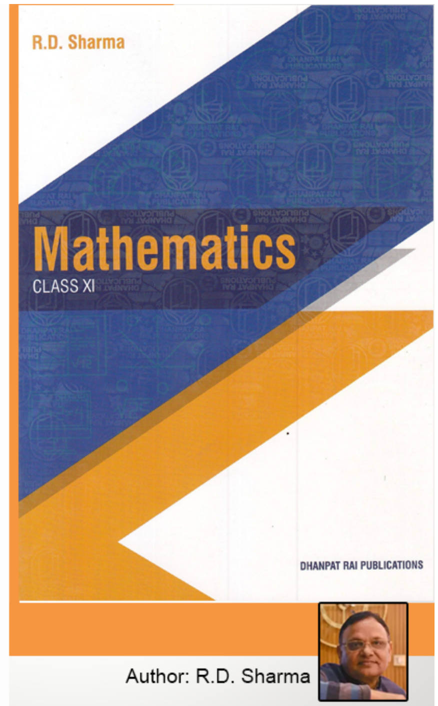
Class 11 - Chapter 26 Ellipse



RD Sharma Solutions for Class 11 Maths Chapter 26–Ellipse

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EXERCISE 26.1 PAGE NO: 26.22

1. Find the equation of the ellipse whose focus is (1, -2), the directrix 3x – 2y + 5 = 0 and eccentricity equal to 1/2.

Solution:

Given:

Focus = (1, -2)

Directrix = 3x – 2y + 5 = 0

Eccentricity = ½

Let P(x, y) be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is given as

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

So,

SP = ePM

$SP^2 = e^2PM^2$

$$(x - 1)^2 + (y - (-2))^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|3x - 2y + 5|}{\sqrt{3^2 + (-2)^2}}\right)^2$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 = \frac{1}{4} \times \frac{(|3x - 2y + 5|)^2}{9 + 4}$$

$$x^2 + y^2 - 2x + 4y + 5 = \frac{1}{52} \times (9x^2 + 4y^2 + 25 - 12xy - 20y + 30x)$$

Upon cross multiplying, we get

$52x^2 + 52y^2 – 104x + 208y + 260 = 9x^2 + 4y^2 – 12xy – 20y + 30x + 25$

$43x^2 + 48y^2 + 12xy – 134x + 228y + 235 = 0$

∴ The equation of the ellipse is $43x^2 + 48y^2 + 12xy - 134x + 228y + 235 = 0$

2. Find the equation of the ellipse in the following cases:

(i) focus is (0, 1), directrix is x + y = 0 and e = ½.

(ii) focus is (- 1, 1), directrix is x – y + 3 = 0 and e = ½.

(iii) focus is (- 2, 3), directrix is 2x + 3y + 4 = 0 and e = 4/5.

(iv) focus is (1, 2), directrix is 3x + 4y – 7 = 0 and e = ½.

Solution:

(i) focus is (0, 1), directrix is x + y = 0 and e = ½

Given:

Focus is (0, 1)

Directrix is x + y = 0

e = ½

Let P(x, y) be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is given as

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

So,

SP = ePM

$SP^2 = e^2PM^2$

$$(x-0)^2 + (y-1)^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|x+y|}{\sqrt{1^2+1^2}}\right)^2$$

$$x^2 + y^2 - 2y + 1 = \frac{1}{4} \times \frac{(|x+y|)^2}{1+1}$$

$$x^2 + y^2 - 2y + 1 = \frac{1}{8} \times (x^2 + y^2 + 2xy)$$

Upon cross multiplying, we get

$8x^2 + 8y^2 – 16y + 8 = x^2 + y^2 + 2xy$

$7x^2 + 7y^2 – 2xy – 16y + 8 = 0$

∴ The equation of the ellipse is $7x^2 + 7y^2 – 2xy – 16y + 8 = 0$

(ii) focus is (- 1, 1), directrix is x – y + 3 = 0 and e = ½

Given:

Focus is (- 1, 1)

