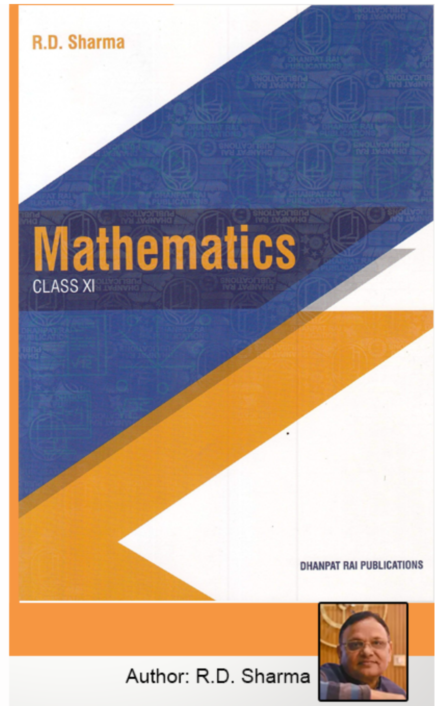
Class 11 - Chapter 3 Functions



RD Sharma Solutions for Class 11 Maths Chapter 3–Functions

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RD Sharma Solutions for Class 11 Maths Chapter 3–Functions

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EXERCISE 3.1 PAGE NO: 3.7

1. Define a function as a set of ordered pairs.

Solution:

Let A and B be two non-empty sets. A relation from A to B, i.e., a subset of A×B, is called a function (or a mapping) from A to B, if

(i) for each a ∈ A there exists b ∈ B such that (a, b) ∈ f

(ii) (a, b) ∈ f and (a, c) ∈ f ⇒ b = c

2. Define a function as a correspondence between two sets.

Solution:

Let A and B be two non-empty sets. Then a function 'f' from set A to B is a rule or method or correspondence which associates elements of set A to elements of set B such that:

(i) all elements of set A are associated to elements in set B.

(ii) an element of set A is associated to a unique element in set B.

3. What is the fundamental difference between a relation and a function? Is every relation a function?

Solution:

Let 'f' be a function and R be a relation defined from set X to set Y.

The domain of the relation R might be a subset of the set X, but the domain of the function f must be equal to X. This is because each element of the domain of a function must have an element associated with it, whereas this is not necessary for a relation.

In relation, one element of X might be associated with one or more elements of Y, while it must be associated with only one element of Y in a function.

Thus, not every relation is a function. However, every function is necessarily a relation.

4. Let A = {–2, –1, 0, 1, 2} and f: A → Z be a function defined by $f(x) = x^2 – 2x – 3$. Find:

(i) range of f i.e. f (A)

(ii) pre-images of 6, –3 and 5

Solution:

Given:

A = {–2, –1, 0, 1, 2}

f : A → Z such that $f(x) = x^2 – 2x – 3$

(i) Range of f i.e. f (A)

A is the domain of the function f. Hence, range is the set of elements f(x) for all x ∈ A.

Substituting x = –2 in f(x), we get

$f(–2) = (–2)^2 – 2(–2) – 3$

= 4 + 4 – 3

= 5

Substituting x = –1 in f(x), we get

$f(–1) = (–1)^2 – 2(–1) – 3$

= 1 + 2 – 3

= 0

Substituting x = 0 in f(x), we get

$f(0) = (0)^2 – 2(0) – 3$

= 0 – 0 – 3

= – 3

Substituting x = 1 in f(x), we get

$f(1) = 1^2 – 2(1) – 3$

= 1 – 2 – 3

= – 4

Substituting x = 2 in f(x), we get

$f(2) = 2^2 – 2(2) – 3$

= 4 – 4 – 3

= –3

Thus, the range of f is {-4, -3, 0, 5}.

(ii) pre-images of 6, –3 and 5

Let x be the pre-image of 6 ⇒ f(x) = 6

$x^2 - 2x - 3 = 6$

$x^2 - 2x - 9 = 0$

$x = [-(-2) \pm \sqrt{((-2)^2 - 4(1)(-9))}] / 2(1)$

$= [2 \pm \sqrt{(4+36)}] / 2$

$= [2 \pm \sqrt{40}] / 2$

$= 1 \pm \sqrt{10}$

However, $1 \pm \sqrt{10} \notin A$

Thus, there exists no pre-image of 6.

Now, let x be the pre-image of –3 ⇒ f(x) = –3

$x^2 - 2x - 3 = -3$

$x^2 - 2x = 0$

$x(x - 2) = 0$

x = 0 or 2

Clearly, both 0 and 2 are elements of A.

