NCERT Solutions for 9th Class Maths : Chapter 8 Quadrilaterals

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NCERT Solutions for 9th Class Maths : Chapter 8 Quadrilaterals

Class 9: Maths Chapter 8 solutions. Complete Class 9 Maths Chapter 8 Notes.

NCERT Solutions for 9th Class Maths : Chapter 8 Quadrilaterals

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Exercise 8.1

1. The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the quadrilateral.

Answer

Let x be the common ratio between the angles.

Sum of the interior angles of the quadrilateral = 360°

Now,

$3x + 5x + 9x + 13x = 360°$

$\Rightarrow 30x = 360°$

$\Rightarrow x = 12°$

Angles of the quadrilateral are:

$3x = 3×12° = 36°$

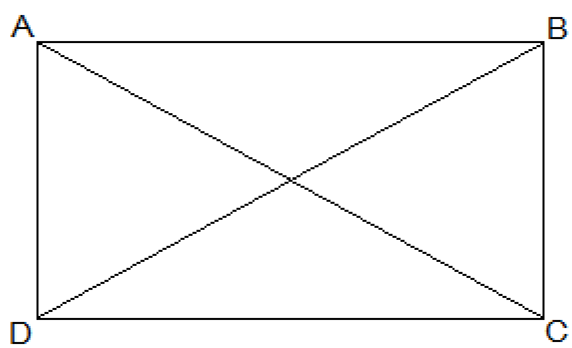
$5x = 5×12° = 60°$

$9x = 9×12° = 108°$

$13x = 13×12° = 156°$

2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Answer



Given,

AC = BD

To show,

To show ABCD is a rectangle we have to prove that one of its interior angle is right angled.

Proof,

In ΔABC and ΔBAD,

BC = BA (Common)

AC = AD (Opposite sides of a parallelogram are equal)

AC = BD (Given)

Therefore, ΔABC ≅ ΔBAD by SSS congruence condition.

$\angle A = \angle B$ (by CPCT)

also,

$\angle A + \angle B = 180°$ (Sum of the angles on the same side of the transversal)

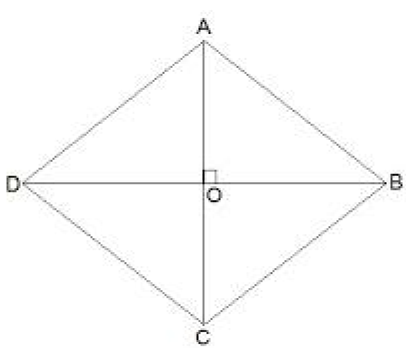
$\Rightarrow 2\angle A = 180°$

$\Rightarrow \angle A = 90° = \angle B$

Thus ABCD is a rectangle.

3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Answer



Let ABCD be a quadrilateral whose diagonals bisect each other at right angles.

Given,

OA = OC, OB = OD and ∠AOB = ∠BOC = ∠OCD = ∠ODA = 90°

To show,

ABCD is parallelogram and AB = BC = CD = AD

Proof,

In ΔAOB and ΔCOB,

OA = OC (Given)

∠AOB = ∠COB (Opposite sides of a parallelogram are equal)

OB = OB (Common)

Therefore, ΔAOB ≅ ΔCOB by SAS congruence condition.

Thus, AB = BC (by CPCT)

Similarly we can prove,

AB = BC = CD = AD

Opposites sides of a quadrilateral are equal hence ABCD is a parallelogram.

Thus, ABCD is rhombus as it is a parallelogram whose diagonals intersect at right angle.

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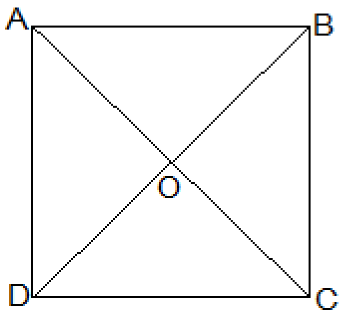
4. Show that the diagonals of a square are equal and bisect each other at right angles.

Answer



Let ABCD be a square and its diagonals AC and BD intersect each other at O.