Directrix is x – y + 3 = 0

e = ½

Let P(x, y) be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is given as

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

So,

SP = ePM

$SP^2 = e^2PM^2$

$$(x-(-1))^2 + (y-1)^2 = \left(\frac{1}{2}\right)^2 \left(\frac{|x-y+3|}{\sqrt{1^2+1^2}}\right)^2$$

$$x^2 + 2x + 1 + y^2 - 2y + 1 = \frac{1}{4} \times \frac{(|x-y+3|)^2}{1+1}$$

$$x^2 + y^2 + 2x - 2y + 2 = \frac{1}{8} \times (x^2 + y^2 + 9 - 2xy - 6y + 6x)$$

Upon cross multiplying, we get

$8x^2 + 8y^2 + 16x – 16y + 16 = x^2 + y^2 – 2xy + 6x – 6y + 9$

$7x^2 + 7y^2 + 2xy + 10x – 10y + 7 = 0$

∴ The equation of the ellipse is $7x^2 + 7y^2 + 2xy + 10x – 10y + 7 = 0$

(iii) focus is (- 2, 3), directrix is 2x + 3y + 4 = 0 and e = 4/5

Focus is (- 2, 3)

Directrix is 2x + 3y + 4 = 0

e = 4/5

Let P(x, y) be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is given as

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

So,

SP = ePM

$SP^2 = e^2PM^2$

$$(x-(-2))^2 + (y-3)^2 = \left(\frac{4}{5}\right)^2 \left(\frac{|2x+3y+4|}{\sqrt{2^2+3^2}}\right)^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = \frac{16}{25} \times \frac{(|2x+3y+4|)^2}{4+9}$$

$$x^2 + y^2 + 4x - 6y + 13 = (16/325) \times (4x^2 + 9y^2 + 16 + 12xy + 16x + 24y)$$

Upon cross multiplying, we get

$325x^2 + 325y^2 + 1300x – 1950y + 4225 = 64x^2 + 144y^2 + 192xy + 256x + 384y + 256$

$261x^2 + 181y^2 – 192xy + 1044x – 2334y + 3969 = 0$

∴ The equation of the ellipse is $261x^2 + 181y^2 – 192xy + 1044x – 2334y + 3969 = 0$

(iv) focus is (1, 2), directrix is 3x + 4y – 7 = 0 and e = ½.

Given:

focus is (1, 2)

directrix is 3x + 4y – 7 = 0

e = ½.

Let P(x, y) be any point on the ellipse.

We know that distance between the points (x_1, y_1) and (x_2, y_2) is given as

$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

We also know that the perpendicular distance from the point (x_1, y_1) to the line ax + by + c = 0 is given as

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

So,

SP = ePM

$SP^2 = e^2PM^2$

$$(x-1)^2 + (y-2)^2 = \left(\frac{1}{2}\right)^2\left(\frac{|3x+4y-5|}{\sqrt{3^2+4^2}}\right)^2$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 = \frac{1}{4} \times \frac{(|3x+4y-5|)^2}{9+16}$$

$$x^2 + y^2 - 2x - 4y + 5 = \frac{1}{100} \times (9x^2 + 16y^2 + 25 + 24xy - 30x - 40y)$$

Upon cross multiplying, we get

$100x^2 + 100y^2 – 200x – 400y + 500 = 9x^2 + 16y^2 + 24xy – 30x – 40y + 25$

$91x^2 + 84y^2 – 24xy – 170x – 360y + 475 = 0$

∴ The equation of the ellipse is $91x^2 + 84y^2 – 24xy – 170x – 360y + 475 = 0$

3. Find the eccentricity, coordinates of foci, length of the latus – rectum of the following ellipse:

(i) $4x^2 + 9y^2 = 1$

(ii) $5x^2 + 4y^2 = 1$

(iii) $4x^2 + 3y^2 = 1$

(iv) $25x^2 + 16y^2 = 1600$

(v) $9x^2 + 25y^2 = 225$

Solution:

(i) $4x^2 + 9y^2 = 1$

Given:

The equation of ellipse => $4x^2 + 9y^2 = 1$

This equation can be expressed as

$$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, a^2 = ¼, b^2 = 1/9

$$= \sqrt{\frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{4}}}$$

$$= \sqrt{\frac{\frac{5}{36}}{\frac{1}{4}}}$$

$$= \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$$

Length of latus rectum = $2b^2/a$

= [2 (1/9)] / (1/2)

= 4/9

Coordinates of foci (±ae, 0)

$$\text{foci} = \left(\pm \frac{1}{2} \times \frac{\sqrt{5}}{3}, 0\right)$$
$$= \left(\pm \frac{\sqrt{5}}{6}, 0\right)$$

∴ The eccentricity is $\frac{\sqrt{5}}{3}$, foci are $\left(\pm \frac{\sqrt{5}}{6}, 0\right)$ and length of the latus rectum is $\frac{4}{9}$.

(ii) $5x^2 + 4y^2 = 1$

Given:

The equation of ellipse => $5x^2 + 4y^2 = 1$

This equation can be expressed as

$$\frac{x^2}{\frac{1}{5}}+\frac{y^2}{\frac{1}{4}}=1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$

Here, $a^2 = 1/5$ and $b^2 = 1/4$

$$e = \sqrt{\frac{\frac{1}{4}-\frac{1}{5}}{\frac{1}{4}}}$$

$$= \sqrt{\frac{\frac{1}{20}}{\frac{1}{4}}}$$

$$= \sqrt{\frac{1}{5}}$$

Length of latus rectum = $2b^2/a$

= [2(1/5)] / (1/2)

= 4/5

Coordinates of foci (±ae, 0)

$$\text{foci} = \left(0, \pm\frac{1}{2}\times\sqrt{\frac{1}{5}}\right)$$

$$= \left(0, \pm\frac{1}{2\sqrt{5}}\right)$$

∴ The eccentricity is $\sqrt{\frac{1}{5}}$, foci are $\left(0, \pm\frac{1}{2\sqrt{5}}\right)$ and length of the latus rectum is $\frac{4}{5}$.

(iii) $4x^2 + 3y^2 = 1$

Given:

The equation of ellipse => $4x^2 + 3y^2 = 1$

This equation can be expressed as

$$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{3}} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, $a^2 = 1/4$ and $b^2 = 1/3$

$$e = \sqrt{1 - \frac{\frac{1}{4}}{\frac{1}{3}}}$$

$$= \sqrt{\frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3}}} \quad = \sqrt{\frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3}}}$$

$$= \sqrt{\frac{\frac{1}{12}}{\frac{1}{3}}} \quad = \sqrt{\frac{\frac{1}{12}}{\frac{1}{3}}}$$

$$= \sqrt{\frac{1}{4}} \quad = \sqrt{\frac{1}{4}}$$

$$= \frac{1}{2} \quad = \frac{1}{2}$$

Length of latus rectum = $2b^2/a$

= [2(1/4)] / (1/√3)

= √3/2

Coordinates of foci (±ae, 0)

$$\text{foci} = \left(0, \pm\frac{1}{\sqrt{3}} \times \frac{1}{2}\right)$$
$$= \left(0, \pm\frac{1}{2\sqrt{3}}\right)$$

∴ The eccentricity is $\frac{\sqrt{3}}{2}$, foci are $\left(0, \pm\frac{1}{2\sqrt{3}}\right)$ and length of the latus rectum is $\frac{\sqrt{3}}{2}$.

(iv) $25x^2 + 16y^2 = 1600$

Given:

The equation of ellipse => $25x^2 + 16y^2 = 1600$

This equation can be expressed as

$$\frac{25x^2}{1600} + \frac{16y^2}{1600} = 1$$
$$\frac{x^2}{64} + \frac{y^2}{100} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, $a^2 = 64$ and $b^2 = 100$

$$e = \sqrt{1 - \frac{64}{100}}$$

$$= \sqrt{\frac{100-64}{100}}$$

$$= \sqrt{\frac{36}{100}} \qquad = \sqrt{\frac{36}{100}}$$

$$= \frac{6}{10} \qquad = \frac{6}{10}$$

$$= \frac{3}{5} \qquad = \frac{3}{5}$$

Length of latus rectum = $2b^2/a$

= [2(64)] / (100)

= 32/25

Coordinates of foci (±ae, 0)

$$\text{foci} = \left(0, \pm 10 \times \frac{3}{5}\right)$$

$$= (0, \pm 6)$$

∴ The eccentricity is $\frac{3}{5}$, foci are (0, ±6) and length of the latus rectum is $\frac{32}{25}$.