Thus, 0 and 2 are the pre-images of –3.

Now, let x be the pre-image of 5 ⇒ f(x) = 5

$x^2 - 2x - 3 = 5$

$x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0$

x(x – 4) + 2(x – 4) = 0

(x + 2)(x – 4) = 0

x = –2 or 4

However, 4 ∉ A but –2 ∈ A

Thus, –2 is the pre-images of 5.

∴ Ø, {0, 2}, -2 are the pre-images of 6, -3, 5

5. If a function f: R → R be defined by

Find: f (1), f (–1), f (0), f (2).

Solution:

Given:

Let us find f (1), f (–1), f (0) and f (2).

When x > 0, f (x) = 4x + 1

Substituting x = 1 in the above equation, we get

f (1) = 4(1) + 1

= 4 + 1

= 5

When x < 0, f(x) = 3x – 2

Substituting x = –1 in the above equation, we get

f (–1) = 3(–1) – 2

= –3 – 2

= –5

When x = 0, f(x) = 1

Substituting x = 0 in the above equation, we get

f (0) = 1

When x > 0, f(x) = 4x + 1

Substituting x = 2 in the above equation, we get

f (2) = 4(2) + 1

= 8 + 1

= 9

∴ f (1) = 5, f (–1) = –5, f (0) = 1 and f (2) = 9.

6. A function f: R → R is defined by f(x) = x^2. Determine

(i) range of f

(ii) {x: f(x) = 4}

(iii) {y: f(y) = –1}

Solution:

Given:

f : R → R and f(x) = x^2.

(i) range of f

Domain of f = R (set of real numbers)

We know that the square of a real number is always positive or equal to zero.

∴ range of f = $R^+ \cup \{0\}$

(ii) {x: f(x) = 4}

Given:

f(x) = 4

we know, $x^2 = 4$

$x^2 - 4 = 0$

$(x - 2)(x + 2) = 0$

∴ $x = \pm 2$

∴ $\{x: f(x) = 4\} = \{-2, 2\}$

(iii) $\{y: f(y) = -1\}$

Given:

$f(y) = -1$

$y^2 = -1$

However, the domain of f is R, and for every real number y, the value of y^2 is non-negative.

Hence, there exists no real y for which $y^2 = -1$.

∴ $\{y: f(y) = -1\} = \varnothing$

7. Let f: $R^+ \rightarrow R$, where R^+ is the set of all positive real numbers, be such that $f(x) = \log_e x$. Determine

(i) the image set of the domain of f

(ii) $\{x: f(x) = -2\}$

(iii) whether $f(xy) = f(x) + f(y)$ holds.

Solution:

Given f: $R^+ \rightarrow R$ and $f(x) = \log_e x$.

(i) the image set of the domain of f

Domain of f = R^+ (set of positive real numbers)

We know the value of logarithm to the base e (natural logarithm) can take all possible real values.

∴ The image set of f = R

(ii) $\{x: f(x) = -2\}$

Given f(x) = –2

$\log_e x = -2$

∴ $x = e^{-2}$ [since, $\log_b a = c \Rightarrow a = b^c$]

∴ {x: f(x) = –2} = $\{e^{-2}\}$

(iii) Whether f (xy) = f (x) + f (y) holds.

We have f (x) = $\log_e x \Rightarrow$ f (y) = $\log_e y$

Now, let us consider f (xy)

F (xy) = $\log_e (xy)$

f (xy) = $\log_e (x \times y)$ [since, $\log_b (a \times c) = \log_b a + \log_b c$]

f (xy) = $\log_e x + \log_e y$

f (xy) = f (x) + f (y)

∴ the equation f (xy) = f (x) + f (y) holds.

8. Write the following relations as sets of ordered pairs and find which of them are functions:

(i) {(x, y): y = 3x, x ∈ {1, 2, 3}, y ∈ {3, 6, 9, 12}}

(ii) {(x, y): y > x + 1, x = 1, 2 and y = 2, 4, 6}

(iii) {(x, y): x + y = 3, x, y ∈ {0, 1, 2, 3}}

Solution:

(i) {(x, y): y = 3x, x ∈ {1, 2, 3}, y ∈ {3, 6, 9, 12}}

When x = 1, y = 3(1) = 3

When x = 2, y = 3(2) = 6

When x = 3, y = 3(3) = 9

∴ R = {(1, 3), (2, 6), (3, 9)}

Hence, the given relation R is a function.

