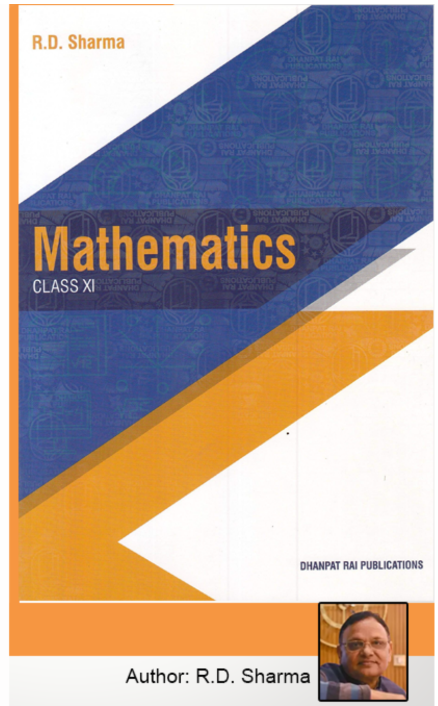
Class 11 - Chapter 12 Mathematical Induction



RD Sharma Solutions for Class 11 Maths Chapter 12–Mathematical Induction

Class 11: Maths Chapter 12 solutions. Complete Class 11 Maths Chapter 12 Notes.

RD Sharma Solutions for Class 11 Maths Chapter 12–Mathematical Induction

RD Sharma 11th Maths Chapter 12, Class 11 Maths Chapter 12 solutions

https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-12-mathematical-induction/

EXERCISE 12.1 PAGE NO: 12.3

1. If P (n) is the statement "n (n + 1) is even", then what is P (3)?

Solution:

Given:

P (n) = n (n + 1) is even.

So,

P (3) = 3 (3 + 1)

= 3 (4)

= 12

Hence, P (3) = 12, P (3) is also even.

2. If P (n) is the statement "n^3 + n is divisible by 3", prove that P (3) is true but P (4) is not true.

Solution:

Given:

P (n) = n^3 + n is divisible by 3

We have P (n) = n^3 + n

So,

P (3) = 3^3 + 3

= 27 + 3

= 30

P (3) = 30, So it is divisible by 3

Now, let's check with P (4)

P (4) = 4^3 + 4

= 64 + 4

= 68

P (4) = 68, so it is not divisible by 3

Hence, P (3) is true and P (4) is not true.

3. If P (n) is the statement "$2^n \geq 3n$", and if P (r) is true, prove that P (r + 1) is true.

Solution:

Given:

P (n) = "$2^n \geq 3n$" and p(r) is true.

We have, P (n) = $2^n \geq 3n$

Since, P (r) is true

So,

$2^r \geq 3r$

Now, let's multiply both sides by 2

$2 \times 2^r \geq 3r \times 2$

$2^{r+1} \geq 6r$

$2^{r+1} \geq 3r + 3r$ [since $3r>3 = 3r + 3r \geq 3 + 3r$]

$\therefore 2^{r+1} \geq 3(r + 1)$

Hence, P (r + 1) is true.

4. If P (n) is the statement "$n^2 + n$" is even", and if P (r) is true, then P (r + 1) is true

Solution:

Given:

P (n) = $n^2 + n$ is even and P (r) is true, then $r^2 + r$ is even

Let us consider $r^2 + r = 2k$ … (i)

Now, $(r + 1)^2 + (r + 1)$

$r^2 + 1 + 2r + r + 1$

$(r^2 + r) + 2r + 2$

$2k + 2r + 2$ [from equation (i)]

$2(k + r + 1)$

2μ

∴ $(r + 1)^2 + (r + 1)$ is Even.

Hence, P (r + 1) is true.

5. Given an example of a statement P (n) such that it is true for all n ϵ N.

Solution:

Let us consider

P (n) = 1 + 2 + 3 + – – – – – – + n = n(n+1)/2

So,

P (n) is true for all natural numbers.

