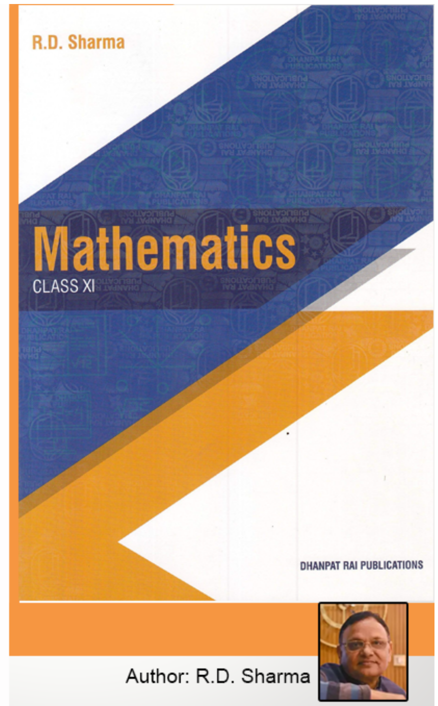
Class 11 - Chapter 30 Derivatives



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EXERCISE 30.1 PAGE NO: 30.3

1. Find the derivative of f(x) = 3x at x = 2

Solution:

Given:

f(x) = 3x

By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \{\text{Where, h is a small positive number}\}$$

Derivative of f(x) = 3x at x = 2 is given as

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h \to 0} \frac{3(2 + h) - 3 \times 2}{h}$$

$$= \lim_{h \to 0} \frac{3h + 6 - 6}{h} = \lim_{h \to 0} \frac{3h}{h}$$

$$= \lim_{h \to 0} 3 = 3$$

Hence,

Derivative of f(x) = 3x at x = 2 is 3

2. Find the derivative of f(x) = x^2 – 2 at x = 10

Solution:

Given:

f(x) = x^2 – 2

By using the derivative formula,

Derivative of $x^2 - 2$ at $x = 10$ is given as

$$f'(10) = \lim_{h \to 0} \frac{f(10+h) - f(10)}{h}$$
$$= \lim_{h \to 0} \frac{(10 + h)^2 - 2 - (10^2 - 2)}{h}$$
$$= \lim_{h \to 0} \frac{100 + h^2 + 20h - 2 - 100 + 2}{h} = \lim_{h \to 0} \frac{h^2 + 20h}{h}$$
$$= \lim_{h \to 0} \frac{h(h + 20)}{h} = \lim_{h \to 0} (h + 20)$$

= 0 + 20 = 20

Hence,

Derivative of f(x) = x^2 – 2 at x = 10 is 20

3. Find the derivative of f(x) = 99x at x = 100.

Solution:

Given:

f(x) = 99x

By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \{\text{Where h is a very small positive number}\}$$

Derivative of 99x at x = 100 is given as

$$f'(100) = \lim_{h \to 0} \frac{f(100+h) - f(100)}{h}$$
$$= \lim_{h \to 0} \frac{99(100 + h) - 99 \times 100}{h}$$
$$= \lim_{h \to 0} \frac{9900 + 99h - 9900}{h} = \lim_{h \to 0} \frac{99h}{h}$$
$$= \lim_{h \to 0} 99 = 99$$

Hence,

Derivative of f(x) = 99x at x = 100 is 99

4. Find the derivative of f(x) = x at x = 1

Solution:

Given:

f(x) = x

By using the derivative formula,

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$ {Where h is a very small positive number}

Derivative of x at x = 1 is given as

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{(1 + h) - 1}{h}$$

$$= \lim_{h\to 0} \frac{1 + h - 1}{h} = \lim_{h\to 0} \frac{h}{h}$$

$$= \lim_{h\to 0} 1 = 1$$

Hence,

Derivative of f(x) = x at x = 1 is 1

5. Find the derivative of f(x) = cos x at x = 0

Solution:

Given:

f(x) = cos x

By using the derivative formula,

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$ {Where h is a very small positive number}

Derivative of cos x at x = 0 is given as

$$f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0}\frac{\cos(h)-\cos 0}{h}$$

$$= \lim_{h\to 0}\frac{\cos h-1}{h}$$

Let us try and evaluate the limit.

We know that 1 – cos x = 2 $\sin^2(x/2)$

So,

$$= \lim_{h\to 0}\frac{-(1-\cos h)}{h} = -\lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h}$$

Divide the numerator and denominator by 2 to get the form (sin x)/x to apply sandwich theorem.

$$= -\lim_{h\to 0}\frac{\frac{2\sin^2\frac{h}{2}}{2}}{\frac{h^2}{2}}\times h$$

By using algebra of limits we get

$$= -\lim_{h\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)\times\lim_{h\to 0} h$$

[By using the formula: $\lim_{x\to 0}\frac{\sin x}{x} = 1$]

f' (0) = – 1×0 = 0

∴ Derivative of f(x) = cos x at x = 0 is 0

6. Find the derivative of f(x) = tan x at x = 0

Solution:

Given:

f(x) = tan x

By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \text{ \{Where h is a small positive number\}}$$

Derivative of cos x at x = 0 is given as

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\tan(h) - \tan 0}{h}$$

$$= \lim_{h \to 0} \frac{\tan h}{h} \text{ [Since it is of indeterminate form]}$$

By using the formula: $\lim_{x \to 0} \frac{\tan x}{x} = 1$ {i.e., sandwich theorem}

f' (0) = 1

∴ Derivative of f(x) = tan x at x = 0 is 1

7. Find the derivatives of the following functions at the indicated points:

(i) sin x at x = π/2

(ii) x at x = 1

(iii) 2 cos x at x = π/2

(iv) sin 2xat x = π/2

Solution:

(i) sin x at x = π/2

Given:

f (x) = sin x

By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ {Where h is a small positive number}

Derivative of sin x at x = π/2 is given as

$$f'\left(\frac{\pi}{2}\right) = \lim_{h \to 0} \frac{f(\frac{\pi}{2} + h) - f(\frac{\pi}{2})}{h}$$

$$= \lim_{h \to 0} \frac{\sin\left(\frac{\pi}{2} + h\right) - \sin\frac{\pi}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\cos h - 1}{h}$$ {∵ sin (π/2 + x) = cos x}

[Since it is of indeterminate form. Let us try to evaluate the limit.]

