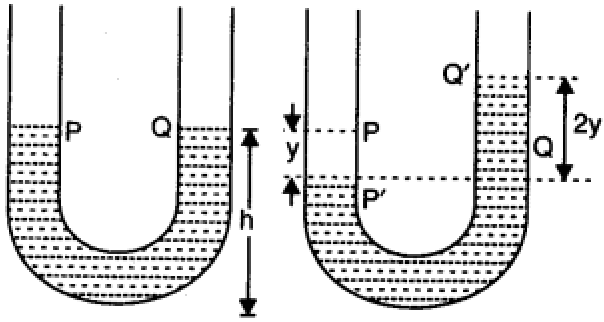
$$F = A(y + \Delta y)\,\rho_l g - Ay\rho_l g = A\rho_l g\Delta y$$

$\therefore$ Acceleration, $a = \dfrac{F}{m} = \dfrac{A\rho_l g\Delta y}{Ah\rho} = \dfrac{\rho_l g}{h\rho}.\,\Delta y$ and the acceleration is directed in a direction opposite to $\Delta y$. Obviously, as $a \propto -\Delta y$, the motion of cork cylinder is SHM, whose time period is given by

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{\Delta y}{a}} = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}.$$




**Question 14. 19. One end of a U-tube containing mercury is connected to a suction pump and the other end to the atmosphere.**




**A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.**

$\therefore \quad F = -2\,Ay\rho g = -(2A\rho g)y$

If $\quad a$ = acceleration produced in the liquid column, Then

$$a = \frac{F}{m}$$

$$= -\frac{(2A\rho g)y}{LA\rho} = -\frac{2A\rho g}{LA}$$

$$= -\frac{2\rho g}{2h\rho}y \qquad \text{...}(i) \quad (\because L = 2h)$$

where $h$ = height of mercury in each limb. Now from eqn. ($i$), it is clear that $a \propto y$ and $-ve$ sign shows that it acts opposite to $y$, so the motion of mercury in $u$-tube is simple harmonic in nature having time period ($T$) given by

$$T = 2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{\frac{2h\rho}{2\rho g}} = 2\pi\sqrt{\frac{h\rho}{\rho g}}$$

$$T = 2\pi\sqrt{\frac{h}{g}}$$

**Answer**

The suction pump creates the pressure difference, thus mercury rises in one limb of the U-tube. When it is removed, a net force acts on the liquid column due to the difference in levels of mercury in the two limbs and hence the liquid column executes S.H.M. which can be expressed as:

Consider the mercury contained in a vertical U-tube upto the level P and Q in its two limbs.

Let P = density of the mercury.

L = Total length of the mercury column in both the limbs.

A = internal cross-sectional area of U-tube. m = mass of mercury in U-tube = LAP.

Assume, the mercury be depressed in left limb to F by a small distance y, then it rises by the same amount in the right limb to position Q'.

.'. Difference in levels in the two limbs = P' Q' = 2y.

:. Volume of mercury contained in the column of length 2y = A X 2y

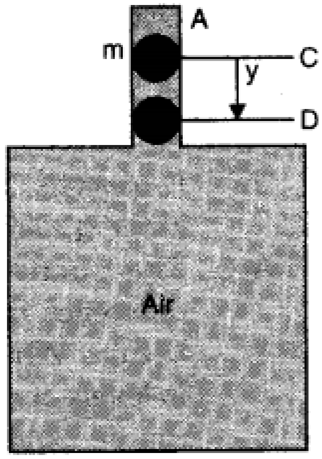
.•. m – A x 2y x ρ.

If W = weight of liquid contained in the column of length 2y.

Then W = mg = A x 2y x ρ x g

This weight produces the restoring force (F) which tends to bring back the mercury to its equilibrium position.

**14. 20. An air chamber of volume V has a neck area of cross-section into which a ball of mass m just fits and can move up and down without any friction (Fig.). Shaw that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal.**




**Answer**

Consider an air chamber of volume V with a long neck of uniform area of cross-section A, and a frictionless ball of mass m fitted smoothly in the neck at position C, Fig. The pressure of air below the ball inside the chamber is equal to the atmospheric pressure.

Increase the pressure on the ball by a little amount p, so that the ball is depressed to position D, where CD = y.

