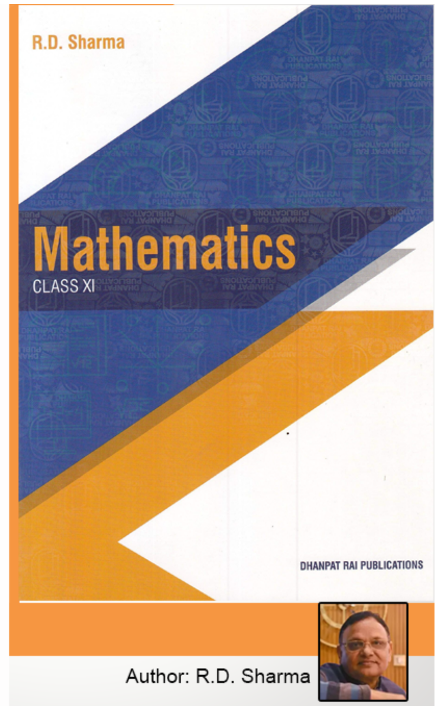
Class 11 - Chapter 13 Complex Numbers



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EXERCISE 13.1 PAGE NO: 13.3

1. Evaluate the following:

(i) i^{457}

(ii) i^{528}

(iii) $1/i^{58}$

(iv) $i^{37} + 1/i^{67}$

(v) $[i^{41} + 1/i^{257}]$

(vi) $(i^{77} + i^{70} + i^{87} + i^{414})^3$

(vii) $i^{30} + i^{40} + i^{60}$

(viii) $i^{49} + i^{68} + i^{89} + i^{110}$

Solution:

(i) i^{457}

Let us simplify we get,

$i^{457} = i^{(456 + 1)}$

$= i^{4(114)} \times i$

$= (1)^{114} \times i$

$= i$ [since $i^4 = 1$]

(ii) i^{528}

Let us simplify we get,

$i^{528} = i^{4(132)}$

$= (1)^{132}$

$= 1$ [since $i^4 = 1$]

(iii) $1/ i^{58}$

Let us simplify we get,

$1/ i^{58} = 1/ i^{56+2}$

$= 1/ i^{56} \times i^2$

$= 1/ (i^4)^{14} \times i^2$

$= 1/ i^2$ [since, $i^4 = 1$]

$= 1/-1$ [since, $i^2 = -1$]

$= -1$

(iv) $i^{37} + 1/i^{67}$

Let us simplify we get,

$i^{37} + 1/i^{67} = i^{36+1} + 1/ i^{64+3}$

$= i + 1/i^3$ [since, $i^4 = 1$]

$= i + i/i^4$

$= i + i$

$= 2i$

(v) $[i^{41} + 1/i^{257}]$

Let us simplify we get,$[i^{41} + 1/i^{257}] = [i^{40+1} + 1/ i^{256+1}]$

$= [i + 1/i]^9$ [since, $1/i = -1$]

$= [i – i]$

$= 0$

(vi) $(i^{77} + i^{70} + i^{87} + i^{414})^3$

Let us simplify we get,

$(i^{77} + i^{70} + i^{87} + i^{414})^3 = (i^{(76 + 1)} + i^{(68 + 2)} + i^{(84 + 3)} + i^{(412 + 2)})^3$

$= (i + i^2 + i^3 + i^2)^3$ [since $i^3 = – i$, $i^2 = – 1$]

$= (i + (– 1) + (– i) + (– 1))^3$

$= (– 2)^3$

$= – 8$

(vii) $i^{30} + i^{40} + i^{60}$

Let us simplify we get,

$i^{30} + i^{40} + i^{60} = i^{(28 + 2) +} i^{40} + i^{60}$

$= (i^4)^7 i^2 + (i^4)^{10} + (i^4)^{15}$

$= i^2 + 1^{10} + 1^{15}$

$= – 1 + 1 + 1$

$= 1$

(viii) $i^{49} + i^{68} + i^{89} + i^{110}$

Let us simplify we get,

$i^{49} + i^{68} + i^{89} + i^{110} = i^{(48 + 1)} + i^{68} + i^{(88 + 1)} + i^{(116 + 2)}$

$= (i^4)^{12}\times i + (i^4)^{17} + (i^4)^{22}\times i + (i^4)^{29}\times i^2$

$= i + 1 + i – 1$

$= 2i$

2. Show that $1 + i^{10} + i^{20} + i^{30}$ is a real number?

Solution:

Given:

$1 + i^{10} + i^{20} + i^{30} = 1 + i^{(8 + 2) +} i^{20} + i^{(28 + 2)}$

$= 1 + (i^4)^2 \times i^2 + (i^4)^5 + (i^4)^7 \times i^2$

$= 1 – 1 + 1 – 1$ [since, $i^4 = 1$, $i^2 = – 1$]

$= 0$

Hence , $1 + i^{10} + i^{20} + i^{30}$ is a real number.

3. Find the values of the following expressions:

(i) $i^{49} + i^{68} + i^{89} + i^{110}$

(ii) $i^{30} + i^{80} + i^{120}$

(iii) $i + i^2 + i^3 + i^4$

(iv) $i^5 + i^{10} + i^{15}$

(v) $[i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}]$

(vi) $1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20}$

(vii) $(1 + i)^6 + (1 – i)^3$

Solution:

(i) $i^{49} + i^{68} + i^{89} + i^{110}$

Let us simplify we get,

$i^{49} + i^{68} + i^{89} + i^{110} = i^{(48 + 1)} + i^{68} + i^{(88 + 1)} + i^{(108 + 2)}$

$= (i^4)^{12} \times i + (i^4)^{17} + (i^4)^{22} \times i + (i^4)^{27} \times i^2$

$= i + 1 + i – 1$ [since $i^4 = 1$, $i^2 = – 1$]

$= 2i$

$\therefore\ i^{49} + i^{68} + i^{89} + i^{110} = 2i$

(ii) $i^{30} + i^{80} + i^{120}$

Let us simplify we get,

$i^{30} + i^{80} + i^{120} = i^{(28+2)} + i^{80} + i^{120}$

$= (i^4)^7 \times i^2 + (i^4)^{20} + (i^4)^{30}$

$= -1 + 1 + 1$ [since $i^4 = 1$, $i^2 = -1$]

$= 1$

$\therefore i^{30} + i^{80} + i^{120} = 1$

(iii) $i + i^2 + i^3 + i^4$

Let us simplify we get,

$i + i^2 + i^3 + i^4 = i + i^2 + i^2 \times i + i^4$

$= i - 1 + (-1) \times i + 1$ [since $i^4 = 1$, $i^2 = -1$]

$= i - 1 - i + 1$

$= 0$

$\therefore i + i^2 + i^3 + i^4 = 0$

(iv) $i^5 + i^{10} + i^{15}$

Let us simplify we get,

$i^5 + i^{10} + i^{15} = i^{(4+1)} + i^{(8+2)} + i^{(12+3)}$

$= (i^4)^1 \times i + (i^4)^2 \times i^2 + (i^4)^3 \times i^3$

$= (i^4)^1 \times i + (i^4)^2 \times i^2 + (i^4)^3 \times i^2 \times i$

$= 1 \times i + 1 \times (-1) + 1 \times (-1) \times i$

$= i - 1 - i$

$= -1$

$\therefore i^5 + i^{10} + i^{15} = -1$

(v) $[i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}]$

Let us simplify we get,$[i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}]$

$= [i^{10} (i^{582} + i^{580} + i^{578} + i^{576} + i^{574}) / (i^{582} + i^{580} + i^{578} + i^{576} + i^{574})]$

$= i^{10}$

$= i^8 i^2$

$= (i^4)^2 i^2$

$= (1)^2 (-1)$ [since $i^4 = 1$, $i^2 = -1$]

$= -1$

$\therefore [i^{592} + i^{590} + i^{588} + i^{586} + i^{584}] / [i^{582} + i^{580} + i^{578} + i^{576} + i^{574}] = -1$

(vi) $1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20}$

Let us simplify we get,

$1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20} = 1 + (-1) + 1 + (-1) + 1 + \ldots + 1$

$= 1$

$\therefore 1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20} = 1$

(vii) $(1 + i)^6 + (1 - i)^3$

Let us simplify we get,

$(1 + i)^6 + (1 - i)^3 = \{(1 + i)^2\}^3 + (1 - i)^2 (1 - i)$

$= \{1 + i^2 + 2i\}^3 + (1 + i^{2\,-}\, 2i)(1 - i)$

$= \{1 - 1 + 2i\}^3 + (1 - 1 - 2i)(1 - i)$

$= (2i)^3 + (-2i)(1 - i)$

$= 8i^3 + (-2i) + 2i^2$

$= -8i - 2i - 2$ [since $i^3 = -i$, $i^2 = -1$]

$= -10i - 2$

$= -2(1 + 5i)$

$= -2 - 10i$

$\therefore (1 + i)^6 + (1 - i)^3 = -2 - 10i$

EXERCISE 13.2 PAGE NO: 13.31

1. Express the following complex numbers in the standard form a + ib:

(i) $(1 + i)(1 + 2i)$

(ii) $(3 + 2i) / (-2 + i)$

(iii) $1/(2 + i)^2$

(iv) $(1 - i) / (1 + i)$

(v) $(2 + i)^3 / (2 + 3i)$

(vi) $[(1 + i)(1 + \sqrt{3}i)] / (1 - i)$

(vii) $(2 + 3i) / (4 + 5i)$

(viii) $(1 - i)^3 / (1 - i^3)$

(ix) $(1 + 2i)^{-3}$

(x) $(3 - 4i) / [(4 - 2i)(1 + i)]$

(xi)

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$$

(xii) $(5 + \sqrt{2}i) / (1 - \sqrt{2}i)$

Solution:

(i) $(1 + i)(1 + 2i)$

Let us simplify and express in the standard form of $(a + ib)$,

$(1 + i)(1 + 2i) = (1+i)(1+2i)$

$= 1(1+2i)+i(1+2i)$

$= 1+2i+i+2i^2$

$= 1+3i+2(-1)$ [since, $i^2 = -1$]

$= 1+3i-2$

$= -1+3i$

∴ The values of a, b are -1, 3.

