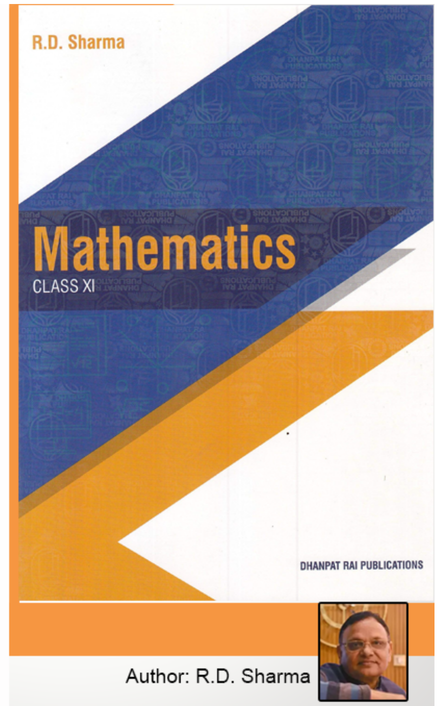
Class 11 - Chapter 11 Trigonometric Equations



RD Sharma Solutions for Class 11 Maths Chapter 11–Trigonometric Equations

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RD Sharma Solutions for Class 11 Maths Chapter 11–Trigonometric Equations

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1. Find the general solutions of the following equations:

(i) sin x = 1/2

(ii) cos x = – √3/2

(iii) cosec x = – √2

(iv) sec x = √2

(v) tan x = -1/√3

(vi) √3 sec x = 2

Solution:

The general solution of any trigonometric equation is given as:

sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

tan x = tan y, implies x = nπ + y, where n ∈ Z.

(i) sin x = 1/2

We know sin 30° = sin π/6 = ½

So,

Sin x = sin π/6

∴ the general solution is

x = nπ + $(-1)^n$ π/6, where n ∈ Z. [since, sin x = sin A => x = nπ + $(-1)^n$ A]

(ii) cos x = – √3/2

We know, cos 150° = (- √3/2) = cos 5π/6

So,

Cos x = cos 5π/6

∴ the general solution is

$x = 2n\pi \pm 5\pi/6$, where n ϵ Z.

(iii) cosec x = – √2

Let us simplify,

1/sin x = – √2 [since, cosec x = 1/sin x]

Sin x = -1/√2

= sin [π + π/4]

= sin 5π/4 or sin (-π/4)

∴ the general solution is

$x = n\pi + (-1)^{n+1}\,\pi/4$, where n ϵ Z.

(iv) sec x = √2

Let us simplify,

1/cos x = √2 [since, sec x = 1/cos x]

Cos x = 1/√2

= cos π/4

∴ the general solution is

$x = 2n\pi \pm \pi/4$, where n ϵ Z.

(v) tan x = -1/√3

Let us simplify,

tan x = -1/√3

tan x = tan (π/6)

= tan (-π/6) [since, tan (-x) = -tan x]

∴ the general solution is

x = nπ + (-π/6), where n ϵ Z.

or x = nπ – π/6, where n ϵ Z.

(vi) √3 sec x = 2

Let us simplify,

sec x = 2/√3

1/cos x = 2/√3

Cos x = √3/2

= cos (π/6)

∴ the general solution is

x = 2nπ ± π/6, where n ϵ Z.

2. Find the general solutions of the following equations:

(i) sin 2x = √3/2

(ii) cos 3x = 1/2

(iii) sin 9x = sin x

(iv) sin 2x = cos 3x

(v) tan x + cot 2x = 0

(vi) tan 3x = cot x

(vii) tan 2x tan x = 1

(viii) tan mx + cot nx = 0

(ix) tan px = cot qx

(x) sin 2x + cos x = 0

(xi) sin x = tan x

(xii) sin 3x + cos 2x = 0

Solution:

The general solution of any trigonometric equation is given as:

sin x = sin y, implies x = nπ + $(– 1)^n$ y, where n ∈ Z.

cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

tan x = tan y, implies x = nπ + y, where n ∈ Z.

(i) sin 2x = √3/2

Let us simplify,

sin 2x = √3/2

= sin (π/3)

∴ the general solution is

2x = nπ + $(-1)^n$ π/3, where n ϵ Z.

x = nπ/2 + $(-1)^n$ π/6, where n ϵ Z.