We know that 1 – cos x = 2 $\sin^2(x/2)$

$$= \lim_{h \to 0} \frac{-(1 - \cos h)}{h} = -\lim_{h \to 0} \frac{2\sin^2\frac{h}{2}}{h}$$

Divide the numerator and denominator by 2 to get the form (sin x)/x to apply sandwich theorem.

$$= -\lim_{h \to 0} \frac{\frac{2\sin^2\frac{h}{2}}{2}}{\frac{h^2}{2}} \times h$$

Using algebra of limits we get

$$= -\lim_{h \to 0} \left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right) \times \lim_{h \to 0} h$$

[By using the formula: $\lim_{x \to 0} \frac{\sin x}{x} = 1$]

f' (π/2) = – 1×0 = 0

∴ Derivative of f(x) = sin x at x = π/2 is 0

(ii) x at x = 1

Given:

f (x) = x

By using the derivative formula,

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$ {Where h is a very small positive number}

Derivative of x at x = 1 is given as

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{(1 + h) - 1}{h}$$

$$= \lim_{h\to 0} \frac{1 + h - 1}{h} = \lim_{h\to 0} \frac{h}{h}$$

$$= \lim_{h\to 0} 1 = 1$$

Hence,

Derivative of f(x) = x at x = 1 is 1

(iii) 2 cos x at x = π/2

Given:

f (x) = 2 cos x

By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ {Where h is a small positive number}

Derivative of 2cos x at x = π/2 is given as

$$f'\left(\frac{\pi}{2}\right) = \lim_{h \to 0} \frac{f(\frac{\pi}{2}+h) - f(\frac{\pi}{2})}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{\pi}{2} + h\right) - 2\cos\frac{\pi}{2}}{h}$$

$$= \lim_{h \to 0} \frac{-2\sin h}{h}$$ {∵ cos (π/2 + x) = – sin x}

[Since it is of indeterminate form]

$$= -2 \lim_{h \to 0} \frac{\sin h}{h}$$

By using the formula: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

f' (π/2) = – 2×1 = – 2

∴ Derivative of f(x) = 2cos x at x = π/2 is – 2

(iv) sin 2xat x = π/2

Solution:

Given:

f (x) = sin 2x

By using the derivative formula,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ {Where h is a small positive number}

Derivative of sin 2x at x = π/2 is given as

$$f'\left(\frac{\pi}{2}\right) = \lim_{h \to 0} \frac{f(\frac{\pi}{2} + h) - f(\frac{\pi}{2})}{h}$$

$$= \lim_{h \to 0} \frac{\sin\left\{2 \times \left(\frac{\pi}{2} + h\right)\right\} - \sin 2 \times \frac{\pi}{2}}{h}$$

$$= \lim_{h \to 0} \frac{\sin(\pi + 2h) - \sin \pi}{h}$$ $\{\because \sin(\pi + x) = -\sin x \ \& \ \sin \pi = 0\}$

$$= \lim_{h \to 0} \frac{-\sin 2h - 0}{h}$$

$$= -\lim_{h \to 0} \frac{\sin 2h}{h}$$

[Since it is of indeterminate form. We shall apply sandwich theorem to evaluate the limit.]

Now, multiply numerator and denominator by 2, we get

EXERCISE 30.2 PAGE NO: 30.25

1. Differentiate each of the following from first principles:

(i) 2/x

(ii) 1/√x

(iii) $1/x^3$

(iv) $[x^2 + 1]/ x$

(v) $[x^2 – 1] / x$

Solution:

(i) 2/x

Given:

f (x) = 2/x

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h\to0}\frac{\frac{2}{x+h}-\frac{2}{x}}{h}$$

$$= \lim_{h\to0}\frac{2x-2x-2h}{hx(x+h)}$$

$$= \lim_{h\to0}\frac{-2h}{hx(x+h)}$$

$$= \lim_{h\to0}\frac{-2}{x(x+h)}$$

When h= 0, we get

$$= \frac{-2}{x^2}$$

$= -2x^{-2}$

∴ Derivative of f(x) = 2/x is $-2x^{-2}$

(ii) 1/√x

Given:

f (x) = 1/√x

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

By using algebra of limits, we get

$$= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \times \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$$

$$= \lim_{h \to 0} \frac{x - x - h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= \lim_{h \to 0} \frac{-h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x} + \sqrt{x+h}\right)}$$

When h = 0, we get

$$= \frac{-1}{\sqrt{x}\sqrt{x}\left(\sqrt{x} + \sqrt{x}\right)}$$

$$= \frac{-1}{x \times 2\sqrt{x}}$$

$$= \frac{-1}{2x^{\frac{3}{2}}}$$

$$= -\frac{1}{2}x^{\frac{-3}{2}}$$

∴ Derivative of f(x) = 1/√x is $-1/2\ x^{-3/2}$

(iii) $1/x^3$

Given:

f (x) = $1/x^3$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$$

$$= \lim_{h\to 0} \frac{x^3 - (x+h)^3}{h(x+h)^3 x^3}$$

By using the formula $[a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$$= \lim_{h\to 0} \frac{x^3 - x^3 - 3x^2h - 3xh^2 - h^3}{h(x+h)^3 x^3}$$

$$= \lim_{h\to 0} \frac{-3x^2h - 3xh^2 - h^3}{h(x+h)^3 x^3}$$

$$= \lim_{h\to 0} \frac{h\left(-3x^2 - 3xh - h^2\right)}{h(x+h)^3 x^3}$$

$$= \lim_{h\to 0} \frac{\left(-3x^2 - 3xh - h^2\right)}{(x+h)^3 x^3}$$

When h = 0, we get

$$= \frac{-3x^2}{x^6}$$

$$= \frac{-3}{x^4}$$

$$= -3x^{-4}$$

∴ Derivative of f(x) = $1/x^3$ is $-3x^{-4}$

(iv) $[x^2 + 1]/ x$

Given:

f (x) = $[x^2 + 1]/ x$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \to 0} \frac{\frac{(x+h)^2+1}{x+h} - \frac{x^2+1}{x}}{h}$$