Hence, P (n) is true for all n ∈ N.

6. If P (n) is the statement "$n^2 – n + 41$ is prime", prove that P (1), P (2) and P (3) are true. Prove also that P (41) is not true.

Solution:

Given:

P(n) = $n^2 – n + 41$ is prime.

$P(n) = n^2 – n + 41$

P (1) = 1 – 1 + 41

= 41

P (1) is Prime.

Similarly,

$P(2) = 2^2 – 2 + 41$

= 4 – 2 + 41

= 43

P (2) is prime.

Similarly,

$P (3) = 3^2 – 3 + 41$

= 9 – 3 + 41

= 47

P (3) is prime

Now,

$P (41) = (41)^2 – 41 + 41$

= 1681

P (41) is not prime

Hence, P (1), P(2), P (3) are true but P (41) is not true.

EXERCISE 12.2 PAGE NO: 12.27

Prove the following by the principle of mathematical induction:

1. 1 + 2 + 3 + … + n = n (n +1)/2 i.e., the sum of the first n natural numbers is n (n + 1)/2.

Solution:

Let us consider P (n) = 1 + 2 + 3 + ….. + n = n (n +1)/2

For, n = 1

LHS of P (n) = 1

RHS of P (n) = 1 (1+1)/2 = 1

So, LHS = RHS

Since, P (n) is true for n = 1

Let us consider P (n) be the true for n = k, so

1 + 2 + 3 + …. + k = k (k+1)/2 … (i)

Now,

(1 + 2 + 3 + … + k) + (k + 1) = k (k+1)/2 + (k+1)

= (k + 1) (k/2 + 1)

= [(k + 1) (k + 2)] / 2

= [(k+1) [(k+1) + 1]] / 2

P (n) is true for n = k + 1

P (n) is true for all n ∈ N

So, by the principle of Mathematical Induction

Hence, P (n) = 1 + 2 + 3 + ….. + n = n (n +1)/2 is true for all n ∈ N.

2. $1^2 + 2^2 + 3^2 + … + n^2$ = [n (n+1) (2n+1)]/6

Solution:

Let us consider P (n) = $1^2 + 2^2 + 3^2 + … + n^2$ = [n (n+1) (2n+1)]/6

For, n = 1

P (1) = [1 (1+1) (2+1)]/6

1 = 1

P (n) is true for n = 1

Let P (n) is true for n = k, so

P (k): $1^2 + 2^2 + 3^2 + \ldots + k^2 = [k (k+1) (2k+1)]/6$

Let's check for P (n) = k + 1, so

P (k) = $1^2 + 2^2 + 3^2 + ------ + k^2 + (k + 1)^2 = [k + 1 (k+2) (2k+3)] /6$

$= 1^2 + 2^2 + 3^2 + ------ + k^2 + (k + 1)^2$

$= [k + 1 (k+2) (2k+3)] /6 + (k + 1)^2$

$= (k +1) [(2k^2 + k)/6 + (k + 1)/1]$

$= (k +1) [2k^2 + k + 6k + 6]/6$

$= (k +1) [2k^2 + 7k + 6]/6$

$= (k +1) [2k^2 + 4k + 3k + 6]/6$

$= (k +1) [2k(k + 2) + 3(k + 2)]/6$

$= [(k +1) (2k + 3) (k + 2)] / 6$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

3. $1 + 3 + 3^2 + \ldots + 3^{n-1} = (3^n – 1)/2$

Solution:

Let P (n) = $1 + 3 + 3^2 + ----- + 3^{n-1} = (3^n – 1)/2$

Now, For n = 1

P (1) = $1 = (3^1 – 1)/2 = 2/2 = 1$

P (n) is true for n = 1

Now, let's check for P (n) is true for n = k

P (k) = 1 + 3 + 3^2 + – – – – + 3^{k-1} = (3^k – 1)/2 … (i)