(ii) cos 3x = 1/2

Let us simplify,

cos 3x = 1/2

= cos (π/3)

∴ the general solution is

3x = 2nπ ± π/3, where n ϵ Z.

x = 2nπ/3 ± π/9, where n ϵ Z.

(iii) sin 9x = sin x

Let us simplify,

Sin 9x – sin x = 0

Using transformation formula,

Sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

So,

= 2 cos (9x+x)/2 sin (9x-x)/2

=> cos 5x sin 4x = 0

Cos 5x = 0 or sin 4x = 0

Let us verify both the expressions,

Cos 5x = 0

Cos 5x = cos π/2

5x = (2n + 1)π/2

x = (2n + 1)π/10, where n ϵ Z.

sin 4x = 0

sin 4x = sin 0

4x = nπ

x = nπ/4, where n ϵ Z.

∴ the general solution is

x = (2n + 1)π/10 or nπ/4, where n ϵ Z.

(iv) sin 2x = cos 3x

Let us simplify,

sin 2x = cos 3x

cos (π/2 – 2x) = cos 3x [since, sin A = cos (π/2 – A)]

π/2 – 2x = 2nπ ± 3x

π/2 – 2x = 2nπ + 3x [or] π/2 – 2x = 2nπ – 3x

5x = π/2 + 2nπ [or] x = 2nπ – π/2

5x = π/2 (1 + 4n) [or] x = π/2 (4n – 1)

x = π/10 (1 + 4n) [or] x = π/2 (4n – 1)

∴ the general solution is

x = π/10 (4n + 1) [or] x = π/2 (4n – 1), where n ϵ Z.

(v) tan x + cot 2x = 0

Let us simplify,

tan x = – cot 2x

tan x = – tan (π/2 – 2x) [since, cot A = tan (π/2 – A)]

tan x = tan (2x – π/2) [since, – tan A = tan -A]

x = nπ + 2x – π/2

x = nπ – π/2

∴ the general solution is

x = nπ – π/2, where n ϵ Z.

(vi) tan 3x = cot x

Let us simplify,

tan 3x = cot x

tan 3x = tan (π/2 – x) [since, cot A = tan (π/2 – A)]

3x = nπ + π/2 – x

$4x = n\pi + \pi/2$

$x = n\pi/4 + \pi/8$

∴ the general solution is

$x = n\pi/4 + \pi/8$, where $n \in Z$.

(vii) tan 2x tan x = 1

Let us simplify,

tan 2x tan x = 1

tan 2x = 1/tan x

= cot x

tan 2x = tan (π/2 – x) [since, cot A = tan (π/2 – A)]

$2x = n\pi + \pi/2 - x$

$3x = n\pi + \pi/2$

$x = n\pi/3 + \pi/6$

∴ the general solution is

$x = n\pi/3 + \pi/6$, where $n \in Z$.

(viii) tan mx + cot nx = 0

Let us simplify,

tan mx + cot nx = 0

tan mx = – cot nx

= – tan (π/2 – nx) [since, cot A = tan (π/2 – A)]

tan mx = tan (nx + π/2) [since, – tan A = tan -A]

$mx = k\pi + nx + \pi/2$

(m – n) x = kπ + π/2

(m – n) x = π (2k + 1)/2

x = π (2k + 1)/2(m – n)

∴ the general solution is

x = π (2k + 1)/2(m – n), where m, n, k ϵ Z.

(ix) tan px = cot qx

Let us simplify,

tan px = cot qx

tan px = tan (π/2 – qx) [since, cot A = tan (π/2 – A)]

px = nπ ± (π/2 – qx)

(p + q) x = nπ + π/2

x = nπ/(p+q) + π/2(p+q)

= π (2n +1)/ 2(p+q)

∴ the general solution is

x = π (2n +1)/ 2(p+q), where n ϵ Z.