(v) $9x^2 + 25y^2 = 225$

Given:

The equation of ellipse => $9x^2 + 25y^2 = 225$

This equation can be expressed as

$$\frac{9x^2}{225} + \frac{25y^2}{225} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

Here, $a^2 = 25$ and $b^2 = 9$

$$e = \sqrt{\frac{25-9}{25}}$$
$$= \sqrt{\frac{16}{25}}$$
$$= \frac{4}{5}$$

Length of latus rectum = $2b^2/a$

= [2(9)] / (5)

= 18/5

Coordinates of foci (±ae, 0)

$$\text{foci} = \left(\pm 5 \times \frac{4}{5}, 0\right)$$
$$= (\pm 4, 0)$$

∴ The eccentricity is $\frac{4}{5}$, foci are $(\pm 4, 0)$ and length of the latus rectum is $\frac{18}{5}$.

4. Find the equation to the ellipse (referred to its axes as the axes of x and y respectively) which passes through the point (-3, 1) and has eccentricity √(2/5).

Solution:

Given:

The point (-3, 1)

Eccentricity = √(2/5)

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

…. (1)

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$
$$\sqrt{\frac{2}{5}} = \sqrt{\frac{a^2 - b^2}{a^2}}$$
$$\frac{2}{5} = 1 - \frac{b^2}{a^2}$$
$$\frac{b^2}{a^2} = \frac{3}{5}$$
$$b^2 = \frac{3a^2}{5} \quad …. (2)$$

Now let us substitute equation (2) in equation (1), we get

$$\frac{x^2}{a^2} + \frac{y^2}{\frac{3a^2}{5}} = 1$$
$$\frac{x^2}{a^2} + \frac{5y^2}{3a^2} = 1$$
$$3x^2 + 5y^2 = 3a^2$$

It is given that the curve passes through the point (-3, 1).

So by substituting the point in the curve we get,

$3(-3)^2 + 5(1)^2 = 3a^2$

$3(9) + 5 = 3a^2$

$32 = 3a^2$

$a^2 = 32/3$

From equation (2)

$b^2 = 3a^2/5$

$= 3(32/3) / 5$

= 32/5

So now, the equation of the ellipse is given as:

$$\frac{x^2}{\frac{32}{3}} + \frac{y^2}{\frac{32}{5}} = 1$$

$$\frac{3x^2}{32} + \frac{5y^2}{32} = 1$$

$3x^2 + 5y^2 = 32$

∴ The equation of the ellipse is $3x^2 + 5y^2 = 32$.

5. Find the equation of the ellipse in the following cases:

(i) eccentricity e = ½ and foci (± 2, 0)

(ii) eccentricity e = 2/3 and length of latus – rectum = 5

(iii) eccentricity e = ½ and semi – major axis = 4

(iv) eccentricity e = ½ and major axis = 12

(v) The ellipse passes through (1, 4) and (- 6, 1)

Solution:

(i) Eccentricity e = ½ and foci (± 2, 0)

Given:

Eccentricity e = ½

Foci (± 2, 0)

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$
$$\frac{1}{2} = \sqrt{\frac{a^2-b^2}{a^2}}$$
$$\frac{1}{4} = 1 - \frac{b^2}{a^2}$$
$$\frac{b^2}{a^2} = \frac{3}{4}$$
$$b^2 = \frac{3a^2}{4}$$

$b^2 = 3a^2/4$

It is given that foci (± 2, 0) =>foci = (±ae, 0)