(ii) $(3 + 2i) / (-2 + i)$

Let us simplify and express in the standard form of $(a + ib)$,

$(3 + 2i) / (-2 + i) = [(3 + 2i) / (-2 + i)] \times (-2-i) / (-2-i)$ [multiply and divide with $(-2-i)$]

$= [3(-2-i) + 2i(-2-i)] / [(-2)^2 – (i)^2]$

$= [-6 -3i – 4i -2i^2] / (4-i^2)$

$= [-6 -7i -2(-1)] / (4 – (-1))$ [since, $i^2 = -1$]

$= [-4 -7i] / 5$

∴ The values of a, b are -4/5, -7/5

(iii) $1/(2 + i)^2$

Let us simplify and express in the standard form of $(a + ib)$,

$1/(2 + i)^2 = 1/(2^2 + i^2 + 2(2)(i))$

$= 1/(4 – 1 + 4i)$ [since, $i^2 = -1$]

$= 1/(3 + 4i)$ [multiply and divide with $(3 – 4i)$]

$= 1/(3 + 4i) \times (3 – 4i)/ (3 – 4i)]$

$= (3-4i)/ (3^2 – (4i)^2)$

$= (3-4i)/ (9 – 16i^2)$

$= (3-4i)/ (9 – 16(-1))$ [since, $i^2 = -1$]

$= (3-4i)/25$

∴ The values of a, b are 3/25, -4/25

(iv) $(1 – i) / (1 + i)$

Let us simplify and express in the standard form of (a + ib),

$(1 – i) / (1 + i) = (1 – i) / (1 + i) \times (1-i)/(1-i)$ [multiply and divide with (1-i)]

$= (1^2 + i^2 – 2(1)(i)) / (1^2 – i^2)$

$= (1 + (-1) -2i) / (1 – (-1))$

$= -2i/2$

$= -i$

∴ The values of a, b are 0, -1

(v) $(2 + i)^3 / (2 + 3i)$

Let us simplify and express in the standard form of (a + ib),

$(2 + i)^3 / (2 + 3i) = (2^3 + i^3 + 3(2)^2(i) + 3(i)^2(2)) / (2 + 3i)$

$= (8 + (i^2.i) + 3(4)(i) + 6i^2) / (2 + 3i)$

$= (8 + (-1)i + 12i + 6(-1)) / (2 + 3i)$

$= (2 + 11i) / (2 + 3i)$[multiply and divide with (2-3i)]

$= (2 + 11i)/(2 + 3i) \times (2-3i)/(2-3i)$

$= [2(2-3i) + 11i(2-3i)] / (2^2 – (3i)^2)$

$= (4 – 6i + 22i – 33i^2) / (4 – 9i^2)$

$= (4 + 16i – 33(-1)) / (4 – 9(-1))$ [since, $i^2 = -1$]

$= (37 + 16i) / 13$

∴ The values of a, b are 37/13, 16/13

(vi) [(1 + i) (1 +√3i)] / (1 – i)

Let us simplify and express in the standard form of (a + ib),[(1 + i) (1 +√3i)] / (1 – i) = [1(1+√3i) + i(1+√3i)] / (1-i)

$= (1 + \sqrt{3}i + i + \sqrt{3}i^2) / (1 – i)$

$= (1 + (\sqrt{3}+1)i + \sqrt{3}(-1)) / (1-i)$ [since, $i^2 = -1$]

$= [(1-\sqrt{3}) + (1+\sqrt{3})i] / (1-i)$[multiply and divide with (1+i)]

$= [(1-\sqrt{3}) + (1+\sqrt{3})i] / (1-i) \times (1+i)/(1+i)$

$= [(1-\sqrt{3}) (1+i) + (1+\sqrt{3})i(1+i)] / (1^2 – i^2)$

$= [1-\sqrt{3}+ (1-\sqrt{3})i + (1+\sqrt{3})i + (1+\sqrt{3})i^2] / (1-(-1))$ [since, $i^2 = -1$]

$= [(1-\sqrt{3})+(1-\sqrt{3}+1+\sqrt{3})i+(1+\sqrt{3})(-1)] / 2$

$= (-2\sqrt{3} + 2i) / 2$

$= -\sqrt{3} + i$

∴ The values of a, b are -√3, 1

(vii) (2 + 3i) / (4 + 5i)

Let us simplify and express in the standard form of (a + ib),

(2 + 3i) / (4 + 5i) = [multiply and divide with (4-5i)]

$= (2 + 3i) / (4 + 5i) \times (4-5i)/(4-5i)$

$= [2(4-5i) + 3i(4-5i)] / (4^2 – (5i)^2)$

$= [8 – 10i + 12i – 15i^2] / (16 – 25i^2)$

= [8+2i-15(-1)] / (16 – 25(-1)) [since, $i^2 = -1$]

= (23 + 2i) / 41

∴ The values of a, b are 23/41, 2/41

(viii) $(1 – i)^3 / (1 – i^3)$

Let us simplify and express in the standard form of (a + ib),

$(1 – i)^3 / (1 – i^3) = [1^3 – 3(1)^2 i + 3(1)(i)^2 – i^3] / (1-i^2.i)$

= $[1 – 3i + 3(-1)-i^2.i] / (1 – (-1)i)$ [since, $i^2 = -1$]

= [-2 – 3i – (-1)i] / (1+i)

= [-2-4i] / (1+i)[Multiply and divide with (1-i)]

= [-2-4i] / (1+i) × (1-i)/(1-i)

= $[-2(1-i)-4i(1-i)] / (1^2 – i^2)$

= $[-2+2i-4i+4i^2] / (1 – (-1))$

= [-2-2i+4(-1)] /2

= (-6-2i)/2

= -3 – i

∴ The values of a, b are -3, -1

(ix) $(1 + 2i)^{-3}$

Let us simplify and express in the standard form of (a + ib),

$(1 + 2i)^{-3} = 1/(1 + 2i)^3$

= $1/(1^3+3(1)^2 (2i)+2(1)(2i)^2 + (2i)^3)$

= $1/(1+6i+4i^2+8i^3)$

= $1/(1+6i+4(-1)+8i^2.i)$ [since, $i^2 = -1$]

$= 1/(-3+6i+8(-1)i)$ [since, $i^2 = -1$]

$= 1/(-3-2i)$

$= -1/(3+2i)$[Multiply and divide with $(3-2i)$]

$= -1/(3+2i) \times (3-2i)/(3-2i)$

$= (-3+2i)/(3^2 – (2i)^2)$

$= (-3+2i) / (9-4i^2)$

$= (-3+2i) / (9-4(-1))$

$= (-3+2i) /13$

∴ The values of a, b are -3/13, 2/13

(x) $(3 – 4i) / [(4 – 2i) (1 + i)]$

Let us simplify and express in the standard form of (a + ib),

$(3 – 4i) / [(4 – 2i) (1 + i)] = (3-4i)/ [4(1+i)-2i(1+i)]$

$= (3-4i)/ [4+4i-2i-2i^2]$

$= (3-4i)/ [4+2i-2(-1)]$ [since, $i^2 = -1$]

$= (3-4i)/ (6+2i)$[Multiply and divide with $(6-2i)$]

$= (3-4i)/ (6+2i) \times (6-2i)/(6-2i)$

$= [3(6-2i)-4i(6-2i)] / (6^2 – (2i)^2)$

$= [18 – 6i – 24i + 8i^2] / (36 – 4i^2)$

$= [18 – 30i + 8 (-1)] / (36 – 4 (-1))$ [since, $i^2 = -1$]

$= [10-30i] / 40$

$= (1 – 3i) / 4$

∴ The values of a, b are 1/4, -3/4

(xi)

$$\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$$

Let us simplify and express in the standard form of (a + ib),

$$\left(\frac{1}{1-4i}-\frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)=\left(\frac{1+i-2(1-4i)}{(1-4i)(1+i)}\right)\left(\frac{3-4i}{5+i}\right)$$

$$=\left(\frac{1+i-2+8i}{1(1+i)-4i(1+i)}\right)\left(\frac{3-4i}{5+i}\right)$$

$$=\left(\frac{-1+9i}{1+i-4i-4i^2}\right)\left(\frac{3-4i}{5+i}\right)$$

$$=\left(\frac{-1+9i}{1-3i-4(-1)}\right)\left(\frac{3-4i}{5+i}\right)$$

$$=\frac{(-1+9i)(3-4i)}{(5-3i)(5+i)}$$

$$=\frac{-1(3-4i)+9i(3-i)}{5(5+i)-3i(5+i)}$$

$$=\frac{-3+4i+27i-9i^2}{25+5i-15i-3i^2}$$

$$=\frac{-3+31i-9(-1)}{25-10i-3(-1)}$$

$$=\frac{6+31i}{28-10i}$$

[Multiply and divide with (28+10i)]

$$=\frac{6+31i}{28-10i}\times\frac{28+10i}{28+10i}$$

$$=\frac{6(28+10i)+31i(28+10i)}{28^2-(10i)^2}$$

$$=\frac{168+60i+868i+310i^2}{784-100i^2}$$

$$=\frac{168+928i+310(-1)}{784-100(-1)}$$

$$=\frac{478+928i}{884}$$

∴ The values of a, b are 478/884, 928/884

(xii) $(5 + \sqrt{2}i) / (1 - \sqrt{2}i)$

Let us simplify and express in the standard form of (a + ib),

$(5 + \sqrt{2}i) / (1 - \sqrt{2}i)$ = [Multiply and divide with $(1+\sqrt{2}i)$]

$= (5 + \sqrt{2}i) / (1 - \sqrt{2}i) \times (1+\sqrt{2}i)/(1+\sqrt{2}i)$

$= [5(1+\sqrt{2}i) + \sqrt{2}i(1+\sqrt{2}i)] / (1^2 - (\sqrt{2})^2)$

$= [5+5\sqrt{2}i + \sqrt{2}i + 2i^2] / (1 - 2i^2)$

$= [5 + 6\sqrt{2}i + 2(-1)] / (1-2(-1))$ [since, $i^2 = -1$]