(x) sin 2x + cos x = 0

Let us simplify,

sin 2x + cos x = 0

cos x = – sin 2x

cos x = – cos (π/2 – 2x) [since, sin A = cos (π/2 – A)]

= cos (π – (π/2 – 2x)) [since, -cos A = cos (π – A)]

= cos (π/2 + 2x)

x = 2nπ ± (π/2 + 2x)

So,

x = 2nπ + (π/2 + 2x) [or] x = 2nπ – (π/2 + 2x)

x = – π/2 – 2nπ [or] 3x = 2nπ – π/2

x = – π/2 (1 + 4n) [or] x = π/6 (4n – 1)

∴ the general solution is

x = – π/2 (1 + 4n), where n ϵ Z. [or] x = π/6 (4n – 1)

x = π/2 (4n – 1), where n ϵ Z. [or] x = π/6 (4n – 1), where n ϵ Z.

(xi) sin x = tan x

Let us simplify,

sin x = tan x

sin x = sin x/cos x

sin x cos x = sin x

sin x (cos x – 1) = 0

So,

Sin x = 0 or cos x – 1 = 0

Sin x = sin 0 [or] cos x = 1

Sin x = sin 0 [or] cos x = cos 0

x = nπ [or] x = 2mπ

∴ the general solution is

x = nπ [or] 2mπ, where n, m ϵ Z.

(xii) sin 3x + cos 2x = 0

Let us simplify,

$\sin 3x + \cos 2x = 0$

$\cos 2x = -\sin 3x$

$\cos 2x = -\cos(\pi/2 - 3x)$ [since, $\sin A = \cos(\pi/2 - A)$]

$\cos 2x = \cos(\pi - (\pi/2 - 3x))$ [since, $-\cos A = \cos(\pi - A)$]

$\cos 2x = \cos(\pi/2 + 3x)$

$2x = 2n\pi \pm (\pi/2 + 3x)$

So,

$2x = 2n\pi + (\pi/2 + 3x)$ [or] $2x = 2n\pi - (\pi/2 + 3x)$

$x = -\pi/2 - 2n\pi$ [or] $5x = 2n\pi - \pi/2$

$x = -\pi/2 (1 + 4n)$ [or] $x = \pi/10 (4n - 1)$

$x = -\pi/2 (4n + 1)$ [or] $\pi/10 (4n - 1)$

∴ the general solution is

$x = -\pi/2 (4n + 1)$ [or] $\pi/10 (4n - 1)$

$x = \pi/2 (4n - 1)$ [or] $\pi/10 (4n - 1)$, where $n \in Z$.

3. Solve the following equations:

(i) $\sin^2 x - \cos x = 1/4$

(ii) $2\cos^2 x - 5\cos x + 2 = 0$

(iii) $2\sin^2 x + \sqrt{3}\cos x + 1 = 0$

(iv) $4\sin^2 x - 8\cos x + 1 = 0$

(v) $\tan^2 x + (1 - \sqrt{3})\tan x - \sqrt{3} = 0$

(vi) $3\cos^2 x - 2\sqrt{3}\sin x \cos x - 3\sin^2 x = 0$

(vii) cos 4x = cos 2x

Solution:

The general solution of any trigonometric equation is given as:

sin x = sin y, implies x = nπ + (– 1)^n y, where n ∈ Z.

cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

tan x = tan y, implies x = nπ + y, where n ∈ Z.

(i) $\sin^2 x - \cos x = 1/4$

Let us simplify,

$\sin^2 x - \cos x = ¼$

$1 - \cos^2 x - \cos x = 1/4$ [since, $\sin^2 x = 1 - \cos^2 x$]

$4 - 4\cos^2 x - 4\cos x = 1$

$4\cos^2 x + 4\cos x - 3 = 0$

Let cos x be 'k'

So,

$4k^2 + 4k - 3 = 0$

$4k^2 - 2k + 6k - 3 = 0$

$2k(2k - 1) + 3(2k - 1) = 0$

$(2k - 1) + (2k + 3) = 0$

$(2k - 1) = 0$ or $(2k + 3) = 0$

k = 1/2 or k = -3/2

cos x = 1/2 or cos x = -3/2

we shall consider only cos x = 1/2. cos x = -3/2 is not possible.

so,

cos x = cos 60° = cos π/3

x = 2nπ ± π/3

∴ the general solution is

x = 2nπ ± π/3, where n ϵ Z.