Where, ae = 2

a(1/2) = 2

a = 4

$a^2 = 16$

We know $b^2 = 3a^2/4$

$b^2 = 3(16)/4$

= 12

So the equation of the ellipse can be given as

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$
$$\frac{3x^2 + 4y^2}{48} = 1$$

$3x^2 + 4y^2 = 48$

∴ The equation of the ellipse is $3x^2 + 4y^2 = 48$

(ii) eccentricity e = 2/3 and length of latus rectum = 5

Given:

Eccentricity e = 2/3

Length of latus – rectum = 5

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{2}{3} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{4}{9} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{5}{9}$$

$$b^2 = \frac{5a^2}{9}$$

By using the formula, length of the latus rectum is $2b^2/a$

$$\frac{2b^2}{a} = 5$$
$$b^2 = \frac{5a}{2}$$
Since, $b^2 = 5a^2/9$
$$\frac{5a^2}{9} = \frac{5a}{2}$$
$$\frac{a}{9} = \frac{1}{2}$$
$$a = \frac{9}{2}$$
$$a^2 = \frac{81}{4}$$
Now, substituting the value of a^2, we get
$$b^2 = \frac{5\left(\frac{81}{4}\right)}{9}$$
$$b^2 = \frac{45}{4}$$

So the equation of the ellipse can be given as

$$\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$$
$$\frac{4x^2}{81} + \frac{4y^2}{45} = 1$$
$$\frac{(20x^2 + 36y^2)}{405} = 1$$

$20x^2 + 36y^2 = 405$

∴ The equation of the ellipse is $20x^2 + 36y^2 = 405$.

(iii) eccentricity e = ½ and semi – major axis = 4

Given:

Eccentricity e = ½

Semi – major axis = 4

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2-b^2}{a^2}}$$

$$\frac{1}{2} = \sqrt{\frac{a^2-b^2}{a^2}} \quad \frac{1}{2} = \sqrt{\frac{a^2-b^2}{a^2}}$$

$$\frac{1}{4} = 1 - \frac{b^2}{a^2} \quad \frac{1}{4} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{3}{4} \quad \frac{b^2}{a^2} = \frac{3}{4}$$

$$b^2 = \frac{3a^2}{4} \quad b^2 = \frac{3a^2}{4}$$

It is given that the length of the semi – major axis is a

a = 4

$a^2 = 16$

We know, $b^2 = 3a^2/4$

$b^2 = 3(16)/4$

= 4

So the equation of the ellipse can be given as

$$\frac{x^2}{16} + \frac{y^2}{12} = 1$$

$$\frac{3x^2 + 4y^2}{48} = 1$$

$3x^2 + 4y^2 = 48$

∴ The equation of the ellipse is $3x^2 + 4y^2 = 48$.

(iv) eccentricity e = ½ and major axis = 12

Given:

Eccentricity e = ½

Major axis = 12

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{1}{2} = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$\frac{1}{4} = 1 - \frac{b^2}{a^2}$$

$$\frac{b^2}{a^2} = \frac{3}{4}$$

$$b^2 = \frac{3a^2}{4}$$

$b^2 = 3a^2/4$

It is given that length of major axis is 2a.

2a = 12

a = 6

$a^2 = 36$

So, by substituting the value of a^2, we get

$b^2 = 3(36)/4$

= 27

So the equation of the ellipse can be given as

$$\frac{x^2}{36} + \frac{y^2}{27} = 1$$
$$\frac{3x^2 + 4y^2}{108} = 1$$

$3x^2 + 4y^2 = 108$

∴ The equation of the ellipse is $3x^2 + 4y^2 = 108$.