(ii) {(x, y): y > x + 1, x = 1, 2 and y = 2, 4, 6}

When x = 1, y > 1 + 1 or y > 2 ⇒ y = {4, 6}

When x = 2, y > 2 + 1 or y > 3 ⇒ y = {4, 6}

∴ R = {(1, 4), (1, 6), (2, 4), (2, 6)}

Hence, the given relation R is not a function.

(iii) {(x, y): x + y = 3, x, y ∈ {0, 1, 2, 3}}

When x = 0, 0 + y = 3 ⇒ y = 3

When x = 1, 1 + y = 3 ⇒ y = 2

When x = 2, 2 + y = 3 ⇒ y = 1

When x = 3, 3 + y = 3 ⇒ y = 0

∴ R = {(0, 3), (1, 2), (2, 1), (3, 0)}

Hence, the given relation R is a function.

9. Let f: R → R and g: C → C be two functions defined as f(x) = x^2 and g(x) = x^2. Are they equal functions?

Solution:

Given:

f: R → R ∈ f(x) = x^2 and g : R → R ∈ g(x) = x^2

f is defined from R to R, the domain of f = R.

g is defined from C to C, the domain of g = C.

Two functions are equal only when the domain and codomain of both the functions are equal.

In this case, the domain of f ≠ domain of g.

∴ f and g are not equal functions.

EXERCISE 3.2 PAGE NO: 3.11

1. If f (x) = x^2 – 3x + 4, then find the values of x satisfying the equation f (x) = f (2x + 1).

Solution:

Given:

$f(x) = x^2 - 3x + 4$.

Let us find x satisfying f (x) = f (2x + 1).

We have,

$f(2x + 1) = (2x + 1)^2 - 3(2x + 1) + 4$

$= (2x)^2 + 2(2x)(1) + 1^2 - 6x - 3 + 4$

$= 4x^2 + 4x + 1 - 6x + 1$

$= 4x^2 - 2x + 2$

Now, f (x) = f (2x + 1)

$x^2 - 3x + 4 = 4x^2 - 2x + 2$

$4x^2 - 2x + 2 - x^2 + 3x - 4 = 0$

$3x^2 + x - 2 = 0$

$3x^2 + 3x - 2x - 2 = 0$

$3x(x + 1) - 2(x + 1) = 0$

$(x + 1)(3x - 2) = 0$

$x + 1 = 0$ or $3x - 2 = 0$

$x = -1$ or $3x = 2$

$x = -1$ or $2/3$

∴ The values of x are –1 and 2/3.

2. If f (x) = $(x – a)^2 (x – b)^2$, find f (a + b).

Solution:

Given:

F (x) = $(x – a)^2(x – b)^2$

Let us find f (a + b).

We have,

f (a + b) = $(a + b – a)^2 (a + b – b)^2$

f (a + b) = $(b)^2 (a)^2$

∴ f (a + b) = a^2b^2

3. If y = f (x) = (ax – b) / (bx – a), show that x = f (y).

Solution:

Given:

y = f (x) = (ax – b) / (bx – a) ⇒ f (y) = (ay – b) / (by – a)

Let us prove that x = f (y).

We have,

y = (ax – b) / (bx – a)

By cross-multiplying,

y(bx – a) = ax – b

bxy – ay = ax – b

bxy – ax = ay – b

x(by – a) = ay – b

x = (ay – b) / (by – a) = f (y)

∴ x = f (y)

Hence proved.

4. If f (x) = 1 / (1 – x), show that f [f {f (x)}] = x.

Solution:

Given:

f (x) = 1 / (1 – x)

Let us prove that f [f {f (x)}] = x.

Firstly, let us solve for f {f (x)}.

f {f (x)} = f {1/(1 – x)}

= 1 / 1 – (1/(1 – x))

= 1 / [(1 – x – 1)/(1 – x)]

= 1 / (-x/(1 – x))

= (1 – x) / -x

= (x – 1) / x

∴ f {f (x)} = (x – 1) / x

Now, we shall solve for f [f {f (x)}]

f [f {f (x)}] = f [(x-1)/x]

= 1 / [1 – (x-1)/x]

= 1 / [(x – (x-1))/x]

= 1 / [(x – x + 1)/x]

= 1 / (1/x)

∴ f [f {f (x)}] = x

Hence proved.

5. If f (x) = (x + 1) / (x – 1), show that f [f (x)] = x.

Solution:

Given:

f (x) = (x + 1) / (x – 1)

Let us prove that f [f (x)] = x.

f [f (x)] = f [(x+1)/(x-1)]

= [(x+1)/(x-1) + 1] / [(x+1)/(x-1) – 1]

= [[(x+1) + (x-1)]/(x-1)] / [[(x+1) – (x-1)]/(x-1)]

= [(x+1) + (x-1)] / [(x+1) – (x-1)]

= (x+1+x-1)/(x+1-x+1)

= 2x/2

= x

∴ f [f (x)] = x

Hence proved.