Upon expansion,

$$= \lim_{h \to 0} \frac{\frac{x^2+2xh+h^2+1}{x+h} - \frac{x^2+1}{x}}{h}$$

By using algebra of limits, we get

$$= \lim_{h \to 0} \frac{x^3 + 2x^2h + h^2x + x - x^3 - x^2h - x - h}{xh(x+h)}$$

$$= \lim_{h \to 0} \frac{x^2h + h^2x - h}{xh(x+h)}$$

$$= \lim_{h \to 0} \frac{h(x^2 + hx - 1)}{xh(x+h)}$$

$$= \lim_{h \to 0} \frac{x^2 + hx - 1}{x(x+h)}$$

When h = 0, we get

$$= \frac{x^2 - 1}{x^2}$$

$= 1 – 1/x^2$

∴ Derivative of f(x) = $1 – 1/x^2$

(v) $[x^2 – 1] / x$

Given:

f (x) = $[x^2 – 1]/ x$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h\to 0}\frac{\frac{(x+h)^2-1}{x+h} - \frac{x^2-1}{x}}{h}$$

Upon expansion,

$$= \lim_{h\to 0}\frac{\frac{x^2+2xh+h^2-1}{x+h} - \frac{x^2-1}{x}}{h}$$

By using algebra of limits, we get

$$= \lim_{h\to 0}\frac{x^3+2x^2h+h^2x-x-x^3-x^2h+x+h}{xh(x+h)}$$

$$= \lim_{h\to 0}\frac{x^2h+h^2x+h}{xh(x+h)}$$

$$= \lim_{h\to 0}\frac{h(x^2+hx+1)}{xh(x+h)}$$

$$= \lim_{h\to 0}\frac{x^2+hx+1}{x(x+h)}$$

When h = 0, we get

$$= \frac{x^2+1}{x^2}$$

$= 1 + 1/x^2$

∴ Derivative of f(x) = $1 + 1/x^2$

2. Differentiate each of the following from first principles:

(i) e^{-x}

(ii) e^{3x}

(iii) e^{ax+b}

Solution:

(i) e^{-x}

Given:

f (x) = e^{-x}

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$\frac{d}{dx}(e^{x}) = \lim_{h\to 0} \frac{e^{-(x+h)} - e^{-x}}{h}$$

$$= \lim_{h\to 0} \frac{e^{-x}e^{-h} - e^{-x}}{h}$$

Taking e^{-x} common, we have

$$= \lim_{h\to 0} \frac{e^{-x}\left(e^{-h} - 1\right)}{h}$$

$$= \lim_{h\to 0} e^{-x} \times \lim_{h\to 0} \frac{e^{-h}-1}{-h} \times (-1)$$

We know that, $\lim_{x\to 0} \frac{e^{x}-1}{x} = \log_e e = 1$

$$= -e^{-x} \lim_{h\to 0} \frac{e^{-h} - 1}{-h}$$

So,

$$= -e^{-x}(1)$$

$$= -e^{-x}$$

∴ Derivative of f (x) = $-e^{-x}$

(ii) e^{3x}

Given:

f (x) = e^{3x}

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$\frac{d}{dx}\left(e^{3x}\right) = \lim_{h\to 0} \frac{e^{3(x+h)} - e^{3x}}{h}$$

$$= \lim_{h\to 0} \frac{e^{3x}e^{3h} - e^{3x}}{h}$$

Taking e^{-x} common, we have

$$= \lim_{h\to 0} \frac{e^{3x}\left(e^{3h} - 1\right)}{3h}$$

By using algebra of limits,

$$= \lim_{h\to 0} e^{3x} \times \lim_{h\to 0} \frac{e^{3h}-1}{h}$$

Since we cannot substitute the value of h directly, we take

$$= \lim_{h\to 0} e^{3x} \times \lim_{h\to 0} \frac{e^{3h}-1}{3h} \times 3$$

We know that, $\lim_{x\to 0} \frac{e^{x}-1}{x} = \log_e e = 1$

$$= 3e^{3x} \lim_{h\to 0} \frac{e^{3h} - 1}{3h}$$

$$= 3e^{3x}(1)$$

$$= 3e^{3x}$$

∴ Derivative of f (x) = $3e^{3x}$

(iii) e^{ax+b}

Given:

f (x) = e^{ax+b}

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

By substituting the values we get,

$$\frac{d}{dx}\left(e^{ax+b}\right) = \lim_{h\to 0}\frac{e^{a(x+h)+b}-e^{ax+b}}{h}$$

$$= \lim_{h\to 0}\frac{e^{ax+b}e^{ah}-e^{ax+b}}{h}$$

Taking e^{ax+b} common, we have

$$= \lim_{h\to 0}\frac{e^{ax+b}\left(e^{ah}-1\right)}{h}$$

By using algebra of limits,

$$= \lim_{h\to 0} e^{ax+b} \times \lim_{h\to 0}\frac{e^{ah}-1}{h}$$

Since we cannot substitute the value of h directly, we take

$$= \lim_{h\to 0} e^{ax+b} \times \lim_{h\to 0}\frac{e^{ah}-1}{ah} \times a$$

We know that, $\lim_{x\to 0}\frac{e^{x}-1}{x} = \log_e e = 1$

$$= ae^{ax+b}\lim_{h\to 0}\frac{e^{ah}-1}{ah}$$

$$= ae^{ax+b}(1)$$

$$= ae^{ax+b}$$

∴ Derivative of f (x) = ae^{ax+b}

3. Differentiate each of the following from first principles:

(i) √(sin 2x)