(v) The ellipse passes through (1, 4) and (- 6, 1)

Given:

The points (1, 4) and (- 6, 1)

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

…. (1)

Let us substitute the point (1, 4) in equation (1), we get

$$\frac{1^2}{a^2} + \frac{4^2}{b^2} = 1$$
$$\frac{1}{a^2} + \frac{16}{b^2} = 1$$
$$\frac{b^2 + 16a^2}{a^2b^2} = 1$$

$b^2 + 16a^2 = a^2 b^2$ …. (2)

Let us substitute the point (-6, 1) in equation (1), we get

$$\frac{(-6)^2}{a^2} + \frac{1^2}{b^2} = 1$$
$$\frac{36}{a^2} + \frac{1}{b^2} = 1$$
$$\frac{36b^2 + a^2}{a^2b^2} = 1$$

$a^2 + 36b^2 = a^2b^2$ …. (3)

Let us multiply equation (3) by 16 and subtract with equation (2), we get

$(16a^2 + 576b^2) – (b^2 + 16a^2) = (16a^2b^2 – a^2b^2)$

$575b^2 = 15a^2b^2$

$15a^2 = 575$

$a^2 = 575/15$

$= 115/3$

So from equation (2),

$$b^2 + 16\left(\frac{115}{3}\right) = b^2\left(\frac{115}{3}\right)$$
$$b^2\left(\frac{112}{3}\right) = \frac{1840}{3}$$
$$b^2 = \frac{115}{7}$$

So the equation of the ellipse can be given as

$$\frac{x^2}{\frac{115}{3}} + \frac{y^2}{\frac{115}{7}} = 1$$
$$\frac{3x^2}{115} + \frac{7y^2}{115} = 1$$

$3x^2 + 7y^2 = 115$

∴ The equation of the ellipse is $3x^2 + 7y^2 = 115$.

6. Find the equation of the ellipse whose foci are (4, 0) and (- 4, 0), eccentricity = 1/3.

Solution:

Given:

Foci are (4, 0) (- 4, 0)

Eccentricity = 1/3.

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

By using the formula,

Eccentricity:

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$
$$\frac{1}{3} = \sqrt{\frac{a^2 - b^2}{a^2}}$$
$$\frac{1}{9} = 1 - \frac{b^2}{a^2}$$
$$\frac{b^2}{a^2} = \frac{8}{9}$$
$$b^2 = \frac{8a^2}{9}$$

$$\frac{b^2}{a^2} = \frac{8}{9}$$
$$b^2 = \frac{8a^2}{9}$$

It is given that foci = (4, 0) (- 4, 0) => foci = (±ae,0)

Where, ae = 4

a(1/3) = 4

a = 12

$a^2 = 144$

By substituting the value of a^2, we get

$b^2 = 8a^2/9$

$b^2 = 8(144)/9$

= 128

So the equation of the ellipse can be given as

$$\frac{x^2}{144} + \frac{y^2}{128} = 1$$

$\therefore$ The equation of the ellipse is $\frac{x^2}{144} + \frac{y^2}{128} = 1$

7. Find the equation of the ellipse in the standard form whose minor axis is equal to the distance between foci and whose latus – rectum is 10.

Solution:

Given:

Minor axis is equal to the distance between foci and whose latus – rectum is 10.

Now let us find the equation to the ellipse.

We know that the equation of the ellipse whose axes are x and y – axis is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

We know that length of the minor axis is 2b and distance between the foci is 2ae.

By using the formula,

Eccentricity:

$e = \sqrt{\frac{a^2-b^2}{a^2}}$

$2b = 2ae$

$b = ae$

$b = a\sqrt{\frac{a^2-b^2}{a^2}}$

$b^2 = a^2 - b^2$

$a^2 = 2b^2$ …. (1)

We know that the length of the latus rectum is $2b^2/a$

It is given that length of the latus rectum = 10

So by equating, we get

$2b^2/a = 10$

$a^2/ a = 10$ [Since, $a^2 = 2b^2$]

a = 10

$a^2 = 100$

Now, by substituting the value of a^2 we get

$2b^2/a = 10$

$2b^2/10 = 10$

$2b^2 = 10(10)$

$b^2 = 100/2$

= 50

So the equation of the ellipse can be given as

$$\frac{x^2}{100} + \frac{y^2}{50} = 1$$

$$\frac{x^2 + 2y^2}{100} = 1$$

$x^2 + 2y^2 = 100$

∴ The equation of the ellipse is $x^2 + 2y^2 = 100$.