$= [3+6\sqrt{2}i]/3$

$= 1+ 2\sqrt{2}i$

∴ The values of a, b are 1, $2\sqrt{2}$

2. Find the real values of x and y, if

(i) (x + iy) (2 – 3i) = 4 + i

(ii) (3x – 2i y) $(2 + i)^2$ = 10(1 + i)

$$\textbf{(iii)}\ \frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$$

(iv) (1 + i) (x + iy) = 2 – 5i

Solution:

(i) (x + iy) (2 – 3i) = 4 + i

Given:

(x + iy) (2 – 3i) = 4 + i

Let us simplify the expression we get,

x(2 – 3i) + iy(2 – 3i) = 4 + i

$2x - 3xi + 2yi - 3yi^2 = 4 + i$

$2x + (-3x+2y)i – 3y (-1) = 4 + i$ [since, $i^2 = -1$]

$2x + (-3x+2y)i + 3y = 4 + i$ [since, $i^2 = -1$]

$(2x+3y) + i(-3x+2y) = 4 + i$

Equating Real and Imaginary parts on both sides, we get

$2x+3y = 4$… (i)

And $-3x+2y = 1$… (ii)

Multiply (i) by 3 and (ii) by 2 and add

On solving we get,

$6x – 6x – 9y + 4y = 12 + 2$

$13y = 14$

$y = 14/13$

Substitute the value of y in (i) we get,

$2x+3y = 4$

$2x + 3(14/13) = 4$

$2x = 4 – (42/13)$

$= (52-42)/13$

$2x = 10/13$

$x = 5/13$

$x = 5/13, y = 14/13$

∴ The real values of x and y are 5/13, 14/13

(ii) $(3x – 2i y) (2 + i)^2 = 10(1 + i)$

Given:

$(3x - 2iy)(2+i)^2 = 10(1+i)$

$(3x - 2yi)(2^2+i^2+2(2)(i)) = 10+10i$

$(3x - 2yi)(4 + (-1)+4i) = 10+10i$ [since, $i^2 = -1$]

$(3x - 2yi)(3+4i) = 10+10i$

Let us divide with 3+4i on both sides we get,

$(3x - 2yi) = (10+10i)/(3+4i)$

= Now multiply and divide with (3-4i)

$= [10(3-4i) + 10i(3-4i)] / (3^2 - (4i)^2)$

$= [30-40i+30i-40i^2] / (9 - 16i^2)$

$= [30-10i-40(-1)] / (9-16(-1))$

$= [70-10i]/25$

Now, equating Real and Imaginary parts on both sides we get

$3x = 70/25$ and $-2y = -10/25$

$x = 70/75$ and $y = 1/5$

$x = 14/15$ and $y = 1/5$

∴ The real values of x and y are 14/15, 1/5

$$\text{(iii)}\ \frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$$

Given:

$$\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$$

$$\frac{\left(\left((1+i)x-2i\right)(3-i)\right)+\left(\left((2-3i)y+i\right)(3+i)\right)}{(3+i)(3-i)}=i$$

$$\frac{\left((1+i)(3-i)x\right)-(2i)(3-i)+\left((2-3i)(3+i)y\right)+(i)(3+i)}{3^2-i^2}=i$$

$$\frac{\left(3-i+3i-i^2\right)x-6i+2i^2+\left(6+2i-9i-3i^2\right)y+3i+i^2}{9-(-1)}=i$$

$$\frac{\left(3+2i-(-1)\right)x-6i+2(-1)+\left(6-7i-3(-1)\right)y+3i+(-1)}{10}=i \text{ [since, } i^2=-1]$$

(4+2i) x-3i-3 + (9-7i)y = 10i

(4x+9y-3) + i(2x-7y-3) = 10i

Now, equating Real and Imaginary parts on both sides we get,

4x+9y-3 = 0 … (i)

And 2x-7y-3 = 10

2x-7y = 13 … (ii)

Multiply (i) by 7 and (ii) by 9 and add

On solving these equations we get

28x + 18x + 63y – 63y = 117 + 21

46x = 117 + 21

46x = 138

x = 138/46

= 3

Substitute the value of x in (i) we get,

4x+9y-3 = 0

9y = -9

y = -9/9

= -1

x = 3 and y = -1

∴ The real values of x and y are 3 and -1

(iv) (1 + i) (x + iy) = 2 – 5i

Given:

(1 + i) (x + iy) = 2 – 5i

Divide with (1+i) on both the sides we get,

(x + iy) = (2 – 5i)/(1+i)

Multiply and divide by (1-i)

= (2 – 5i)/(1+i) × (1-i)/(1-i)

= [2(1-i) – 5i (1-i)] / (1^2 – i^2)

= [2 – 7i + 5(-1)] / 2 [since, i^2 = -1]

= (-3-7i)/2

Now, equating Real and Imaginary parts on both sides we get

x = -3/2 and y = -7/2

∴ Thee real values of x and y are -3/2, -7/2

3. Find the conjugates of the following complex numbers:

(i) 4 – 5i

(ii) 1 / (3 + 5i)

(iii) 1 / (1 + i)

(iv) $(3 – i)^2$ / (2 + i)

(v) [(1 + i) (2 + i)] / (3 + i)

(vi) [(3 – 2i) (2 + 3i)] / [(1 + 2i) (2 – i)]

Solution:

(i) 4 – 5i

Given:

4 – 5i

We know the conjugate of a complex number (a + ib) is (a – ib)

So,

∴ The conjugate of (4 – 5i) is (4 + 5i)

(ii) 1 / (3 + 5i)

Given:

1 / (3 + 5i)

Since the given complex number is not in the standard form of (a + ib)

Let us convert to standard form by multiplying and dividing with (3 – 5i)

We get,

$$\frac{1}{3+5i} = \frac{1}{3+5i} \times \frac{3-5i}{3-5i}$$

$$= \frac{3-5i}{3^2-(5i)^2}$$

$$= \frac{3-5i}{9-25i^2}$$

$$= \frac{3-5i}{9-25(-1)} \text{ [Since, } i^2 = -1]$$

$$= \frac{3-5i}{34}$$

We know the conjugate of a complex number (a + ib) is (a – ib)

So,

∴ The conjugate of (3 – 5i)/34 is (3 + 5i)/34

(iii) 1 / (1 + i)

Given:

1 / (1 + i)

Since the given complex number is not in the standard form of (a + ib)

Let us convert to standard form by multiplying and dividing with (1 – i)

We get,

$$\frac{1}{1+i} = \frac{1}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{1-i}{1^2-i^2}$$

$$= \frac{1-i}{1-(-1)} \text{ [since, } i^2 = -1]$$

$$= \frac{1-i}{2}$$

We know the conjugate of a complex number (a + ib) is (a – ib)

So,

∴ The conjugate of (1-i)/2 is (1+i)/2

(iv) $(3 – i)^2$ / (2 + i)

Given:

$(3 – i)^2$ / (2 + i)

Since the given complex number is not in the standard form of (a + ib)

Let us convert to standard form,

$$\frac{(3-i)^2}{2+i} = \frac{3^2+i^2-2(3)(i)}{2+i}$$

$$= \frac{9+(-1)-6i}{2+i} \quad [\text{Since, } i^2 = -1]$$

$$= \frac{8-6i}{2+i}$$

Now, let us multiply and divide with (2 - i) we get,

$$\frac{8-6i}{2+i} = \frac{8-6i}{2+i} \times \frac{2-i}{2-i}$$

$$= \frac{8(2-i)-6i(2-i)}{2^2-i^2}$$

$$= \frac{16-8i-12i+6i^2}{4-(-1)} \quad [\text{Since, } i^2 = -1]$$

$$= \frac{16-20i+6(-1)}{5}$$

$$= \frac{10-20i}{5}$$

$$= 10/5 – 20i/5$$

$$= 2 – 4i$$

We know the conjugate of a complex number (a + ib) is (a – ib)

So,

∴ The conjugate of (2 – 4i) is (2 + 4i)

(v) [(1 + i) (2 + i)] / (3 + i)

Given:[(1 + i) (2 + i)] / (3 + i)

Since the given complex number is not in the standard form of (a + ib)

Let us convert to standard form,

$$\frac{(1+i)(2+i)}{3+i} = \frac{1(2+i)+i(2+i)}{3+i}$$

$$= \frac{2+i+2i+i^2}{3+i}$$

$$= \frac{2+3i+(-1)}{3+i} \quad [\text{Since, } i^2 = -1]$$

$$= \frac{1+3i}{3+i}$$

Now, let us multiply and divide with (3 - i) we get,

$$\frac{1+3i}{3+i} = \frac{1+3i}{3+i} \times \frac{3-i}{3-i}$$

$$= \frac{1(3-i)+3i(3-i)}{3^2-i^2}$$

$$= \frac{3-i+9i-3i^2}{9-(-1)} \quad [\text{Since, } i^2 = -1]$$

$$= \frac{3+8i-3(-1)}{10}$$

$$= \frac{6+8i}{10}$$

$$= \frac{3}{5} + \frac{4i}{5}$$

We know the conjugate of a complex number (a + ib) is (a – ib)

So,

∴ The conjugate of (3 + 4i)/5 is (3 – 4i)/5

(vi) [(3 – 2i) (2 + 3i)] / [(1 + 2i) (2 – i)]

Given:[(3 – 2i) (2 + 3i)] / [(1 + 2i) (2 – i)]

Since the given complex number is not in the standard form of (a + ib)

Let us convert to standard form,

$$\frac{(3-2i)(2+3i)}{(1+2i)(2-i)} = \frac{3(2+3i)-2i(2+3i)}{1(2-i)+2i(2-i)}$$

$$= \frac{6+9i-4i-6i^2}{2-i+4i-2i^2}$$

$$= \frac{6+5i-6(-1)}{2+3i-2(-1)}$$

$$= \frac{12+5i}{4+3i}$$

Now, let us multiply and divide with (4 - 3i) we get,

$$\frac{12+5i}{4+3i} = \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$$

$$= \frac{12(4-3i)+5i(4-3i)}{4^2-(3i)^2}$$

$$= \frac{48-36i+20i-15i^2}{16-9i^2}$$

$$= \frac{48-16i-15(-1)}{16-9(-1)}$$

$$= \frac{63-16i}{25}$$

We know the conjugate of a complex number (a + ib) is (a – ib)