6. If

$$f(x) = \begin{cases} x^2, \text{when } x < 0 \\ x, \text{when } 0 \leq x < 1 \\ \frac{1}{x}, \text{when } x \geq 1 \end{cases}$$

Find:

(i) f (1/2)

(ii) f (-2)

(iii) f (1)

(iv) f (√3)

(v) f (√-3)

Solution:

(i) f (1/2)

When, $0 \le x \le 1$, f(x) = x

∴ f (1/2) = ½

(ii) f (-2)

When, x < 0, f(x) = x^2

f (–2) = $(–2)^2$

= 4

∴ f (–2) = 4

(iii) f (1)

When, x ≥ 1, f (x) = 1/x

f (1) = 1/1

∴ f(1) = 1

(iv) f (√3)

We have √3 = 1.732 > 1

When, x ≥ 1, f (x) = 1/x

∴ f (√3) = 1/√3

(v) f (√-3)

We know √-3 is not a real number and the function f(x) is defined only when x ∈ R.

∴ f (√-3) does not exist.

EXERCISE 3.3 PAGE NO: 3.18

1. Find the domain of each of the following real valued functions of real variable:

(i) f (x) = 1/x

(ii) f (x) = 1/(x-7)

(iii) f (x) = (3x-2)/(x+1)

(iv) f (x) = (2x+1)/(x^2-9)

(v) f (x) = (x^2+2x+1)/(x^2-8x+12)

Solution:

(i) f (x) = 1/x

We know, f (x) is defined for all real values of x, except for the case when x = 0.

∴ Domain of f = R – {0}

(ii) f (x) = 1/(x-7)

We know, f (x) is defined for all real values of x, except for the case when x – 7 = 0 or x = 7.

∴ Domain of f = R – {7}

(iii) f (x) = (3x-2)/(x+1)

We know, f(x) is defined for all real values of x, except for the case when x + 1 = 0 or x = –1.

∴ Domain of f = R – {–1}

(iv) f (x) = (2x+1)/(x^2-9)

We know, f (x) is defined for all real values of x, except for the case when $x^2 – 9 = 0$.

$x^2 – 9 = 0$

$x^2 – 3^2 = 0$

(x + 3)(x – 3) = 0

x + 3 = 0 or x – 3 = 0

x = ± 3

∴ Domain of f = R – {–3, 3}

(v) f (x) = $(x^2+2x+1)/(x^2-8x+12)$

We know, f(x) is defined for all real values of x, except for the case when $x^2 - 8x + 12 = 0$.

$x^2 - 8x + 12 = 0$

$x^2 - 2x - 6x + 12 = 0$

$x(x - 2) - 6(x - 2) = 0$

$(x - 2)(x - 6) = 0$

$x - 2 = 0$ or $x - 6 = 0$

x = 2 or 6

∴ Domain of f = R – {2, 6}

2. Find the domain of each of the following real valued functions of real variable:

(i) f (x) = $\sqrt{(x-2)}$

(ii) f (x) = $1/(\sqrt{(x^2-1)})$

(iii) f (x) = $\sqrt{(9-x^2)}$

(iv) f (x) = $\sqrt{(x-2)/(3-x)}$

Solution:

(i) f (x) = $\sqrt{(x-2)}$

We know the square of a real number is never negative.

f (x) takes real values only when $x - 2 \geq 0$

$x \geq 2$

∴ $x \in [2, \infty)$

∴ Domain (f) = $[2, \infty)$

(ii) f (x) = $1/(\sqrt{(x^2-1)})$

We know the square of a real number is never negative.

f (x) takes real values only when $x^2 – 1 \geq 0$

$x^2 – 1^2 \geq 0$

$(x + 1) (x – 1) \geq 0$

$x \leq –1$ or $x \geq 1$

$\therefore x \in (–\infty, –1] \cup [1, \infty)$

In addition, f (x) is also undefined when $x^2 – 1 = 0$ because denominator will be zero and the result will be indeterminate.

