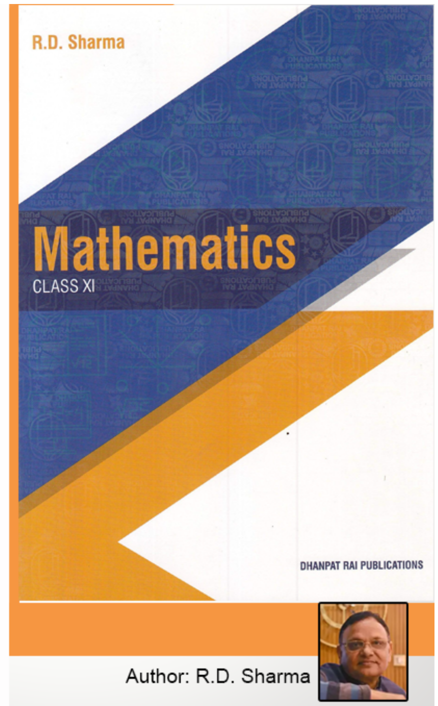
Class 11 - Chapter 29 Limits



RD Sharma Solutions for Class 11 Maths Chapter 29–Limits

Class 11: Maths Chapter 29 solutions. Complete Class 11 Maths Chapter 29 Notes.

RD Sharma Solutions for Class 11 Maths Chapter 29–Limits

RD Sharma 11th Maths Chapter 29, Class 11 Maths Chapter 29 solutions

EXERCISE 29.1 PAGE NO: 29.11

1. Show that does not exist.

Solution:

Firstly let us consider LHS:

$$\lim_{x\to 0^-}\left(\frac{x}{|x|}\right)$$

So, let x = 0 – h, where, h = 0

$$\lim_{x\to 0}\frac{x}{|x|} = \lim_{h\to 0}\left(\frac{0-h}{|0-h|}\right)$$

$$= \lim_{h\to 0}\left(\frac{-h}{h}\right)$$

$$= -1$$

Now, let us consider RHS:

$$\lim_{x\to 0^+}\frac{(x)}{|x|}$$

So, let x = 0 + h, where, h = 0

$$\lim_{x\to 0}\frac{x}{|x|} = \lim_{h\to 0}\left(\frac{0+h}{|0+h|}\right)$$

$$= \lim_{h\to 0}\left(\frac{h}{h}\right)$$

$$= 1$$

Since LHS $\neq$ RHS

$\therefore$ Limit does not exist.

2. Find k so that $\lim_{x\to 2} f(x)$ **may exist, where** $f(x) = \begin{cases} 2x+3, x \le 2 \\ x+k,\ x > 2 \end{cases}$

Solution:

Firstly let us consider LHS:

$$\lim_{x\to 2^-} f(x)$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(2x+3)$$

So, let x = 2 – h, where h = 0

Substituting the value of x, we get

$$\lim_{h \to 0} [2(2-h)+3]$$

=> 2 (2 - 0) + 3 = 7

Now let us consider RHS:

$$\lim_{x \to 2^+} f(x)$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x+k)$$

So, let x = 2 + h, where, h = 0

$$\lim_{h \to 0} (2+h+k)$$

=> 2 + 0 + k = 2 + k

Since, Limit exists, LHS = RHS

7 = 2 + k

k = 7 – 2

= 5

∴ Value of k is 5.

3. Show that $\lim_{x \to 0} \frac{1}{x}$ does not exist.

Solution:

Firstly let us consider LHS:

$$\lim_{x \to 0^-} \left(\frac{1}{x}\right)$$

So, let x = 0 – h, where h = 0

$$\lim_{x \to 0^-} \left(\frac{1}{x}\right) = \lim_{h \to 0} \left(\frac{1}{0-h}\right)$$

$$= -\infty$$

Now, let us consider RHS:

$$\lim_{x \to 0^+} \left(\frac{1}{x}\right)$$

So, let x = 0 + h, where h = 0

$$\lim_{x \to 0^+} \left(\frac{1}{x}\right) = \lim_{h \to 0} \left(\frac{1}{0+h}\right)$$

$$= \infty$$

Since, LHS ≠ RHS

∴ Limit does not exist.

4. Let f(x) be a function defined by $f(x)=\begin{cases}\dfrac{3x}{|x|+2x}, & x\neq 0\\ 0, & x=0\end{cases}$

Show that $\lim_{x\to 0} f(x)$ **does not exist.**

Solution:

Firstly let us consider LHS:

$$\lim_{x\to 0^-}\left[\frac{3x}{|x|+2x}\right]$$

So, let x = 0 – h, where h = 0

Substituting the value of x, we get

$$\lim_{x\to 0^-}\left[\frac{3x}{|x|+2x}\right]=\lim_{h\to 0}\left[\frac{3(-h)}{|-h|+2(-h)}\right]$$

$$=\lim_{h\to 0}\left[\frac{-3h}{h-2h}\right]$$

$$=\lim_{h\to 0}\left[\frac{-3h}{-h}\right]$$

$$=3$$

Now, let us consider RHS:

$$\lim_{x\to 0^+}\left(\frac{3x}{|x|+2x}\right)$$

So, let x = 0 + h, where h = 0

Substituting the value of x, we get

$$\lim_{x\to 0^+}\left(\frac{3x}{|x|+2x}\right)=\lim_{h\to 0}\left(\frac{3h}{|h|+2h}\right)$$

$$=\lim_{h\to 0}\left(\frac{3h}{h+2h}\right)$$

$$=1$$

Since, LHS ≠ RHS

∴ Limit does not exist.

5. Let $f(x)=\begin{cases}x+1, \text{if } x>0\\ x-1, \text{if } x<0\end{cases}$ **. Prove that** $\lim_{x\to 0} f(x)$ **does not exist.**

Solution:

Firstly let us consider LHS:

$$\lim_{x\to 0^-} f(x)$$

So, let x = 0 – h, where h = 0

Substituting the value of x, we get

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0}(0-h-1)$$

$$= -1$$

Now, let us consider RHS

$$\lim_{x\to 0^+} f(x)$$

So, let x = 0 + h, where h = 0

Substituting the value of x, we get

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0}(x+1)$$

$$= \lim_{h\to 0}(0+h+1)$$

$$= 1$$

Since, LHS ≠ RHS

∴ Limit does not exist.

EXERCISE 29.2 PAGE NO: 29.18

Evaluate the following limits:

1. $\lim_{x\to1}\frac{x^2+1}{x+1}$

Solution:
Given:

$$\lim_{x\to1}\frac{x^2+1}{x+1}$$

Let us substitute the value of x directly in the given limit, we get

$$\lim_{x\to1}\frac{x^2+1}{x+1} = \frac{1^2+1}{1+1}$$

$= 2/2$

$= 1$

∴ The value of the given limit is 1.

2. $\lim_{x\to0}\frac{2x^2+3x+4}{x^2+3x+2}$

Solution:
Given:

$$\lim_{x\to0}\frac{2x^2+3x+4}{x^2+3x+2}$$

Let us substitute the value of x directly in the given limit, we get

$$\lim_{x\to0}\frac{2x^2+3x+4}{x^2+3x+2} = \frac{2(0^2)+3(0)+4}{0^2+3(0)+2}$$

$= 4/2$

$= 2$

∴ The value of the given limit is 2.

3. $\lim_{x\to3}\frac{\sqrt{2x+3}}{x+3}$

Solution:
Given:

$$\lim_{x \to 3} \frac{\sqrt{2x+3}}{x+3}$$

Let us substitute the value of x directly in the given limit, we get

$$\lim_{x \to 3} \frac{\sqrt{2x+3}}{x+3} = \frac{\sqrt{2(3)+3}}{3+3}$$

$= \sqrt{9} / 6$

$= 3 / 6$

$= 1 / 2$

∴ The value of the given limit is 1/2.