Now, we have to show P (n) is true for n = k + 1

P (k + 1) = 1 + 3 + 3^2 + – – – – + 3^k = (3^{k+1} – 1)/2

Then, {1 + 3 + 3^2 + – – – – + 3^{k-1}} + 3^{k+1-1}

= (3k – 1)/2 + 3^k using equation (i)

= (3k – 1 + 2×3^k)/2

= (3×3 k – 1)/2

= (3^{k+1} – 1)/2

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

4. 1/1.2 + 1/2.3 + 1/3.4 + … + 1/n(n+1) = n/(n+1)

Solution:

Let P (n) = 1/1.2 + 1/2.3 + 1/3.4 + … + 1/n(n+1) = n/(n+1)

For, n = 1

P (n) = 1/1.2 = 1/1+1

1/2 = 1/2

P (n) is true for n = 1

Let's check for P (n) is true for n = k,

1/1.2 + 1/2.3 + 1/3.4 + … + 1/k(k+1) + k/(k+1) (k+2) = (k+1)/(k+2)

Then,

1/1.2 + 1/2.3 + 1/3.4 + … + 1/k(k+1) + k/(k+1) (k+2)

= 1/(k+1)/(k+2) + k/(k+1)

= 1/(k+1) [k(k+2)+1]/(k+2)

= 1/(k+1) [k^2 + 2k + 1]/(k+2)

=1/(k+1) [(k+1) (k+1)]/(k+2)

= (k+1) / (k+2)

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

5. 1 + 3 + 5 + … + (2n – 1) = n^2 i.e., the sum of first n odd natural numbers is n^2.

Solution:

Let P (n): 1 + 3 + 5 + … + (2n – 1) = n^2

Let us check P (n) is true for n = 1

P (1) = 1 =1^2

1 = 1

P (n) is true for n = 1

Now, Let's check P (n) is true for n = k

P (k) = 1 + 3 + 5 + … + (2k – 1) = k^2 … (i)

We have to show that

1 + 3 + 5 + … + (2k – 1) + 2(k + 1) – 1 = $(k + 1)^2$

Now,

1 + 3 + 5 + … + (2k – 1) + 2(k + 1) – 1

= k^2 + (2k + 1)

$= k^2 + 2k + 1$

$= (k + 1)^2$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

6. 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3n-1) (3n+2) = n/(6n+4)

Solution:

Let P (n) = 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3n-1) (3n+2) = n/(6n+4)

Let us check P (n) is true for n = 1

P (1): 1/2.5 = 1/6.1+4 => 1/10 = 1/10

P (1) is true.

Now,

Let us check for P (k) is true, and have to prove that P (k + 1) is true.

P (k): 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3k-1) (3k+2) = k/(6k+4)

P (k +1): 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3k-1)(3k+2) + 1/(3k+3-1)(3k+3+2)

: k/(6k+4) + 1/(3k+2)(3k+5)

: [k(3k+5)+2] / [2(3k+2)(3k+5)]

: (k+1) / (6(k+1)+4)

P (k + 1) is true.

Hence proved by mathematical induction.

7. 1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3n-2)(3n+1) = n/3n+1

Solution:

Let P (n) = 1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3n-2)(3n+1) = n/3n+1

Let us check for n = 1,

P (1): 1/1.4 = 1/4

1/4 = 1/4

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k) = 1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3k-2)(3k+1) = k/3k+1 … (i)

So,[1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3k-2)(3k+1)]+ 1/(3k+1)(3k+4)

= k/(3k+1) + 1/(3k+1)(3k+4)

= 1/(3k+1) [k/1 + 1/(3k+4)]

= 1/(3k+1) [k(3k+4)+1]/(3k+4)

= 1/(3k+1) [$3k^2$ + 4k + 1]/ (3k+4)

= 1/(3k+1) [$3k^2$ + 3k+k+1]/(3k+4)