There will be decrease in volume and hence increase in pressure of air inside the chamber. The decrease in volume of the air inside the chamber, $\Delta V = Ay$

$$\text{Volumetric strain} = \frac{\text{change in volume}}{\text{original volume}}$$

$$= \frac{\Delta V}{V} = \frac{Ay}{V}$$

∴ Bulk Modulus of elasticity $E$, will be

$$E = \frac{\text{stress (or increase in pressure)}}{\text{volumetric strain}}$$

$$= \frac{-p}{Ay/V} = \frac{-pV}{Ay}$$

Here, negative sign shows that the increase in pressure will decrease the volume of air in the chamber.

Now, $$p = \frac{-E\,Ay}{V}$$

Due to this excess pressure, the restoring force acting on the ball is

$$F = p \times A = \frac{-E\,Ay}{V} \cdot A = \frac{-E\,A^2}{V} y \quad \text{...}(i)$$

Since $F \propto y$ and negative sign shows that the force is directed towards equilibrium position. If the applied increased pressure is removed from the ball, the ball will start executing linear SHM in the neck of chamber with C as mean position.

In S.H.M., the restoring force,

$$F = -ky \quad \text{...}(ii)$$

Comparing ($i$) and ($ii$), we have

Spring factor, $k = EA^2/V$

Here, inertia factor = mass of ball = $m$.

$$\text{Period, } T = 2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$$

$$= 2\pi\sqrt{\frac{m}{EA^2/V}} = \frac{2\pi}{A}\sqrt{\frac{mV}{E}}$$

$$\therefore \text{ Frequency, } v = \frac{1}{T} = \frac{A}{2\pi}\sqrt{\frac{E}{mV}}$$

**Note.** If the ball oscillates in the neck of chamber under isothermal conditions, thru E = P = picture of air inside the chamber, when ball is at equilibrium position. If the ball oscillate in the neck of chamber under adiabatic conditions, then E = gP. where g = $C_p/C_v$.

**14. 21. You are riding an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg. g = 10 m/s².**

**Answer**

(*a*) Here, mass, M = 300 kg, displacement, $x$ = 15 cm = 0.15 m, g = 10 m/s$^2$. There are four spring systems. If $k$ is the spring constant of each spring, then total spring constant of all the four springs in parallel is

$$K_p = 4k \quad \therefore \quad M_g = k_p x = 4kx$$

$$\Rightarrow \qquad K = \frac{Mg}{4x} = \frac{3000 \times 10}{4 \times 0.15} = 5 \times 10^4 \text{ N.}$$

(*b*) For each spring system supporting 750 kg of weight,

$$t = 2\pi\sqrt{\frac{m}{k}} = 2 \times 3.14 \times \sqrt{\frac{750}{5 \times 10^4}} = 0.77 \text{ sec.}$$

$\therefore$ Using $\qquad x = x_0 e^{-\frac{bt}{2m}}$, we get

$$\frac{50}{100} x_0 = x_0 e^{-\frac{b \times 0.77}{2 \times 750}} \quad \text{or} \quad e^{\frac{0.77b}{1500}} = 2$$

Taking logarithm of both sides,

$$\frac{0.77b}{1500} = \ln 2 = 2.303 \log 2$$

$$\therefore \qquad b = \frac{1500}{0.77} \times 2.303 \times 0.3010$$

$$= 1350.4 \text{ kg s}^{-1}$$

**14. 22. Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.**

**Answer**

Let the particle executing SHM starts oscillating from its mean position. Then displacement equation is

$$x = A \sin \omega t$$

$\therefore$ Particle velocity, $v = A\omega \cos \omega t$

$\therefore$ Instantaneous K.E., $K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2 \cos^2 \omega t$

$\therefore$ Average value of K.E. over one complete cycle

$$K_{av} = \frac{1}{T}\int_0^T \frac{1}{2}mA^2\omega^2 \cos^2 \omega t\, dt = \frac{mA^2\omega^2}{2T}\int_0^T \cos^2 \omega t\, dt$$

$$= \frac{mA^2\omega^2}{2T}\int_0^T \frac{(1+\cos 2\omega t)}{2} dt$$

$$= \frac{mA^2\omega^2}{4T}\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{mA^2\omega^2}{4T}\left[(T-0) + \left(\frac{\sin 2\omega t - \sin 0}{2\omega}\right)\right]$$

$$= \frac{1}{4}mA^2w^2 \quad \text{...}(i)$$

Again instantaneous P.E., $U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2 \sin^2 \omega t$

