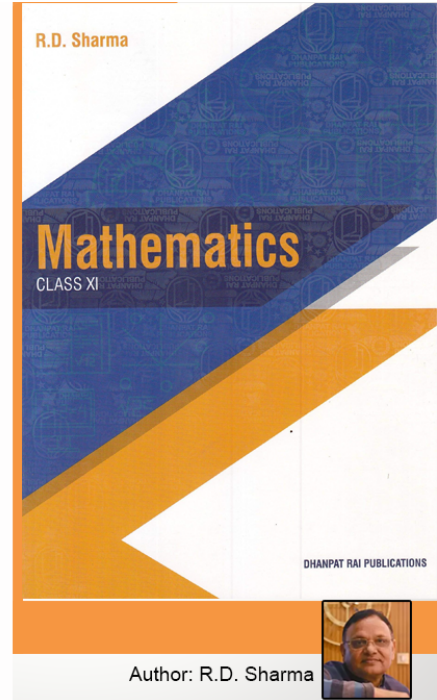


Class 11 - Chapter 27 Hyperbola



RD Sharma Solutions for Class 11 Maths Chapter 27–Hyperbola

Class 11: Maths Chapter 27 solutions. Complete Class 11 Maths Chapter 27 Notes.

RD Sharma Solutions for Class 11 Maths Chapter 27–Hyperbola

RD Sharma 11th Maths Chapter 27, Class 11 Maths Chapter 27 solutions

EXERCISE 27.1 PAGE NO: 27.13

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>



1. The equation of the directrix of a hyperbola is $x - y + 3 = 0$. Its focus is $(-1, 1)$ and eccentricity 3. Find the equation of the hyperbola.

Solution:

Given:

The equation of the directrix of a hyperbola $\Rightarrow x - y + 3 = 0$.

Focus = $(-1, 1)$ and

Eccentricity = 3

Now, let us find the equation of the hyperbola

Let 'M' be the point on directrix and P(x, y) be any point of the hyperbola.

By using the formula,

$$e = PF/PM$$

PF = ePM [where, e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix]

So,

$$\sqrt{(x+1)^2 + (y-1)^2} = 3 \left| \frac{(x-y+3)}{\sqrt{1^2 + (-1)^2}} \right|$$

$$\sqrt{(x+1)^2 + (y-1)^2} = 3 \left| \frac{(x-y+3)}{\sqrt{1+1}} \right|$$

By squaring on both sides we get

$$\left(\sqrt{(x+1)^2 + (y-1)^2} \right)^2 = \left(3 \left| \frac{(x-y+3)}{\sqrt{1+1}} \right| \right)^2$$

$$(x+1)^2 + (y-1)^2 = \frac{3^2(x-y+3)^2}{2}$$

[We know that $(a-b)^2 = a^2 + b^2 + 2ab$ & $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$]

$$\text{So, } 2\{x^2 + 1 + 2x + y^2 + 1 - 2y\} = 9\{x^2 + y^2 + 9 + 6x - 6y - 2xy\}$$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$2x^2 + 2 + 4x + 2y^2 + 2 - 4y = 9x^2 + 9y^2 + 81 + 54x - 54y - 18xy$$

$$2x^2 + 4 + 4x + 2y^2 - 4y - 9x^2 - 9y^2 - 81 - 54x + 54y + 18xy = 0$$

$$-7x^2 - 7y^2 - 50x + 50y + 18xy - 77 = 0$$

$$7(x^2 + y^2) - 18xy + 50x - 50y + 77 = 0$$

∴ The equation of hyperbola is $7(x^2 + y^2) - 18xy + 50x - 50y + 77 = 0$

2. Find the equation of the hyperbola whose

(i) focus is (0, 3), directrix is $x + y - 1 = 0$ and eccentricity = 2

(ii) focus is (1, 1), directrix is $3x + 4y + 8 = 0$ and eccentricity = 2

(iii) focus is (1, 1) directrix is $2x + y = 1$ and eccentricity = $\sqrt{3}$

(iv) focus is (2, -1), directrix is $2x + 3y = 1$ and eccentricity = 2

(v) focus is (a, 0), directrix is $2x + 3y = 1$ and eccentricity = 2

(vi) focus is (2, 2), directrix is $x + y = 9$ and eccentricity = 2

Solution:

(i) focus is (0, 3), directrix is $x + y - 1 = 0$ and eccentricity = 2

Given:

Focus = (0, 3)

Directrix $\Rightarrow x + y - 1 = 0$

Eccentricity = 2

Now, let us find the equation of the hyperbola

Let 'M' be the point on directrix and P(x, y) be any point of the hyperbola.