$x^2 – 1 = 0 \Rightarrow x = \pm 1$

So, $x \in (–\infty, –1] \cup [1, \infty) – \{–1, 1\}$

$x \in (–\infty, –1) \cup (1, \infty)$

$\therefore$ Domain (f) = $(–\infty, –1) \cup (1, \infty)$

(iii) f (x) = $\sqrt{(9-x^2)}$

We know the square of a real number is never negative.

f (x) takes real values only when $9 – x^2 \geq 0$

$9 \geq x^2$

$x^2 \leq 9$

$x^2 – 9 \leq 0$

$x^2 – 3^2 \leq 0$

$(x + 3)(x – 3) \leq 0$

$x \geq –3$ and $x \leq 3$

$x \in [–3, 3]$

$\therefore$ Domain (f) = $[–3, 3]$

(iv) f (x) = √(x-2)/(3-x)

We know the square root of a real number is never negative.

f (x) takes real values only when x – 2 and 3 – x are both positive and negative.

(a) Both x – 2 and 3 – x are positive

$x - 2 \geq 0$

$x \geq 2$

$3 - x \geq 0$

$x \leq 3$

Hence, $x \geq 2$ and $x \leq 3$

∴ $x \in [2, 3]$

(b) Both x – 2 and 3 – x are negative

$x - 2 \leq 0$

$x \leq 2$

$3 - x \leq 0$

$x \geq 3$

Hence, $x \leq 2$ and $x \geq 3$

However, the intersection of these sets is null set. Thus, this case is not possible.

Hence, $x \in [2, 3] - \{3\}$

$x \in [2, 3]$

∴ Domain (f) = [2, 3]

3. Find the domain and range of each of the following real valued functions:

(i) f (x) = (ax+b)/(bx-a)

(ii) f (x) = (ax-b)/(cx-d)

(iii) f (x) = √(x-1)

(iv) f (x) = √(x-3)

(v) f (x) = (x-2)/(2-x)

(vi) f (x) = |x-1|

(vii) f (x) = -|x|

(viii) f (x) = √(9-x^2)

Solution:

(i) f (x) = (ax+b)/(bx-a)

f(x) is defined for all real values of x, except for the case when bx – a = 0 or x = a/b.

Domain (f) = R – (a/b)

Let f (x) = y

(ax+b)/(bx-a) = y

ax + b = y(bx – a)

ax + b = bxy – ay

ax – bxy = –ay – b

x(a – by) = –(ay + b)

∴ x = – (ay+b)/(a-by)

When a – by = 0 or y = a/b

Hence, f(x) cannot take the value a/b.

∴ Range (f) = R – (a/b)

(ii) f (x) = (ax-b)/(cx-d)

f(x) is defined for all real values of x, except for the case when cx – d = 0 or x = d/c. Domain (f) = R – (d/c)

Let f (x) = y

(ax-b)/(cx-d) = y

ax – b = y(cx – d)

ax – b = cxy – dy

ax – cxy = b – dy

x(a – cy) = b – dy

∴ x = (b-dy)/(a-cy)

When a – cy = 0 or y = a/c,

Hence, f(x) cannot take the value a/c.

∴ Range (f) = R – (a/c)

(iii) f (x) = √(x-1)

We know the square of a real number is never negative.

f(x) takes real values only when x – 1 ≥ 0

x ≥ 1

∴ x ∈ [1, ∞)

Thus, domain (f) = [1, ∞)

When x ≥ 1, we have x – 1 ≥ 0

Hence, √(x-1) ≥ 0 ⇒ f (x) ≥ 0

f(x) ∈ [0, ∞)

∴ Range (f) = [0, ∞)

(iv) f (x) = √(x-3)

We know the square of a real number is never negative.

f (x) takes real values only when x – 3 ≥ 0 x ≥ 3

∴ x ∈ [3, ∞)

Domain (f) = [3, ∞)

When x ≥ 3, we have x – 3 ≥ 0

Hence, √(x-3) ≥ 0 ⇒ f (x) ≥ 0

f(x) ∈ [0, ∞)

∴ Range (f) = [0, ∞)

(v) f (x) = (x-2)/(2-x)

f(x) is defined for all real values of x, except for the case when 2 – x = 0 or x = 2.

Domain (f) = R – {2}

We have, f (x) = (x-2)/(2-x)

f (x) = -(2-x)/(2-x)

= –1

When x ≠ 2, f(x) = –1

∴ Range (f) = {–1}

(vi) f (x) = |x-1|

$$we\ know\ |x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$$

Now we have,

$$|x-1| = \begin{cases} -(x-1), x-1 < 0 \\ x-1, x-1 \geq 0 \end{cases} \therefore f(x) = |x-1| = \begin{cases} 1-x, x < 1 \\ x-1, x \geq 1 \end{cases}$$

Hence, f(x) is defined for all real numbers x.