(ii) $2 \cos^2 x – 5 \cos x + 2 = 0$

Let us simplify,

$2 \cos^2 x – 5 \cos x + 2 = 0$

Let cos x be ‘k’

$2k^2 – 5k + 2 = 0$

$2k^2 – 4k – k +2 = 0$

2k(k – 2) -1(k -2) = 0

(k – 2) (2k – 1) = 0

k = 2 or k = 1/2

cos x = 2 or cos x = 1/2

we shall consider only cos x = 1/2. cos x = 2 is not possible.

so,

cos x = cos 60° = cos π/3

x = 2nπ ± π/3

∴ the general solution is

x = 2nπ ± π/3, where n ϵ Z.

(iii) $2 \sin^2 x + \sqrt{3} \cos x + 1 = 0$

Let us simplify,

$2\sin^2 x + \sqrt{3}\cos x + 1 = 0$

$2(1 - \cos^2 x) + \sqrt{3}\cos x + 1 = 0$ [since, $\sin^2 x = 1 - \cos^2 x$]

$2 - 2\cos^2 x + \sqrt{3}\cos x + 1 = 0$

$2\cos^2 x - \sqrt{3}\cos x - 3 = 0$

Let cos x be ‘k’

$2k^2 - \sqrt{3}k - 3 = 0$

$2k^2 - 2\sqrt{3}k + \sqrt{3}k - 3 = 0$

$2k(k - \sqrt{3}) + \sqrt{3}(k - \sqrt{3}) = 0$

$(2k + \sqrt{3})(k - \sqrt{3}) = 0$

$k = \sqrt{3}$ or $k = -\sqrt{3}/2$

$\cos x = \sqrt{3}$ or $\cos x = -\sqrt{3}/2$

we shall consider only $\cos x = -\sqrt{3}/2$. $\cos x = \sqrt{3}$ is not possible.

so,

$\cos x = -\sqrt{3}/2$

$\cos x = \cos 150° = \cos 5\pi/6$

$x = 2n\pi \pm 5\pi/6$, where $n \in Z$.

(iv) $4\sin^2 x - 8\cos x + 1 = 0$

Let us simplify,

$4\sin^2 x - 8\cos x + 1 = 0$

$4(1 - \cos^2 x) - 8\cos x + 1 = 0$ [since, $\sin^2 x = 1 - \cos^2 x$]

$4 - 4\cos^2 x - 8\cos x + 1 = 0$

$4 \cos^2 x + 8 \cos x – 5 = 0$

Let cos x be ‘k’

$4k^2 + 8k – 5 = 0$

$4k^2 -2k + 10k – 5 = 0$

2k(2k – 1) + 5(2k – 1) = 0

(2k + 5) (2k – 1) = 0

k = -5/2 = -2.5 or k = 1/2

cos x = -2.5 or cos x = 1/2

we shall consider only cos x = 1/2. cos x = -2.5 is not possible.

so,

cos x = cos 60° = cos π/3

x = 2nπ ± π/3

∴ the general solution is

x = 2nπ ± π/3, where n ϵ Z.

(v) $\tan^2 x + (1 – \sqrt{3}) \tan x – \sqrt{3} = 0$

Let us simplify,

$\tan^2 x + (1 – \sqrt{3}) \tan x – \sqrt{3} = 0$

$\tan^2 x + \tan x – \sqrt{3} \tan x – \sqrt{3} = 0$

tan x (tan x + 1) – √3 (tan x + 1) = 0

(tan x + 1) (tan x – √3) = 0

tan x = -1 or tan x = √3

As, tan x ϵ (-∞ , ∞) so both values are valid and acceptable.

tan x = tan (-π/4) or tan x = tan (π/3)

x = mπ – π/4 or x = nπ + π/3

∴ the general solution is

x = mπ – π/4 or nπ + π/3, where m, n ϵ Z.