By using the formula,

$$e = PF/PM$$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

PF = ePM [where, e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix]

So,

$$\sqrt{(x-0)^2 + (y-3)^2} = 2 \left| \frac{(x+y-1)}{\sqrt{1^2+1^2}} \right|$$

$$\sqrt{(x-0)^2 + (y-3)^2} = 2 \left| \frac{(x+y-1)}{\sqrt{1+1}} \right|$$

By squaring on both sides we get

$$\left(\sqrt{(x-0)^2 + (y-3)^2} \right)^2 = \left(2 \left| \frac{(x+y-1)}{\sqrt{1+1}} \right| \right)^2$$

$$(x-0)^2 + (y-3)^2 = \frac{2^2(x+y-1)^2}{2}$$

[We know that $(a-b)^2 = a^2 + b^2 + 2ab$ & $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$]

$$\text{So, } 2\{x^2 + y^2 + 9 - 6y\} = 4\{x^2 + y^2 + 1 - 2x - 2y + 2xy\}$$

$$2x^2 + 2y^2 + 18 - 12y = 4x^2 + 4y^2 + 4 - 8x - 8y + 8xy$$

$$2x^2 + 2y^2 + 18 - 12y - 4x^2 - 4y^2 - 4 - 8x + 8y - 8xy = 0$$

$$-2x^2 - 2y^2 - 8x - 4y - 8xy + 14 = 0$$

$$-2(x^2 + y^2 - 4x + 2y + 4xy - 7) = 0$$

$$x^2 + y^2 - 4x + 2y + 4xy - 7 = 0$$

∴ The equation of hyperbola is $x^2 + y^2 - 4x + 2y + 4xy - 7 = 0$

(ii) focus is (1, 1), directrix is $3x + 4y + 8 = 0$ and eccentricity = 2

Focus = (1, 1)

Directrix $\Rightarrow 3x + 4y + 8 = 0$

Eccentricity = 2

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

Now, let us find the equation of the hyperbola

Let 'M' be the point on directrix and P(x, y) be any point of the hyperbola.

By using the formula,

$$e = PF/PM$$

PF = ePM [where, e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix]

So,

$$\sqrt{(x-0)^2 + (y-3)^2} = 2 \left| \frac{(x+y-1)}{\sqrt{1^2+1^2}} \right|$$
$$\sqrt{(x-0)^2 + (y-3)^2} = 2 \left| \frac{(x+y-1)}{\sqrt{1+1}} \right|$$

By squaring on both sides we get

$$\left(\sqrt{(x-0)^2 + (y-3)^2} \right)^2 = \left(2 \left| \frac{(x+y-1)}{\sqrt{1+1}} \right| \right)^2$$
$$(x-0)^2 + (y-3)^2 = \frac{2^2(x+y-1)^2}{2}$$

[We know that $(a-b)^2 = a^2 + b^2 + 2ab$ & $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$]

$$25\{x^2 + 1 - 2x + y^2 + 1 - 2y\} = 4\{9x^2 + 16y^2 + 64 + 24xy + 64y + 48x\}$$

$$25x^2 + 25 - 50x + 25y^2 + 25 - 50y = 36x^2 + 64y^2 + 256 + 96xy + 256y + 192x$$

$$25x^2 + 25 - 50x + 25y^2 + 25 - 50y - 36x^2 - 64y^2 - 256 - 96xy - 256y - 192x = 0$$

$$-11x^2 - 39y^2 - 242x - 306y - 96xy - 206 = 0$$

$$11x^2 + 96xy + 39y^2 + 242x + 306y + 206 = 0$$

∴ The equation of hyperbola is $11x^2 + 96xy + 39y^2 + 242x + 306y + 206 = 0$

(iii) focus is (1, 1) directrix is $2x + y = 1$ and eccentricity $=\sqrt{3}$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

Given:

$$\text{Focus} = (1, 1)$$

$$\text{Directrix} \Rightarrow 2x + y = 1$$

$$\text{Eccentricity} = \sqrt{3}$$

Now, let us find the equation of the hyperbola

Let 'M' be the point on directrix and P(x, y) be any point of the hyperbola.

By using the formula,

$$e = PF/PM$$

PF = ePM [where, e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix]

So,

$$\sqrt{(x-1)^2 + (y-1)^2} = \sqrt{3} \left| \frac{(2x+y-1)}{\sqrt{2^2+1^2}} \right|$$
$$\sqrt{(x-1)^2 + (y-1)^2} = \sqrt{3} \left| \frac{(2x+y-1)}{\sqrt{4+1}} \right|$$

By squaring on both sides we get

$$\left(\sqrt{(x-1)^2 + (y-1)^2} \right)^2 = \left(\sqrt{3} \left| \frac{(2x+y-1)}{\sqrt{5}} \right| \right)^2$$
$$(x-1)^2 + (y-1)^2 = \frac{3(2x+y-1)^2}{5}$$

[We know that $(a-b)^2 = a^2 + b^2 - 2ab$ & $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$]

$$5\{x^2 + 1 - 2x + y^2 + 1 - 2y\} = 3\{4x^2 + y^2 + 1 + 4xy - 2y - 4x\}$$

$$5x^2 + 5 - 10x + 5y^2 + 5 - 10y = 12x^2 + 3y^2 + 3 + 12xy - 6y - 12x$$

$$5x^2 + 5 - 10x + 5y^2 + 5 - 10y - 12x^2 - 3y^2 - 3 - 12xy + 6y + 12x = 0$$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$-7x^2 + 2y^2 + 2x - 4y - 12xy + 7 = 0$$

$$7x^2 + 12xy - 2y^2 - 2x + 4y - 7 = 0$$

∴ The equation of hyperbola is $7x^2 + 12xy - 2y^2 - 2x + 4y - 7 = 0$

(iv) focus is (2, -1), directrix is $2x + 3y = 1$ and eccentricity = 2

Given:

Focus = (2, -1)

Directrix $\Rightarrow 2x + 3y = 1$

Eccentricity = 2

Now, let us find the equation of the hyperbola

Let 'M' be the point on directrix and P(x, y) be any point of the hyperbola.