4. $\lim_{x \to 1} \frac{\sqrt{x+8}}{\sqrt{x}}$

Solution:

Given:

$\lim_{x \to 1} \frac{\sqrt{x+8}}{\sqrt{x}}$

Let us substitute the value of x directly in the given limit, we get

$$\lim_{x \to 1} \frac{\sqrt{x+8}}{\sqrt{x}} = \frac{\sqrt{1+8}}{1}$$
$$= \frac{\sqrt{9}}{1}$$
$$= 3$$

∴ The value of the given limit is 3.

5. $\lim_{x \to a} \frac{\sqrt{x}+\sqrt{a}}{x+a}$

Solution:

Given:

$\lim_{x \to a} \frac{\sqrt{x}+\sqrt{a}}{x+a}$

Let us substitute the value of x directly in the given limit, we get

$$\lim_{x \to a} \frac{\sqrt{x}+\sqrt{a}}{x+a} = \frac{\sqrt{a}+\sqrt{a}}{a+a}$$
$$= \frac{2\sqrt{a}}{2a}$$
$$= \frac{1}{\sqrt{a}}$$

∴ The value of the given limit is $1/\sqrt{a}$.

EXERCISE 29.3 PAGE NO: 29.23

Evaluate the following limits:

1. $\lim_{x\to -5} \frac{2x^2+9x-5}{x+5}$

Solution:

Given:

The limit $\lim_{x\to -5} \frac{2x^2+9x-5}{x+5}$

By substituting the value of x, we get

$$\lim_{x\to -5} \frac{2x^2+9x-5}{x+5} = \frac{2(-5)^2+9(-5)-5}{(-5)+5}$$

$$= \frac{50-50}{(-5)+5}$$

$$= \frac{0}{0}$$ [Since, it is of the form indeterminate]

By using factorization method:

$$\lim_{x\to -5} \frac{2x^2+9x-5}{x+5} = \lim_{x\to -5} \frac{2x^2 + 9x-5}{x + 5}$$

$$= \lim_{x\to -5} \frac{2x^2 + 10x-x-5}{x + 5}$$

$$= \lim_{x\to -5} \frac{2x(x + 5)-(x + 5)}{x + 5}$$

$$= \lim_{x\to -5} \frac{(2x-1)(x + 5)}{x + 5}$$

$$= \lim_{x\to -5} 2x-1$$

= 2 (- 5) - 1

= - 11

∴ The value of the given limit is – 11.

2. $\lim_{x\to 3} \frac{x^2-4x+3}{x^2-2x-3}$

Solution:

Given:

The limit $\lim_{x\to 3} \frac{x^2-4x+3}{x^2-2x-3}$

∴ The value of the given limit is ½.

3. $\lim_{x\to 3} \frac{x^4 - 81}{x^2 - 9}$

Solution:

Given:

The limit $\lim_{x\to 3} \frac{x^4 - 81}{x^2 - 9}$

By substituting the value of x, we get

$$\lim_{x\to 3} \frac{x^4 - 81}{x^2 - 9} = \frac{(3)^4 - 81}{(3)^2 - 9}$$

$$= \frac{81 - 81}{(-9) + 9}$$

$$= \frac{0}{0}$$ [Since, it is of the form indeterminate]

By using factorization method:

$$\lim_{x\to 3} \frac{x^4 - 81}{x^2 - 9} = \lim_{x\to 3} \frac{(x^4 - 81)}{(x^2 - 9)}$$

$$= \lim_{x\to 3} \frac{(x^4 - 3^4)}{(x^2 - 3^2)}$$

$$= \lim_{x\to 3} \frac{((x^2)^2 - (3^2)^2)}{(x^2 - 3^2)}$$ [Since $a^2 - b^2 = (a + b)(a - b)$]

So,

$$= \lim_{x\to 3} \frac{(x^2 - 3^2)(x^2 + 3^2)}{(x^2 - 3^2)}$$

$$= \lim_{x\to 3} (x^2 + 3^2)$$

$$= 3^2 + 3^2$$

$$= 18$$

∴ The value of the given limit is 18.

4. $\lim_{x\to 2} \frac{x^3 - 8}{x^2 - 4}$

Solution:

Given:

The limit $\lim_{x\to 2} \frac{x^3 - 8}{x^2 - 4}$

By substituting the value of x, we get

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4} = \frac{(2)^3-8}{(2)^2-4}$$
$$= \frac{8-8}{(4)-4}$$
$$= \frac{0}{0}$$ [Since, it is of the form indeterminate]

By using factorization method:

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4} = \lim_{x\to 2}\frac{(x^3-8)}{(x^2-4)}$$
$$= \lim_{x\to 2}\frac{(x^3-2^3)}{(x^2-2^2)}$$
$$= \lim_{x\to 2}\frac{(x-2)(x^2+2^2+2x)}{(x+2)(x-2)}$$

[By using the formula, $(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$ & $(a^2 - b^2) = (a + b)(a - b)$]

$$= \lim_{x\to 2}\frac{(x^2+2^2+2x)}{(x+2)}$$
$$= \frac{(2^2+2^2+2(2))}{(2+2)}$$
$$= \frac{3.4}{(4)}$$
$$= 3$$

∴ The value of the given limit is 3.

5. $\lim_{x\to -1/2} \frac{8x^3+1}{2x+1}$

Solution:

Given:

The limit $\lim_{x\to -1/2} \frac{8x^3+1}{2x+1}$

By substituting the value of x, we get

$$\lim_{x\to -1/2} \frac{8x^3+1}{2x+1} = \frac{8\left(-\frac{1}{2}\right)^3+1}{2\left(-\frac{1}{2}\right)+1}$$

$$= \frac{-1+1}{-1+1}$$

$$= \frac{0}{0}$$ [Since, it is of the form indeterminate]

By using factorization method:

$$\lim_{x\to -1/2} \frac{8x^3+1}{2x+1} = \lim_{x\to -\frac{1}{2}} \frac{8x^3 + 1}{2x + 1}$$

$$= \lim_{x\to -\frac{1}{2}} \frac{(2x)^3 + (1)^3}{2x + 1}$$

[By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$]

$$= \lim_{x\to -\frac{1}{2}} \frac{(2x + 1)((2x)^2 + (1)^2 - 2x)}{2x + 1}$$

$$= \lim_{x\to -\frac{1}{2}} (2x)^2 + (1)^2 - 2x$$

$$= (2(\frac{-1}{2}))^2 + (1)^2 - 2(-\frac{1}{2})$$

$$= 1 + 1 + 1$$

$$= 3$$

∴ The value of the given limit is 3.

EXERCISE 29.4 PAGE NO: 29.28

Evaluate the following limits:

1. $\lim_{x\to0}\frac{\sqrt{1+x+x^2}-1}{x}$

Solution:

Given:

The limit $\lim_{x\to0}\frac{\sqrt{1+x+x^2}-1}{x}$

We need to find the limit of the given equation when x => 0

Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.

Let us rationalizing the given equation, we get

$$\lim_{x\to0}\frac{\sqrt{1+x+x^2}-1}{x}=\lim_{x\to0}\frac{(\sqrt{1+x+x^2}-1)}{x}\frac{(\sqrt{1+x+x^2}+1)}{(\sqrt{1+x+x^2}+1)}$$

[By using the formula: $(a+b)(a-b)=a^2-b^2$]

$$=\lim_{x\to0}\frac{1+x+x^2-1}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$=\lim_{x\to0}\frac{x(1+x)}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$=\lim_{x\to0}\frac{(1+x)}{\left(\sqrt{1+x+x^2}+1\right)}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of x as 0, we get

$$\lim_{x\to0}\frac{\sqrt{1+x+x^2}-1}{x}=\frac{1}{1+1}$$

$$=\frac{1}{2}$$

∴ The value of the given limit is ½.