So,

∴ The conjugate of (63 – 16i)/25 is (63 + 16i)/25

4. Find the multiplicative inverse of the following complex numbers:

(i) 1 – i

(ii) (1 + i √3)^2

(iii) 4 – 3i

(iv) √5 + 3i

Solution:

(i) 1 – i

Given:

1 – i

We know the multiplicative inverse of a complex number (Z) is Z^{-1} or 1/Z

So,

$$Z = 1 - i$$

$$Z^{-1} = \frac{1}{1-i}$$

Let us multiply and divide by (1 + i) we get,

$$= \frac{1}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{1+i}{1^2-(i)^2}$$

$$= \frac{1+i}{1-(-1)} \text{ [Since, } i^2 = -1]$$

$$= \frac{1+i}{2}$$

∴ The multiplicative inverse of (1 – i) is (1 + i)/2

(ii) $(1 + i\sqrt{3})^2$

Given:

$(1 + i\sqrt{3})^2$

$Z = (1 + i\sqrt{3})^2$

$= 1^2 + (i\sqrt{3})^2 + 2(1)(i\sqrt{3})$

$= 1 + 3i^2 + 2i\sqrt{3}$

$= 1 + 3(-1) + 2i\sqrt{3}$ [since, $i^2 = -1$]

$= 1 - 3 + 2i\sqrt{3}$

$= -2 + 2i\sqrt{3}$

We know the multiplicative inverse of a complex number (Z) is Z^{-1} or 1/Z

So,

Z = -2 + 2 i√3

$$Z^{-1} = \frac{1}{-2+2i\sqrt{3}}$$

Let us multiply and divide by -2 - 2 i√3, we get

$$= \frac{1}{-2+2\sqrt{3}i} \times \frac{-2-2\sqrt{3}i}{-2-2\sqrt{3}i}$$

$$= \frac{-2-2\sqrt{3}i}{(-2)^2-\left(2\sqrt{3}i\right)^2}$$

$$= \frac{-2-2\sqrt{3}i}{4-12i^2}$$

$$= \frac{-2-2\sqrt{3}i}{4-12(-1)}$$

$$= \frac{-2-2\sqrt{3}i}{16}$$

$$= \frac{-1}{8}\frac{-i\sqrt{3}}{8}$$

∴ The multiplicative inverse of $(1 + i\sqrt{3})^2$ is (-1-i√3)/8

(iii) 4 – 3i

Given:

4 – 3i

We know the multiplicative inverse of a complex number (Z) is Z^{-1} or 1/Z

So,

Z = 4 – 3i

$$Z^{-1} = \frac{1}{4-3i}$$

Let us multiply and divide by (4 + 3i), we get

$$= \frac{1}{4-3i} \times \frac{4+3i}{4+3i}$$

$$= \frac{4+3i}{4^2-(3i)^2}$$

$$= \frac{4+3i}{16-9i^2}$$

$$= \frac{4+3i}{16-9(-1)}$$

$$= \frac{4+3i}{25}$$

∴ The multiplicative inverse of (4 – 3i) is (4 + 3i)/25

(iv) √5 + 3i

Given:

√5 + 3i

We know the multiplicative inverse of a complex number (Z) is Z^{-1} or 1/Z

So,

Z = √5 + 3i

$$Z^{-1} = \frac{1}{\sqrt{5}+3i}$$

Let us multiply and divide by (√5 – 3i)

$$= \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$$

$$= \frac{\sqrt{5}-3i}{\left(\sqrt{5}\right)^2-(3i)^2}$$

$$= \frac{\sqrt{5}-3i}{5-9i^2}$$

$$= \frac{\sqrt{5}-3i}{5-9(-1)}$$

$$= \frac{\sqrt{5}-3i}{14}$$

∴ The multiplicative inverse of (√5 + 3i) is (√5 – 3i)/14

5. If $z_1 = 2 – i$, $z_2 = 1 + i$, find

$$\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$$

Solution:

Given:

$z_1 = (2 – i)$ and $z_2 = (1 + i)$

We know that, |a/b| = |a| / |b|

So,

$$\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right| = \frac{|z_1+z_2+1|}{|z_1-z_2+i|}$$

$$= \frac{|2-i+1+i+1|}{|2-i-(1+i)+i|}$$

$$= \frac{|4|}{|1-i|}$$

We know, |a + ib| is $\sqrt{a^2+b^2}$

So now,

$$= \frac{\sqrt{4^2+0^2}}{\sqrt{1^2+(-1)^2}}$$

$$= \frac{4}{\sqrt{2}}$$

$$= 2\sqrt{2}$$

∴ The value of $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$ is $2\sqrt{2}$

6. If z_1 = (2 – i), z_2 = (-2 + i), find

$$(\mathbf{i})\mathbf{Re}\left(\frac{\mathbf{z_1 z_2}}{\bar{\mathbf{z_1}}}\right)$$

$$(\mathbf{ii})\mathbf{Im}\left(\frac{\mathbf{1}}{\mathbf{z_1}\bar{\mathbf{z_1}}}\right)$$

Solution:

Given:

$z_1 = (2 – i)$ and $z_2 = (-2 + i)$

$(\mathbf{i})\mathbf{Re}\left(\frac{\mathbf{z_1 z_2}}{\bar{\mathbf{z_1}}}\right)$

We shall rationalise the denominator, we get

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{z_1 z_2}{z_1} \times \frac{z_1}{z_1}$$

$$= \frac{(z_1)^2 z_2}{z_1 z_1}$$

$$= \frac{(2-i)^2(-2+i)}{|z_1|^2} [since, z\bar{z} = |z|^2]$$

$$= \frac{(2^2 + i^2 - 2 \times 2 \times i)(-2+i)}{|2-i|^2}$$

$$= \frac{(4-1-4i)(-2+i)}{2^2+(-1)^2}$$

$$= \frac{(3-4i)(-2+i)}{4+i}$$

$$= \frac{3(-2+i)-4i(-2+i)}{4+i}$$

$$= \frac{-6+3i+8i+4}{5}$$

$$= \frac{-2+11i}{5}$$

$$(\mathrm{i})\mathrm{Re}\left(\frac{\mathbf{z_1 z_2}}{\bar{\mathbf{z_1}}}\right)$$

We shall rationalise the denominator, we get

$$\begin{aligned}
\frac{z_1 z_2}{\bar{z_1}} &= \frac{z_1 z_2}{\bar{z_1}} \times \frac{z_1}{z_1} \\
&= \frac{(z_1)^2 z_2}{\bar{z_1} z_1} \\
&= \frac{(2-i)^2(-2+i)}{|z_1|^2} [since, z\bar{z} = |z|^2] \\
&= \frac{(2^2+i^2-2\times 2\times i)(-2+i)}{|2-i|^2} \\
&= \frac{(4-1-4i)(-2+i)}{2^2+(-1)^2} \\
&= \frac{(3-4i)(-2+i)}{4+i} \\
&= \frac{3(-2+i)-4i(-2+i)}{4+i} \\
&= \frac{-6+3i+8i+4}{5} \\
&= \frac{-2+11i}{5}
\end{aligned}$$

7. Find the modulus of [(1 + i)/(1 – i)] – [(1 – i)/(1 + i)]

Solution:

Given:[(1 + i)/(1 – i)] – [(1 – i)/(1 + i)]

So,

Z = [(1 + i)/(1 – i)] – [(1 – i)/(1 + i)]

Let us simplify, we get

= [(1+i) (1+i) – (1-i) (1-i)] / (1^2 – i^2)

= [1^2 + i^2 + 2(1)(i) – (1^2 + i^2 – 2(1)(i))] / (1 – (-1)) [Since, i^2 = -1]

= 4i/2

= 2i

We know that for a complex number Z = (a+ib) it's magnitude is given by |z| = $\sqrt{(a^2 + b^2)}$

So,

|Z| = $\sqrt{(0^2 + 2^2)}$

= 2

∴ The modulus of [(1 + i)/(1 – i)] – [(1 – i)/(1 + i)] is 2.

8. If x + iy = (a+ib)/(a-ib), prove that $x^2 + y^2 = 1$

Solution:

Given:

x + iy = (a+ib)/(a-ib)

We know that for a complex number Z = (a+ib) it's magnitude is given by |z| = $\sqrt{(a^2 + b^2)}$

So,

|a/b| is |a| / |b|

Applying Modulus on both sides we get,

$$|x+iy| = \left|\frac{a+ib}{a-ib}\right|$$

$$\sqrt{x^2+y^2} = \frac{|a+ib|}{|a-ib|}$$

$$= \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+(-b)^2}}$$

$$= \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}}$$

$$= 1$$

Squaring on both sides we get,

$$\left(\sqrt{x^2+y^2}\right)^2 = 1^2$$

$x^2+y^2=1$

∴ Hence Proved.

9. Find the least positive integral value of n for which $[(1+i)/(1-i)]^n$ is real.

Solution:

Given:$[(1+i)/(1-i)]^n$

$Z = [(1+i)/(1-i)]^n$

Now let us multiply and divide by (1+i), we get

$$= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{(1+i)^2}{1^2-i^2}$$

$$= \frac{1^2+i^2+2(1)(i)}{1-(-1)}$$

$$= \frac{1-1+2i}{2}$$

$$= \frac{2i}{2}$$

= i [which is not real]

For n = 2, we have$[(1+i)/(1-i)]^2 = i^2$

= -1 [which is real]

So, the smallest positive integral 'n' that can make $[(1+i)/(1-i)]^n$ real is 2.

∴ The smallest positive integral value of 'n' is 2.