By using the formula,

$$e = PF/PM$$

$PF = ePM$ [where, e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix]

So,

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$\sqrt{(x-2)^2 + (y+1)^2} = 2 \left| \frac{(2x+3y-1)}{\sqrt{2^2+3^2}} \right|$$

$$\sqrt{(x-2)^2 + (y+1)^2} = 2 \left| \frac{(2x+3y-1)}{\sqrt{4+9}} \right|$$

By squaring on both sides we get

$$\left(\sqrt{(x-2)^2 + (y+1)^2} \right)^2 = \left(2 \left| \frac{(2x+3y-1)}{\sqrt{13}} \right| \right)^2$$

$$(x-2)^2 + (y+1)^2 = \frac{4(2x+3y-1)^2}{13}$$

$$(x-2)^2 + (y+1)^2 = \frac{4(2x+3y-1)^2}{13}$$

[We know that $(a-b)^2 = a^2 + b^2 + 2ab$ & $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$]

$$13\{x^2 + 4 - 4x + y^2 + 1 + 2y\} = 4\{4x^2 + 9y^2 + 1 + 12xy - 6y - 4x\}$$

$$13x^2 + 52 - 52x + 13y^2 + 13 + 26y = 16x^2 + 36y^2 + 4 + 48xy - 24y - 16x$$

$$13x^2 + 52 - 52x + 13y^2 + 13 + 26y - 16x^2 - 36y^2 - 4 - 48xy + 24y + 16x = 0$$

$$-3x^2 - 23y^2 - 36x + 50y - 48xy + 61 = 0$$

$$3x^2 + 23y^2 + 48xy + 36x - 50y - 61 = 0$$

∴ The equation of hyperbola is $3x^2 + 23y^2 + 48xy + 36x - 50y - 61 = 0$

(v) focus is (a, 0), directrix is $2x + 3y = 1$ and eccentricity = 2

Given:

Focus = (a, 0)

Directrix $\Rightarrow 2x + 3y = 1$

Eccentricity = 2

Now, let us find the equation of the hyperbola

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

Let 'M' be the point on directrix and P(x, y) be any point of the hyperbola.

By using the formula,

$$e = PF/PM$$

PF = ePM [where, e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix]

So,

$$\sqrt{(x-a)^2 + (y-0)^2} = \frac{4}{3} \left| \frac{(2x-y+a)}{\sqrt{2^2 + (-1)^2}} \right|$$

$$\sqrt{(x-a)^2 + (y)^2} = \frac{4}{3} \left| \frac{(2x-y+a)}{\sqrt{4+1}} \right|$$

By squaring on both sides we get

$$\left(\sqrt{(x-a)^2 + (y)^2} \right)^2 = \left(\frac{4}{3} \left| \frac{(2x-y+a)}{\sqrt{5}} \right| \right)^2$$

$$(x-a)^2 + (y)^2 = \frac{16(2x-y+a)^2}{9 \times 5}$$

[We know that $(a-b)^2 = a^2 + b^2 + 2ab$ & $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$]

$$45\{x^2 + a^2 - 2ax + y^2\} = 16\{4x^2 + y^2 + a^2 - 4xy - 2ay + 4ax\}$$

$$45x^2 + 45a^2 - 90ax + 45y^2 = 64x^2 + 16y^2 + 16a^2 - 64xy - 32ay + 64ax$$

$$45x^2 + 45a^2 - 90ax + 45y^2 - 64x^2 - 16y^2 - 16a^2 + 64xy + 32ay - 64ax = 0$$

$$19x^2 - 29y^2 + 154ax - 32ay - 64xy - 29a^2 = 0$$

∴ The equation of hyperbola is $19x^2 - 29y^2 + 154ax - 32ay - 64xy - 29a^2 = 0$

(vi) focus is (2, 2), directrix is $x + y = 9$ and eccentricity = 2

Given:

Focus = (2, 2)

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

Directrix $\Rightarrow x + y = 9$

Eccentricity = 2

Now, let us find the equation of the hyperbola

Let 'M' be the point on directrix and P(x, y) be any point of the hyperbola.