2. $\lim_{x\to0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$

Solution:

Given:

The limit $\lim_{x\to0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$

We need to find the limit of the given equation when x => 0

Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.

Let us rationalizing the given equation, we get

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}=\lim_{x\to 0}\frac{2x}{(\sqrt{a+x}-\sqrt{a-x})}\frac{(\sqrt{a+x}+\sqrt{a-x})}{(\sqrt{a+x}+\sqrt{a-x})}$$

[By using the formula: $(a + b)(a - b) = a^2 - b^2$]

$$=\lim_{x\to 0}\frac{2x(\sqrt{a+x}+\sqrt{a-x})}{a+x-a+x}$$

$$=\lim_{x\to 0}\frac{2x(\sqrt{a+x}+\sqrt{a-x})}{2x}$$

$$=\lim_{x\to 0}\frac{(\sqrt{a+x}+\sqrt{a-x})}{1}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of x as 0, we get

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}=\sqrt{a}+\sqrt{a}$$

$$=2\sqrt{a}$$

∴ The value of the given limit is $2\sqrt{a}$

3. $\lim_{x\to 0}\frac{\sqrt{a^2+x^2}-a}{x^2}$

Solution:

Given:

The limit $\lim_{x\to 0}\frac{\sqrt{a^2+x^2}-a}{x^2}$

We need to find the limit of the given equation when $x \Rightarrow 0$

Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.

Let us rationalizing the given equation, we get

$$\lim_{x\to 0}\frac{\sqrt{a^2+x^2}-a}{x^2}=\lim_{x\to 0}\frac{(\sqrt{a^2+x^2}-a)}{x^2}\frac{(\sqrt{a^2+x^2}+a)}{(\sqrt{a^2+x^2}+a)}$$

[By using the formula: $(a + b)(a - b) = a^2 - b^2$]

$$=\lim_{x\to 0}\frac{(a^2+x^2-a^2)}{x^2(\sqrt{a^2+x^2}+a)}$$

$$=\lim_{x\to 0}\frac{x^2}{x^2(\sqrt{a^2+x^2}+a)}$$

$$=\lim_{x\to 0}\frac{1}{(\sqrt{a^2+x^2}+a)}$$

Now we can see that the indeterminate form is removed,
So, now we can substitute the value of x as 0, we get

$$\lim_{x\to 0}\frac{\sqrt{a^2+x^2}-a}{x^2}=\frac{1}{a+a}$$
$$=\frac{1}{2a}$$

∴ The value of the given limit is $\frac{1}{2a}$.

4. $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$

Solution:

Given:
The limit $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$

We need to find the limit of the given equation when x => 0
Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.
Let us rationalizing the given equation, we get

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{(1-x)}}{2x}=\lim_{x\to 0}\frac{\left(\sqrt{1+x}-\sqrt{(1-x)}\right)}{2x}\frac{\left(\sqrt{1+x}+\sqrt{(1-x)}\right)}{\left(\sqrt{1+x}+\sqrt{(1-x)}\right)}$$

[By using the formula: $(a + b)(a - b) = a^2 - b^2$]

$$=\lim_{x\to 0}\frac{1+x-1+x}{2x\left(\sqrt{1+x}+\sqrt{(1-x)}\right)}$$
$$=\lim_{x\to 0}\frac{2x}{2x\left(\sqrt{1+x}+\sqrt{(1-x)}\right)}$$
$$=\lim_{x\to 0}\frac{1}{\left(\sqrt{1+x}+\sqrt{(1-x)}\right)}$$

Now we can see that the indeterminate form is removed,
So, now we can substitute the value of x as 0, we get

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{(1-x)}}{2x}=\frac{1}{1+1}$$
$$=\frac{1}{2}$$

∴ The value of the given limit is ½.

5. $\lim_{x\to 2}\frac{\sqrt{3-x}-1}{2-x}$

Solution:

Given:

The limit $\lim_{x\to 2}\frac{\sqrt{3-x}-1}{2-x}$

We need to find the limit of the given equation when x => 0

Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.

Let us rationalizing the given equation, we get

$$\lim_{x\to 2}\frac{\sqrt{3-x}-1}{2-x}=\lim_{x\to 2}\frac{(\sqrt{3-x}-1)}{(2-x)}\frac{(\sqrt{3-x}+1)}{(\sqrt{3-x}+1)}$$

[By using the formula: $(a + b)(a - b) = a^2 - b^2$]

$$=\lim_{x\to 2}\frac{(3-x-1)}{(2-x)(\sqrt{3-x}+1)}$$
$$=\lim_{x\to 2}\frac{(2-x)}{(2-x)(\sqrt{3-x}+1)}$$
$$=\lim_{x\to 2}\frac{1}{(\sqrt{3-x}+1)}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of x as 0, we get

$$\lim_{x\to 2}\frac{\sqrt{3-x}-1}{2-x}=\frac{1}{1+1}$$
$$=\frac{1}{2}$$

∴ The value of the given limit is ½.

EXERCISE 29.5 PAGE NO: 29.33

Evaluate the following limits:

1. $\lim_{x\to a} \frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a}$

Solution:

Given:

The limit $\lim_{x\to a} \frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a}$

When x = a, the expression $\lim_{x\to a} \frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a}$ assumes the form (0/0).

So let, $Z = \lim_{x\to a} \frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a}$

By using the formula: $\lim_{x\to a} \frac{(x)^n-(a)^n}{x-a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

$Z = \lim_{x\to a} \frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a}$

$Z = \lim_{x\to a} \frac{(x+2)^{5/2}-(a+2)^{5/2}}{x+2-(a+2)}$

Let x + 2 = y and a + 2 = k

As x → a; y → k

So,

$Z = \lim_{y\to k} \frac{(y)^{5/2}-(k)^{5/2}}{y-k}$

By using the formula: $\lim_{x\to a} \frac{(x)^n-(a)^n}{x-a} = na^{n-1}$

$Z = \frac{5}{2} k^{\frac{5}{2}-1}$

$= \frac{5}{2} k^{\frac{3}{2}}$

$= \frac{5}{2} (a+2)^{\frac{3}{2}}$

$\therefore \lim_{x\to a} \frac{(x+2)^{5/2}-(a+2)^{5/2}}{x-a} = \frac{5}{2} (a+2)^{\frac{3}{2}}$

2. $\lim\limits_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$

Solution:

Given:

The limit $\lim\limits_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$

When x = a, the expression $\lim\limits_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$ assumes the form (0/0).

So let, $Z = \lim\limits_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$

By using the formula: $\lim\limits_{x\to a}\frac{(x)^n-(a)^n}{x-a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

$Z = \lim\limits_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$

$Z = \lim\limits_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x+2-(a+2)}$

Let x + 2 = y and a+2 = k

As x → a; y → k

$Z = \lim\limits_{y\to k}\frac{(y)^{3/2}-(k)^{3/2}}{y-k}$

By using the formula: $\lim\limits_{x\to a}\frac{(x)^n-(a)^n}{x-a} = na^{n-1}$

$Z = \frac{3}{2}\, k^{\frac{3}{2}-1}$

$= \frac{3}{2}\, k^{\frac{1}{2}}$

$= \frac{3}{2}(a+2)^{\frac{1}{2}}$

$\therefore \lim\limits_{x\to a}\frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a} = \frac{3}{2}\sqrt{a+2}$

3. $\lim\limits_{x\to a}\frac{(1+x)^6-1}{(1+x)^2-1}$

Solution:

Given:

The limit $\lim\limits_{x\to a}\frac{(1+x)^6-1}{(1+x)^2-1}$

4. $\lim_{x \to a} \frac{x^{2/7} - a^{2/7}}{x - a}$

Solution:

Given: $\lim_{x \to a} \frac{x^{2/7} - a^{2/7}}{x - a}$

The limit

When x = a, the expression $\lim_{x \to a} \frac{x^{2/7} - a^{2/7}}{x - a}$ assumes the form (0/0).