(vi) $3 \cos^2 x – 2\sqrt{3} \sin x \cos x – 3 \sin^2 x = 0$

Let us simplify,

$3 \cos^2 x – 2\sqrt{3} \sin x \cos x – 3 \sin^2 x = 0$

$3 \cos^2 x – 3\sqrt{3} \sin x \cos x + \sqrt{3} \sin x \cos x – 3 \sin^2 x = 0$

3 cos x (cos x – √3sin x) + √3 sin x (cos x – √3 sin x) = 0

√3 (cos x – √3 sin x) (√3 cos x + sin x) = 0

cos x – √3 sin x = 0 or sin x + √3 cos x = 0

cos x = √3 sin x or sin x = -√3 cos x

tan x = 1/√3 or tan x = -√3

As, tan x ϵ (-∞ , ∞) so both values are valid and acceptable.

tan x = tan (π/6) or tan x = tan (-π/3)

x = mπ + π/6 or x = nπ – π/3

∴ the general solution is

x = mπ + π/6 or nπ – π/3, where m, n ϵ Z.

(vii) cos 4x = cos 2x

Let us simplify,

cos 4x = cos 2x

4x = 2nπ ± 2x

So,

4x = 2nπ + 2x [or] 4x = 2nπ – 2x

2x = 2nπ [or] 6x = 2nπ

x = nπ [or] x = nπ/3

∴ the general solution is

x = nπ [or] nπ/3, where n ϵ Z.

4. Solve the following equations:

(i) cos x + cos 2x + cos 3x = 0

(ii) cos x + cos 3x – cos 2x = 0

(iii) sin x + sin 5x = sin 3x

(iv) cos x cos 2x cos 3x = 1/4

(v) cos x + sin x = cos 2x + sin 2x

(vi) sin x + sin 2x + sin 3x = 0

(vii) sin x + sin 2x + sin 3x + sin 4x = 0

(viii) sin 3x – sin x = 4 $\cos^2$ x – 2

(ix) sin 2x – sin 4x + sin 6x = 0

Solution:

The general solution of any trigonometric equation is given as:

sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.

cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

tan x = tan y, implies x = nπ + y, where n ∈ Z.

(i) cos x + cos 2x + cos 3x = 0

Let us simplify,

cos x + cos 2x + cos 3x = 0

we shall rearrange and use transformation formula

cos 2x + (cos x + cos 3x) = 0

by using the formula, cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

cos 2x + 2 cos (3x+x)/2 cos (3x-x)/2 = 0

cos 2x + 2cos 2x cos x = 0

cos 2x (1 + 2 cos x) = 0

cos 2x = 0 or 1 + 2cos x = 0

cos 2x = cos 0 or cos x = -1/2

cos 2x = cos π/2 or cos x = cos (π – π/3)

cos 2x = cos π/2 or cos x = cos (2π/3)

2x = (2n + 1) π/2 or x = 2mπ ± 2π/3

x = (2n + 1) π/4 or x = 2mπ ± 2π/3

∴ the general solution is

x = (2n + 1) π/4 or 2mπ ± 2π/3, where m, n ϵ Z.

(ii) cos x + cos 3x – cos 2x = 0

Let us simplify,

cos x + cos 3x – cos 2x = 0

we shall rearrange and use transformation formula

cos x – cos 2x + cos 3x = 0

– cos 2x + (cos x + cos 3x) = 0

By using the formula, cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2

– cos 2x + 2 cos (3x+x)/2 cos (3x-x)/2 = 0

– cos 2x + 2cos 2x cos x = 0

cos 2x (-1 + 2 cos x) = 0

cos 2x = 0 or -1 + 2cos x = 0

cos 2x = cos 0 or cos x = 1/2

cos 2x = cos π/2 or cos x = cos (π/3)

2x = (2n + 1) π/2 or x = 2mπ ± π/3

x = (2n + 1) π/4 or x = 2mπ ± π/3

∴ the general solution is

x = (2n + 1) π/4 or 2mπ ± π/3, where m, n ϵ Z.

(iii) sin x + sin 5x = sin 3x

Let us simplify,

sin x + sin 5x = sin 3x

sin x + sin 5x – sin 3x = 0

we shall rearrange and use transformation formula

– sin 3x + sin x + sin 5x = 0

– sin 3x + (sin 5x + sin x) = 0

By using the formula, sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

– sin 3x + 2 sin (5x+x)/2 cos (5x-x)/2 = 0

2sin 3x cos 2x – sin 3x = 0

sin 3x (2cos 2x – 1) = 0

sin 3x = 0 or 2cos 2x – 1 = 0

sin 3x = sin 0 or cos 2x = 1/2

sin 3x = sin 0 or cos 2x = cos π/3

3x = nπ or 2x = 2mπ ± π/3

x = nπ/3 or x = mπ ± π/6

∴ the general solution is

x = nπ/3 or mπ ± π/6, where m, n ϵ Z.