To show,

AC = BD, AO = OC and ∠AOB = 90°

Proof,

In ΔABC and ΔBAD,

BC = BA (Common)

∠ABC = ∠BAD = 90°

AC = AD (Given)

Therefore, ΔABC ≅ ΔBAD by SAS congruence condition.

Thus, AC = BD by CPCT. Therefore, diagonals are equal.

Now,

In ΔAOB and ΔCOD,

∠BAO = ∠DCO (Alternate interior angles)

∠AOB = ∠COD (Vertically opposite)

AB = CD (Given)

Therefore, ΔAOB ≅ ΔCOD by AAS congruence condition.

Thus, AO = CO by CPCT. (Diagonal bisect each other.)

Now,

In ΔAOB and ΔCOB,

OB = OB (Given)

AO = CO (diagonals are bisected)

AB = CB (Sides of the square)

Therefore, ΔAOB ≅ ΔCOB by SSS congruence condition.

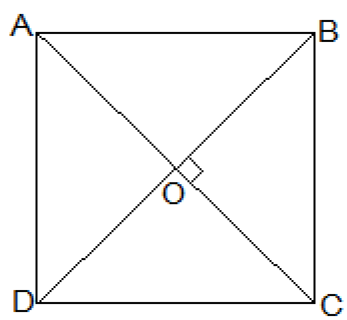
also, ∠AOB = ∠COB

∠AOB + ∠COB = 180° (Linear pair)

Thus, ∠AOB = ∠COB = 90° (Diagonals bisect each other at right angles)

5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Answer



Given,

Let ABCD be a quadrilateral in which diagonals AC and BD bisect each other at right angle at O.

To prove,

Quadrilateral ABCD is a square.

Proof,

In ΔAOB and ΔCOD,

AO = CO (Diagonals bisect each other)

$\angle AOB = \angle COD$ (Vertically opposite)

OB = OD (Diagonals bisect each other)

Therefore, $\Delta AOB \cong \Delta COD$ by SAS congruence condition.

Thus, AB = CD by CPCT. --- (i)

also,

$\angle OAB = \angle OCD$ (Alternate interior angles)

$\Rightarrow AB \parallel CD$

Now,

In ΔAOD and ΔCOD,

AO = CO (Diagonals bisect each other)

$\angle AOD = \angle COD$ (Vertically opposite)

OD = OD (Common)

Therefore, $\Delta AOD \cong \Delta COD$ by SAS congruence condition.

Thus, AD = CD by CPCT. --- (ii)

also,

AD = BC and AD = CD

$\Rightarrow AD = BC = CD = AB$ --- (ii)

also, $\angle ADC = \angle BCD$ by CPCT.

and $\angle ADC + \angle BCD = 180°$ (co-interior angles)

$\Rightarrow 2\angle ADC = 180°$

⇒ ∠ADC = 90° --- (iii)

One of the interior ang is right angle.

Thus, from (i), (ii) and (iii) given quadrilateral ABCD is a square.

6. Diagonal AC of a parallelogram ABCD bisects ∠A (see Fig. 8.19). Show that

(i) it bisects ∠C also,

(ii) ABCD is a rhombus.

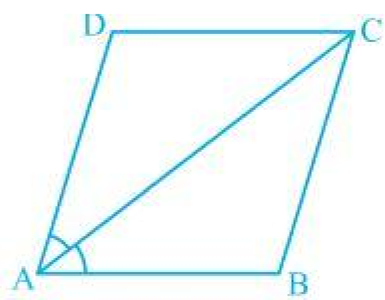


Fig. 8.19

Answer

(i)

In ΔADC and ΔCBA,

AD = CB (Opposite sides of a ||gm)

DC = BA (Opposite sides of a ||gm)

AC = CA (Common)

Therefore, ΔADC ≅ ΔCBA by SSS congruence condition.

Thus,

∠ACD = ∠CAB by CPCT

and ∠CAB = ∠CAD (Given)

⇒ ∠ACD = ∠BCA

Thus, AC bisects ∠C also.