So let, $Z = \lim_{x \to a} \frac{x^{2/7} - a^{2/7}}{x - a}$

By using the formula: $\lim_{x \to a} \frac{(x)^n - (a)^n}{x - a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

$Z = \lim_{x \to a} \frac{x^{2/7} - a^{2/7}}{x - a}$

By using the formula: $\lim_{x \to a} \frac{(x)^n - (a)^n}{x - a} = na^{n-1}$

$Z = \frac{2}{7} a^{\frac{2}{7} - 1}$

$= \frac{2}{7} a^{-\frac{5}{7}}$

$\therefore \lim_{x \to a} \frac{x^{2/7} - a^{2/7}}{x - a} = \frac{2}{7} a^{-\frac{5}{7}}$

5. $\lim_{x \to a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$

Solution:

Given: $\lim_{x \to a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$

The limit When x = a, the expression $\lim_{x \to a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$ assumes the form (0/0).

So let, $Z = \lim_{x \to a} \frac{x^{5/7} - a^{5/7}}{x^{2/7} - a^{2/7}}$

By using the formula: $\lim_{x \to a} \frac{(x)^n - (a)^n}{x - a} = na^{n-1}$

Since, Z is not of the form as described above.

Let us simplify, we get

$Z = \lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x^{\frac{2}{7}} - a^{\frac{2}{7}}}$

Let us divide the numerator and denominator by (x – a), we get

$Z = \lim_{x \to a} \frac{\frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}}$

By using algebra of limits, we have

$Z = \frac{\lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\lim_{x \to a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}}$

So now again, by using the formula: $\lim_{x \to a} \frac{(x)^n - (a)^n}{x - a} = na^{n-1}$

$Z = \frac{\frac{5}{7} a^{\frac{5}{7} - 1}}{\frac{2}{7} a^{\frac{2}{7} - 1}}$

$= \frac{5a^{-\frac{2}{7}}}{2a^{-\frac{5}{7}}}$

$= \frac{5}{2} a^{\frac{3}{7}}$

$\therefore \lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x^{\frac{2}{7}} - a^{\frac{2}{7}}} = \frac{5}{2} a^{\frac{3}{7}}$

EXERCISE 29.6 PAGE NO: 29.38

Evaluate the following limits:

1. $\lim_{x\to\infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)}$

Solution:

Given:

The limit $\lim_{x\to\infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)}$

Let us simplify the expression, we get

$$\lim_{x\to\infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)} = \lim_{x\to\infty} \frac{(12x^2-10x+2)}{(x^2+9x-8)}$$

$$= \lim_{x\to\infty} \left(\frac{12-\frac{10}{x}+\frac{2}{x^2}}{1+\frac{9}{x}-\frac{8}{x^2}} \right)$$

When substituting the value of x as $x\to\infty$ and $\frac{1}{x}\to 0$ then,

$$= \frac{12-0+0}{1}$$

$$= 12$$

$\therefore \lim_{x\to\infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)} = 12$

2. $\lim_{x\to\infty} \frac{3x^3-4x^2+6x-1}{2x^3+x^2-5x+7}$

Solution:

Given:

The limit $\lim_{x\to\infty} \frac{3x^3-4x^2+6x-1}{2x^3+x^2-5x+7}$

Let us simplify the expression, we get

$$\lim_{x\to\infty} \frac{3x^3-4x^2+6x-1}{2x^3+x^2-5x+7} = \lim_{x\to\infty} \frac{3-\frac{4}{x}+\frac{6}{x^2}-\frac{1}{x^3}}{2+\frac{1}{x}-\frac{5}{x^2}+\frac{7}{x^3}}$$

When substituting the value of x as $x\to\infty$ and $\frac{1}{x}\to 0$ then,

$$= \frac{3-0+0-0}{2+0-0+0}$$

= 3 / 2

$\therefore \lim_{x\to\infty} \frac{3x^3-4x^2+6x-1}{2x^3+x^2-5x+7} = \frac{3}{2}$

3. $\lim_{x\to\infty} \frac{5x^3-6}{\sqrt{9+4x^6}}$

Solution:

Given:

The limit $\lim_{x\to\infty} \frac{5x^3-6}{\sqrt{9+4x^6}}$

Let us simplify the expression, we get

$$\lim_{x\to\infty} \frac{5x^3-6}{\sqrt{(9+4x^6)}} = \lim_{x\to\infty} \frac{5-\frac{6}{x^3}}{\sqrt{(\frac{9}{x^6}+\frac{4x^6}{x^6}}}$$

$$= \lim_{x\to\infty} \frac{\left(5-\frac{6}{x^3}\right)}{\sqrt{\frac{9}{x^6}+4}}$$

When substituting the value of x as $x \to \infty$ and $\frac{1}{x} \to 0$ then,

$$= \frac{5}{\sqrt{4}}$$

= 5 / 2

$\therefore \lim_{x\to\infty} \frac{5x^3-6}{\sqrt{(9+4x^6)}} = \frac{5}{2}$

4. $\lim_{x \to \infty} \sqrt{x^2 + cx} - x$

Solution:

Given:

The limit $\lim_{x \to \infty} \sqrt{x^2 + cx} - x$

Let us simplify the expression by rationalizing the numerator, we get

$$\lim_{x \to \infty} \sqrt{x^2 + cx} - x = \lim_{x \to \infty} \left(\sqrt{x^2 + cx} - x\right).\frac{\sqrt{x^2 + cx} + x}{\sqrt{x^2 + cx} + x}$$

$$= \lim_{x \to \infty} \frac{(x^2 + cx - x^2)}{\sqrt{x^2 + cx} + x}$$

$$= \lim_{x \to \infty} \frac{cx}{\sqrt{x^2 + cx} + x}$$

By taking 'x' as common from both numerator and denominator, we get

$$= \lim_{x\to\infty} \frac{c}{\sqrt{1+\frac{c}{x}}+1}$$

When substituting the value of x as $x \to \infty$ and $\frac{1}{x} \to 0$ then,

$$= \frac{c}{1+1}$$

$$= \frac{c}{2}$$

$$\therefore \lim_{x\to\infty} \sqrt{x^2+cx} - x = \frac{c}{2}$$

5. $\lim_{x\to\infty} \sqrt{x+1} - \sqrt{x}$

Solution:

Given: $\lim_{x\to\infty} \sqrt{x+1} - \sqrt{x}$

The limit

Let us simplify the expression by rationalizing the numerator, we get

On rationalizing the numerator we get,

$$\lim_{x\to\infty} \sqrt{x+1} - \sqrt{x} = \lim_{x\to\infty} \sqrt{x+1} - \sqrt{x}.\frac{\sqrt{x+1}+\sqrt{x}}{(\sqrt{x+1}+\sqrt{x})}$$

$$= \lim_{x\to\infty} \frac{(x+1-x)}{\sqrt{x+1}+\sqrt{x}}$$

$$= \lim_{x\to\infty} \left(\frac{1}{\sqrt{x+1}+\sqrt{x}}\right)$$

When substituting the value of x as $x \to \infty$ and $\frac{1}{x} \to 0$ then,

$$= \frac{1}{\infty}$$

$$= 0$$

$$\therefore \lim_{x\to\infty} \sqrt{x+1} - \sqrt{x} = 0$$

EXERCISE 29.7 PAGE NO: 29.49

Evaluate the following limits:

1. $\lim_{x\to0}\frac{\sin 3x}{5x}$

Solution:

Given:

The limit $\lim_{x\to0}\frac{\sin 3x}{5x}$

Let us consider the limit:

$$\lim_{x\to0}\frac{\sin 3x}{5x}=\frac{1}{5}\lim_{x\to0}\frac{\sin 3x}{x}$$

Now let us multiply and divide the expression by 3, we get

$$=\frac{1}{5}\lim_{x\to0}\frac{\sin 3x}{3x}\times 3$$

$$=\frac{3}{5}\lim_{x\to0}\frac{\sin 3x}{3x}$$

Now, put $3x = y$

$$=\frac{3}{5}\lim_{y\to0}\frac{\sin y}{y}$$ [We know that, $\lim_{y\to0}\frac{\sin y}{y}=1$]

So,

$$\lim_{x\to0}\frac{\sin 3x}{5x}=\frac{3}{5}\lim_{y\to0}\frac{\sin y}{y}$$

$$=\frac{3}{5}\times 1$$

$$=\frac{3}{5}$$

∴ The value of $\lim_{x\to0}\frac{\sin 3x}{5x}=\frac{3}{5}$

2. $\lim_{x\to0}\frac{\sin x^0}{x}$

Solution:

Given:

The limit $\lim_{x\to0}\frac{\sin x^0}{x}$

We know, $1° = \frac{\pi}{180}$ radians

So,

$x^\circ = \frac{\pi x}{180}$ radians

Let us consider the limit,

$$\lim_{x\to 0}\frac{\sin x^\circ}{x} = \lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{x}$$

Now let us multiply and divide the expression by $\frac{\pi}{180}$, we get

$$= \lim_{x\to 0}\frac{\sin\frac{\pi x}{180}\times\frac{\pi}{180}}{x\times\frac{\pi}{180}}$$

$$= \frac{\pi}{180}\lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}}$$

Now, put $\frac{\pi x}{180} = y$

$$= \frac{\pi}{180}\lim_{y\to 0}\frac{\sin y}{y}$$ [We know that, $\lim_{y\to 0}\frac{\sin y}{y} = 1$]

$$\lim_{x\to 0}\frac{\sin x^\circ}{x} = \frac{\pi}{180}\lim_{y\to 0}\frac{\sin y}{y}$$

$$= \frac{\pi}{180}\times 1$$

$$= \frac{\pi}{180}$$

∴ The value of $\lim_{x\to 0}\frac{\sin x^\circ}{x} = \frac{\pi}{180}$

3. $\lim_{x\to 0} \frac{x^2}{\sin x^2}$

Solution:

Given:

The limit $\lim_{x\to 0} \frac{x^2}{\sin x^2}$

Let us consider the limit and divide the expression by x^2, we get

$$\lim_{x\to 0} \frac{x^2}{\sin x^2} = \lim_{x\to 0} \frac{1}{\frac{\sin x^2}{x^2}}$$

Now, put $x^2 = y$

$$\lim_{x\to 0} \frac{1}{\frac{\sin x^2}{x^2}} = \frac{1}{\lim_{y\to 0} \frac{\sin y}{y}}$$ [We know that, $\lim_{y\to 0} \frac{\sin y}{y} = 1$]

$$= \frac{1}{1}$$

$$= 1$$

∴ The value of $\lim_{x\to 0} \frac{x^2}{\sin x^2} = 1$

4. $\lim_{x\to 0} \frac{\sin x \cos x}{3x}$

Solution:

Given:

The limit $\lim_{x\to 0} \frac{\sin x \cos x}{3x}$

Let us consider the limit

$$\lim_{x\to 0} \frac{\sin x \cos x}{3x} = \frac{1}{3}\lim_{x\to 0} \frac{\sin x \cos x}{x}$$

$$= \frac{1}{3}\lim_{x\to 0} \left(\frac{\sin x}{x}\right) \cos x$$

We know,

$$\lim_{x\to 0} A(x).B(x) = \lim_{x\to 0} A(x) \times \lim_{x\to 0} B(x)$$

So,

$$= \frac{1}{3}\lim_{x\to 0} \frac{\sin x}{x} \times \lim_{x\to 0} \cos x$$ [We know that, $\lim_{x\to 0} \frac{\sin x}{x} = 1$]

$$\lim_{x\to 0} \frac{\sin x \cos x}{3x} = \frac{1}{3} \times 1 \times \cos 0$$

$$= \frac{1}{3} \times 1 \times 1$$ [Since, cos 0 = 1]

$$= \frac{1}{3}$$

∴ The value of $\lim_{x\to 0} \frac{\sin x \cos x}{3x} = \frac{1}{3}$

5. $\lim_{x\to 0}\frac{3\sin x-4\sin^3 x}{x}$

Solution:

Given: $\lim_{x\to 0}\frac{3\sin x-4\sin^3 x}{x}$

The limit

We know that, $\sin 3x = 3\sin x - 4\sin^3 x$

So,

$$\lim_{x\to 0}\frac{3\sin x-4\sin^3 x}{x}=\lim_{x\to 0}\frac{\sin 3x}{x}$$

Now multiply and divide the expression by 3, we get

$$\lim_{x\to 0}\frac{\sin 3x}{x}=\lim_{x\to 0}\frac{\sin 3x\times 3}{3x}$$

$$=3\lim_{x\to 0}\frac{\sin 3x}{3x}$$

Now, put $3x = y$

$$=3\lim_{y\to 0}\frac{\sin y}{y}\quad \left[\text{We know that, }\lim_{y\to 0}\frac{\sin y}{y}=1\right]$$

$$\lim_{x\to 0}\frac{3\sin x-4\sin^3 x}{x}=3\lim_{y\to 0}\frac{\sin y}{y}$$

$$=3\times 1$$

$$=3$$

∴ The value of $\lim_{x\to 0}\frac{3\sin x-4\sin^3 x}{x}=3$

EXERCISE 29.8 PAGE NO: 29.62

Evaluate the following limits:

1. $\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x$

Solution:

Given:

The limit $\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x$

Let us assume, $y = \frac{\pi}{2} - x$

So,

$x \to \frac{\pi}{2}, y \to 0$

$$\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x = \lim_{y \to 0} y \tan\left(\frac{\pi}{2} - y\right)$$

$$= \lim_{y \to 0} y \frac{\sin\left(\frac{\pi}{2} - y\right)}{\cos\left(\frac{\pi}{2} - y\right)}$$ [We know that, tan = sin/cos]

$$= \lim_{y \to 0} y \frac{\cos y}{\sin y}$$

Upon simplification, we get

$$= \lim_{y \to 0} \cos y - \lim_{y \to 0} \frac{y}{\sin y}$$

Substituting the value of $y = 0$, then

$$= \cos 0 - \frac{0}{\sin 0}$$

$$= 1 - 0$$

$$= 1$$

$\therefore$ The value of $\lim_{x \to \frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) = 1$

2. $\lim_{x \to \pi/2} \frac{\sin 2x}{\cos x}$

Solution:

Given:

The limit $\lim_{x \to \pi/2} \frac{\sin 2x}{\cos x}$

We know, $\sin 2x = 2\sin x.\cos x$

So,

$$\lim_{x\to\pi/2} \frac{\sin 2x}{\cos x} = \lim_{x\to\pi/2} \frac{2\sin x \cos x}{\cos x}$$