By using the formula,

$$e = PF/PM$$

PF = ePM [where, e is eccentricity, PM is perpendicular from any point P on hyperbola to the directrix]

So,

$$\sqrt{(x-2)^2 + (y-2)^2} = 2 \left| \frac{(x+y-9)}{\sqrt{1^2 + 1^2}} \right|$$

$$\sqrt{(x-2)^2 + (y-2)^2} = 2 \left| \frac{(x+y-9)}{\sqrt{1+1}} \right|$$

By squaring on both sides we get

$$\left(\sqrt{(x-2)^2 + (y-2)^2} \right)^2 = \left(2 \left| \frac{(x+y-9)}{\sqrt{2}} \right| \right)^2$$

$$(x-2)^2 + (y-2)^2 = \frac{4(x+y-9)^2}{2}$$

[We know that $(a-b)^2 = a^2 + b^2 + 2ab$ & $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$]

$$x^2 + 4 - 4x + y^2 + 4 - 4y = 2\{x^2 + y^2 + 81 + 2xy - 18y - 18x\}$$

$$x^2 - 4x + y^2 + 8 - 4y = 2x^2 + 2y^2 + 162 + 4xy - 36y - 36x$$

$$x^2 - 4x + y^2 + 8 - 4y - 2x^2 - 2y^2 - 162 - 4xy + 36y + 36x = 0$$

$$-x^2 - y^2 + 32x + 32y + 4xy - 154 = 0$$

$$x^2 + 4xy + y^2 - 32x - 32y + 154 = 0$$

$\therefore$ The equation of hyperbola is $x^2 + 4xy + y^2 - 32x - 32y + 154 = 0$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

3. Find the eccentricity, coordinates of the foci, equations of directrices and length of the latus-rectum of the hyperbola.

(i) $9x^2 - 16y^2 = 144$

(ii) $16x^2 - 9y^2 = -144$

(iii) $4x^2 - 3y^2 = 36$

(iv) $3x^2 - y^2 = 4$

(v) $2x^2 - 3y^2 = 5$

Solution:

(i) $9x^2 - 16y^2 = 144$

Given:

The equation $\Rightarrow 9x^2 - 16y^2 = 144$

The equation can be expressed as:

$$\frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

$$\frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$$

The obtained equation is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where, $a^2 = 16$, $b^2 = 9$ i.e., $a = 4$ and $b = 3$

Eccentricity is given by:

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$\begin{aligned}e &= \sqrt{1 + \frac{a^2}{b^2}} \\&= \sqrt{1 + \frac{9}{16}} \\&= \sqrt{\frac{25}{16}} \\&= \frac{5}{4}\end{aligned}$$

Foci: The coordinates of the foci are $(0, \pm be)$

$$(0, \pm be) = (0, \pm 4(5/4))$$

$$= (0, \pm 5)$$

The equation of directrices is given as:

$$\begin{aligned}x &= \pm \frac{a}{e} \\&\Rightarrow x = \pm \frac{4}{5} \\&\Rightarrow x = \pm \frac{16}{5} \\&\Rightarrow 5x = \pm 16\end{aligned}$$

$$5x \mp 16 = 0$$

The length of latus-rectum is given as:

$$2b^2/a$$

$$= 2(9)/4$$

$$= 9/2$$

$$(ii) 16x^2 - 9y^2 = -144$$

Given:

$$\text{The equation } \Rightarrow 16x^2 - 9y^2 = -144$$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

The equation can be expressed as:

$$\begin{aligned}\frac{9y^2}{144} - \frac{16x^2}{144} &= 1 \\ \frac{y^2}{16} - \frac{x^2}{9} &= 1 \\ \frac{x^2}{3^2} - \frac{y^2}{4^2} &= -1\end{aligned}$$

The obtained equation is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where, $a^2 = 9$, $b^2 = 16$ i.e., $a = 3$ and $b = 4$

Eccentricity is given by:

$$\begin{aligned}e &= \sqrt{1 + \frac{a^2}{b^2}} \\ &= \sqrt{1 + \frac{9}{16}} \\ &= \sqrt{\frac{25}{16}} \\ &= \frac{5}{4}\end{aligned}$$

Foci: The coordinates of the foci are $(0, \pm be)$

$$(0, \pm be) = (0, \pm 4(5/4))$$

$$= (0, \pm 5)$$

The equation of directrices is given as:

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$\begin{aligned}y &= \pm \frac{b}{e} \\ \Rightarrow y &= \pm \frac{4}{\frac{5}{4}} \\ \Rightarrow y &= \pm \frac{16}{5} \\ \Rightarrow 5y &= \pm 16 \\ 5y \mp 16 &= 0\end{aligned}$$

The length of latus-rectum is given as:

$$\begin{aligned}2a^2/b \\ = 2(9)/4 \\ = 9/2\end{aligned}$$

$$(iii) 4x^2 - 3y^2 = 36$$

Given:

$$\text{The equation } \Rightarrow 4x^2 - 3y^2 = 36$$

The equation can be expressed as:

$$\begin{aligned}\frac{4x^2}{36} - \frac{3y^2}{36} &= 1 \\ \frac{x^2}{9} - \frac{y^2}{12} &= 1 \\ \frac{x^2}{3^2} - \frac{y^2}{(\sqrt{12})^2} &= 1\end{aligned}$$

The obtained equation is of the form

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where, $a^2 = 9$, $b^2 = 12$ i.e., $a = 3$ and $b = \sqrt{12}$