(iv) cos x cos 2x cos 3x = 1/4

Let us simplify,

cos x cos 2x cos 3x = 1/4

4 cos x cos 2x cos 3x – 1 = 0

By using the formula,

2 cos A cos B = cos (A + B) + cos (A – B)

2(2cos x cos 3x) cos 2x – 1 = 0

2(cos 4x + cos 2x) cos2x – 1 = 0

2($2\cos^2$ 2x – 1 + cos 2x) cos 2x – 1 = 0 [using cos 2A = $2\cos^2 A$ – 1]

$4\cos^3$ 2x – 2cos 2x + $2\cos^2$ 2x – 1 = 0

$2\cos^2$ 2x (2cos 2x + 1) -1(2cos 2x + 1) = 0

($2\cos^2$ 2x – 1) (2 cos 2x + 1) = 0

So,

2cos 2x + 1 = 0 or ($2\cos^2$ 2x – 1) = 0

cos 2x = -1/2 or cos 4x = 0 [using cos 2θ = $2\cos^2\theta$ – 1] cos 2x = cos (π – π/3) or cos 4x = cos π/2

cos 2x = cos 2π/3 or cos 4x = cos π/2

2x = 2mπ ± 2π/3 or 4x = (2n + 1) π/2

x = mπ ± π/3 or x = (2n + 1) π/8

∴ the general solution is

x = mπ ± π/3 or (2n + 1) π/8, where m, n ϵ Z.

(v) cos x + sin x = cos 2x + sin 2x

Let us simplify,

cos x + sin x = cos 2x + sin 2x

upon rearranging we get,

cos x – cos 2x = sin 2x – sin x

By using the formula,

sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

cos A – cos B = – 2 sin (A+B)/2 sin (A-B)/2

So,

-2 sin (2x+x)/2 sin (2x-x)/2 = 2 cos (2x+x)/2 sin (2x-x)/2

2 sin 3x/2 sin x/2 = 2 cos 3x/2 sin x/2

Sin x/2 (sin 3x/2 – cos 3x/2) = 0

So,

Sin x/2 = 0 or sin 3x/2 = cos 3x/2

Sin x/2 = sin mπ or sin 3x/2 / cos 3x/2 = 0

Sin x/2 = sin mπ or tan 3x/2 = 1

Sin x/2 = sin mπ or tan 3x/2 = tan π/4

x/2 = mπ or 3x/2 = nπ + π/4

x = 2mπ or x = 2nπ/3 + π/6

∴ the general solution is

x = 2mπ or 2nπ/3 + π/6, where m, n ϵ Z.

(vi) sin x + sin 2x + sin 3x = 0

Let us simplify,

sin x + sin 2x + sin 3x = 0

we shall rearrange and use transformation formula

sin 2x + sin x + sin 3x = 0

By using the formula,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

So,

Sin 2x + 2 sin (3x+x)/2 cos (3x-x)/2 = 0

Sin 2x + 2sin 2x cos x = 0

Sin 2x (2 cos x + 1) = 0

Sin 2x = 0 or 2cos x + 1 = 0

Sin 2x = sin 0 or cos x = -1/2

Sin 2x = sin 0 or cos x = cos (π – π/3)

Sin 2x = sin 0 or cos x = cos 2π/3

2x = nπ or x = 2mπ ± 2π/3

x = nπ/2 or x = 2mπ ± 2π/3

∴ the general solution is

x = nπ/2 or 2mπ ± 2π/3, where m, n ϵ Z.