= [3k(k+1) + (k+1)] / [(3k+4) (3k+1)]

= [(3k+1)(k+1)] / [(3k+4) (3k+1)]

= (k+1) / (3k+4)

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

8. 1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2n+1)(2n+3) = n/3(2n+3)

Solution:

Let P (n) = 1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2n+1)(2n+3) = n/3(2n+3)

Let us check for n = 1,

P (1): 1/3.5 = 1/3(2.1+3)

: 1/15 = 1/15

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k) = 1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2k+1)(2k+3) = k/3(2k+3) … (i)

So,

1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2k+1)(2k+3) + 1/[2(k+1)+1][2(k+1)+3]

1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2k+1)(2k+3) + 1/(2k+3)(2k+5)

Now substituting the value of P (k) we get,

= k/3(2k+3) + 1/(2k+3)(2k+5)

= [k(2k+5)+3] / [3(2k+3)(2k+5)]

= (k+1) / [3(2(k+1)+3)]

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

9. 1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4n-1)(4n+3) = n/3(4n+3)

Solution:

Let P (n) = 1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4n-1)(4n+3) = n/3(4n+3)

Let us check for n = 1,

P (1): 1/3.7 = 1/(4.1-1)(4+3)

: 1/21 = 1/21

P (n) is true for n =1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4k-1)(4k+3) = k/3(4k+3) …. (i)

So,

1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4k-1)(4k+3) + 1/(4k+3)(4k+7)

Substituting the value of P (k) we get,

= k/(4k+3) + 1/(4k+3)(4k+7)

= 1/(4k+3) [k(4k+7)+3] / [3(4k+7)]

= 1/(4k+3) [4k^2 + 7k +3]/ [3(4k+7)]

= 1/(4k+3) [4k^2 + 3k+4k+3] / [3(4k+7)]

= 1/(4k+3) [4k(k+1)+3(k+1)]/ [3(4k+7)]

= 1/(4k+3) [(4k+3)(k+1)] / [3(4k+7)]

= (k+1) / [3(4k+7)]

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

10. 1.2 + 2.2^2 + 3.2^3 + … + n.2^n = (n–1) 2^{n+1} + 2

Solution:

Let P (n) = 1.2 + 2.2^2 + 3.2^3 + … + n.2^n = (n–1) 2^{n+1} + 2

Let us check for n = 1,

P (1):1.2 = 0.2^0 + 2

: 2 = 2

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 1.2 + 2.2^2 + 3.2^3 + … + k.2^k = (k–1) 2^{k+1} + 2 …. (i)

So,

$\{1.2 + 2.2^2 + 3.2^3 + \ldots + k.2^k\} + (k + 1)2^{k+1}$

Now, substituting the value of P (k) we get,

$= [(k – 1)2^{k+1} + 2] + (k + 1)2^{k+1}$ using equation (i)

$= (k – 1)2^{k+1} + 2 + (k + 1)2^{k+1}$

$= 2^{k+1}(k – 1 + k + 1) + 2$

$= 2^{k+1} \times 2k + 2$

$= k \times 2^{k+2} + 2$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

11. 2 + 5 + 8 + 11 + … + (3n – 1) = 1/2 n (3n + 1)

Solution:

Let P (n) = 2 + 5 + 8 + 11 + … + (3n – 1) = 1/2 n (3n + 1)

Let us check for n = 1,

P (1): 2 = 1/2 × 1 × 4

: 2 = 2

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k) = 2 + 5 + 8 + 11 + … + (3k – 1) = 1/2 k (3k + 1) … (i)

So,

2 + 5 + 8 + 11 + … + (3k – 1) + (3k + 2)

Now, substituting the value of P (k) we get,

= 1/2 × k (3k + 1) + (3k + 2) by using equation (i)