Domain (f) = R

When, $x < 1$, we have $x – 1 < 0$ or $1 – x > 0$.

$|x – 1| > 0 \Rightarrow f(x) > 0$

When, $x \geq 1$, we have $x – 1 \geq 0$.

$|x – 1| \geq 0 \Rightarrow f(x) \geq 0$

$\therefore$ $f(x) \geq 0$ or $f(x) \in [0, \infty)$

Range (f) = $[0, \infty)$

(vii) $f(x) = -|x|$

$$we\ know\ |x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$$

Now we have,

$$-|x| = \begin{cases} -(-x), x < 0 \\ -x, x \geq 0 \end{cases} \therefore f(x) = -|x| = \begin{cases} x, x < 0 \\ -x, x \geq 0 \end{cases}$$

Hence, f(x) is defined for all real numbers x.

Domain (f) = R

When, $x < 0$, we have $–|x| < 0$

$f(x) < 0$

When, $x \geq 0$, we have $–x \leq 0$.

$–|x| \leq 0 \Rightarrow f(x) \leq 0$

$\therefore$ $f(x) \leq 0$ or $f(x) \in (–\infty, 0]$

Range (f) = $(–\infty, 0]$

(viii) $f(x) = \sqrt{(9-x^2)}$

We know the square of a real number is never negative.

f(x) takes real values only when $9 – x^2 \geq 0$

$9 \geq x^2$

$x^2 \leq 9$

$x^2 - 9 \leq 0$

$x^2 - 3^2 \leq 0$

$(x + 3)(x - 3) \leq 0$

$x \geq -3$ and $x \leq 3$

$\therefore x \in [-3, 3]$

Domain (f) = [–3, 3]

When, $x \in [-3, 3]$, we have $0 \leq 9 - x^2 \leq 9$

$0 \leq \sqrt{(9-x^2)} \leq 3 \Rightarrow 0 \leq f(x) \leq 3$

$\therefore f(x) \in [0, 3]$

Range (f) = [0, 3]

EXERCISE 3.4 PAGE NO: 3.38

1. Find f + g, f – g, cf ($c \in R$, $c \neq 0$), fg, 1/f and f/g in each of the following:

(i) f (x) = x^3 + 1 and g (x) = x + 1

(ii) f (x) = $\sqrt{(x-1)}$ and g (x) = $\sqrt{(x+1)}$

Solution:

(i) f (x) = $x^3 + 1$ and g(x) = x + 1

We have f(x): R → R and g(x): R → R

(a) f + g

We know, (f + g) (x) = f(x) + g(x)

$(f + g)(x) = x^3 + 1 + x + 1$

$= x^3 + x + 2$

So, (f + g) (x): R → R

∴ f + g: R → R is given by $(f + g)(x) = x^3 + x + 2$

(b) f – g

We know, (f – g) (x) = f(x) – g(x)

$(f – g)(x) = x^3 + 1 – (x + 1)$

$= x^3 + 1 – x – 1$

$= x^3 – x$

So, (f – g) (x): R → R

∴ f – g: R → R is given by $(f – g)(x) = x^3 – x$

(c) cf ($c \in R$, $c \neq 0$)

We know, (cf) (x) = c × f(x)

$(cf)(x) = c(x^3 + 1)$

$= cx^3 + c$

So, (cf) (x) : R → R

∴ cf: R → R is given by $(cf)(x) = cx^3 + c$

(d) fg

We know, (fg) (x) = f(x) g(x)

$(fg)(x) = (x^3 + 1)(x + 1)$

$= (x + 1)(x^2 – x + 1)(x + 1)$

$= (x + 1)^2 (x^2 – x + 1)$

So, (fg) (x): R → R

∴ fg: R → R is given by $(fg)(x) = (x + 1)^2(x^2 – x + 1)$

(e) 1/f

We know, (1/f) (x) = 1/f (x)

1/f (x) = 1 / (x^3 + 1)

Observe that 1/f(x) is undefined when f(x) = 0 or when x = – 1.

So, 1/f: R – {–1} → R is given by 1/f (x) = 1 / (x^3 + 1)

(f) f/g

We know, (f/g) (x) = f(x)/g(x)

(f/g) (x) = (x^3 + 1) / (x + 1)

Observe that (x^3 + 1) / (x + 1) is undefined when g(x) = 0 or when x = –1.