(ii) sin x/x

Solution:

(i) √(sin 2x)

Given:

f (x) = √(sin 2x)

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h\to0}\frac{\sqrt{\sin(2x+2h)}-\sqrt{\sin 2x}}{h}$$

Multiply numerator and denominator by √(sin 2(x + h)) + √(sin 2x), we have

$$= \lim_{h\to0}\frac{\sqrt{\sin(2x+2h)}-\sqrt{\sin 2x}}{h}\times\frac{\sqrt{\sin(2x+2h)}+\sqrt{\sin 2x}}{\sqrt{\sin(2x+2h)}+\sqrt{\sin 2x}}$$

By using $a^2 - b^2 = (a + b)(a - b)$, we get

$$= \lim_{h\to0}\frac{\sin(2x+2h)-\sin 2x}{h\left(\sqrt{\sin(2x+2h)}+\sqrt{\sin 2x}\right)}$$

By using the formula,

$$sinC - sinD = 2cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{2x+2h+2x}{2}\right)\sin\left(\frac{2x+2h-2x}{2}\right)}{h\left(\sqrt{\sin(2x+2h)}+\sqrt{\sin 2x}\right)}$$

$$= \lim_{h \to 0} \frac{2\cos(2x+h)\sin h}{h\left(\sqrt{\sin(2x+2h)}+\sqrt{\sin 2x}\right)}$$

By applying limits to each term, we get

$$= \lim_{h \to 0} 2\cos(2x+h) \lim_{h \to 0} \frac{\sin h}{h} \lim_{h \to 0} \frac{1}{\left(\sqrt{\sin(2x+2h)}+\sqrt{\sin 2x}\right)}$$

$$= 2\cos 2x\,(1)\,\frac{1}{\sqrt{\sin 2x}+\sqrt{\sin 2x}}$$

$$= \frac{2\cos 2x}{2\sqrt{\sin 2x}}$$

$$= \frac{\cos 2x}{\sqrt{\sin 2x}}$$

∴ Derivative of f (x) = cos 2x / √(sin 2x)

(ii) sin x/x

Given:

f (x) = sin x/x

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h\to 0} \frac{\frac{\sin(x+h)}{x+h} - \frac{\sin x}{x}}{h}$$

$$= \lim_{h\to 0} \frac{x\sin(x+h) - (x+h)\sin x}{hx(x+h)}$$

By using algebra of limits,

$$= \lim_{h\to 0} \frac{x(\sin x\cos h + \cos x\sin h) - x\sin x - h\sin x}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{x\sin x\cos h + x\cos x\sin h - x\sin x - h\sin x}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{x\sin x\cos h - x\sin x + x\cos x\sin h - h\sin x}{hx(x+h)}$$

By applying limits to each term, we get

$$= x\sin x \lim_{h\to 0} \frac{\cos h - 1}{h} + \frac{x\cos x}{x} \lim_{h\to 0} \frac{\sin h}{h} \lim_{h\to 0} \frac{1}{x+h} - \frac{\sin x}{x} \lim_{h\to 0} \frac{1}{x+h}$$

$$= x\sin x \lim_{h\to 0} \frac{-2\sin^2\frac{h}{2}}{h} + \frac{x\cos x}{x} \lim_{h\to 0} \frac{\sin h}{h} \lim_{h\to 0} \frac{1}{x+h} - \frac{\sin x}{x} \lim_{h\to 0} \frac{1}{x+h}$$

$$= x\sin x \lim_{h\to 0} \frac{-2\sin^2\frac{h}{2}}{\frac{h^2}{4}} \times \frac{h}{4} + \frac{x\cos x}{x} \lim_{h\to 0} \frac{\sin h}{h} \lim_{h\to 0} \frac{1}{x+h} - \frac{\sin x}{x} \lim_{h\to 0} \frac{1}{x+h}$$

$$= -x\sin x \times \lim_{h\to 0} \frac{h}{2} + \frac{x\cos x}{x} \lim_{h\to 0} \frac{\sin h}{h} \lim_{h\to 0} \frac{1}{x+h} - \frac{\sin x}{x} \lim_{h\to 0} \frac{1}{x+h}$$

When h = 0, we get

$$= -x \sin x \left(\frac{1}{2}\right)(0) + \frac{cosx}{x} - \frac{sinx}{x^2}$$

$$= \frac{\cos x}{x} - \frac{\sin x}{x^2}$$

By taking LCM, we get

$$= \frac{x \cos x - \sin x}{x^2}$$

∴ Derivative of f (x) = [x cos x – sin x]/x^2

4. Differentiate the following from first principles:

(i) $\tan^2 x$

(ii) tan (2x + 1)

Solution:

(i) $\tan^2 x$

Given:

f (x) = $\tan^2 x$

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h\to 0}\frac{\tan^2(x+h)-\tan^2 x}{h}$$

By using (a + b) (a – b) = $a^2 - b^2$, we have

$$= \lim_{h\to 0}\frac{[\tan(x+h)+\tan x]\,[\tan(x+h)-\tan x]}{h}$$

Replacing tan with sin/cos,

$$= \lim_{h\to 0}\frac{\left[\frac{\sin(x+h)}{\cos(x+h)}+\frac{\sin x}{\cos x}\right]\left[\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin x}{\cos x}\right]}{h}$$