Eccentricity is given by:

$$\begin{aligned} e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{12}{9}} \\ &= \sqrt{\frac{21}{9}} \end{aligned}$$

$$= \sqrt{\frac{7}{3}}$$

Foci: The coordinates of the foci are $(\pm ae, 0)$

$$\begin{aligned} \pm ae &= \pm 3 \times \sqrt{\frac{7}{3}} \\ &= \pm 3 \times \frac{\sqrt{7}}{\sqrt{3}} \\ &= \pm \sqrt{3} \times \sqrt{7} \\ &= \pm \sqrt{21} \end{aligned}$$

$$(\pm ae, 0) = (\pm \sqrt{21}, 0)$$

The equation of directrices is given as:

$$\begin{aligned} x &= \frac{\pm a}{e} \\ x &= \pm 3 \times \frac{1}{\frac{\sqrt{7}}{\sqrt{3}}} \\ &= \pm \frac{3\sqrt{3}}{\sqrt{7}} \\ \sqrt{7}x \mp 3\sqrt{3} &= 0 \end{aligned}$$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

The length of latus-rectum is given as:

$$2b^2/a$$

$$= 2(12)/3$$

$$= 24/3$$

$$= 8$$

$$(iv) 3x^2 - y^2 = 4$$

Given:

$$\text{The equation } \Rightarrow 3x^2 - y^2 = 4$$

The equation can be expressed as:

$$\frac{3x^2}{4} - \frac{y^2}{4} = 1$$

$$\frac{x^2}{\frac{4}{3}} - \frac{y^2}{4} = 1$$

$$\frac{x^2}{\left(\frac{2}{\sqrt{3}}\right)^2} - \frac{y^2}{(2)^2} = 1$$

The obtained equation is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where, $a = 2/\sqrt{3}$ and $b = 2$

Eccentricity is given by:

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$\begin{aligned}e &= \sqrt{1 + \frac{b^2}{a^2}} \\&= \sqrt{1 + \frac{4}{\frac{4}{3}}} \\&= \sqrt{1 + 3} \\&= \sqrt{4} \\&= 2\end{aligned}$$

Foci: The coordinates of the foci are $(\pm ae, 0)$

$$(\pm ae, 0) = \pm(2/\sqrt{3})(2) = \pm 4/\sqrt{3}$$

$$(\pm ae, 0) = (\pm 4/\sqrt{3}, 0)$$

The equation of directrices is given as:

$$\begin{aligned}x &= \pm \frac{a}{e} \\&\Rightarrow x = \pm \frac{2}{\frac{2}{\sqrt{3}}} \\&\Rightarrow x = \pm \frac{1}{\sqrt{3}} \\&\Rightarrow \sqrt{3}x = \pm 1 \\&\sqrt{3}x \mp 1 = 0\end{aligned}$$

The length of latus-rectum is given as:

$$2b^2/a$$

$$= 2(4)/[2/\sqrt{3}]$$

$$= 4\sqrt{3}$$

$$(v) 2x^2 - 3y^2 = 5$$

Given:

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

The equation $\Rightarrow 2x^2 - 3y^2 = 5$

The equation can be expressed as:

$$\begin{aligned}\frac{2x^2}{5} - \frac{3y^2}{5} &= 1 \\ \frac{x^2}{\frac{5}{2}} - \frac{y^2}{\frac{5}{3}} &= 1 \\ \frac{x^2}{\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2} - \frac{y^2}{\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2} &= 1\end{aligned}$$

The obtained equation is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where, $a = \sqrt{5}/\sqrt{2}$ and $b = \sqrt{5}/\sqrt{3}$

Eccentricity is given by:

$$\begin{aligned}e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{\frac{5}{3}}{\frac{5}{2}}} \\ &= \sqrt{1 + \frac{5}{3} \times \frac{2}{5}} \\ &= \sqrt{1 + \frac{2}{3}} \\ &= \sqrt{\frac{5}{3}}\end{aligned}$$

Foci: The coordinates of the foci are $(\pm ae, 0)$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$\begin{aligned}\pm ae &= \pm \sqrt{\frac{5}{2}} \times \sqrt{\frac{5}{3}} \\ &= \pm \frac{5}{\sqrt{6}}\end{aligned}$$

$$(\pm ae, 0) = (\pm 5/\sqrt{6}, 0)$$

The equation of directrices is given as:

$$\begin{aligned}x &= \pm \frac{a}{e} \\ \Rightarrow x &= \pm \frac{\frac{\sqrt{5}}{\sqrt{2}}}{\frac{\sqrt{5}}{\sqrt{3}}} \\ \Rightarrow x &= \pm \frac{1}{\sqrt{6}} \\ \Rightarrow \sqrt{6}x &= \pm 1 \\ \sqrt{6}x \mp 1 &= 0\end{aligned}$$

The length of latus-rectum is given as:

$$2b^2/a$$

$$\begin{aligned}&= \frac{2 \left(\frac{\sqrt{5}}{\sqrt{3}} \right)^2}{\frac{\sqrt{5}}{\sqrt{2}}} \\ &= \frac{2 \times \frac{5}{3}}{\frac{\sqrt{5}}{\sqrt{2}}} \\ &= \frac{2\sqrt{10}}{3}\end{aligned}$$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

4. Find the axes, eccentricity, latus-rectum and the coordinates of the foci of the hyperbola $25x^2 - 36y^2 = 225$.

Solution:

Given:

The equation $\Rightarrow 25x^2 - 36y^2 = 225$

The equation can be expressed as:

$$\frac{25x^2}{225} - \frac{36y^2}{225} = 1$$
$$\frac{x^2}{\left(\frac{15}{5}\right)^2} - \frac{y^2}{\left(\frac{15}{6}\right)^2} = 1 \quad \frac{x^2}{3^2} - \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$$

The obtained equation is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where, $a = 3$ and $b = 5/2$

Eccentricity is given by:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$
$$= \sqrt{1 + \frac{25}{9}}$$
$$= \sqrt{1 + \frac{25}{36}}$$
$$= \sqrt{\frac{61}{36}}$$
$$= \frac{\sqrt{61}}{6}$$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

Foci: The coordinates of the foci are $(\pm ae, 0)$

$$(\pm ae, 0) = \pm 3 (\sqrt{61}/6) = \pm \sqrt{61}/2$$

$$(\pm ae, 0) = (\pm \sqrt{61}/2, 0)$$

The equation of directrices is given as:

$$\begin{aligned}x &= \pm \frac{a}{e} \\ \Rightarrow x &= \pm \frac{3}{\frac{\sqrt{61}}{6}} \\ \Rightarrow x &= \pm \frac{18}{\sqrt{61}} \\ \Rightarrow \sqrt{61}x &= \pm 18 \\ \sqrt{61}x \mp 18 &= 0\end{aligned}$$

The length of latus-rectum is given as:

$$2b^2/a$$

$$\begin{aligned}&= \frac{2 \left(\frac{5}{2}\right)^2}{3} \\ &= \frac{2 \times \frac{25}{4}}{3} \\ &= \frac{25}{6}\end{aligned}$$

$\therefore$ Transverse axis = 6, conjugate axis = 5, $e = \sqrt{61}/6$, LR = 25/6, foci = $(\pm \sqrt{61}/2, 0)$

5. Find the centre, eccentricity, foci and directions of the hyperbola

(i) $16x^2 - 9y^2 + 32x + 36y - 164 = 0$

(ii) $x^2 - y^2 + 4x = 0$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

(iii) $x^2 - 3y^2 - 2x = 8$

Solution:

(i) $16x^2 - 9y^2 + 32x + 36y - 164 = 0$

Given:

The equation $\Rightarrow 16x^2 - 9y^2 + 32x + 36y - 164 = 0$

Let us find the centre, eccentricity, foci and directions of the hyperbola

By using the given equation

$$16x^2 - 9y^2 + 32x + 36y - 164 = 0$$

$$16x^2 + 32x + 16 - 9y^2 + 36y - 36 - 16 + 36 - 164 = 0$$

$$16(x^2 + 2x + 1) - 9(y^2 - 4y + 4) - 16 + 36 - 164 = 0$$

$$16(x^2 + 2x + 1) - 9(y^2 - 4y + 4) - 144 = 0$$

$$16(x + 1)^2 - 9(y - 2)^2 = 144$$

$$\frac{16(x + 1)^2}{144} - \frac{9(y - 2)^2}{144} = 1$$

$$\frac{(x + 1)^2}{9} - \frac{(y - 2)^2}{16} = 1$$

$$\frac{(x + 1)^2}{3^2} - \frac{(y - 2)^2}{4^2} = 1$$

Here, center of the hyperbola is $(-1, 2)$

So, let $x + 1 = X$ and $y - 2 = Y$

The obtained equation is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where, $a = 3$ and $b = 4$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

Eccentricity is given by:

$$\begin{aligned}e &= \sqrt{1 + \frac{b^2}{a^2}} \\&= \sqrt{1 + \frac{16}{9}} \\&= \sqrt{\frac{25}{9}} \\&= \frac{5}{3}\end{aligned}$$

Foci: The coordinates of the foci are $(\pm ae, 0)$

$$X = \pm 5 \text{ and } Y = 0$$

$$x + 1 = \pm 5 \text{ and } y - 2 = 0$$

$$x = \pm 5 - 1 \text{ and } y = 2$$

$$x = 4, -6 \text{ and } y = 2$$

So, Foci: $(4, 2)$ $(-6, 2)$

Equation of directrix are:

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$\begin{aligned}
 X &= \pm \frac{a}{e} \\
 \Rightarrow X &= \pm \frac{3}{\frac{5}{3}} \\
 \Rightarrow X &= \pm \frac{9}{5} \\
 \Rightarrow 5X &= \pm 9 \\
 \Rightarrow 5X \mp 9 &= 0 \\
 \Rightarrow 5(x+1) \mp 9 &= 0 \\
 \Rightarrow 5x+5 \mp 9 &= 0 \\
 \Rightarrow 5x+5-9 &= 0 \text{ and } 5x+5+9=0 \\
 5x-4 &= 0 \text{ and } 5x+14=0
 \end{aligned}$$

$\therefore$ The center is $(-1, 2)$, eccentricity $(e) = 5/3$, Foci = $(4, 2)$ $(-6, 2)$, Equation of directrix = $5x - 4 = 0$ and $5x + 14 = 0$

(ii) $x^2 - y^2 + 4x = 0$

Given:

The equation $\Rightarrow x^2 - y^2 + 4x = 0$

Let us find the centre, eccentricity, foci and directions of the hyperbola

By using the given equation

$$x^2 - y^2 + 4x = 0$$

$$x^2 + 4x + 4 - y^2 - 4 = 0$$

$$(x+2)^2 - y^2 = 4$$

$$\frac{(x+2)^2}{4} - \frac{y^2}{4} = 1$$

$$\frac{(x+2)^2}{2^2} - \frac{y^2}{2^2} = 1$$

Here, center of the hyperbola is $(2, 0)$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

So, let $x - 2 = X$

The obtained equation is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where, $a = 2$ and $b = 2$

Eccentricity is given by:

$$\begin{aligned} e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{4}{4}} \\ &= \sqrt{1 + 1} \\ &= \sqrt{2} \end{aligned}$$

Foci: The coordinates of the foci are $(\pm ae, 0)$

$$X = \pm 2\sqrt{2} \text{ and } Y = 0$$

$$X + 2 = \pm 2\sqrt{2} \text{ and } Y = 0$$

$$X = \pm 2\sqrt{2} - 2 \text{ and } Y = 0$$

$$\text{So, Foci} = (\pm 2\sqrt{2} - 2, 0)$$

Equation of directrix are:

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$X = \pm \frac{a}{e}$$

$$\Rightarrow X = \pm \frac{2}{\sqrt{2}}$$

$$\Rightarrow X = \pm \frac{2}{\sqrt{2}}$$

$$\Rightarrow X = \pm \sqrt{2}$$

$$\Rightarrow X \mp \sqrt{2} = 0$$

$$\Rightarrow x + 2 \mp \sqrt{2} = 0$$

$$x + 2 - \sqrt{2} = 0 \text{ and } x + 2 + \sqrt{2} = 0$$

$\therefore$ The center is $(-2, 0)$, eccentricity $(e) = \sqrt{2}$, Foci $= (-2 \pm 2\sqrt{2}, 0)$, Equation of directrix =

$$x + 2 = \pm \sqrt{2}$$

$$\text{(iii) } x^2 - 3y^2 - 2x = 8$$

Given:

$$\text{The equation } \Rightarrow x^2 - 3y^2 - 2x = 8$$

Let us find the centre, eccentricity, foci and directions of the hyperbola

By using the given equation

$$x^2 - 3y^2 - 2x = 8$$

$$x^2 - 2x + 1 - 3y^2 - 1 = 8$$

$$(x - 1)^2 - 3y^2 = 9$$

$$\frac{(x - 1)^2}{9} - \frac{3y^2}{9} = 1$$

$$\frac{(x - 1)^2}{3^2} - \frac{y^2}{(\sqrt{3})^2} = 1$$

Here, center of the hyperbola is $(1, 0)$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

So, let $x - 1 = X$

The obtained equation is of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Where, $a = 3$ and $b = \sqrt{3}$

Eccentricity is given by:

$$\begin{aligned} e &= \sqrt{1 + \frac{b^2}{a^2}} \\ &= \sqrt{1 + \frac{3}{9}} \\ &= \sqrt{1 + \frac{1}{3}} \\ &= \sqrt{\frac{4}{3}} \\ &= \frac{2}{\sqrt{3}} \\ &= \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \\ &= \frac{2\sqrt{3}}{3} \end{aligned}$$

Foci: The coordinates of the foci are $(\pm ae, 0)$

$$X = \pm 2\sqrt{3} \text{ and } Y = 0$$

$$X - 1 = \pm 2\sqrt{3} \text{ and } Y = 0$$

$$X = \pm 2\sqrt{3} + 1 \text{ and } Y = 0$$

$$\text{So, Foci} = (1 \pm 2\sqrt{3}, 0)$$

Equation of directrix are:

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$X = \pm \frac{a}{e}$$

$$\Rightarrow X = \pm \frac{3}{\frac{2\sqrt{3}}{3}}$$

$$\Rightarrow X = \pm \frac{9}{2\sqrt{3}}$$

$$X = \pm \frac{9}{2\sqrt{3}} + 1$$

$$x = \pm \frac{9}{2\sqrt{3}}$$

$\therefore$ The center is (1, 0), eccentricity (e) = $2\sqrt{3}/3$, Foci = $(1 \pm 2\sqrt{3}, 0)$, Equation of directrix =

$$X = 1 \pm 9/2\sqrt{3}$$

6. Find the equation of the hyperbola, referred to its principal axes as axes of coordinates, in the following cases:

(i) the distance between the foci = 16 and eccentricity = $\sqrt{2}$

(ii) conjugate axis is 5 and the distance between foci = 13

(iii) conjugate axis is 7 and passes through the point (3, -2)

Solution:

(i) the distance between the foci = 16 and eccentricity = $\sqrt{2}$

Given:

Distance between the foci = 16

Eccentricity = $\sqrt{2}$

Let us compare with the equation of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between the foci is $2ae$ and $b^2 = a^2(e^2 - 1)$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