(ii) ∠ACD = ∠CAD (Proved)

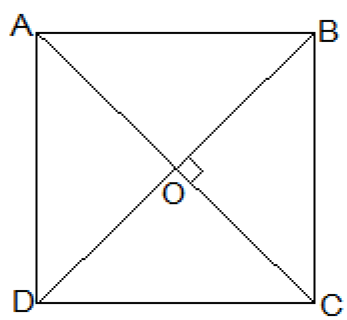
⇒ AD = CD (Opposite sides of equal angles of a triangle are equal)

Also, AB = BC = CD = DA (Opposite sides of a ||gm)

Thus, ABCD is a rhombus.

7. ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

Answer



Let ABCD is a rhombus and AC and BD are its diagonals.

Proof,

AD = CD (Sides of a rhombus)

∠DAC = ∠DCA (Angles opposite of equal sides of a triangle are equal.)

also, AB || CD

⇒ ∠DAC = ∠BCA (Alternate interior angles)

⇒ ∠DCA = ∠BCA

Therefore, AC bisects ∠C.

Similarly, we can prove that diagonal AC bisects ∠A.

Also, by preceding above method we can prove that diagonal BD bisects ∠B as well as ∠D.

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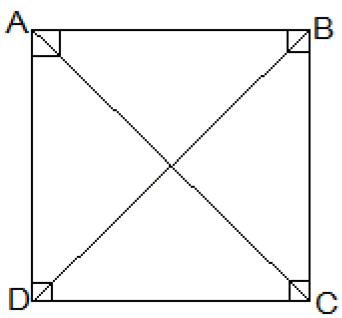
8. ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:

(i) ABCD is a square

(ii) diagonal BD bisects ∠B as well as ∠D.

Answer



(i)∠DAC = ∠DCA (AC bisects ∠A as well as ∠C)

⇒ AD = CD (Sides opposite to equal angles of a triangle are equal)

also, CD = AB (Opposite sides of a rectangle)

Therefore, AB = BC = CD = AD

Thus, ABCD is a square.

(ii) In ΔBCD,

BC = CD

⇒ ∠CDB = ∠CBD (Angles opposite to equal sides are equal)

also, ∠CDB = ∠ABD (Alternate interior angles)

⇒ ∠CBD = ∠ABD

Thus, BD bisects ∠B

Now,

∠CBD = ∠ADB

⇒ ∠CDB = ∠ADB

Thus, BD bisects ∠D

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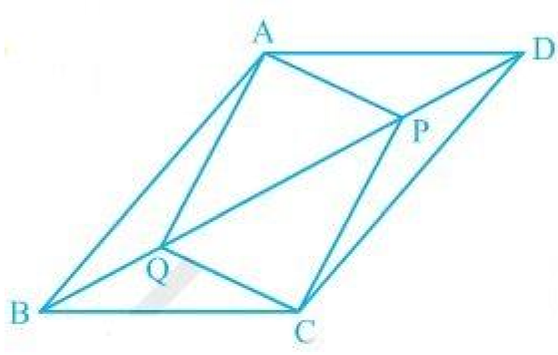

Fig. 8.20

9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig. 8.20). Show that:

(i) ΔAPD ≅ ΔCQB

(ii) AP = CQ

(iii) ΔAQB ≅ ΔCPD

(iv) AQ = CP

(v) APCQ is a parallelogram

Answer

(i) In ΔAPD and ΔCQB,

DP = BQ (Given)

∠ADP = ∠CBQ (Alternate interior angles)

AD = BC (Opposite sides of a ||gm)

Thus, ΔAPD ≅ ΔCQB by SAS congruence condition.

(ii) AP = CQ by CPCT as ΔAPD ≅ ΔCQB.

(iii) In ΔAQB and ΔCPD,

BQ = DP (Given)

∠ABQ = ∠CDP (Alternate interior angles)

AB = BCCD (Opposite sides of a ||gm)

Thus, ΔAQB ≅ ΔCPD by SAS congruence condition.

(iv) AQ = CP by CPCT as ΔAQB ≅ ΔCPD.

(v) From (ii) and (iv), it is clear that APCQ has equal opposite sides also it has equal opposite angles. Thus, APCQ is a ||gm.

10. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.21). Show that

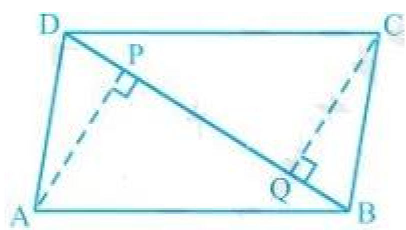

Fig. 8.21

(i) ΔAPB ≅ ΔCQD

(ii) AP = CQ

Answer

(i) In ΔAPB and ΔCQD,

∠ABP = ∠CDQ (Alternate interior angles)

∠APB = ∠CQD (equal to right angles as AP and CQ are perpendiculars)

AB = CD (ABCD is a parallelogram)

Thus, ΔAPB ≅ ΔCQD by AAS congruence condition.

(ii) AP = CQ by CPCT as ΔAPB ≅ ΔCQD.

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11. In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see Fig. 8.22).

Show that

(i) quadrilateral ABED is a parallelogram

(ii) quadrilateral BEFC is a parallelogram

(iii) AD || CF and AD = CF

(iv) quadrilateral ACFD is a parallelogram

(v) AC = DF

(vi) ΔABC ≅ ΔDEF.

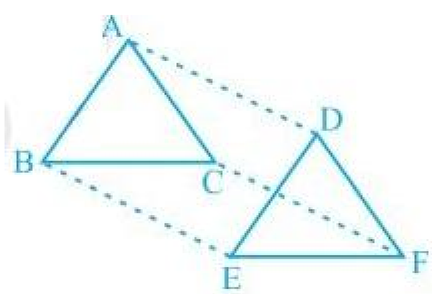

Fig. 8.22

Answer

(i) AB = DE and AB || DE (Given)

Thus, quadrilateral ABED is a parallelogram because two opposite sides of a quadrilateral are equal and parallel to each other.

(ii) Again BC = EF and BC || EF.

Thus, quadrilateral BEFC is a parallelogram.

(iii) Since ABED and BEFC are parallelograms.

⇒ AD = BE and BE = CF (Opposite sides of a parallelogram are equal)

Thus, AD = CF.

Also, AD || BE and BE || CF (Opposite sides of a parallelogram are parallel)

Thus, AD || CF

(iv) AD and CF are opposite sides of quadrilateral ACFD which are equal and parallel to each other. Thus, it is a parallelogram.

(v) AC || DF and AC = DF because ACFD is a parallelogram.

(vi) In ΔABC and ΔDEF,

AB = DE (Given)

BC = EF (Given)

AC = DF (Opposite sides of a parallelogram)

Thus, ΔABC ≅ ΔDEF by SSS congruence condition.

12. ABCD is a trapezium in which AB || CD and

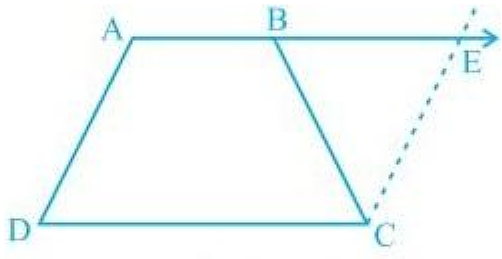

Fig. 8.23

AD = BC (see Fig. 8.23). Show that

(i) ∠A = ∠B

(ii) ∠C = ∠D

(iii) ΔABC ≅ ΔBAD

(iv) diagonal AC = diagonal BD[Hint : Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

Answer

Construction: Draw a line through C parallel to DA intersecting AB produced at E.

(i) CE = AD (Opposite sides of a parallelogram)

AD = BC (Given)

Therefor, BC = CE

⇒ ∠CBE = ∠CEB

also,

∠A +

∠CBE = 180° (Angles on the same side of transversal and ∠CBE = ∠CEB)

∠B + ∠CBE = 180° (Linear pair)

⇒ ∠A = ∠B

(ii) ∠A +

∠D = ∠B +

∠C = 180° (Angles on the same side of transversal)

⇒ ∠A + ∠D = ∠A + ∠C (∠A = ∠B)

⇒ ∠D = ∠C

(iii) In ΔABC and ΔBAD,

AB = AB (Common)

∠DBA = ∠CBA

AD = BC (Given)

Thus, ΔABC ≅ ΔBAD by SAS congruence condition.