(vii) sin x + sin 2x + sin 3x + sin 4x = 0

Let us simplify,

sin x + sin 2x + sin 3x + sin 4x = 0

we shall rearrange and use transformation formula

sin x + sin 3x + sin 2x + sin 4x = 0

By using the formula,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

So,

2 sin (3x+x)/2 cos (3x-x)/2 + 2 sin (4x+2x)/2 cos (4x-2x)/2 = 0

2 sin 2x cos x + 2 sin 3x cos x = 0

2cos x (sin 2x + sin 3x) = 0

Again by using the formula,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

we get,

2cos x (2 sin (3x+2x)/2 cos (3x-2x)/2) = 0

2cos x (2 sin 5x/2 cos x/2) = 0

4 cos x sin 5x/2 cos x/2 = 0

So,

Cos x = 0 or sin 5x/2 = 0 or cos x/2 = 0

Cos x = cos 0 or sin 5x/2 = sin 0 or cos x/2 = cos 0

Cos x = cos π/2 or sin 5x/2 = kπ or cos x/2 = cos (2p + 1) π/2

x = (2n + 1) π/2 or 5x/2 = kπ or x/2 = (2p + 1) π/2

x = (2n + 1) π/2 or x = 2kπ/5 or x = (2p + 1)

x = nπ + π/2 or x = 2kπ/5 or x = (2p + 1)

∴ the general solution is

x = nπ + π/2 or x = 2kπ/5 or x = (2p + 1), where n, k, p ϵ Z.

(viii) sin 3x – sin x = 4 $\cos^2$ x – 2

Let us simplify,

sin 3x – sin x = 4 $\cos^2$ x – 2

sin 3x – sin x = 2(2 $\cos^2$ x – 1)

sin 3x – sin x = 2 cos 2x [since, cos 2A = $2\cos^2$ A – 1]

By using the formula,

Sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2

So,

2 cos (3x+x)/2 sin (3x-x)/2 = 2 cos 2x

2 cos 2x sin x – 2 cos 2x = 0

2 cos 2x (sin x – 1) = 0

Then,

2 cos 2x = 0 or sin x – 1 = 0

Cos 2x = 0 or sin x = 1

Cos 2x = cos 0 or sin x = sin 1

Cos 2x = cos 0 or sin x = sin π/2

$2x = (2n + 1)\pi/2$ or $x = m\pi + (-1)^m \pi/2$

$x = (2n + 1)\pi/4$ or $x = m\pi + (-1)^m \pi/2$

∴ the general solution is

$x = (2n + 1)\pi/4$ or $m\pi + (-1)^m \pi/2$, where m, n ϵ Z.

(ix) sin 2x – sin 4x + sin 6x = 0

Let us simplify,

sin 2x – sin 4x + sin 6x = 0

we shall rearrange and use transformation formula

– sin 4x + sin 6x + sin 2x = 0

By using the formula,

sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2

we get,

– sin 4x + 2 sin (6x+2x)/2 cos (6x-2x)/2 = 0

– sin 4x + 2 sin 4x cos 2x = 0

Sin 4x (2 cos 2x – 1) = 0

So,

Sin 4x = 0 or 2 cos 2x – 1 = 0

Sin 4x = sin 0 or cos 2x = 1/2

Sin 4x = sin 0 or cos 2x = π/3

4x = nπ or 2x = 2mπ ± π/3

x = nπ/4 or x = mπ ± π/6

∴ the general solution is

$x = n\pi/4$ or $m\pi \pm \pi/6$, where m, n ϵ Z.

5. Solve the following equations:

(i) tan x + tan 2x + tan 3x = 0

(ii) tan x + tan 2x = tan 3x

(iii) tan 3x + tan x = 2 tan 2x

Solution:

The general solution of any trigonometric equation is given as:

sin x = sin y, implies $x = n\pi + (-1)^n y$, where n ∈ Z.

cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.

tan x = tan y, implies x = nπ + y, where n ∈ Z.