Upon simplification, we get

$$= \lim_{x\to\pi/2} 2\sin x$$

Substitute the value of x, we get

$$= 2\sin\frac{\pi}{2}$$

$$= 2 \times 1$$

$$= 2$$

$\therefore$ The value of $\lim_{x\to\pi/2} \frac{\sin 2x}{\cos x} = 2$

3. $\lim_{x\to\pi/2} \frac{\cos^2 x}{1-\sin x}$

Solution:

Given:

The limit $\lim_{x\to\pi/2} \frac{\cos^2 x}{1-\sin x}$

We know that, $\cos^2 x = 1-\sin^2 x$

So, by substituting this value we get,

$$\lim_{x\to\pi/2} \frac{\cos^2 x}{1-\sin x} = \lim_{x\to\pi/2} \frac{1-\sin^2 x}{1-\sin x}$$

Upon expansion,

$$= \lim_{x\to\pi/2} \frac{(1-\sin x)(1+\sin x)}{1-\sin x}$$

When simplified, we get

$$= \lim_{x\to\pi/2} 1+\sin x$$

Now, substitute the value of x, we get

$$= 1 + \sin\frac{\pi}{2}$$

$$= 1 + 1$$

$$= 2$$

$\therefore$ The value of $\lim_{x\to\pi/2} \frac{\cos^2 x}{1-\sin x} = 2$

4. $\lim_{x\to\pi/3} \frac{\sqrt{1-\cos 6x}}{\sqrt{2}(\pi/3-x)}$

Solution:

Given:

The limit $\lim_{x\to\pi/3} \frac{\sqrt{1-\cos 6x}}{\sqrt{2}(\pi/3-x)}$

We know that, $1 - \cos 2x = 2\sin^2 x$

So,

$$\lim_{x\to\frac{\pi}{3}} \frac{\sqrt{1-\cos 6x}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)} = \lim_{x\to\frac{\pi}{3}} \frac{\sqrt{2\sin^2 3x}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)}$$

$$= \lim_{x\to\frac{\pi}{3}} \frac{\sqrt{2}\sin 3x}{\sqrt{2}\left(\frac{\pi}{3}-x\right)}$$

$$= \lim_{x\to\frac{\pi}{3}} \frac{\sin 3x}{\left(\frac{\pi}{3}-x\right)}$$

$$= \lim_{x\to\frac{\pi}{3}} \frac{3\sin 3x}{\pi-3x}$$

We know that, $\sin x = \sin(\pi - x)$

So,

$$\lim_{x\to\frac{\pi}{3}} \frac{\sqrt{1-\cos 6x}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)} = \lim_{x\to\frac{\pi}{3}} \frac{3\sin(\pi-3x)}{\pi-3x}$$ [We know that, $\lim_{x\to 0}\frac{\sin x}{x} = 1$]

$= 3$

∴ The value of $\lim_{x\to\frac{\pi}{3}} \frac{\sqrt{1-\cos 6x}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)} = 3$

5. $\lim_{x\to a} \frac{\cos x - \cos a}{x - a}$

Solution:

Given:

The limit $\lim_{x\to a} \frac{\cos x - \cos a}{x - a}$

We know that,

$$\left[\quad \cos A - \cos B = 2\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)\right]$$

By substituting in the formula, we get

$$\lim_{x\to a}\frac{\cos x - \cos a}{x - a} = \lim_{x\to a}\frac{\left(-2\sin\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)\right)}{x-a}$$

$$= -2\lim_{x\to a}\sin\left(\frac{x+a}{a}\right)\lim_{x\to a}\frac{\sin\left(\frac{x-a}{a}\right)}{x-a}$$

Upon simplification, we get

$$= -2\sin\left(\frac{a+a}{a}\right)\left(\lim_{x\to a}\frac{\sin\left(\frac{x-a}{a}\right)}{x-a}\right)\times\frac{1}{2}$$

$$= -2\sin a \times 1 \times \frac{1}{2}$$

$$= -\sin a$$

∴ The value of $\lim_{x\to a}\frac{\cos x - \cos a}{x-a} = -\sin a$

EXERCISE 29.9 PAGE NO: 29.65

Evaluate the following limits:

1. $\lim\limits_{x\to\pi} \frac{1+\cos x}{\tan^2 x}$

Solution:

Given: $\lim\limits_{x\to\pi} \frac{1+\cos x}{\tan^2 x}$

The limit $\lim\limits_{x\to\pi} \frac{1+\cos x}{\tan^2 x}$

When $x = \pi$, the expression assumes the form (0/0).

So, let us multiply the expression by $\cos^2 x$

$$\lim_{x\to\pi} \frac{1+\cos x}{\tan^2 x} = \lim_{x\to\pi}\left[\frac{(1+\cos x)}{\sin^2 x}\times\cos^2 x\right]$$

$$= \lim_{x\to\pi}\left[\frac{(1+\cos x)}{1-\cos^2 x}\times\cos^2 x\right]$$

Upon expansion, we get

$$= \lim_{x\to\pi}\left[\frac{(1+\cos x)}{(1-\cos x)(1+\cos x)}\times\cos^2 x\right]$$

$$= \lim_{x\to\pi}\left[\frac{\cos^2 x}{(1-\cos x)}\right]$$

Now, substitute the value of x, we get

$$= \frac{\cos^2 \pi}{1-\cos\pi}$$

$$= \frac{(-1)^2}{1-(-1)}$$

$$= \frac{1}{2}$$

∴ The value of $\lim\limits_{x\to\pi} \frac{1+\cos x}{\tan^2 x} = \frac{1}{2}$

2. $\lim_{x \to \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$

Solution:

Given: $\lim_{x \to \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$

The limit

When x = π/4, the expression $\lim_{x \to \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$ assumes the form (0/0).

So,

$$\lim_{x \to \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1} = \lim_{x \to \frac{\pi}{4}} \left[\frac{1 + \cot^2 x - 2}{\cot x - 1}\right]$$ [Since, $\operatorname{cosec}^2 x = 1 + \cot^2 x$]

$$= \lim_{x \to \frac{\pi}{4}} \left[\frac{\cot^2 x - 1}{\cot x - 1}\right]$$

Upon expansion, we get

$$= \lim_{x \to \frac{\pi}{4}} \left[\frac{(\cot x - 1)(\cot x + 1)}{(\cot x - 1)}\right]$$

Now, substitute the value of x, we get

$$= \cot \frac{\pi}{4} + 1$$

$$= 2$$

∴ The value of $\lim_{x \to \frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1} = 2$

3. $\lim_{x \to \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$

Solution:

Given: $\lim_{x \to \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$

The limit

When x = π/6, the expression $\lim_{x \to \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$ assumes the form (0/0).

$$\lim_{x \to \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2} = \lim_{x \to \frac{\pi}{6}} \left[\frac{cosec^2 x - 1 - 3}{cosec x - 2}\right]$$ [Since, $\cot^2 x = \operatorname{cosec}^2 x - 1$]

4. $\lim_{x \to \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x}$

Solution:

Given: $\lim_{x \to \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x}$

The limit $\lim_{x \to \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x}$

When x = π/4, the expression assumes the form (0/0).

So,

$$\lim_{x \to \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x} = \lim_{x \to \frac{\pi}{4}} \left[\frac{2 - (1 + \cot^2 x)}{1 - \cot x}\right]$$ [Since, $\operatorname{cosec}^2 x = 1 + \cot^2 x$]

$$= \lim_{x \to \frac{\pi}{4}} \left[\frac{1 - \cot^2 x}{1 - \cot x}\right]$$

Upon expansion, we get

$$= \lim_{x \to \frac{\pi}{4}} \left[\frac{(1 - \cot x)(1 + \cot x)}{(1 - \cot x)}\right]$$

Now, substitute the value of x, we get

$$= 1 + \cot\left(\frac{\pi}{4}\right)$$

$$= 1 + 1$$

$$= 2$$

∴ The value of $\lim_{x \to \frac{\pi}{4}} \frac{2 - \operatorname{cosec}^2 x}{1 - \cot x} = 2$

5. $\lim_{x \to \pi} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$

Solution:

Given: $\lim_{x \to \pi} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$

The limit

When x = π, the expression $\lim_{x \to \pi} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$ assumes the form (0/0).