$= [3k^2 + k + 2 (3k + 2)] / 2$

$= [3k^2 + k + 6k + 2] / 2$

$= [3k^2 + 7k + 2] / 2$

$= [3k^2 + 4k + 3k + 2] / 2$

= [3k (k+1) + 4(k+1)] / 2

= [(k+1) (3k+4)] /2

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

12. 1.3 + 2.4 + 3.5 + … + n. (n+2) = 1/6 n (n+1) (2n+7)

Solution:

Let P (n): 1.3 + 2.4 + 3.5 + … + n. (n+2) = 1/6 n (n+1) (2n+7)

Let us check for n = 1,

P (1): 1.3 = 1/6 × 1 × 2 × 9

: 3 = 3

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 1.3 + 2.4 + 3.5 + … + k. (k+2) = 1/6 k (k+1) (2k+7) … (i)

So,

1.3 + 2.4 + 3.5 + … + k. (k+2) + (k+1) (k+3)

Now, substituting the value of P (k) we get,

= 1/6 k (k+1) (2k+7) + (k+1) (k+3) by using equation (i)

= (k+1) [{k(2k+7)/6} + {(k+3)/1}]

= (k+1) [(2k^2 + 7k + 6k + 18)] / 6

= (k+1) [2k^2 + 13k + 18] / 6

= (k+1) [2k^2 + 9k + 4k + 18] / 6

= (k+1) [2k(k+2) + 9(k+2)] / 6

= (k+1) [(2k+9) (k+2)] / 6

= 1/6 (k+1) (k+2) (2k+9)

P (n) is true for n = k + 1

Hence, P (n) is true for all n $\in$ N.

13. 1.3 + 3.5 + 5.7 + … + (2n – 1) (2n + 1) = n(4n^2 + 6n – 1)/3

Solution:

Let P (n): 1.3 + 3.5 + 5.7 + … + (2n – 1) (2n + 1) = n(4n^2 + 6n – 1)/3

Let us check for n = 1,

P (1): (2.1 – 1) (2.1 + 1) = 1(4.1^2 + 6.1 -1)/3

: 1×3 = 1(4+6-1)/3

: 3 = 3

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 1.3 + 3.5 + 5.7 + … + (2k – 1) (2k + 1) = k(4k^2 + 6k – 1)/3 … (i)

So,

1.3 + 3.5 + 5.7 + … + (2k – 1) (2k + 1) + (2k + 1) (2k + 3)

Now, substituting the value of P (k) we get,

= k(4k^2 + 6k – 1)/3 + (2k + 1) (2k + 3) by using equation (i)

= [k(4k^2 + 6k-1) + 3 (4k^2 + 6k + 2k + 3)] / 3

= [4k^3 + 6k^2 – k + 12k^2 + 18k + 6k + 9] /3

= [4k^3 + 18k^2 + 23k + 9] /3

= [4k^3 + 4k^2 + 14k^2 + 14k +9k + 9] /3

= [(k+1) (4k^2 + 8k +4 + 6k + 6 – 1)] / 3

= [(k+1) 4[(k+1)^2 + 6(k+1) -1]] /3

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

14. 1.2 + 2.3 + 3.4 + … + n(n+1) = [n (n+1) (n+2)] / 3

Solution:

Let P (n): 1.2 + 2.3 + 3.4 + … + n(n+1) = [n (n+1) (n+2)] / 3

Let us check for n = 1,

P (1): 1(1+1) = [1(1+1) (1+2)] /3

: 2 = 2

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 1.2 + 2.3 + 3.4 + … + k(k+1) = [k (k+1) (k+2)] / 3 … (i)

So,

1.2 + 2.3 + 3.4 + … + k(k+1) + (k+1) (k+2)

Now, substituting the value of P (k) we get,

= [k (k+1) (k+2)] / 3 + (k+1) (k+2) by using equation (i)

= (k+2) (k+1) [k/2 + 1]

= [(k+1) (k+2) (k+3)] /3

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

15. $1/2 + 1/4 + 1/8 + … + 1/2^n = 1 – 1/2^n$

Solution:

Let P (n): $1/2 + 1/4 + 1/8 + … + 1/2^n = 1 – 1/2^n$

Let us check for n = 1,

P (1): $1/2^1 = 1 – 1/2^1$

: 1/2 = 1/2

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

Let P (k): $1/2 + 1/4 + 1/8 + … + 1/2^k = 1 – 1/2^k$ … (i)

So,

$1/2 + 1/4 + 1/8 + … + 1/2^k + 1/2^{k+1}$

Now, substituting the value of P (k) we get,

$= 1 – 1/2^k + 1/2^{k+1}$ by using equation (i)

$= 1 – ((2-1)/2^{k+1})$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

16. $1^2 + 3^2 + 5^2 + … + (2n – 1)^2 = 1/3\ n\ (4n^2 – 1)$

Solution:

Let P (n): $1^2 + 3^2 + 5^2 + … + (2n – 1)^2 = 1/3\ n\ (4n^2 – 1)$

Let us check for n = 1,

P (1): $(2.1 - 1)^2 = 1/3 \times 1 \times (4 - 1)$

: 1 = 1

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $1^2 + 3^2 + 5^2 + \ldots + (2k - 1)^2 = 1/3\ k\ (4k^2 - 1)$ … (i)

So,

$1^2 + 3^2 + 5^2 + \ldots + (2k - 1)^2 + (2k + 1)^2$

Now, substituting the value of P (k) we get,

$= 1/3\ k\ (4k^2 - 1) + (2k + 1)^2$ by using equation (i)

$= 1/3\ k\ (2k + 1)\ (2k - 1) + (2k + 1)^2$

$= (2k + 1)\ [\{k(2k-1)/3\} + (2k+1)]$

$= (2k + 1)\ [2k^2 - k + 3(2k+1)] / 3$

$= (2k + 1)\ [2k^2 - k + 6k + 3] / 3$

$= [(2k+1)\ 2k^2 + 5k + 3] /3$

$= [(2k+1)\ (2k(k+1)) + 3\ (k+1)] /3$

$= [(2k+1)\ (2k+3)\ (k+1)] /3$

$= (k+1)/3\ [4k^2 + 6k + 2k + 3]$

$= (k+1)/3\ [4k^2 + 8k - 1]$

$= (k+1)/3\ [4(k+1)^2 - 1]$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

17. $a + ar + ar^2 + \ldots + ar^{n-1} = a[(r^n - 1)/(r - 1)], r \neq 1$

Solution:

Let P (n): $a + ar + ar^2 + \ldots + ar^{n-1} = a[(r^n - 1)/(r - 1)]$

Let us check for n = 1,

P (1): $a = a(r^1 - 1)/(r-1)$

: a = a

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $a + ar + ar^2 + \ldots + ar^{k-1} = a[(r^k - 1)/(r - 1)]$ … (i)

So,

$a + ar + ar^2 + \ldots + ar^{k-1} + ar^k$

Now, substituting the value of P (k) we get,

$= a[(r^k - 1)/(r - 1)] + ar^k$ by using equation (i)

$= a[r^k - 1 + r^k(r-1)] / (r-1)$

$= a[r^k - 1 + r^{k+1} - r^{-k}] / (r-1)$

$= a[r^{k+1} - 1] / (r-1)$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

18. a + (a + d) + (a + 2d) + … + (a + (n-1)d) = n/2 [2a + (n-1)d]

Solution:

Let P (n): a + (a + d) + (a + 2d) + … + (a + (n-1)d) = n/2 [2a + (n-1)d]

Let us check for n = 1,

P (1): a = ½ [2a + (1-1)d]

: a = a

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): a + (a + d) + (a + 2d) + … + (a + (k-1)d) = k/2 [2a + (k-1)d] … (i)

So,

a + (a + d) + (a + 2d) + … + (a + (k-1)d) + (a + (k)d)