(i) tan x + tan 2x + tan 3x = 0

Let us simplify,

tan x + tan 2x + tan 3x = 0

tan x + tan 2x + tan (x + 2x) = 0

By using the formula,

tan (A+B) = [tan A + tan B] / [1 – tan A tan B]

So,

tan x + tan 2x + [[tan x + tan 2x]/[1- tan x tan 2x]] = 0

(tan x + tan 2x) (1 + 1/(1- tan x tan 2x)) = 0

(tan x + tan 2x) ([2 – tan x tan 2x] / [1 – tan x tan 2x]) = 0

Then,

(tan x + tan 2x) = 0 or ([2 – tan x tan 2x] / [1 – tan x tan 2x]) = 0 (tan x + tan 2x) = 0 or [2 – tan x tan 2x] = 0

tan x = tan (-2x) or tan x tan 2x = 2

x = nπ + (-2x) or tax x [2tan x/(1 – tan^2 x)] = 2 [Using, tan 2x = 2 tan x / 1-tan^2 x]

3x = nπ or 2 tan^2 x / (1-tan^2 x) = 2

3x = nπ or 2 tan^2 x = 2(1 – tan^2 x)

3x = nπ or 2 tan^2 x = 2 – 2tan^2 x

3x = nπ or 4 tan^2 x = 2

x = nπ/3 or tan^2 x = 2/4

x = nπ/3 or tan^2 x = 1/2

x = nπ/3 or tan x = 1/√2

x = nπ/3 or x = tan α [let 1/√2 be 'α']

x = nπ/3 or x = mπ + α

∴ the general solution is

x = nπ/3 or mπ + α, where α = tan^{-1}1/√2, m, n ∈ Z.

(ii) tan x + tan 2x = tan 3x

Let us simplify,

tan x + tan 2x = tan 3x

tan x + tan 2x – tan 3x = 0

tan x + tan 2x – tan (x + 2x) = 0

By using the formula,

tan (A+B) = [tan A + tan B] / [1 – tan A tan B]

So,

tan x + tan 2x – [[tan x + tan 2x]/[1- tan x tan 2x]] = 0

(tan x + tan 2x) (1 – 1/(1- tan x tan 2x)) = 0

(tan x + tan 2x) ([– tan x tan 2x] / [1 – tan x tan 2x]) = 0

Then,

(tan x + tan 2x) = 0 or ([– tan x tan 2x] / [1 – tan x tan 2x]) = 0

(tan x + tan 2x) = 0 or [– tan x tan 2x] = 0

tan x = tan (-2x) or -tan x tan 2x = 0

tan x = tan (-2x) or $2\tan^2 x / (1 - \tan^2 x) = 0$ [Using, $\tan 2x = 2 \tan x / 1-\tan^2 x$]

x = nπ + (-2x) or x = mπ + 0

3x = nπ or x = mπ

x = nπ/3 or x = mπ

∴ the general solution is

x = nπ/3 or mπ, where m, n ∈ Z.

(iii) tan 3x + tan x = 2 tan 2x

Let us simplify,

tan 3x + tan x = 2 tan 2x

tan 3x + tan x = tan 2x + tan 2x

upon rearranging we get,

tan 3x – tan 2x = tan 2x – tan x

By using the formula,

tan (A-B) = [tan A – tan B] / [1 + tan A tan B]

so,[(tan 3x – tan 2x) (1+tan 3x tan 2x)] / [1 + tan 3x tan 2x] = [(tan 2x-tan x) (1+tan x tan 2x)] / [1 + tan 2x tan x]

$\tan(3x - 2x)(1 + \tan 3x \tan 2x) = \tan(2x - x)(1 + \tan x \tan 2x)$

$\tan x [1 + \tan 3x \tan 2x - 1 - \tan 2x \tan x] = 0$

$\tan x \tan 2x (\tan 3x - \tan x) = 0$

so,

$\tan x = 0$ or $\tan 2x = 0$ or $(\tan 3x - \tan x) = 0$

$\tan x = 0$ or $\tan 2x = 0$ or $\tan 3x = \tan x$

$x = n\pi$ or $2x = m\pi$ or $3x = k\pi + x$

$x = n\pi$ or $x = m\pi/2$ or $2x = k\pi$

$x = n\pi$ or $x = m\pi/2$ or $x = k\pi/2$

∴ the general solution is

$x = n\pi$ or $m\pi/2$ or $k\pi/2$, where, $m, n, k \in Z$.

Chapterwise RD Sharma Solutions for Class 11 Maths :

- Chapter 1–Sets
- Chapter 2–Relations
- Chapter 3–Functions
- Chapter 4–Measurement of Angles
- Chapter 5–Trigonometric Functions
- Chapter 6–Graphs of Trigonometric Functions
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About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.