So,

$$2ae = 16$$

$$ae = 16/2$$

$$a\sqrt{2} = 8$$

$$a = 8/\sqrt{2}$$

$$a^2 = 64/2$$

$$= 32$$

We know that, $b^2 = a^2(e^2 - 1)$

$$\text{So, } b^2 = 32 [(\sqrt{2})^2 - 1]$$

$$= 32 (2 - 1)$$

$$= 32$$

The Equation of hyperbola is given as

$$\frac{x^2}{32} - \frac{y^2}{32} = 1$$

$$x^2 - y^2 = 32$$

∴ The Equation of hyperbola is $x^2 - y^2 = 32$

(ii) conjugate axis is 5 and the distance between foci = 13

Given:

Conjugate axis = 5

Distance between foci = 13

Let us compare with the equation of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

Distance between the foci is $2ae$ and $b^2 = a^2(e^2 - 1)$

Length of conjugate axis is $2b$

So,

$$2b = 5$$

$$b = 5/2$$

$$b^2 = 25/4$$

We know that, $2ae = 13$

$$ae = 13/2$$

$$a^2e^2 = 169/4$$

$$b^2 = a^2(e^2 - 1)$$

$$b^2 = a^2e^2 - a^2$$

$$25/4 = 169/4 - a^2$$

$$a^2 = 169/4 - 25/4$$

$$= 144/4$$

$$= 36$$

The Equation of hyperbola is given as

$$\begin{aligned} &\Rightarrow \frac{x^2}{36} - \frac{y^2}{\frac{25}{4}} = 1 \\ &\Rightarrow \frac{x^2}{36} - \frac{4y^2}{25} = 1 \\ &\Rightarrow \frac{25x^2 - 144y^2}{900} = 1 \\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 &\Rightarrow 25x^2 - 144y^2 = 900 \end{aligned}$$

$\therefore$ The Equation of hyperbola is $25x^2 - 144y^2 = 900$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

(iii) conjugate axis is 7 and passes through the point (3, -2)

Given:

Conjugate axis = 7

Passes through the point (3, -2)

Conjugate axis is $2b$

So,

$$2b = 7$$

$$b = 7/2$$

$$b^2 = 49/4$$

The Equation of hyperbola is given as

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since it passes through points (3, -2)

$$\Rightarrow \frac{(3)^2}{a^2} - \frac{(-2)^2}{\frac{49}{4}} = 1$$

$$\Rightarrow \frac{9}{a^2} - \frac{4(4)}{49} = 1$$

$$\Rightarrow \frac{9}{a^2} - \frac{16}{49} = 1$$

$$\Rightarrow \frac{9}{a^2} = 1 + \frac{16}{49}$$

$$\Rightarrow \frac{9}{a^2} = \frac{49 + 16}{49}$$

$$\Rightarrow \frac{9}{a^2} = \frac{65}{49}$$

$$\Rightarrow a^2 = \frac{49}{65} \times 9$$

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

$$a^2 = 441/65$$

The equation of hyperbola is given as:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$a^2 = 441/65 \text{ and } b^2 = 49/4$$

$$\Rightarrow \frac{x^2}{\frac{441}{65}} - \frac{y^2}{\frac{49}{4}} = 1$$

$$\Rightarrow \frac{65x^2}{441} - \frac{4y^2}{49} = 1$$

$$\Rightarrow \frac{65x^2 - 36y^2}{441} = 1$$

$$\Rightarrow 65x^2 - 36y^2 = 441$$

∴ The Equation of hyperbola is $65x^2 - 36y^2 = 441$



Chapterwise RD Sharma Solutions for Class 11 Maths :

- Chapter 1–Sets
- Chapter 2–Relations
- Chapter 3–Functions
- Chapter 4–Measurement of Angles
- Chapter 5–Trigonometric Functions
- Chapter 6–Graphs of Trigonometric Functions
- Chapter 7–Values of Trigonometric Functions at Sum or Difference of Angles
- Chapter 8–Transformation Formulae
- Chapter 9–Values of Trigonometric Functions at Multiples and Submultiples of an Angle
- Chapter 10–Sine and Cosine Formulae and their Applications
- Chapter 11–Trigonometric Equations
- Chapter 12–Mathematical Induction
- Chapter 13–Complex Numbers
- Chapter 14–Quadratic Equations
- Chapter 15–Linear Inequations
- Chapter 16–Permutations
- Chapter 17–Combinations
- Chapter 18–Binomial Theorem
- Chapter 19–Arithmetic Progressions
- Chapter 20–Geometric Progressions

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbolic/>

- Chapter 21–Some Special Series
- Chapter 22–Brief review of Cartesian System of Rectangular Coordinates
- Chapter 23–The Straight Lines
- Chapter 24–The Circle
- Chapter 25–Parabola
- Chapter 26–Ellipse
- Chapter 27–Hyperbola
- Chapter 28–Introduction to Three Dimensional Coordinate Geometry
- Chapter 29–Limits
- Chapter 30–Derivatives
- Chapter 31–Mathematical Reasoning
- Chapter 32–Statistics
- Chapter 33–Probability

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>

About RD Sharma

RD Sharma isn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many CBSE students lose their dread of maths. Sunday Times profiles the tutor turned internet star

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.

<https://www.indcareer.com/schools/rd-sharma-solutions-for-class-11-maths-chapter-27-hyperbola/>