10. Find the real values of θ for which the complex number (1 + i cos θ) / (1 – 2i cos θ) is purely real.

Solution:

Given:

(1 + i cos θ) / (1 – 2i cos θ)

Z = (1 + i cos θ) / (1 – 2i cos θ)

Let us multiply and divide by (1 + 2i cos θ)

$$= \frac{1+icos\theta}{1-2icos\theta} \times \frac{1+2icos\theta}{1+2icos\theta}$$

$$= \frac{1(1+2icos\theta)+icos\theta(1+2icos\theta)}{1^2-(2icos\theta)^2}$$

$$= \frac{1+2icos\theta+icos\theta+2i^2\cos^2\theta}{1-4i^2\cos^2\theta}$$

$$= \frac{1+3icos\theta+2(-1)\cos^2\theta}{1-4(-1)\cos^2\theta}$$

$$= \frac{1-2\cos^2\theta+3icos\theta}{1+4\cos^2\theta}$$

For a complex number to be purely real, the imaginary part should be equal to zero.

So,

$$\frac{3cos\theta}{1+4\cos^2\theta} = 0$$

3cos θ = 0 (since, $1 + 4\cos^2\theta \geq 1$)

cos θ = 0

cos θ = cos π/2

θ = [(2n+1)π] / 2, for n ∈ Z

= 2nπ ± π/2, for n ∈ Z

∴ The values of θ to get the complex number to be purely real is 2nπ ± π/2, for n ∈ Z

11. Find the smallest positive integer value of n for which $(1+i)^n / (1-i)^{n-2}$ is a real number.

Solution:

Given:

$(1+i)^n / (1-i)^{n-2}$

$Z = (1+i)^n / (1-i)^{n-2}$

Let us multiply and divide by $(1 - i)^2$

$$= \frac{(1+i)^n}{(1-i)^{n-2}} \times \frac{(1-i)^2}{(1-i)^2}$$

$$= \left(\frac{1+i}{1-i}\right)^n \times (1-i)^2$$

$$= \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^n \times \left(1^2 + i^2 - 2(1)(i)\right)$$

$$= \left(\frac{(1+i)^2}{1^2-i^2}\right)^n \times (1 + i^2 - 2i)$$

$$= \left(\frac{1^2+i^2+2(1)(i)}{1-(-1)}\right)^n \times (1 + (-1) - 2i)$$

$$= \left(\frac{1-1+2i}{2}\right)^n \times (-2i)$$

$$= \left(\frac{2i}{2}\right)^n \times (-2i)$$

$$= i^n \times (-2i)$$

$$= -2i^{n+1}$$

For n = 1,

Z = $-2i^{1+1}$

= $-2i^2$

= 2, which is a real number.

∴ The smallest positive integer value of n is 1.

12. If $[(1+i)/(1-i)]^3 - [(1-i)/(1+i)]^3 = x + iy$, find (x, y)

Solution:

Given: $[(1+i)/(1-i)]^3 - [(1-i)/(1+i)]^3 = x + iy$

Let us rationalize the denominator, we get

$$\left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^3-\left(\frac{1-i}{1+i}\times\frac{1-i}{1-i}\right)^3=x+iy$$

$$\left(\frac{(1+i)^2}{1^2-i^2}\right)^3-\left(\frac{(1-i)^2}{1^2-i^2}\right)^3=x+iy$$

$$\left(\frac{1^2+i^2+2(1)(i)}{1-(-1)}\right)^3-\left(\frac{1^2+i^2-2(1)(i)}{1-(-1)}\right)^3=x+iy$$

$$\left(\frac{1-1+2i}{2}\right)^3-\left(\frac{1-1-2i}{2}\right)^3=x+iy$$

$$\left(\frac{2i}{2}\right)^3-\left(\frac{-2i}{2}\right)^3=x+iy$$

$i^3-(-i)^3 = x + iy$

$2i^3 = x + iy$

$2i^2.i = x + iy$

2(-1)I = x + iy

-2i = x + iy

Equating Real and Imaginary parts on both sides we get x = 0 and y = -2

∴ The values of x and y are 0 and -2.

13. If $(1+i)^2$ / (2-i) = x + iy, find x + y

Solution:

Given:

$(1+i)^2 / (2-i) = x + iy$

Upon expansion we get,

$$\frac{1^2+i^2+2(1)(i)}{2-i} = x + iy$$

$$\frac{1+(-1)+2i}{2-i} = x + iy$$

$$\frac{2i}{2-i} = x + iy$$

Now, let us multiply and divide by (2+i), we get

$$\frac{2i}{2-i} \times \frac{2+i}{2+i} = x + iy$$

$$\frac{4i+2i^2}{2^2-i^2} = x + iy$$

$$\frac{2(-1)+4i}{4-(-1)} = x + iy$$

$$\frac{-2+4i}{5} = x + iy$$

Let us equate real and imaginary parts on both sides we get,

x = -2/5 and y = 4/5

so,

x + y = -2/5 + 4/5

= (-2+4)/5

= 2/5

∴ The value of (x + y) is 2/5

EXERCISE 13.3 PAGE NO: 13.39

1. Find the square root of the following complex numbers.

(i) – 5 + 12i

(ii) -7 – 24i

(iii) 1 – i

(iv) – 8 – 6i

(v) 8 – 15i

(vi) -11 – 60√-1

(vii) 1 + 4√-3

(viii) 4i

(ix) -i

Solution:

$$if\ b > 0, \sqrt{a+ib} = \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right]$$

$$if\ b < 0, \sqrt{a+ib} = \pm\left[\left(\frac{a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-a+\sqrt{a^2+b^2}}{2}\right)^{\frac{1}{2}}\right]$$

(i) – 5 + 12i

Given:

– 5 + 12i

We know, Z = a + ib

So, √(a + ib) = √(-5+12i)

Here, b > 0

Let us simplify now,

$$\sqrt{-5+12i} = \pm\left[\left(\frac{-5+\sqrt{(-5)^2+12^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+\sqrt{(-5)^2+12^2}}{2}\right)^{\frac{1}{2}}\right] \text{ [Since, b > 0]}$$

$$= \pm\left[\left(\frac{-5+\sqrt{25+144}}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+\sqrt{25+144}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-5+\sqrt{169}}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+\sqrt{169}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-5+13}{2}\right)^{\frac{1}{2}} + i\left(\frac{5+13}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{8}{2}\right)^{\frac{1}{2}} + i\left(\frac{18}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[4^{\frac{1}{2}} + i9^{\frac{1}{2}}\right]$$

$$= \pm[2+3i]$$

∴ Square root of (– 5 + 12i) is **±**[2 + 3i]

(ii) -7 – 24i

Given:

-7 – 24i

We know, Z = -7 – 24i

So, √(a + ib) = √(-7 – 24i)

Here, b < 0

Let us simplify now,

$$\sqrt{-7-24i} = \pm\left[\left(\frac{-7+\sqrt{(-7)^2+(-24)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+\sqrt{(-7)^2+(-24)^2}}{2}\right)^{\frac{1}{2}}\right] \text{[Since, b < 0]}$$

$$= \pm\left[\left(\frac{-7+\sqrt{49+576}}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+\sqrt{49+576}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-7+\sqrt{625}}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+\sqrt{625}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-7+25}{2}\right)^{\frac{1}{2}} - i\left(\frac{7+25}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{18}{2}\right)^{\frac{1}{2}} - i\left(\frac{32}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm[9^{1/2} - i16^{1/2}]$$

$$= \pm[3-4i]$$

∴ Square root of (-7 – 24i) is **±** [3 – 4i]

(iii) 1 – i

Given:

1 – i

We know, Z = (1 – i)

So, √(a + ib) = √(1 – i)

Here, b < 0

Let us simplify now,

$$\sqrt{1-i} = \pm\left[\left(\frac{1+\sqrt{(1)^2+(-1)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-1+\sqrt{(1)^2+(-1)^2}}{2}\right)^{\frac{1}{2}}\right] \text{[Since, b < 0]}$$

$$= \pm\left[\left(\frac{1+\sqrt{1+1}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-1+\sqrt{1+1}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{1+\sqrt{2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-1+\sqrt{2}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\sqrt{\frac{\sqrt{2}+1}{2}}\right) - i\left(\sqrt{\frac{\sqrt{2}-1}{2}}\right)\right]$$

∴ Square root of (1 – i) is **±** [(√(√2+1)/2) – i (√(√2-1)/2)]

(iv) -8 -6i

Given:

-8 -6i

We know, Z = -8 -6i

So, √(a + ib) = -8 -6i

Here, b < 0

Let us simplify now,

$$\sqrt{-8-6i} = \pm\left[\left(\frac{-8+\sqrt{(-8)^2+(-6)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+\sqrt{(-8)^2+(-6)^2}}{2}\right)^{\frac{1}{2}}\right] \text{[Since, b < 0]}$$

$$= \pm\left[\left(\frac{-8+\sqrt{64+36}}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+\sqrt{64+36}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-8+\sqrt{100}}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+\sqrt{100}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-8+10}{2}\right)^{\frac{1}{2}} - i\left(\frac{8+10}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{2}{2}\right)^{\frac{1}{2}} - i\left(\frac{18}{2}\right)^{\frac{1}{2}}\right]$$

= [$1^{1/2}$ – i $9^{1/2}$]

= **±** [1 – 3i]

∴ Square root of (-8 -6i) is **±** [1 – 3i]

(v) 8 – 15i

Given:

8 – 15i

We know, Z = 8 – 15i

So, √(a + ib) = 8 – 15i

Here, b < 0

Let us simplify now,

$$\sqrt{8-15i} = \pm\left[\left(\frac{8+\sqrt{8^2+(-15)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+\sqrt{8^2+(-15)^2}}{2}\right)^{\frac{1}{2}}\right] \text{ [Since, b < 0]}$$

$$= \pm\left[\left(\frac{8+\sqrt{64+225}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+\sqrt{64+225}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{8+\sqrt{289}}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+\sqrt{289}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{8+17}{2}\right)^{\frac{1}{2}} - i\left(\frac{-8+17}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{25}{2}\right)^{\frac{1}{2}} - i\left(\frac{9}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\frac{5}{\sqrt{2}} - \frac{i3}{\sqrt{2}}\right]$$