By taking LCM,

$$= \lim_{h\to 0}\frac{[\sin(x+h)\cos x+\cos(x+h)\sin x]\,[\sin(x+h)\cos x-\cos(x+h)\sin x]}{h\cos^2 x\cos^2(x+h)}$$

$$= \lim_{h\to 0}\frac{[\sin(2x+h)]\,[\sin h]}{h\cos^2 x\cos^2(x+h)}$$

By applying limits to each term, we get

$$= \frac{1}{\cos^2 x}\lim_{h\to 0}\sin(2x+h)\lim_{h\to 0}\frac{\sin h}{h}\lim_{h\to 0}\frac{1}{\cos^2(x+h)}$$

When h = 0, we get

$$= \frac{1}{\cos^2 x}\sin(2x)\,(1)\,\frac{1}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}2\sin x\cos x\frac{1}{\cos^2 x}$$

$$= 2\times\frac{\sin x}{\cos x}\times\frac{1}{\cos^2 x}$$

$$= 2\tan x\sec^2 x$$

∴ Derivative of f (x) = $2\tan x\sec^2 x$

(ii) tan (2x + 1)

Given:

f (x) = tan (2x + 1)

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \to 0} \frac{\tan(2x+2h+1) - \tan(2x+1)}{h}$$

Replacing tan with sin/cos,

$$= \lim_{h \to 0} \frac{\frac{sin(2x+2h+1)}{\cos(2x+2h+1)} - \frac{\sin(2x+1)}{\cos(2x+1)}}{h}$$

By taking LCM,

$$= \lim_{h \to 0} \frac{sin(2x+2h+1)\cos(2x+1) - \cos(2x+2h+1)\sin(2x+1)}{h\cos(2x+2h+1)\cos(2x+1)}$$

$$= \lim_{h \to 0} \frac{\sin(2x+2h+1-2x-1)}{h\cos(2x+2h+1)\cos(2x+1)}$$

By applying limits to each term, we get

$$= \frac{1}{\cos(2x+1)} \lim_{h \to 0} \frac{\sin(2h)}{2h} \times 2 \lim_{h \to 0} \frac{1}{\cos(2x+2h+1)}$$

When h = 0, we get

$$= \frac{1}{\cos(2x+1)} \times 2 \times \frac{1}{\cos(2x+1)}$$

$$= \frac{2}{\cos^2(2x+1)}$$

$$= 2\sec^2(2x+1)$$

∴ Derivative of f (x) = 2 $\sec^2$ (2x + 1)

5. Differentiate the following from first principles:

(i) sin √2x

(ii) cos √x

Solution:

(i) sin √2x

Given:

f (x) = sin √2x

f (x + h) = sin √2(x+h)

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h\to 0} \frac{\sin\sqrt{2x+2h} - \sin\sqrt{2x}}{h}$$

By using the formula,

$$sinC - sinD = 2sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)$$

$$= \lim_{h\to 0} \frac{2\sin\left(\sqrt{2x+2h} - \sqrt{2x}\right)\cos\left(\sqrt{2x+2h} - \sqrt{2x}\right)}{h}$$

By using algebra of limits,

$$= \lim_{h\to 0} \frac{2\times 2\sin\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)\cos\left(\frac{\sqrt{2x+2h}+\sqrt{2x}}{2}\right)}{2h+2x-2x}$$

To use the sandwich theorem to evaluate the limit, we need $\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}$ in denominator.

$$= \lim_{h\to 0} \frac{2\times 2\sin\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)\cos\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)}{\left(\sqrt{2x+2h}-\sqrt{2x}\right)\sqrt{2x+2h}+\sqrt{2x}}$$

$$= \lim_{h\to 0} \frac{2\times 2\sin\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)\cos\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)}{2\times\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)\left(\sqrt{2x+2h}+\sqrt{2x}\right)}$$

By applying limits to each term, we get

$$= \lim_{h\to 0} \frac{\sin\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)}{\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)} \lim_{h\to 0} \frac{2\cos\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)}{\sqrt{2x+2h}+\sqrt{2x}}$$

When h = 0, we get

$$= 1\times\frac{2\cos\sqrt{2x}}{2\sqrt{2x}}\left[\because \lim_{h\to 0}\frac{\sin\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)}{\left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{2}\right)} = 1\right]$$

$$= \frac{\cos\sqrt{2x}}{\sqrt{2x}}$$

∴ Derivative of f (x) = cos √2x / √2x

(ii) cos √x

Given:

f (x) = cos √x

f (x + h) = cos √(x+h)

By using the formula,

$$\frac{d}{dx}(f(x)) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

By substituting the values we get,

$$= \lim_{h \to 0} \frac{\cos\sqrt{x+h} - \cos\sqrt{x}}{h}$$

By using the formula,

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{h}$$

By using algebra of limits, we get

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{x+h-x}$$

To use the sandwich theorem to evaluate the limit, we need $\frac{\sqrt{x+h}-\sqrt{x}}{2}$ in denominator.

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{2 \times \left(\sqrt{x+h}+\sqrt{x}\right)\frac{\left(\sqrt{x+h}-\sqrt{x}\right)}{2}}$$

By applying limits to each term, we get

$$= \lim_{h \to 0} \frac{\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{\frac{\sqrt{x+h}-\sqrt{x}}{2}} \lim_{h \to 0} \frac{-\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)}{\sqrt{x+h}+\sqrt{x}}$$

When h = 0, we get

$$= 1 \times \frac{-\sin\sqrt{x}}{2\sqrt{x}} \left[\because \lim_{h\to 0} \frac{\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{\frac{\sqrt{x+h}-\sqrt{x}}{2}} = 1\right]$$

$$= \frac{-\sin\sqrt{x}}{2\sqrt{x}}$$

∴ Derivative of f (x) = - sin √x / 2√x

EXERCISE 30.3 PAGE NO: 30.33

Differentiate the following with respect to x:

1. x^4 – 2sin x + 3 cos x

Solution:

Given:

f (x) = x^4 – 2sin x + 3 cos x

Differentiate on both the sides with respect to x, we get

$$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(x^4 - 2\sin x + 3\cos x)$$

By using algebra of derivatives,

$$f' = \frac{d}{dx}(x^4) - 2\frac{d}{dx}(\sin x) + 3\frac{d}{dx}(\cos x)$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

So,

$= 4x^{4-1} - 2\cos x + 3(-\sin x)$

$= 4x^3 - 2\cos x - 3\sin x$

∴ Derivative of f (x) is $4x^3 - 2\cos x - 3\sin x$

2. $3^x + x^3 + 3^3$

Solution:

Given:

f (x) = $3^x + x^3 + 3^3$

Differentiate on both the sides with respect to x, we get

$$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(3^x + x^3 + 3^3)$$

By using algebra of derivatives,

$$f' = \frac{d}{dx}(3^x) + \frac{d}{dx}(x^3) + \frac{d}{dx}(3^3)$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(a^x) = a^x \log a$$

$$\frac{d}{dx}(\text{constant}) = 0$$

$$f' = 3^x \log_e 3 + 3x^{3-1} + 0$$

$$= 3^x \log_e 3 + 3x^2$$

$\therefore$ Derivative of f (x) is $3^x \log_e 3 + 3x^2$

3. $\frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2}$

Solution:

Given:

$$f(x) = \frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2}$$

Differentiate on both the sides with respect to x, we get

$$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left(\frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2}\right)$$

By using algebra of derivatives,

$$f' = \frac{d}{dx}\left(\frac{x^3}{3}\right) - 2\frac{d}{dx}(\sqrt{x}) + 5\frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= \frac{1}{3}\frac{d}{dx}(x^3) - 2\frac{d}{dx}(x^{\frac{1}{2}}) + 5\frac{d}{dx}(x^{-2})$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$f' = \frac{1}{3}(3x^{3-1}) - 2\times\frac{1}{2}x^{\frac{1}{2}-1} + 5(-2)x^{-2-1}$$

$$= 3\times\frac{1}{3}x^2 - x^{-\frac{1}{2}} - 10x^{-3}$$

$$= x^2 - x^{(-1/2)} - 10x^{-3}$$

$\therefore$ Derivative of f (x) is $x^2 - x^{(-1/2)} - 10x^{-3}$

4. $e^{x \log a} + e^{a \log x} + e^{a \log a}$

Solution:

Given:

$f(x) = e^{x \log a} + e^{a \log x} + e^{a \log a}$

We know that,

$e^{\log f(x)} = f(x)$

So,

$f(x) = a^x + x^a + a^a$

Differentiate on both the sides with respect to x, we get

$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(a^x + x^a + a^a)$

By using algebra of derivatives,

$f' = \frac{d}{dx}(a^x) + \frac{d}{dx}(x^a) + \frac{d}{dx}(a^a)$

We know that,

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(a^x) = a^x \log a$

$\frac{d}{dx}(\text{constant}) = 0$

$f' = a^x \log_e a - ax^{a-1} + 0$

$= a^x \log a - ax^{a-1}$

∴ Derivative of f (x) is $a^x \log a - ax^{a-1}$

5. (2x^2 + 1) (3x + 2)

Solution:

Given:

f (x) = (2x^2 + 1) (3x + 2)

= 6x^3 + 4x^2 + 3x + 2

Differentiate on both the sides with respect to x, we get

$$\frac{d}{dx}\{f(x)\} = \frac{d}{dx}(6x^3 + 4x^2 + 3x + 2)$$

By using algebra of derivatives,

$$f' = 6\frac{d}{dx}(x^3) + 4\frac{d}{dx}(x^2) + 3\frac{d}{dx}(x) + \frac{d}{dx}(2)$$

We know that,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\text{constant}) = 0$$

$$f' = 6(3x^{3-1}) + 4(2x^{2-1}) + 3(x^{1-1}) + 0$$

$$= 18x^2 + 8x + 3 + 0$$

$$= 18x^2 + 8x + 3$$

∴ Derivative of f (x) is $18x^2 + 8x + 3$

EXERCISE 30.4 PAGE NO: 30.39

Differentiate the following functions with respect to x:

1. x^3 sin x

Solution:

Let us consider y = x^3 sin x

We need to find dy/dx

We know that y is a product of two functions say u and v where,

u = x^3 and v = sin x

∴ y = uv

Now let us apply product rule of differentiation.

By using product rule, we get

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ … Equation (1)

As, $u = x^3$

$\frac{du}{dx} = 3x^{3-1} = 3x^2$ … Equation (2) {Since, $\frac{d}{dx}(x^n) = nx^{n-1}$}

As, $v = \sin x$

$\frac{dv}{dx} = \frac{d}{dx}(\sin x) = \cos x$ … Equation (3) {Since, $\frac{d}{dx}(\sin x) = \cos x$}

From equation (1), we can find dy/dx

$\frac{dy}{dx} = x^3\frac{dv}{dx} + \sin x\frac{du}{dx}$

$\frac{dy}{dx} = x^3\cos x + 3x^2\sin x$ {Using equation 2 & 3}

$\therefore \frac{dy}{dx} = x^3\cos x + 3x^2\sin x$

2. x^3 e^x

Solution:

Let us consider $y = x^3 e^x$

We need to find dy/dx

We know that y is a product of two functions say u and v where,

$u = x^3$ and $v = e^x$

$\therefore$ y = uv

Now let us apply product rule of differentiation.