(iv) Diagonal AC = diagonal BD by CPCT as ΔABC ≅ ΔBA.

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Exercise 8.2

1. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see Fig 8.29). AC is a diagonal. Show that :

(i) SR || AC and SR = 1/2 AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

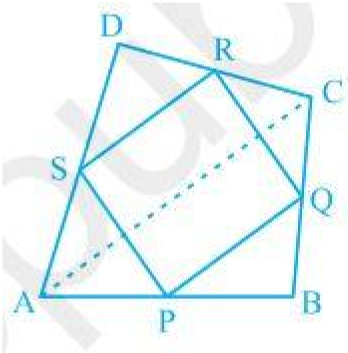


Fig. 8.29

Answer

(i) In ΔDAC,

R is the mid point of DC and S is the mid point of DA.

Thus by mid point theorem, SR || AC and SR = 1/2 AC

(ii) In ΔBAC,

P is the mid point of AB and Q is the mid point of BC.

Thus by mid point theorem, PQ || AC and PQ = 1/2 AC

also, SR = 1/2 AC

Thus, PQ = SR

(iii) SR || AC - from (i)

and, PQ || AC - from (ii)

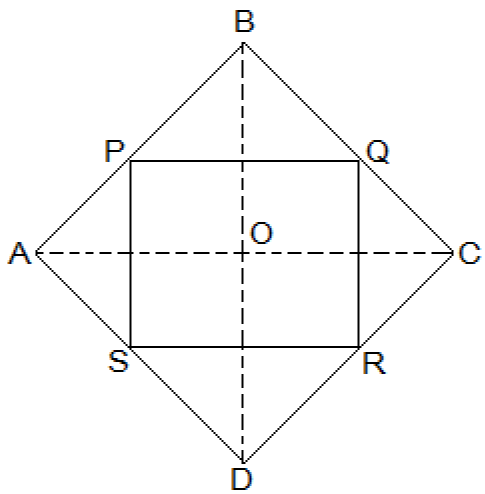
⇒ SR || PQ - from (i) and (ii)

also, PQ = SR

Thus, PQRS is a parallelogram.

2. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Answer



Given,

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.

To Prove,

PQRS is a rectangle.

Construction,

AC and BD are joined.

Proof,

In ΔDRS and ΔBPQ,

DS = BQ (Halves of the opposite sides of the rhombus)

∠SDR = ∠QBP (Opposite angles of the rhombus)

DR = BP (Halves of the opposite sides of the rhombus)

Thus, ΔDRS ≅ ΔBPQ by SAS congruence condition.

RS = PQ by CPCT --- (i)

In ΔQCR and ΔSAP,

RC = PA (Halves of the opposite sides of the rhombus)

∠RCQ = ∠PAS (Opposite angles of the rhombus)

CQ = AS (Halves of the opposite sides of the rhombus)

Thus, ΔQCR ≅ ΔSAP by SAS congruence condition.

RQ = SP by CPCT --- (ii)

Now,

In ΔCDB,

R and Q are the mid points of CD and BC respectively.

⇒ QR || BD

also,

P and S are the mid points of AD and AB respectively.

⇒ PS || BD

⇒ QR || PS

Thus, PQRS is a parallelogram.

also, ∠PQR = 90°

Now,

In PQRS,

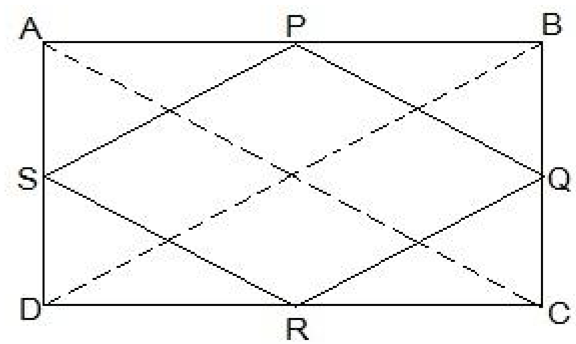
RS = PQ and RQ = SP from (i) and (ii)

∠Q = 90°

Thus, PQRS is a rectangle.