Now, substituting the value of P (k) we get,

= k/2 [2a + (k-1)d] + (a + kd) by using equation (i)

= [2ka + k(k-1)d + 2(a+kd)] / 2

= [2ka + k^2d – kd + 2a + 2kd] / 2

= [2ka + 2a + k^2d + kd] / 2

= [2a(k+1) + d(k^2 + k)] / 2

= (k+1)/2 [2a + kd]

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

19. 5^{2n} – 1 is divisible by 24 for all n ϵ N

Solution:

Let P (n): 5^{2n} – 1 is divisible by 24

Let us check for n = 1,

P (1): 5^2 – 1 = 25 – 1 = 24

P (n) is true for n = 1. Where, P (n) is divisible by 24

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $5^{2k} – 1$ is divisible by 24

: $5^{2k} – 1 = 24λ$ … (i)

We have to prove,

$5^{2k + 1} – 1$ is divisible by 24

$5^{2(k + 1)} – 1 = 24μ$

So,

$= 5^{2(k + 1)} – 1$

$= 5^{2k}.5^2 – 1$

$= 25.5^{2k} – 1$

= 25.(24λ + 1) – 1 by using equation (1)

= 25.24λ + 24

= 24λ

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

20. 3^{2n} + 7 is divisible by 8 for all n ϵ N

Solution:

Let P (n): $3^{2n} + 7$ is divisible by 8

Let us check for n = 1,

P (1): $3^2 + 7 = 9 + 7 = 16$

P (n) is true for n = 1. where, P (n) is divisible by 8

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $3^{2k} + 7$ is divisible by 8

: $3^{2k} + 7 = 8λ$

: $3^{2k} = 8λ – 7$ … (i)

We have to prove,

$3^{2(k + 1)} + 7$ is divisible by 8

$3^{2k + 2} + 7 = 8μ$

So,

$= 3^{2(k + 1)} + 7$

$= 3^{2k}.3^2 + 7$

$= 9.3^{2k} + 7$

$= 9.(8λ – 7) + 7$ by using equation (i)

$= 72λ – 63 + 7$

$= 72λ – 56$

$= 8(9λ – 7)$

$= 8μ$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

21. $5^{2n + 2} – 24n – 25$ is divisible by 576 for all n ϵ N

Solution:

Let P (n): $5^{2n + 2} – 24n – 25$ is divisible by 576

Let us check for n = 1,

P (1): $5^{2.1+2} – 24.1 – 25$

: 625 – 49

: 576

P (n) is true for n = 1. Where, P (n) is divisible by 576

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $5^{2k+2} - 24k - 25$ is divisible by 576

: $5^{2k+2} - 24k - 25 = 576\lambda$ …. (i)

We have to prove,

$5^{2k+4} - 24(k + 1) - 25$ is divisible by 576

$5^{(2k+2)+2} - 24(k + 1) - 25 = 576\mu$

So,

$= 5^{(2k+2)+2} - 24(k + 1) - 25$

$= 5^{(2k+2)}.5^2 - 24k - 24 - 25$

$= (576\lambda + 24k + 25)25 - 24k - 49$ by using equation (i)

$= 25.\ 576\lambda + 576k + 576$

$= 576(25\lambda + k + 1)$

$= 576\mu$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

22. $3^{2n+2} - 8n - 9$ is divisible by 8 for all n ϵ N

Solution:

Let P (n): $3^{2n+2} - 8n - 9$ is divisible by 8

Let us check for n = 1,

P (1): $3^{2.1 + 2} – 8.1 – 9$

: 81 – 17

: 64

P (n) is true for n = 1. Where, P (n) is divisible by 8

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $3^{2k + 2} – 8k – 9$ is divisible by 8

: $3^{2k + 2} – 8k – 9 = 8λ$ … (i)