$= \pm 1/\sqrt{2}\ (5 - 3i)$

∴ Square root of (8 – 15i) is **±** 1/√2 (5 – 3i)

(vi) -11 – 60√-1

Given:

-11 – 60√-1

We know, Z = -11 – 60√-1

So, √(a + ib) = -11 – 60√-1

= -11 – 60i

Here, b < 0

Let us simplify now,

$$\sqrt{-11-60i} = \pm\left[\left(\frac{-11+\sqrt{(-11)^2+(-60)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+\sqrt{(-11)^2+(60)^2}}{2}\right)^{\frac{1}{2}}\right] \text{[Since, b < 0]}$$

$$= \pm\left[\left(\frac{-11+\sqrt{121+3600}}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+\sqrt{121+3600}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-11+\sqrt{3721}}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+\sqrt{3721}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{-11+61}{2}\right)^{\frac{1}{2}} - i\left(\frac{11+61}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{50}{2}\right)^{\frac{1}{2}} - i\left(\frac{72}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[25^{\frac{1}{2}} - i36^{\frac{1}{2}}\right]$$

$$= \pm(5-6i)$$

∴ Square root of (-11 – 60√-1) is **±** (5 – 6i)

(vii) 1 + 4√-3

Given:

1 + 4√-3

We know, Z = 1 + 4√-3

So, √(a + ib) = 1 + 4√-3

= 1 + 4(√3) (√-1)

= 1 + 4√3i

Here, b > 0

Let us simplify now,

$$\sqrt{1+4\sqrt{3}i} = \pm\left[\left(\frac{1+\sqrt{1^2+(4\sqrt{3})^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+\sqrt{1^2+(4\sqrt{3})^2}}{2}\right)^{\frac{1}{2}}\right] \text{[Since, b > 0]}$$

$$= \pm\left[\left(\frac{1+\sqrt{1+48}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+\sqrt{1+48}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{1+\sqrt{49}}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+\sqrt{49}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{1+7}{2}\right)^{\frac{1}{2}} + i\left(\frac{-1+7}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{8}{2}\right)^{\frac{1}{2}} + i\left(\frac{6}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[4^{\frac{1}{2}} + i3^{\frac{1}{2}}\right]$$

$$= \pm\left[2+\sqrt{3}i\right]$$

∴ Square root of (1 + 4√-3) is **±** (2 + √3i)

(viii) 4i

Given:

4i

We know, Z = 4i

So, √(a + ib) = 4i

Here, b > 0

Let us simplify now,

$$\sqrt{4i} = \pm\left[\left(\frac{0+\sqrt{0^2+4^2}}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+\sqrt{0^2+4^2}}{2}\right)^{\frac{1}{2}}\right] \text{[Since, b > 0]}$$

$$= \pm\left[\left(\frac{0+\sqrt{0+16}}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+\sqrt{0+16}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{0+\sqrt{16}}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+\sqrt{16}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{0+4}{2}\right)^{\frac{1}{2}} + i\left(\frac{0+4}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{4}{2}\right)^{\frac{1}{2}} + i\left(\frac{4}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[2^{\frac{1}{2}} + i2^{\frac{1}{2}}\right]$$

$$= \pm\left[\sqrt{2} + \sqrt{2}i\right]$$

$$= \pm\sqrt{2}\,(1 + i)$$

∴ Square root of 4i is **±** √2 (1 + i)

(ix) –i

Given:

-i

We know, Z = -i

So, √(a + ib) = -i

Here, b < 0

Let us simplify now,

$$\sqrt{-i} = \pm\left[\left(\frac{0+\sqrt{0^2+(-1)^2}}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+\sqrt{0^2+(-1)^2}}{2}\right)^{\frac{1}{2}}\right] \text{[Since, b < 0]}$$

$$= \pm\left[\left(\frac{0+\sqrt{0+1}}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+\sqrt{0+1}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{0+\sqrt{1}}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+\sqrt{1}}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{0+1}{2}\right)^{\frac{1}{2}} - i\left(\frac{0+1}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\left(\frac{1}{2}\right)^{\frac{1}{2}} - i\left(\frac{1}{2}\right)^{\frac{1}{2}}\right]$$

$$= \pm\left[\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right]$$

$$= \pm 1/\sqrt{2}\ (1 - i)$$

∴ Square root of –i is ± 1/√2 (1 – i)

EXERCISE 13.4 PAGE NO: 13.57

1. Find the modulus and arguments of the following complex numbers and hence express each of them in the polar form:

(i) 1 + i

(ii) √3 + i

(iii) 1 – i

(iv) (1 – i) / (1 + i)

(v) 1/(1 + i)

(vi) (1 + 2i) / (1 – 3i)

(vii) sin 120° – i cos 120°

(viii) -16 / (1 + i√3)

Solution:

We know that the polar form of a complex number Z = x + iy is given by Z = |Z| (cos θ + i sin θ)

Where,

|Z| = modulus of complex number = $\sqrt{(x^2 + y^2)}$

θ = arg (z) = argument of complex number = $\tan^{-1}$ (|y| / |x|)

(i) 1 + i

Given: Z = 1 + i

So now,

|Z| = $\sqrt{(x^2 + y^2)}$

= $\sqrt{(1^2 + 1^2)}$

= √(1 + 1)

= √2

θ = $\tan^{-1}$ (|y| / |x|)

= $\tan^{-1}$ (1 / 1)

= $\tan^{-1}$ 1

Since x > 0, y > 0 complex number lies in 1^{st} quadrant and the value of θ is $0^0 \le \theta \le 90^0$.

θ = π/4

Z = √2 (cos (π/4) + i sin (π/4))

∴ Polar form of (1 + i) is √2 (cos (π/4) + i sin (π/4))

(ii) √3 + i

Given: $Z = \sqrt{3} + i$

So now,

$|Z| = \sqrt{(x^2 + y^2)}$

$= \sqrt{((\sqrt{3})^2 + 1^2)}$

$= \sqrt{(3 + 1)}$

$= \sqrt{4}$

$= 2$

$\theta = \tan^{-1} (|y| / |x|)$

$= \tan^{-1} (1 / \sqrt{3})$

Since $x > 0$, $y > 0$ complex number lies in 1st quadrant and the value of θ is $0^0 \le \theta \le 90^0$.

$\theta = \pi/6$

$Z = 2 (\cos (\pi/6) + i \sin (\pi/6))$

∴ Polar form of $(\sqrt{3} + i)$ is $2 (\cos (\pi/6) + i \sin (\pi/6))$

(iii) $1 – i$

Given: $Z = 1 – i$

So now,

$|Z| = \sqrt{(x^2 + y^2)}$

$= \sqrt{(1^2 + (-1)^2)}$

$= \sqrt{(1 + 1)}$

$= \sqrt{2}$

$\theta = \tan^{-1} (|y| / |x|)$

$= \tan^{-1} (1 / 1)$

= $\tan^{-1} 1$

Since x > 0, y < 0 complex number lies in 4th quadrant and the value of θ is $-90^0 \le \theta \le 0^0$.

θ = -π/4

Z = √2 (cos (-π/4) + i sin (-π/4))

= √2 (cos (π/4) – i sin (π/4))

∴ Polar form of (1 – i) is √2 (cos (π/4) – i sin (π/4))

(iv) (1 – i) / (1 + i)

Given: Z = (1 – i) / (1 + i)

Let us multiply and divide by (1 – i), we get

$$
\begin{aligned}
z &= \frac{1-i}{1+i} \times \frac{1-i}{1-i} \\
&= \frac{(1-i)^2}{1^2-i^2} \\
&= \frac{1^2+i^2-2(1)(i)}{1-(-1)} \\
&= \frac{1+(-1)-2i}{2} \\
&= \frac{-2i}{2}
\end{aligned}
$$

= 0 – i

So now,

$|Z| = \sqrt{(x^2 + y^2)}$

$= \sqrt{(0^2 + (-1)^2)}$

= √(0 + 1)

= √1

θ = $\tan^{-1}$ (|y| / |x|)

$= \tan^{-1} (1 / 0)$

$= \tan^{-1} \infty$

Since $x \geq 0$, $y < 0$ complex number lies in 4th quadrant and the value of θ is $-90^0 \leq \theta \leq 0^0$.

$\theta = -\pi/2$

$Z = 1 (\cos (-\pi/2) + i \sin (-\pi/2))$

$= 1 (\cos (\pi/2) - i \sin (\pi/2))$

∴ Polar form of $(1 - i) / (1 + i)$ is $1 (\cos (\pi/2) - i \sin (\pi/2))$

(v) $1/(1 + i)$

Given: $Z = 1 / (1 + i)$

Let us multiply and divide by $(1 - i)$, we get

$$z = \frac{1}{1+i} \times \frac{1-i}{1-i}$$
$$= \frac{1-i}{1^2-i^2}$$
$$= \frac{1-i}{1-(-1)}$$
$$= \frac{1-i}{2}$$

So now,

$|Z| = \sqrt{(x^2 + y^2)}$

$= \sqrt{((1/2)^2 + (-1/2)^2)}$

$= \sqrt{(1/4 + 1/4)}$

$= \sqrt{(2/4)}$

$= 1/\sqrt{2}$

$\theta = \tan^{-1} (|y| / |x|)$

$= \tan^{-1} ((1/2) / (1/2))$

$= \tan^{-1} 1$

Since $x > 0$, $y < 0$ complex number lies in 4^{th} quadrant and the value of θ is $-90^0 \le \theta \le 0^0$.