So, let us rationalize the numerator, we get

$$\lim_{x \to \pi} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2} = \lim_{x \to \pi}\left[\frac{\left(\sqrt{2+\cos x}-1\right)\times\left(\sqrt{2+\cos x}+1\right)}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}\right]$$

Let us simplify the above expression, we get

$$= \lim_{x \to \pi}\left[\frac{2+\cos x-1}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}\right]$$

$$= \lim_{x \to \pi}\left[\frac{1+\cos x}{(\pi-x)^2\left[\sqrt{2+\cos x}+1\right]}\right]$$

Now, let x = π – h

When x = π, then h = 0

So,

$$= \lim_{h \to 0}\left[\frac{1+\cos(\pi-h)}{[\pi-(\pi-h)]^2\left[\sqrt{2+\cos(\pi-h)}+1\right]}\right]$$

$$= \lim_{h \to 0} \left[\frac{1 - \cos h}{h^2 \left[\sqrt{2 - \cos h} + 1 \right]} \right] \{\because \cos(\pi - \theta) = -\cos\theta\}$$

Let us simplify further,

$$= \lim_{h \to 0} \left[\frac{2\sin^2\left(\frac{h}{2}\right)}{4 \times \frac{h^2}{4}\left[\sqrt{2 - \cos h} + 1\right]} \right]$$

$$= \frac{1}{2} \lim_{h \to 0} \left[\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}} \right)^2 \times \frac{1}{\left[\sqrt{2 - \cos h} + 1\right]} \right]$$

Now, substitute the value of h, we get

$$= \frac{1}{2} \times 1 \times \frac{1}{\left(\sqrt{2 - \cos 0} + 1\right)}$$

$$= \frac{1}{2} \times \frac{1}{\left(\sqrt{1} + 1\right)}$$

$$= \frac{1}{2 \times 2}$$

$$= \frac{1}{4}$$

∴ The value of $\lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} = \frac{1}{4}$

EXERCISE 29.10 PAGE NO: 29.71

Evaluate the following limits:

1. $\lim\limits_{x\to 0} \frac{5^x - 1}{\sqrt{4+x} - 2}$

Solution:

Given:

The limit $\lim\limits_{x\to 0} \frac{5^x - 1}{\sqrt{4+x} - 2}$

When x = 0, the expression $\lim\limits_{x\to 0} \frac{5^x - 1}{\sqrt{4+x} - 2}$ assumes the form (0/0).

So,

As Z = $\lim\limits_{x\to 0} \frac{5^x - 1}{\sqrt{4+x} - 2}$

Now, multiply both numerator and denominator by √(4+x) +2 so that we can remove the indeterminate form.

$$Z = \lim_{x\to 0} \frac{(5^x-1)\sqrt{4+x}+2}{(\sqrt{4+x})^2-2^2}$$

$$= \lim_{x\to 0} \frac{5^x-1}{\sqrt{4+x}-2} \times \frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$$

{By using $a^2 - b^2 = (a + b)(a - b)$}

$$Z = \lim_{x\to 0} \frac{(5^x-1)\sqrt{4+x}+2}{4+x-4}$$

$$= \lim_{x\to 0} \frac{(5^x-1)\sqrt{4+x}+2}{x}$$

By using basic algebra of limits, we get

$$Z = \lim_{x\to 0} \frac{(5^x-1)}{x} \times \lim_{x\to 0} \sqrt{4+x} + 2 = \{\sqrt{4+0}+2\}\lim_{x\to 0} \frac{(5^x-1)}{x}$$

$$= 4 \lim_{x\to 0} \frac{(5^x-1)}{x}$$ [By using the formula: $\lim\limits_{x\to 0} \frac{(a^x-1)}{x} = \log a$]

Z = 4log 5

∴ The value of $\lim\limits_{x\to 0} \frac{5^x-1}{\sqrt{4+x}-2} = 4\log 5$

2. $\lim_{x \to 0} \frac{\log(1+x)}{3^x - 1}$

Solution:

Given: $\lim_{x \to 0} \frac{\log(1+x)}{3^x - 1}$

The limit

When x = 0, the expression $\lim_{x \to 0} \frac{\log(1+x)}{3^x - 1}$ assumes the form (0/0).

So,

As $Z = \lim_{x \to 0} \frac{\log(1+x)}{3^x - 1}$

Let us divide numerator and denominator by x, we get

$Z = \lim_{x \to 0} \frac{\frac{\log(1+x)}{x}}{\frac{3^x - 1}{x}} = \frac{\lim_{x \to 0} \frac{\log(1+x)}{x}}{\lim_{x \to 0} \frac{3^x - 1}{x}}$ {by using basic limit algebra}

[By using the formula: $\lim_{x \to 0} \frac{(a^x - 1)}{x} = \log a$]

$= \frac{1}{\log 3}$

∴ The value of $\lim_{x \to 0} \frac{\log(1+x)}{3^x - 1} = \frac{1}{\log 3}$

3. $\lim_{x \to 0} \frac{a^x + a^{-x} - 2}{x^2}$

Solution:

Given: $\lim_{x \to 0} \frac{a^x + a^{-x} - 2}{x^2}$

The limit

When x = 0, the expression $\lim_{x \to 0} \frac{a^x + a^{-x} - 2}{x^2}$ assumes the form (0/0).

So,

As $Z = \lim_{x \to 0} \frac{a^x + a^{-x} - 2}{x^2}$

$= \lim_{x \to 0} \frac{a^{-x}(a^{2x} - 2a^x + 1)}{x^2}$

$= \lim_{x \to 0} \frac{(a^{2x} - 2a^x + 1)}{a^x x^2}$

$= \lim_{x \to 0} \frac{(a^x - 1)^2}{a^x x^2}$ {By using $(a + b)^2 = a^2 + b^2 + 2ab$}

Let us use algebra of limit, we get

$Z = \lim_{x\to 0}\left(\frac{a^x - 1}{x}\right)^2 \times \lim_{x\to 0}\frac{1}{a^x}$

[By using the formula: $\lim_{x\to 0}\frac{(a^x - 1)}{x} = \log a$]

$Z = (\log a)^2 \frac{1}{a^0} = (\log a)^2$

∴ The value of $\lim_{x\to 0}\frac{a^x + a^{-x} - 2}{x^2} = (\log a)^2$

4. $\lim_{x\to 0} \frac{a^{mx}-1}{b^{nx}-1}, n \neq 0$

Solution:

Given: $\lim_{x\to 0} \frac{a^{mx}-1}{b^{nx}-1}, n \neq 0$

The limit $\lim_{x\to 0} \frac{a^{mx}-1}{b^{nx}-1}, n \neq 0$

When x = 0, the expression $\lim_{x\to 0} \frac{a^{mx}-1}{b^{nx}-1}, n \neq 0$ assumes the form (0/0).