By using product rule, we get

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ … Equation (1)

As, $u = x^3$

$\frac{du}{dx} = 3x^{3-1} = 3x^2$ … Equation (2) $\{\frac{d}{dx}(x^n) = nx^{n-1}\}$

As, $v = e^x$

$\frac{dv}{dx} = \frac{d}{dx}(e^x) = e^x$ … Equation (3) {Since, $\frac{d}{dx}(e^x) = e^x$}

Now from equation (1), we can find dy/dx

$\frac{dy}{dx} = x^3\frac{dv}{dx} + e^x\frac{du}{dx}$

$\frac{dy}{dx} = x^3e^x + 3x^2e^x$ {Using equation 2 & 3}

$\therefore \frac{dy}{dx} = x^2e^x(3 + x)$

3. x^2 e^x log x

Solution:

Let us consider y = x^2 e^x log x

We need to find dy/dx

We know that y is a product of two functions say u and v where,

u = x^2 and v = e^x

∴ y = uv

Now let us apply product rule of differentiation.

By using product rule, we get

$\frac{dy}{dx} = uw\frac{dv}{dx} + vw\frac{du}{dx} + uv\frac{dw}{dx}$ …equation 1

As, $u = x^2$

$\frac{du}{dx} = 2x^{2-1} = 2x$ … Equation (2) {Since, $\frac{d}{dx}(x^n) = nx^{n-1}$}

As, $v = e^x$

$\frac{dv}{dx} = \frac{d}{dx}(e^x) = e^x$ … Equation (3) {Since, $\frac{d}{dx}(e^x) = e^x$}

As, $w = \log x$

$\frac{dw}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$ … Equation (4) {Since, $\frac{d}{dx}(\log_e x) = \frac{1}{x}$}

Now, from equation 1, we can find dy/dx

$\frac{dy}{dx} = x^2 \log x \frac{dv}{dx} + e^x \log x \frac{du}{dx} + x^2 e^x \frac{dw}{dx}$

$\frac{dy}{dx} = x^2 e^x \log x + 2xe^x \log x + x^2 e^x \frac{1}{x}$ {Using equation 2, 3 & 4}

$\therefore \frac{dy}{dx} = xe^x(1 + x \log x + 2 \log x)$

4. x^n tan x

Solution:

Let us consider y = x^n tan x

We need to find dy/dx

We know that y is a product of two functions say u and v where,

u = x^n and v = tan x

∴ y = uv

Now let us apply product rule of differentiation.

By using product rule, we get

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ … Equation 1

As, $u = x^n$

$\frac{du}{dx} = nx^{n-1}$ … Equation 2 {Since, $\frac{d}{dx}(x^n) = nx^{n-1}$}

As, $v = \tan x$

$\frac{dv}{dx} = \frac{d}{dx}(\tan x) = \sec^2 x$ … Equation 3 {Since, $\frac{d}{dx}(\tan x) = \sec^2 x$}

Now, from equation 1, we can find dy/dx

$\frac{dy}{dx} = x^n\frac{dv}{dx} + \tan x\frac{du}{dx}$

$\frac{dy}{dx} = x^n \sec^2 x + nx^{n-1}\tan x$ {Using equation 2 & 3}

$\therefore \frac{dy}{dx} = x^{n-1}(n \tan x + x \sec^2 x)$

5. $x^n \log_a x$

Solution:

Let us consider $y = x^n \log_a x$

We need to find dy/dx

We know that y is a product of two functions say u and v where,

$u = x^n$ and $v = \log_a x$

$\therefore y = uv$

Now let us apply product rule of differentiation.

By using product rule, we get

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ … Equation (1)

As, $u = x^n$

$\frac{du}{dx} = nx^{n-1}$ … Equation (2) {Since, $\frac{d}{dx}(x^n) = nx^{n-1}$}

As, $v = \log_a x$

$\frac{dv}{dx} = \frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$ … Equation (3) {Since, $\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$}

Now, from equation 1, we can find dy/dx

$\frac{dy}{dx} = x^n \frac{dv}{dx} + \log_a x \frac{du}{dx}$

$\frac{dy}{dx} = x^n \frac{1}{x \log_e a} + nx^{n-1}\log_a x$ {Using equation 2 & 3}

$\therefore \frac{dy}{dx} = x^{n-1}\left(n \log_a x + \frac{1}{\log a}\right)$

EXERCISE 30.5 PAGE NO: 30.44

Differentiate the following functions with respect to x:

$\mathbf{1.} \frac{x^2+1}{x+1}$

Solution:

Let us consider

y =

$\frac{x^2+1}{x+1}$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$u = x^2 + 1$ and $v = x + 1$

$\therefore y = u/v$

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \ldots \text{Equation (1)}$$

As, $u = x^2 + 1$

$$\frac{du}{dx} = 2x^{2-1} + 0 = 2x \ldots \text{Equation (2)} \left\{\text{Since, } \frac{d}{dx}(x^n) = nx^{n-1}\right\}$$

As, $v = x + 1$

$$\frac{dv}{dx} = \frac{d}{dx}(x + 1) = 1 \ldots \text{Equation (3)} \left\{\text{Since, } \frac{d}{dx}(x^n) = nx^{n-1}\right\}$$

Now, from equation 1, we can find dy/dx

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(x+1)(2x) - (x^2+1)(1)}{(x+1)^2} \{\text{Using equation 2 and 3}\}$$

$$= \frac{2x^2 + 2x - x^2 - 1}{(x+1)^2}$$

$$= \frac{x^2 + 2x - 1}{(x+1)^2}$$

$$\therefore \frac{dy}{dx} = \frac{x^2 + 2x - 1}{(x + 1)^2}$$

$$2. \frac{2x-1}{x^2+1}$$

Solution:

Let us consider

y =

$$\frac{2x-1}{x^2+1}$$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$u = 2x – 1$ and $v = x^2 + 1$

∴ y = u/v

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \text{ … Equation (1)}$$

As, u = 2x – 1

$$\frac{du}{dx} = 2x^{1-1} - 0 = 2 \text{ … Equation (2) \{Since, } \frac{d}{dx}(x^n) = nx^{n-1}\}$$