$\therefore$ Average value of P.E. over one complete cycle

$$U_{av} = \frac{1}{T}\int_0^T \frac{1}{2}m\omega^2A^2 \sin^2 \omega t = \frac{m\omega^2A^2}{2T}\int_0^T \sin^2 \omega t\, dt$$

$$= \frac{m\omega^2A^2}{2T}\int_0^T \frac{(1-\cos 2\omega t)}{2} dt$$

$$= \frac{m\omega^2A^2}{4T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{m\omega^2A^2}{4T}\left[(T-0) - \frac{(\sin 2\omega t - \sin 0)}{2\omega}\right]$$

$$= \frac{1}{4}m\omega^2A^2 \quad \text{...}(ii)$$

Simple comparison of (*i*) and (*ii*), shows that

$$K_{av} = U_{av} = \frac{1}{4}m\omega^2A^2$$

**14. 23. A circular disc, of mass 10 kg, is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations of found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant) a is defined by the relation J = – αθ, where J is the restoring couple and θ the angle of twist).**

**Answer:**

$$T = 2\pi\sqrt{\frac{I}{\alpha}} \quad \text{or} \quad T^2 = \frac{4\pi^2 I}{\alpha}$$

or $$\alpha = \frac{4\pi^2 I}{T^2} \quad \text{or} \quad \alpha = \frac{4\pi^2}{T^2}\left(\frac{1}{2}MR^2\right)$$

or $$\alpha = \frac{2\pi^2 MR^2}{T^2}$$

or $$\alpha = \frac{2(3.14)^2 \times 10 \times (0.15)^2}{(1.5)^2} \text{ Nm rad}^{-1}$$

$$= \mathbf{1.97\ Nm\ rad^{-1}}.$$

**Question 14. 24. A body describes simple harmonic motion with an amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (a) 5 cm (b) 3 cm (c) 0 cm**

**Answer**

Here, $r = 5$ cm $= 0.05$ m; T = 0.2 s; $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi$ rad/s

When displacement is $y$, then

$$\text{acceleration, } A = -\omega^2 y$$

$$\text{velocity, } V = \omega\sqrt{r^2 - y^2}$$

Case (*a*) When $y = 5 \text{ cm} = 0.05 \text{ m}$

$$A = -(10\pi)^2 \times 0.05 = -5\pi^2 \text{ m/s}^2$$

$$V = 10\pi\sqrt{(0.05)^2 - (0.05)^2} = 0.$$

Case (*b*) When $y = 3 \text{ cm} = 0.03 \text{ m}$

$$A = -(10\pi)^2 \times 0.03 = -3\pi^2 \text{ m/s}^2$$

$$V = 10\pi\sqrt{(0.05)^2 - (0.03)^2} = 10\pi \times 0.04 = 0.4\pi \text{ m/s}$$

Case (c) When $y = 0, \quad A = -(10\pi)^2 \times 0 = 0$

$$V = 10\pi\sqrt{(0.05)^2 - 0^2} = 10\pi \times 0.05 = 0.5\pi \text{ m/s.}$$

**14. 25. A mass attached to a spring is free to oscillate, with angular velocity w, in a horizontal plane without friction or damping. It is pulled to a distance $x_o$ and pushed towards the centre with a velocity $v_o$ at time t = 0. Determine the amplitude of the resulting oscillations in terms of the parameters w, $x_o$ and $v_o$.**

**Answer**

$$x = \alpha \cos(\omega t + \theta)$$

$$v = \frac{dx}{dt} = -a\omega \sin(\omega t + \theta)$$

When $t = 0, \quad x = x_0 \quad \text{and} \quad \frac{dx}{dt} = -v_0$

$\therefore \quad x_0 = a \cos\theta \qquad$ ...(i)

and $\quad -v_0 = -a\,\omega \sin\theta \quad \text{or} \quad a \sin\theta = \frac{v_0}{\omega} \qquad$ ...(ii)

Squaring and adding (i) and (ii), we get

$$a^2(\cos^2\theta + \sin^2\theta) = {x_0}^2 + \frac{v_0^2}{\omega^2}$$

or

$$a = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}.$$

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