So, let us include mx and nx as follows:

$$Z = \lim_{x\to 0} \frac{a^{mx}-1}{b^{nx}-1} = \lim_{x\to 0} \frac{\frac{a^{mx}-1}{mx}\times mx}{\frac{b^{nx}-1}{nx}\times nx}$$

$$= \frac{m}{n} \lim_{x\to 0} \frac{\frac{a^{mx}-1}{mx}}{\frac{b^{nx}-1}{nx}}$$

By using algebra of limits, we get

$$Z = \frac{m}{n} \frac{\lim_{x\to 0} \frac{a^{mx}-1}{mx}}{\lim_{x\to 0} \frac{b^{nx}-1}{nx}}$$

[By using the formula: $\lim_{x\to 0} \frac{(a^x-1)}{x} = \log a$]

$$Z = \frac{m}{n} \frac{\log a}{\log b}, n \neq 0$$

∴ The value of $\lim_{x\to 0} \frac{a^{mx}-1}{b^{nx}-1} = \frac{m}{n} \frac{\log a}{\log b}, n \neq 0$

5. $\lim_{x\to 0} \frac{a^x + b^x - 2}{x}$

Solution:

Given: $\lim_{x\to 0} \frac{a^x + b^x - 2}{x}$

The limit

When x = 0, the expression $\lim_{x\to 0} \frac{a^x + b^x - 2}{x}$ assumes the form (0/0).

So,

As $Z = \lim_{x\to 0} \frac{a^x + b^x - 2}{x}$

$= \lim_{x\to 0} \frac{a^x - 1 + b^x - 1}{x}$

By using algebra of limits, we get

$Z = \lim_{x\to 0} \frac{a^x - 1}{x} + \lim_{x\to 0} \frac{b^x - 1}{x}$

[By using the formula: $\lim_{x\to 0} \frac{(a^x - 1)}{x} = \log a$]

$Z = \log a + \log b = \log ab$

$\therefore$ The value of $\lim_{x\to 0} \frac{a^x + b^x - 2}{x} = \log ab$

EXERCISE 29.11 PAGE NO: 29.71

Evaluate the following limits:

1. $\lim_{x\to\pi}\left(1-\frac{x}{\pi}\right)^{\pi}$

Solution:

Given:

The limit $\lim_{x\to\pi}\left(1-\frac{x}{\pi}\right)^{\pi}$

Let us substitute the value of x = π directly, we get

$Z = \lim_{x\to\pi}\left(1-\frac{x}{\pi}\right)^{\pi} = \left(1-\frac{\pi}{\pi}\right)^{\pi} = (1-1)^{\pi} = 0^{\pi} = 0$

Since, it is not of indeterminate form.

Z = 0

∴ The value of $\lim_{x\to\pi}\left(1-\frac{x}{\pi}\right)^{\pi} = 0$

2. $\lim_{x\to0^+}\left\{1+\tan^{\sqrt{x}}\right\}^{1/2x}$

Solution:

Given:

The limit $\lim_{x\to0^+}\left\{1+\tan^{\sqrt{x}}\right\}^{1/2x}$

Let us use the theorem given below

If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ such that $\lim_{x\to a}\frac{f(x)}{g(x)}$ exists, then $\lim_{x\to a}[1+f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x\to a}\frac{f(x)}{g(x)}}$.

So here,

$f(x) = \tan^2 \sqrt{x}$

$g(x) = 2x$

Then,

$$\lim_{x\to0^+}\left\{1+\tan^{\sqrt{x}}\right\}^{1/2x} = e^{\lim_{x\to0^+}}\left(\frac{\tan^2\sqrt{x}}{2x}\right)$$

$$= e^{\lim_{x\to0^+}}\left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)\times\left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)\times\frac{1}{2}$$

$$= e^{1\times1\times\frac{1}{2}}$$

$$= \sqrt{e}$$

∴ The value of $\lim_{x\to0^+}\left\{1+\tan^{\sqrt{x}}\right\}^{1/2x} = \sqrt{e}$

3. $\lim_{x\to 0} (\cos x)^{1/\sin x}$

Solution:

Given:

The limit $\lim_{x\to 0} (\cos x)^{1/\sin x}$

Let us use the theorem given below

If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ such that $\lim_{x\to a} \frac{f(x)}{g(x)}$ exists, then $\lim_{x\to a}[1+f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x\to a} \frac{f(x)}{g(x)}}$.

So here,

f (x) = cos x - 1

g (x) = sin x

Then,

$$\lim_{x\to 0} (\cos x)^{1/\sin x} = e^{\lim_{x\to 0}\left(\frac{\cos x - 1}{\sin x}\right)}$$

$$= e^{\lim_{x\to 0}\left(\frac{-2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)}$$

$$= e^{\lim_{x\to 0}\left(-\tan\frac{x}{2}\right)}$$

$$= e^0$$

$$= 1$$

∴ The value of $\lim_{x\to 0} (\cos x)^{1/\sin x} = 1$

4. $\lim_{x\to 0} (\cos x + \sin x)^{1/x}$

Solution:

Given:

The limit $\lim_{x\to 0} (\cos x + \sin x)^{1/x}$

Let us add and subtract '1' to the given expression, we get

$$\lim_{x\to 0}[1+\cos x+\sin x-1]^{\frac{1}{x}}$$

Let us use the theorem given below

If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ such that $\lim_{x\to a} \frac{f(x)}{g(x)}$ exists, then $\lim_{x\to a}[1+f(x)]^{\frac{1}{g(x)}} = e^{\lim_{x\to a} \frac{f(x)}{g(x)}}$.

So here,

f (x) = cos x + sin x – 1

g (x) = x

Then,

$$\lim_{x\to 0}(\cos x+\sin x)^{1/x} = e^{\lim_{x\to 0}}\left(\frac{\cos x+\sin x-1}{x}\right)$$

Upon computing, we get

$$= e^{\lim_{x\to 0}}\left[\frac{\sin x}{x}-\frac{(1-\cos x)}{x}\right]$$

$$= e^{\lim_{x\to 0}}\left(\frac{\sin x}{x}-\frac{2\sin^2\frac{x}{2}}{x}\right)$$

$$= e^{\lim_{x\to 0}}\left(\frac{\sin x}{x}-\frac{2\sin\left(\frac{x}{2}\right)\times\sin\left(\frac{x}{2}\right)}{2\times\frac{x}{2}}\right)$$

Now, substitute the value of x, we get

$= e^{1-0}$

$= e^1$

$= e$

∴ The value of $\lim_{x\to 0}(\cos x+\sin x)^{1/x} = e$

5. $\lim_{x\to 0}(\cos x + a\sin bx)^{1/x}$

Solution:

Given:

The limit $\lim_{x\to 0}(\cos x + a\sin bx)^{1/x}$

Let us add and subtract '1' to the given expression, we get

$$\lim_{x\to 0}[1+\cos x+a\sin bx-1]^{\frac{1}{x}}$$

Let us use the theorem given below

If $\lim_{x\to a} f(x)=\lim_{x\to a} g(x)=0$ such that $\lim_{x\to a}\frac{f(x)}{g(x)}$ exists, then $\lim_{x\to a}[1+f(x)]^{\frac{1}{g(x)}}=e^{\lim_{x\to a}}\frac{f(x)}{g(x)}$.

So here,

f (x) = cos x + a sin bx - 1

g (x) = x

Then,

$$\lim_{x\to 0}(\cos x + a\sin bx)^{1/x} = e^{\lim_{x\to 0}}\left[\frac{\cos x+a\sin bx-1}{x}\right]$$

Let us compute now, we get

$$= e^{\lim_{x \to 0}} \left[\frac{b \times a \sin bx}{bx} - \frac{(1 - \cos x)}{x} \right]$$

$$= e^{\lim_{x \to 0}} \left(\frac{ab \sin bx}{bx} - \frac{2 \sin^2 \frac{x}{2}}{x} \right)$$

Now, substitute the value of x, we get

$$= e^{ab}$$

∴ The value of $\lim_{x \to 0} (\cos x + a \sin bx)^{1/x} = e^{ab}$

Chapterwise RD Sharma Solutions for Class 11 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.