Using x^3 + 1 = (x + 1) (x^2 – x + 1), we have

(f/g) (x) = [(x+1) (x^2– x+1)/(x+1)]

= x^2 – x + 1

∴ f/g: R – {–1} → R is given by (f/g) (x) = x^2 – x + 1

(ii) f (x) = √(x-1) and g (x) = √(x+1)

We have f(x): [1, ∞) → R^+ and g(x): [–1, ∞) → R^+ as real square root is defined only for non-negative numbers.

(a) f + g

We know, (f + g) (x) = f(x) + g(x)

(f+g) (x) = √(x-1) + √(x+1)

Domain of (f + g) = Domain of f ∩ Domain of g

Domain of (f + g) = [1, ∞) ∩ [–1, ∞)

Domain of (f + g) = [1, ∞)

∴ f + g: [1, ∞) → R is given by (f+g) (x) = √(x-1) + √(x+1)

(b) f – g

We know, (f – g) (x) = f(x) – g(x)

(f-g) (x) = √(x-1) – √(x+1)

Domain of (f – g) = Domain of f ∩ Domain of g

Domain of (f – g) = [1, ∞) ∩ [–1, ∞)

Domain of (f – g) = [1, ∞)

∴ f – g: [1, ∞) → R is given by (f-g) (x) = √(x-1) – √(x+1)

(c) cf (c ∈ R, c ≠ 0)

We know, (cf) (x) = c × f(x)

(cf) (x) = c√(x-1)

Domain of (cf) = Domain of f

Domain of (cf) = [1, ∞)

∴ cf: [1, ∞) → R is given by (cf) (x) = c√(x-1)

(d) fg

We know, (fg) (x) = f(x) g(x)

(fg) (x) = √(x-1) √(x+1)

= √(x^2 -1)

Domain of (fg) = Domain of f ∩ Domain of g

Domain of (fg) = [1, ∞) ∩ [–1, ∞)

Domain of (fg) = [1, ∞)

∴ fg: [1, ∞) → R is given by (fg) (x) = √(x^2 -1)

(e) 1/f

We know, (1/f) (x) = 1/f(x)

(1/f) (x) = 1/√(x-1)

Domain of (1/f) = Domain of f

Domain of (1/f) = [1, ∞)

Observe that 1/√(x-1) is also undefined when x – 1 = 0 or x = 1.

∴ 1/f: (1, ∞) → R is given by (1/f) (x) = 1/√(x-1)

(f) f/g

We know, (f/g) (x) = f(x)/g(x)

(f/g) (x) = √(x-1)/√(x+1)

(f/g) (x) = √[(x-1)/(x+1)]

Domain of (f/g) = Domain of f ∩ Domain of g

Domain of (f/g) = [1, ∞) ∩ [–1, ∞)

Domain of (f/g) = [1, ∞)

∴ f/g: [1, ∞) → R is given by (f/g) (x) = √[(x-1)/(x+1)]

2. Let f(x) = 2x + 5 and g(x) = x^2 + x. Describe

(i) f + g

(ii) f – g

(iii) fg

(iv) f/g

Find the domain in each case.

Solution:

Given:

f(x) = 2x + 5 and g(x) = x^2 + x

Both f(x) and g(x) are defined for all x ∈ R.

So, domain of f = domain of g = R

(i) f + g

We know, (f + g)(x) = f(x) + g(x)

(f + g)(x) = 2x + 5 + x^2 + x

= x^2 + 3x + 5

(f + g)(x) Is defined for all real numbers x.

∴ The domain of (f + g) is R

(ii) f – g

We know, (f – g)(x) = f(x) – g(x)

(f – g)(x) = 2x + 5 – (x^2 + x)

= 2x + 5 – x^2 – x

= 5 + x – x^2

(f – g)(x) is defined for all real numbers x.

∴ The domain of (f – g) is R

(iii) fg

We know, (fg)(x) = f(x)g(x)

(fg)(x) = (2x + 5)(x^2 + x)

= 2x(x^2 + x) + 5(x^2 + x)

= $2x^3$ + $2x^2$ + $5x^2$ + 5x

= $2x^3$ + $7x^2$ + 5x

(fg)(x) is defined for all real numbers x.

∴ The domain of fg is R

(iv) f/g

We know, (f/g) (x) = f(x)/g(x)

(f/g) (x) = $(2x+5)/(x^2+x)$

(f/g) (x) is defined for all real values of x, except for the case when $x^2 + x = 0$.

$x^2 + x = 0$

x(x + 1) = 0

x = 0 or x + 1 = 0

x = 0 or –1

When x = 0 or –1, (f/g) (x) will be undefined as the division result will be indeterminate.