We have to prove,

$3^{2k + 4} – 8(k + 1) – 9$ is divisible by 8

$3^{(2k + 2) + 2} – 8(k + 1) – 9 = 8μ$

So,

$= 3^{2(k + 1)}.3^2 – 8(k + 1) – 9$

= (8λ + 8k + 9)9 – 8k – 8 – 9

= 72λ + 72k + 81 – 8k – 17 using equation (1)

= 72λ + 64k + 64

= 8(9λ + 8k + 8)

= 8μ

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

23. $(ab)^n = a^n b^n$ for all n ϵ N

Solution:

Let P (n): $(ab)^n = a^n b^n$

Let us check for n = 1,

P (1): $(ab)^{1} = a^{1} b^{1}$

: ab = ab

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $(ab)^{k} = a^{k} b^{k}$ … (i)

We have to prove,

$(ab)^{k+1} = a^{k+1}.b^{k+1}$

So,

$= (ab)^{k+1}$

$= (ab)^{k} (ab)$

$= (a^{k} b^{k}) (ab)$ using equation (1)

$= (a^{k+1}) (b^{k+1})$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

24. n (n + 1) (n + 5) is a multiple of 3 for all n ϵ N.

Solution:

Let P (n): n (n + 1) (n + 5) is a multiple of 3

Let us check for n = 1,

P (1): 1 (1 + 1) (1 + 5)

: 2 × 6

: 12

P (n) is true for n = 1. Where, P (n) is a multiple of 3

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): k (k + 1) (k + 5) is a multiple of 3

: k(k + 1) (k + 5) = 3λ … (i)

We have to prove,

(k + 1)[(k + 1) + 1][(k + 1) + 5] is a multiple of 3

(k + 1)[(k + 1) + 1][(k + 1) + 5] = 3μ

So,

= (k + 1) [(k + 1) + 1] [(k + 1) + 5]

= (k + 1) (k + 2) [(k + 1) + 5]

= [k (k + 1) + 2(k + 1)] [(k + 5) + 1]

= k (k + 1) (k + 5) + k(k + 1) + 2(k + 1) (k + 5) + 2(k + 1)

= $3\lambda + k^2 + k + 2(k^2 + 6k + 5) + 2k + 2$

= $3\lambda + k^2 + k + 2k^2 + 12k + 10 + 2k + 2$

= $3\lambda + 3k^2 + 15k + 12$

= $3(\lambda + k^2 + 5k + 4)$

= 3μ

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

25. $7^{2n} + 2^{3n-3}$. 3n – 1 is divisible by 25 for all n ϵ N

Solution:

Let P (n): $7^{2n} + 2^{3n-3}$. 3n – 1 is divisible by 25

Let us check for n = 1,

P (1): $7^2 + 2^0.3^0$

: 49 + 1

: 50

P (n) is true for n = 1. Where, P (n) is divisible by 25

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $7^{2k} + 2^{3k-3}$. 3k – 1 is divisible by 25

: $7^{2k} + 2^{3k-3}. 3^{k-1} = 25\lambda$ … (i)

We have to prove that:

$7^{2k+1} + 2^{3k}. 3^k$ is divisible by 25

$7^{2k+2} + 2^{3k}. 3^k = 25\mu$

So,

$= 7^{2(k+1)} + 2^{3k}. 3^k$

$= 7^{2k}.7^1 + 2^{3k}. 3^k$

$= (25\lambda – 2^{3k-3}. 3^{k-1})\ 49 + 2^{3k}$. 3k by using equation (i)

$= 25\lambda. 49 – 2^{3k}/8. 3^k/3. 49 + 2^{3k}. 3^k$

$= 24×25×49\lambda – 2^{3k} . 3^k . 49 + 24 . 2^{3k}.3^k$

$= 24×25×49\lambda – 25 . 2^{3k}. 3^k$

$= 25(24 . 49\lambda – 2^{3k}. 3^k)$

$= 25\mu$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

Chapterwise RD Sharma Solutions for Class 11 Maths :

IndCareer

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.