As, v = x² + 1

$$\frac{dv}{dx} = \frac{d}{dx}(x^2 + 1) = 2x \text{ … Equation (3) \{Since, } \frac{d}{dx}(x^n) = nx^{n-1}\}$$

Now, from equation 1, we can find dy/dx

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(x^2+1)(2) - (2x-1)(2x)}{(x^2+1)^2} \text{ \{Using equation 2 and 3\}}$$

$$= \frac{2x^2 + 2 - 4x^2 + 2x}{(x^2+1)^2}$$

$$= \frac{-2x^2 + 2x + 2}{(x^2+1)^2}$$

$$\therefore \frac{dy}{dx} = \frac{2(1 + x - x^2)}{(x^2+1)^2}$$

3. $\frac{x + e^x}{1 + \log x}$

Solution:

Let us consider

y =

$$\frac{x + e^x}{1 + \log x}$$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

u = x + e^x and v = 1 + log x

∴ y = u/v

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad \ldots \text{Equation 1}$$

As, $u = x + e^x$

$$\frac{du}{dx} = \frac{d}{dx}(x + e^x) \left\{\text{Since, } \frac{d}{dx}(x^n) = nx^{n-1} \,\&\, \frac{d}{dx}(e^x) = e^x\right\}$$

$$\frac{du}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(e^x) = 1 + e^x \quad \ldots \text{Equation 2}$$

As, $v = 1 + \log x$

$$\frac{dv}{dx} = \frac{d}{dx}(\log x + 1)$$

$$= \frac{d}{dx}(1) + \frac{d}{dx}(\log x)$$

$$\frac{dv}{dx} = 0 + \frac{1}{x} = \frac{1}{x} \ldots \text{Equation 3} \left\{\text{Since, } \frac{d}{dx}(\log x) = \frac{1}{x}\right\}$$

Now, from equation 1, we can find dy/dx

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{(1 + \log x)(1 + e^x) - (x + e^x)\left(\frac{1}{x}\right)}{(\log x + 1)^2} \quad \{\text{Using equation 2 and 3}\}$$

$$= \frac{1 + e^x + \log x + e^x \log x - 1 - \frac{e^x}{x}}{(\log x + 1)^2}$$

$$= \frac{x\log x(1 + e^x) + e^x(x-1)}{x(\log x + 1)^2}$$

$$\therefore \frac{dy}{dx} = \frac{x \log x(1 + e^x) - e^x(1 - x)}{x(1 + \log x)^2}$$

4. $\dfrac{e^x - \tan x}{\cot x - x^n}$

Solution:

Let us consider

y =

$$\frac{e^x - \tan x}{\cot x - x^n}$$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

u = e^x – tan x and v = cot x – x^n

∴ y = u/v

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ ... Equation (1)

As, $u = e^x - \tan x$

$\frac{du}{dx} = \frac{d}{dx}(e^x - \tan x)$ {Since, $\frac{d}{dx}(\tan x) = \sec^2 x$ & $\frac{d}{dx}(e^x) = e^x$}

$\frac{du}{dx} = -\frac{d}{dx}(\tan x) + \frac{d}{dx}(e^x) = \sec^2 x + e^x$ ... Equation (2)

As, $v = \cot x - x^n$

$\frac{dv}{dx} = \frac{d}{dx}(\cot x - x^n)$

$= \frac{d}{dx}(\cot x) - \frac{d}{dx}(x^n)$ {Since, $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$ & $\frac{d}{dx}(x^n) = nx^{n-1}$}

$\frac{dv}{dx} = -\mathrm{cosec}^2 x - nx^{n-1}$ ... Equation (3)

Now, from equation 1, we can find dy/dx

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ {Using equation 2 and 3, we get}

$\frac{dy}{dx} = \frac{(\cot x - x^n)(\sec^2 x + e^x) - (e^x - \tan x)(-\mathrm{cosec}^2 x - nx^{n-1})}{(\cot x - x^n)^2}$

$\therefore \frac{dy}{dx} = \frac{(\cot x - x^n)(e^x - \sec^2 x) + (e^x - \tan x)(\mathrm{cosec}^2 x + nx^{n-1})}{(\cot x - x^n)^2}$

5. $\frac{ax^2 + bx + c}{px^2 + qx + r}$

Solution:

Let us consider

y =

$$\frac{ax^2 + bx + c}{px^2 + qx + r}$$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where, u = ax² + bx + c and v = px² + qx + r

∴ y = u/v

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \quad \ldots \text{Equation (1)}$$

As, $u = ax^2 + bx + c$

$$\frac{du}{dx} = 2ax + b \quad \ldots \text{Equation (2)} \left\{\text{Since, } \frac{d}{dx}(x^n) = nx^{n-1}\right\}$$

As, $v = px^2 + qx + r$

$$\frac{dv}{dx} = \frac{d}{dx}(px^2 + qx + r) = 2px + q \quad \ldots \text{Equation (3)}$$

Now, from equation 1, we can find dy/dx

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(px^2 + qx + r)(2ax + b) - (ax^2 + bx + c)(2px + q)}{(px^2 + qx + r)^2} \quad \{\text{Using equation 2 and 3}\}$$

$$= \frac{2apx^3 + bpx^2 + 2aqx^2 + bqx + 2arx + br - 2apx^3 - aqx^2 - 2bpx^2 - bqx - 2pcx - qc}{(px^2 + qx + r)^2}$$

$$= \frac{aqx^2 - bpx^2 + 2arx + br - 2pcx - qc}{(px^2 + qx + r)^2}$$

$$= \frac{x^2(aq - bp) + 2x(ar - pc) + br - qc}{(px^2 + qx + r)^2}$$

$$\therefore \frac{dy}{dx} = \frac{x^2(aq - bp) + 2x(ar - pc) + br - qc}{(px^2 + qx + r)^2}$$

Chapterwise RD Sharma Solutions for Class 11 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.