$\theta = -\pi/4$

$Z = 1/\sqrt{2} (\cos (-\pi/4) + i \sin (-\pi/4))$

$= 1/\sqrt{2} (\cos (\pi/4) – i \sin (\pi/4))$

∴ Polar form of $1/(1 + i)$ is $1/\sqrt{2} (\cos (\pi/4) – i \sin (\pi/4))$

(vi) (1 + 2i) / (1 – 3i)

Given: Z = (1 + 2i) / (1 – 3i)

Let us multiply and divide by (1 + 3i), we get

$$
\begin{aligned}
z &= \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} \\
&= \frac{1(1+3i)+2i(1+3i)}{1^2-(3i)^2} \\
&= \frac{1+3i+2i+6i^2}{1-9i^2} \\
&= \frac{1+5i+6(-1)}{1-9(-1)} \\
&= \frac{-5+5i}{10} \\
&= \frac{-1+i}{2}
\end{aligned}
$$

So now,

$|Z| = \sqrt{(x^2 + y^2)}$

$= \sqrt{((-1/2)^2 + (1/2)^2)}$

$= \sqrt{(1/4 + 1/4)}$

$= \sqrt{(2/4)}$

$= 1/\sqrt{2}$

$\theta = \tan^{-1} (|y| / |x|)$

$= \tan^{-1} ((1/2) / (1/2))$

$= \tan^{-1} 1$

Since $x < 0$, $y > 0$ complex number lies in 2nd quadrant and the value of θ is $90^0 \leq \theta \leq 180^0$.

$\theta = 3\pi/4$

$Z = 1/\sqrt{2} (\cos (3\pi/4) + i \sin (3\pi/4))$

∴ Polar form of $(1 + 2i) / (1 - 3i)$ is $1/\sqrt{2} (\cos (3\pi/4) + i \sin (3\pi/4))$

(vii) $\sin 120^\circ - i \cos 120^\circ$

Given: $Z = \sin 120^\circ - i \cos 120^\circ$

$= \sqrt{3}/2 - i (-1/2)$

$= \sqrt{3}/2 + i (1/2)$

So now,

$|Z| = \sqrt{(x^2 + y^2)}$

$= \sqrt{((\sqrt{3}/2)^2 + (1/2)^2)}$

$= \sqrt{(3/4 + 1/4)}$

$= \sqrt{(4/4)}$

$= \sqrt{1}$

$= 1$

$\theta = \tan^{-1} (|y| / |x|)$

$= \tan^{-1} ((1/2) / (\sqrt{3}/2))$

$= \tan^{-1} (1/\sqrt{3})$

Since x > 0, y > 0 complex number lies in 1st quadrant and the value of θ is $0^0 \le \theta \le 90^0$.

θ = π/6

Z = 1 (cos (π/6) + i sin (π/6))

∴ Polar form of √3/2 + i (1/2) is 1 (cos (π/6) + i sin (π/6))

(viii) -16 / (1 + i√3)

Given: Z = -16 / (1 + i√3)

Let us multiply and divide by (1 – i√3), we get

$$
\begin{aligned}
z &= \frac{-16}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}} \\
&= \frac{-16+i16\sqrt{3}}{1^2-(i\sqrt{3})^2} \\
&= \frac{-16+i16\sqrt{3}}{1-3i^2} \\
&= \frac{-16+i16\sqrt{3}}{1-3(-1)} \\
&= \frac{-16+i16\sqrt{3}}{4} \\
&= -4 + i\,4\sqrt{3}
\end{aligned}
$$

So now,

$|Z| = \sqrt{(x^2 + y^2)}$

$= \sqrt{((-4)^2 + (4\sqrt{3})^2)}$

= √(16 + 48)

= √(64)

= 8

$\theta = \tan^{-1}(|y| / |x|)$

$= \tan^{-1}((4\sqrt{3}) / 4)$

$= \tan^{-1} (\sqrt{3})$

Since $x < 0$, $y > 0$ complex number lies in 2^{nd} quadrant and the value of θ is $90^0 \leq \theta \leq 180^0$.

$\theta = 2\pi/3$

$Z = 8 (\cos (2\pi/3) + i \sin (2\pi/3))$

∴ Polar form of $-16 / (1 + i\sqrt{3})$ is $8 (\cos (2\pi/3) + i \sin (2\pi/3))$

2. Write $(i^{25})^3$ in polar form.

Solution:

Given: $Z = (i^{25})^3$

$= i^{75}$

$= i^{74}. i$

$= (i^2)^{37}. i$

$= (-1)^{37}. i$

$= (-1). i$

$= – i$

$= 0 – i$

So now,

$|Z| = \sqrt{(x^2 + y^2)}$

$= \sqrt{(0^2 + (-1)^2)}$

$= \sqrt{(0 + 1)}$

$= \sqrt{1}$

$\theta = \tan^{-1} (|y| / |x|)$

$= \tan^{-1} (1 / 0)$

$= \tan^{-1} \infty$

Since $x \geq 0$, $y < 0$ complex number lies in 4^{th} quadrant and the value of θ is $-90^0 \leq \theta \leq 0^0$.

$\theta = -\pi/2$

$Z = 1 (\cos (-\pi/2) + i \sin (-\pi/2))$

$= 1 (\cos (\pi/2) – i \sin (\pi/2))$

∴ Polar form of $(i^{25})^3$ is $1 (\cos (\pi/2) – i \sin (\pi/2))$

3. Express the following complex numbers in the form r (cos θ + i sin θ):

(i) 1 + i tan α

(ii) tan α – i

(iii) 1 – sin α + i cos α

(iv) (1 – i) / (cos π/3 + i sin π/3)

Solution:

(i) 1 + i tan α

Given: $Z = 1 + i \tan \alpha$

We know that the polar form of a complex number $Z = x + iy$ is given by $Z = |Z| (\cos \theta + i \sin \theta)$

Where,

$|Z|$ = modulus of complex number = $\sqrt{(x^2 + y^2)}$

$\theta = \arg (z)$ = argument of complex number = $\tan^{-1} (|y| / |x|)$

We also know that tan α is a periodic function with period π.

So α is lying in the interval $[0, \pi/2) \cup (\pi/2, \pi]$.

Let us consider case 1:

$\alpha \in [0, \pi/2)$

So now,

$|Z| = r = \sqrt{(x^2 + y^2)}$

$= \sqrt{(1^2 + \tan^2 \alpha)}$

$= \sqrt{(\sec^2 \alpha)}$

$= |\sec \alpha|$ since, $\sec \alpha$ is positive in the interval $[0, \pi/2)$

$\theta = \tan^{-1} (|y| / |x|)$

$= \tan^{-1} (\tan \alpha / 1)$

$= \tan^{-1} (\tan \alpha)$

$= \alpha$ since, $\tan \alpha$ is positive in the interval $[0, \pi/2)$

∴ Polar form is $Z = \sec \alpha (\cos \alpha + i \sin \alpha)$

Let us consider case 2:

$\alpha \in (\pi/2, \pi]$

So now,

$|Z| = r = \sqrt{(x^2 + y^2)}$

$= \sqrt{(1^2 + \tan^2 \alpha)}$

$= \sqrt{(\sec^2 \alpha)}$

$= |\sec \alpha|$

$= -\sec \alpha$ since, $\sec \alpha$ is negative in the interval $(\pi/2, \pi]$

$\theta = \tan^{-1} (|y| / |x|)$

$= \tan^{-1} (\tan \alpha / 1)$

$= \tan^{-1} (\tan \alpha)$

$= -\pi + \alpha$ since, $\tan \alpha$ is negative in the interval $(\pi/2, \pi]$

$\theta = -\pi + \alpha$ [since, θ lies in 4th quadrant]

$Z = -\sec\alpha\,(\cos(\alpha - \pi) + i\sin(\alpha - \pi))$

∴ Polar form is $Z = -\sec\alpha\,(\cos(\alpha - \pi) + i\sin(\alpha - \pi))$

(ii) $\tan\alpha - i$

Given: $Z = \tan\alpha - i$

We know that the polar form of a complex number $Z = x + iy$ is given by $Z = |Z|(\cos\theta + i\sin\theta)$

Where,

$|Z|$ = modulus of complex number = $\sqrt{(x^2 + y^2)}$

$\theta = \arg(z)$ = argument of complex number = $\tan^{-1}(|y| / |x|)$

We also know that $\tan\alpha$ is a periodic function with period π.

So α is lying in the interval $[0, \pi/2) \cup (\pi/2, \pi]$.

Let us consider case 1:

$\alpha \in [0, \pi/2)$

So now,

$|Z| = r = \sqrt{(x^2 + y^2)}$

$= \sqrt{(\tan^2\alpha + 1^2)}$

$= \sqrt{(\sec^2\alpha)}$

$= |\sec\alpha|$ since, $\sec\alpha$ is positive in the interval $[0, \pi/2)$

$= \sec\alpha$

$\theta = \tan^{-1}(|y| / |x|)$

$= \tan^{-1}(1/\tan\alpha)$

$= \tan^{-1}(\cot\alpha)$ since, $\cot\alpha$ is positive in the interval $[0, \pi/2)$

$= \alpha - \pi/2$ [since, θ lies in 4^{th} quadrant]

$Z = \sec \alpha (\cos (\alpha - \pi/2) + i \sin (\alpha - \pi/2))$

∴ Polar form is $Z = \sec \alpha (\cos (\alpha - \pi/2) + i \sin (\alpha - \pi/2))$

Let us consider case 2:

$\alpha \in (\pi/2, \pi]$

So now,

$|Z| = r = \sqrt{(x^2 + y^2)}$

$= \sqrt{(\tan^2 \alpha + 1^2)}$

$= \sqrt{(\sec^2 \alpha)}$

$= |\sec \alpha|$

$= - \sec \alpha$ since, $\sec \alpha$ is negative in the interval $(\pi/2, \pi]$

$\theta = \tan^{-1} (|y| / |x|)$

$= \tan^{-1} (1/\tan \alpha)$

$= \tan^{-1} (\cot \alpha)$

$= \pi/2 + \alpha$ since, $\cot \alpha$ is negative in the interval $(\pi/2, \pi]$

$\theta = \pi/2 + \alpha$ [since, θ lies in 3^{th} quadrant]

$Z = -\sec \alpha (\cos (\pi/2 + \alpha) + i \sin (\pi/2 + \alpha))$

∴ Polar form is $Z = -\sec \alpha (\cos (\pi/2 + \alpha) + i \sin (\pi/2 + \alpha))$