3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Answer



Given,

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively.

Construction,

AC and BD are joined.

To Prove,

PQRS is a rhombus.

Proof,

In ΔABC

P and Q are the mid-points of AB and BC respectively

Thus, PQ || AC and PQ = 1/2 AC (Mid point theorem) --- (i)

In ΔADC,

SR || AC and SR = 1/2 AC (Mid point theorem) --- (ii)

So, PQ || SR and PQ = SR

As in quadrilateral PQRS one pair of opposite sides is equal and parallel to each other, so, it is a parallelogram.

PS || QR and PS = QR (Opposite sides of parallelogram) --- (iii)

Now,

In ΔBCD,

Q and R are mid points of side BC and CD respectively.

Thus, QR || BD and QR = 1/2 BD (Mid point theorem) --- (iv)

AC = BD (Diagonals of a rectangle are equal) --- (v)

From equations (i), (ii), (iii), (iv) and (v),

PQ = QR = SR = PS

So, PQRS is a rhombus.

4. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig. 8.30). Show that F is the mid-point of BC.

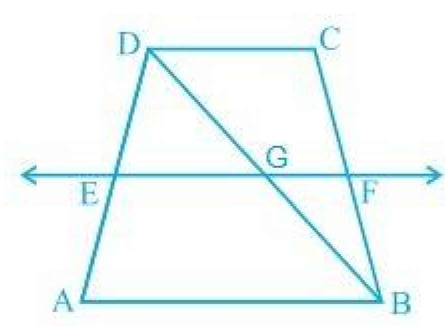


Fig. 8.30

Answer

Given,

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD.

To prove,

F is the mid-point of BC.

Proof,

BD intersected EF at G.

In ΔBAD,

E is the mid point of AD and also EG || AB.

Thus, G is the mid point of BD (Converse of mid point theorem)

Now,

In ΔBDC,

G is the mid point of BD and also GF || AB || DC.

Thus, F is the mid point of BC (Converse of mid point theorem)

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5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig. 8.31). Show that the line segments AF and EC trisect the diagonal BD.

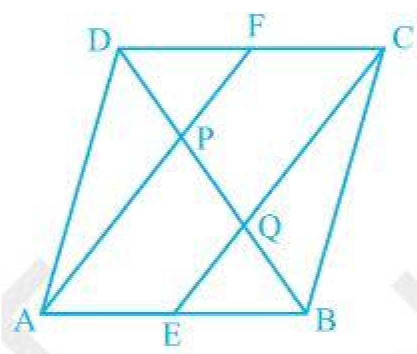


Fig. 8.31

Answer

Given,

ABCD is a parallelogram. E and F are the mid-points of sides AB and CD respectively.

To show,

AF and EC trisect the diagonal BD.

Proof,

ABCD is a parallelogram

Therefor, AB || CD

also, AE || FC

Now,

AB = CD (Opposite sides of parallelogram ABCD)

⇒ 1/2 AB = 1/2 CD

⇒ AE = FC (E and F are midpoints of side AB and CD)

AECF is a parallelogram (AE and CF are parallel and equal to each other)

AF || EC (Opposite sides of a parallelogram)

Now,

In ΔDQC,

F is mid point of side DC and FP || CQ (as AF || EC).

P is the mid-point of DQ (Converse of mid-point theorem)

⇒ DP = PQ --- (i)

Similarly,

In APB,

E is mid point of side AB and EQ || AP (as AF || EC).

Q is the mid-point of PB (Converse of mid-point theorem)

⇒ PQ = QB --- (ii)

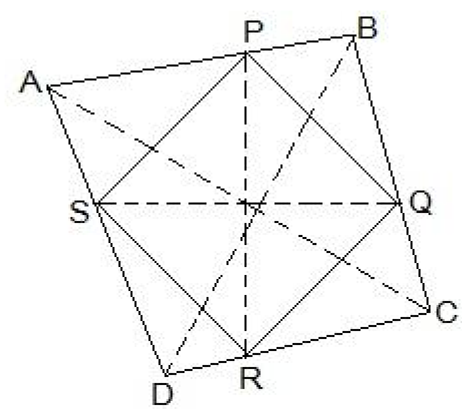
From equations (i) and (i),

DP = PQ = BQ

Hence, the line segments AF and EC trisect the diagonal BD.

6. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Answer



Let ABCD be a quadrilateral and P, Q, R and S are the mid points of AB, BC, CD and DA respectively.

Now,

In ΔACD,

R and S are the mid points of CD and DA respectively.

Thus, SR || AC.

Similarly we can show that,

PQ || AC

PS || BD

QR || BD

Thus, PQRS is parallelogram.

PR and QS are the diagonals of the parallelogram PQRS. So, they will bisect each other.

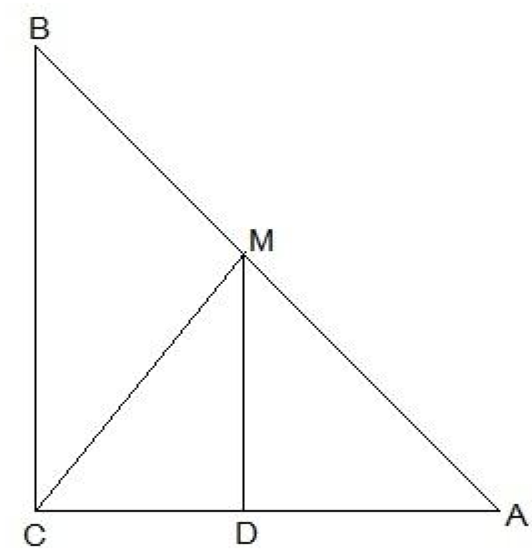
7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) MD ⊥ AC

(iii) CM = MA = 1/2 AB

Answer



(i) In ΔACB,

M is the mid point of AB and MD || BC

Thus, D is the mid point of AC (Converse of mid point theorem)

(ii) ∠ACB = ∠ADM (Corresponding angles)

also, ∠ACB = 90°

Thus, ∠ADM = 90° and MD ⊥ AC

(iii) In ΔAMD and ΔCMD,

AD = CD (D is the midpoint of side AC)

∠ADM = ∠CDM (Each 90°)

DM = DM (common)

Thus, ΔAMD ≅ ΔCMD by SAS congruence condition.

AM = CM by CPCT

also, AM = 1/2 AB (M is mid point of AB)

Hence, CM = MA = 1/2 AB

Chapter 8 Quadrilaterals NCERT Solutions is very important for the preparation of exams. A figure formed by joining four points in an order is called a quadrilateral. A quadrilateral has four sides, four angles and four vertices. In this chapter, we will be discussing different types of quadrilaterals, their properties and about parallelograms.

• Angle Sum Property of a Quadrilateral: The sum of the angles of a quadrilateral is 360º. This can be verified by drawing a diagonal and dividing the quadrilateral into two triangles.

• Types of Quadrilaterals:

A square is a rectangle and also a rhombus.

A parallelogram is a trapezium.

A kite is not a parallelogram.

A trapezium is not a parallelogram (as only one pair of opposite sides is parallel in a trapezium and we require both pairs to be parallel in a parallelogram).

A rectangle or a rhombus is not a square.

• Properties of a Parallelogram:

1. A diagonal of a parallelogram divides it into two congruent triangles.

2. In a parallelogram, opposite sides are equal.

3. If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.

4. In a parallelogram, opposite angles are equal.

5. If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.

6. The diagonals of a parallelogram bisect each other.

7. If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

• Another Condition for a Quadrilateral to be a Parallelogram: A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.

• The Mid-point Theorem:

The line segment joining the mid-points of two sides of a triangle is parallel to the third side.

The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

There are only two exercises in Chapter 8 Quadrilaterals NCERT Solutions which are provided below which can be helpful in completing your homework on time.

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NCERT 9th Maths Chapter 8, class 9 Maths Chapter 8 solutions

Chapterwise NCERT Solutions for Class 9 Maths :

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