8. Find the equation of the ellipse whose centre is (-2, 3) and whose semi – axis are 3 and 2 when the major axis is (i) parallel to x – axis (ii) parallel to the y – axis.

Solution:

Given:

Centre = (-2, 3)

Semi – axis are 3 and 2

(i) When major axis is parallel to x-axis

Now let us find the equation to the ellipse.

We know that the equation of the ellipse with centre (p, q) is given by

$$\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$$

Since major axis is parallel to x – axis

So, a = 3 and b = 2.

$a^2 = 9$

$b^2 = 4$

So the equation of the ellipse can be given as

$$\frac{(x+2)^2}{9} + \frac{(y-3)^2}{4} = 1$$
$$\frac{4(x+2)^2 + 9(y-3)^2}{36} = 1$$

$4(x^2 + 4x + 4) + 9(y^2 – 6y + 9) = 36$

$4x^2 + 16x + 16 + 9y^2 – 54y + 81 = 36$

$4x^2 + 9y^2 + 16x – 54y + 61 = 0$

∴ The equation of the ellipse is $4x^2 + 9y^2 + 16x – 54y + 61 = 0$.

(ii) When major axis is parallel to y-axis

Now let us find the equation to the ellipse.

We know that the equation of the ellipse with centre (p, q) is given by

$$\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$$

Since major axis is parallel to y – axis

So, a = 2 and b = 3.

$a^2 = 4$

$b^2 = 9$

So the equation of the ellipse can be given as

$$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$$
$$\frac{9(x+2)^2 + 4(y-3)^2}{36} = 1$$

$9(x^2 + 4x + 4) + 4(y^2 – 6y + 9) = 36$

$9x^2 + 36x + 36 + 4y^2 – 24y + 36 = 36$

$9x^2 + 4y^2 + 36x – 24y + 36 = 0$

∴ The equation of the ellipse is $9x^2 + 4y^2 + 36x – 24y + 36 = 0$.

9. Find the eccentricity of an ellipse whose latus – rectum is

(i) Half of its minor axis

(ii) Half of its major axis

Solution:

Given:

We need to find the eccentricity of an ellipse.

(i) If latus – rectum is half of its minor axis

We know that the length of the semi – minor axis is b and the length of the latus – rectum is $2b^2/a$.

$2b^2/a = b$

$a = 2b$ …. (1)

By using the formula,

We know that eccentricity of an ellipse is given as

(ii) If latus – rectum is half of its major axis

We know that the length of the semi – major axis is a and the length of the latus – rectum is $2b^2/a$.

$2b^2/a$

$a^2 = 2b^2$ …. (1)

By using the formula,

We know that eccentricity of an ellipse is given as

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

From equation (1)

$$e = \sqrt{\frac{a^2 - b^2}{a^2}}$$

$$= \sqrt{\frac{2b^2 - b^2}{2b^2}}$$

$$= \sqrt{\frac{b^2}{2b^2}}$$

$$= \sqrt{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2}}$$

Chapterwise RD Sharma Solutions for Class 11 Maths :

- Chapter 1–Sets
- Chapter 2–Relations
- Chapter 3–Functions
- Chapter 4–Measurement of Angles
- Chapter 5–Trigonometric Functions
- Chapter 6–Graphs of Trigonometric Functions
- Chapter 7–Values of Trigonometric Functions at Sum or Difference of Angles
- Chapter 8–Transformation Formulae
- Chapter 9–Values of Trigonometric Functions at Multiples and Submultiples of an Angle
- Chapter 10–Sine and Cosine Formulae and their Applications
- Chapter 11–Trigonometric Equations
- Chapter 12–Mathematical Induction
- Chapter 13–Complex Numbers
- Chapter 14–Quadratic Equations
- Chapter 15–Linear Inequations
- Chapter 16–Permutations
- Chapter 17–Combinations
- Chapter 18–Binomial Theorem
- Chapter 19–Arithmetic Progressions
- Chapter 20–Geometric Progressions

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.