(iii) $1 - \sin \alpha + i \cos \alpha$

Given: $Z = 1 - \sin \alpha + i \cos \alpha$

By using the formulas,

$\text{Sin}^2 \theta + \cos^2 \theta = 1$

Sin 2θ = 2 sin θ cos θ

Cos 2θ = $\cos^2 θ – \sin^2 θ$

So,

z= $(\sin^2(α/2) + \cos^2(α/2) – 2 \sin(α/2) \cos(α/2)) + i (\cos^2(α/2) – \sin^2(α/2))$

= $(\cos(α/2) – \sin(α/2))^2 + i (\cos^2(α/2) – \sin^2(α/2))$

We know that the polar form of a complex number Z = x + iy is given by Z = |Z| (cos θ + i sin θ)

Where,

|Z| = modulus of complex number = $\sqrt{(x^2 + y^2)}$

θ = arg (z) = argument of complex number = $\tan^{-1}$ (|y| / |x|)

Now,

$$
\begin{aligned}
|z| &= \sqrt{(1-sin\alpha)^2+\cos^2\alpha} \\
&= \sqrt{1+\sin^2\alpha-2sin\alpha+\cos^2\alpha} \\
&= \sqrt{1+1-2sin\alpha} \\
&= \sqrt{(2)(1-sin\alpha)} \\
&= \sqrt{(2)\left(\sin^2\left(\frac{\alpha}{2}\right)+\cos^2\left(\frac{\alpha}{2}\right)-2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)\right)} \\
&= \sqrt{(2)\left(\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\right)^2} \\
&= \left|\sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\right)\right|
\end{aligned}
$$

$$
\begin{aligned}
\theta &= \tan^{-1}\left(\frac{\cos^2\left(\frac{\alpha}{2}\right)-\sin^2\left(\frac{\alpha}{2}\right)}{\left(\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\right)^2}\right) \\
&= \tan^{-1}\left(\frac{\left(\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\right)\left(\cos\left(\frac{\alpha}{2}\right)+\sin\left(\frac{\alpha}{2}\right)\right)}{\left(\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)\right)^2}\right) \\
&= \tan^{-1}\left(\frac{\cos\left(\frac{\alpha}{2}\right)+\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)-\sin\left(\frac{\alpha}{2}\right)}\right) \\
&= \tan^{-1}\left(\frac{\cos\left(\frac{\alpha}{2}\right)\left(1+\tan\left(\frac{\alpha}{2}\right)\right)}{\cos\left(\frac{\alpha}{2}\right)\left(1-\tan\left(\frac{\alpha}{2}\right)\right)}\right) \\
&= \tan^{-1}\left(\frac{\tan\left(\frac{\pi}{4}\right)+\tan\left(\frac{\alpha}{2}\right)}{1-\tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\alpha}{2}\right)}\right) \\
&= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right)
\end{aligned}
$$

We know that sine and cosine functions are periodic with period 2π

Here we have 3 intervals:

$0 \le \alpha \le \pi/2$

$\pi/2 \le \alpha \le 3\pi/2$

$3\pi/2 \le \alpha \le 2\pi$

Let us consider case 1:

In the interval $0 \le \alpha \le \pi/2$

Cos $(\alpha/2) > \sin(\alpha/2)$ and also $0 < \pi/4 + \alpha/2 < \pi/2$

So,

$$|z| = \left|\sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)\right|$$
$$= \sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)$$
$$\theta = \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right)$$
$$= \pi/4 + \alpha/2 \text{ [since, } \theta \text{ lies in } 1^{st} \text{ quadrant]}$$

∴ Polar form is Z = √2 (cos (α/2) – sin (α/2)) (cos (π/4 + α/2) + i sin (π/4 + α/2))

Let us consider case 2:

In the interval $\pi/2 \le \alpha \le 3\pi/2$

Cos $(\alpha/2) < \sin(\alpha/2)$ and also $\pi/2 < \pi/4 + \alpha/2 < \pi$

So,

Since, $(1 - \sin\alpha) > 0$ and $\cos\alpha < 0$ [Z lies in 4^{th} quadrant]

$= \alpha/2 - 3\pi/4$

∴ Polar form is Z = –√2 (cos (α/2) – sin (α/2)) (cos (α/2 – 3π/4) + i sin (α/2 – 3π/4))

Let us consider case 3:

In the interval $3\pi/2 \le \alpha \le 2\pi$

Cos $(\alpha/2) < \sin(\alpha/2)$ and also $\pi < \pi/4 + \alpha/2 < 5\pi/4$

So,

$$|z| = \left|\sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)\right|$$
$$= -\sqrt{2}\left(\cos\left(\frac{\alpha}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right)$$
$$= \sqrt{2}\left(\sin\left(\frac{\alpha}{2}\right) - \cos\left(\frac{\alpha}{2}\right)\right)$$

θ = $\tan^{-1}$ (tan (π/4 + α/2))

= π – (π/4 + α/2) [since, θ lies in 1st quadrant and tan's period is π]

= α/2 – 3π/4

∴ Polar form is Z = –√2 (cos (α/2) – sin (α/2)) (cos (α/2 – 3π/4) + i sin (α/2 – 3π/4))

(iv) (1 – i) / (cos π/3 + i sin π/3)

Given: Z = (1 – i) / (cos π/3 + i sin π/3)

Let us multiply and divide by (1 – i√3), we get

$$Z = \frac{1-i}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}$$
$$= 2 \times \frac{1-i}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$
$$= 2 \times \frac{1+i^2\sqrt{3}-i(1+\sqrt{3})}{1-i^2 3}$$
$$= 2 \times \frac{(1+(-\sqrt{3})-i(1+\sqrt{3}))}{1-(-3)}$$
$$= 2 \times \frac{(1-\sqrt{3})-i(1+\sqrt{3})}{4}$$
$$= \frac{(1-\sqrt{3})-i(1+\sqrt{3})}{2}$$

We know that the polar form of a complex number Z = x + iy is given by Z = |Z| (cos θ + i sin θ)

Where,

|Z| = modulus of complex number = $\sqrt{(x^2 + y^2)}$

θ = arg (z) = argument of complex number = $\tan^{-1}$ (|y| / |x|)

Now,

$$|z| = \sqrt{\left(\frac{1-\sqrt{3}}{2}\right)^2 + \left(\frac{-1-\sqrt{3}}{2}\right)^2}$$
$$= \sqrt{\frac{1+3-2\sqrt{3}+1+2+2\sqrt{3}}{4}}$$
$$= \sqrt{\frac{8}{4}}$$
$$= \sqrt{2}$$

$$\theta = \tan^{-1}\left(\left|\frac{\frac{1+\sqrt{3}}{2}}{\frac{1-\sqrt{3}}{2}}\right|\right)$$
$$= \tan^{-1}\left(\left|\frac{1+\sqrt{3}}{1-\sqrt{3}}\right|\right)$$
$$= \tan^{-1}\left(\left|\frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}\right|\right)$$
$$= \tan^{-1}\left(\left|\frac{1+3+2\sqrt{3}}{1-3}\right|\right)$$
$$= \tan^{-1}\left(\frac{4+2\sqrt{3}}{2}\right)$$

Since x < 0, y < 0 complex number lies in 3rd quadrant and the value of θ is $180^0 \leq \theta \leq -90^0$.

= $\tan^{-1}$ (2 + √3)

= -7π/12

Z = √2 (cos (-7π/12) + i sin (-7π/12))

= √2 (cos (7π/12) – i sin (7π/12))

∴ Polar form of (1 – i) / (cos π/3 + i sin π/3) is √2 (cos (7π/12) – i sin (7π/12))

4. If z_1 and z_2 are two complex number such that $|z_1| = |z_2|$ and arg (z_1) + arg (z_2) = π, then show that

Solution:

Given:

$|z_1| = |z_2|$ and arg (z_1) + arg (z_2) = π

Let us assume arg (z_1) = θ

arg (z_2) = π – θ

We know that in the polar form, z = |z| (cos θ + i sin θ)

$z_1 = |z_1|$ (cos θ + i sin θ) ………… (i)

$z_2 = |z_2|$ (cos (π – θ) + i sin (π – θ))

$= |z_2|$ (-cos θ + i sin θ)

$= – |z_2|$ (cos θ – i sin θ)

Now let us find the conjugate of

$\bar{z}_2$

$= – |z_2|$ (cos θ + i sin θ) …… (ii) (since, |‾‾‾‾‾‾Z2|=|Z2||Z2‾|=|Z2|)

Now,

z_1 /

$\bar{z}_2$

$= [|z_1|$ (cos θ + i sin θ)] / [-$|z_2|$ (cos θ + i sin θ)]

$= – |z_1| / |z_2|$ [since, $|z_1| = |z_2|$]

= -1

When we cross multiply we get,

$z_1 = –$

$\bar{z}_2$

Hence proved.

5. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, prove that arg (z_1/z_4) + arg (z_2/z_3) = 0

Solution:

Given:

$z_1 = \bar{z}_2$
$z_3 = \bar{z}_4$
We know that arg (z_1/z_2) = arg (z_1) – arg (z_2)
So,
arg (z_1/z_4) + arg (z_2/z_3) = arg (z_1) – arg (z_4) + arg (z_2) – arg (z_3)
= arg ($\bar{z}_2$) – arg (z_4) + arg (z_2) – arg ($\bar{z}_4$)
= [arg (z_2) + arg ($\bar{z}_2$)] – [arg (z_4) + arg ($\bar{z}_4$)]
= 0 – 0 [since, arg (z) + arg ($\bar{z}$) = 0]
= 0

Hence proved.

6. Express sin π/5 + i (1 – cos π/5) in polar form.

Solution:

Given:

Z = sin π/5 + i (1 – cos π/5)

By using the formula,

sin 2θ = 2 sin θ cos θ

1- cos 2θ = 2 $\sin^2$ θ

So,

Z = 2 sin π/10 cos π/10 + i (2 $\sin^2$ π/10)

= 2 sin π/10 (cos π/10 + i sin π/10)

∴ The polar form of sin π/5 + i (1 – cos π/5) is 2 sin π/10 (cos π/10 + i sin π/10)

Chapterwise RD Sharma Solutions for Class 11 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.