∴ The domain of f/g = R – {–1, 0}

3. If f(x) be defined on [–2, 2] and is given by

$$f(x)=\begin{cases}-1, -2 \leq x \leq 0 \\ x-1, 0 < x \leq 2\end{cases}$$

and g(x) = f(|x|) + |f(x)|. Find g(x).

Solution:

Given:

$$f(x)=\begin{cases}-1,-2\le x\le 0\\ x-1,0<x\le 2\end{cases}$$ and

g(x) = f(|x|) + |f(x)|

Now we have,

$$f(|x|)=\begin{cases}-1,-2\le |x|\le 0\\ |x|-1,0<|x|\le 2\end{cases}$$

However, |x| ≥ 0 ⇒ f(|x|) = |x| – 1 when 0 < |x| ≤ 2

We also have,

$$|f(x)|=\begin{cases}|-1|,-2\le x\le 0\\ |x-1|,0<x\le 2\end{cases}$$
$$=\begin{cases}1,-2\le x\le 0\\ |x-1|,0<x\le 2\end{cases}$$

We also know,

$$|x-1|=\begin{cases}-(x-1),x-1<0\\ x-1,x-1\ge 0\end{cases}$$
$$=\begin{cases}-(x-1),x<1\\ x-1,x\ge 1\end{cases}$$

Here, we shall only the range between [0, 2].

$$|x-1|=\begin{cases}-(x-1),0<x<1\\ x-1,1\le x\le 2\end{cases}$$

Substituting this value of |x – 1| in |f(x)|, we get

$$|f(x)|=\begin{cases}1,-2\le x\le 0\\ -(x-1),0<x<1\\ x-1,1\le x\le 2\end{cases}$$
$$=\begin{cases}1,-2\le x\le 0\\ 1-x,0<x<1\\ x-1,1\le x\le 2\end{cases}$$

Now, we need to find g(x)

g(x) = f(|x|) + |f(x)|

= |x| – 1 when 0 < |x| ≤ 2 + $\begin{cases}1,-2\le x\le 0\\ 1-x,0<x<1\\ x-1,1\le x\le 2\end{cases}$

$$g(x)=\begin{cases}-x-1,-2\le x\le 0\\ x-1,0<x<1\\ x-1,1\le x\le 2\end{cases}+\begin{cases}1,-2\le x\le 0\\ 1-x,0<x<1\\ x-1,1\le x\le 2\end{cases}$$
$$=\begin{cases}-x-1+1,-2\le x\le 0\\ x-1+1-x,0<x<1\\ x-1+x-1,1\le x\le 2\end{cases}$$
$$=\begin{cases}-x,-2\le x\le 0\\ 0,0<x<1\\ 2(x-1),1\le x\le 2\end{cases}$$

∴ $g(x) = f(|x|) + |f(x)|$ |
$$=\begin{cases}-x,-2\le x\le 0\\ 0,0<x<1\\ 2(x-1),1\le x\le 2\end{cases}$$

4. Let f, g be two real functions defined by f(x) = √(x+1) and g(x) = √(9-x²). Then, describe each of the following functions.

(i) f + g

(ii) g – f

(iii) fg

(iv) f/g

(v) g/f

(vi) 2f – √5g

(vii) f² + 7f

(viii) 5/g

Solution:

Given:

$f(x) = \sqrt{(x+1)}$ and $g(x) = \sqrt{(9-x^2)}$

We know the square of a real number is never negative.

So, f(x) takes real values only when $x + 1 \geq 0$

$x \geq -1$, $x \in [-1, \infty)$

Domain of f = $[-1, \infty)$

Similarly, g(x) takes real values only when $9 - x^2 \geq 0$

$9 \geq x^2$

$x^2 \leq 9$

$x^2 - 9 \leq 0$

$x^2 - 3^2 \leq 0$

$(x + 3)(x - 3) \leq 0$

$x \geq -3$ and $x \leq 3$

$\therefore x \in [-3, 3]$

Domain of g = $[-3, 3]$

(i) f + g

We know, $(f + g)(x) = f(x) + g(x)$

$(f + g)(x) = \sqrt{(x+1)} + \sqrt{(9-x^2)}$

Domain of f + g = Domain of f ∩ Domain of g

$= [-1, \infty) \cap [-3, 3]$

$= [-1, 3]$

$\therefore$ f + g: $[-1, 3] \rightarrow R$ is given by $(f + g)(x) = f(x) + g(x) = \sqrt{(x+1)} + \sqrt{(9-x^2)}$

(ii) g – f

We know, $(g - f)(x) = g(x) - f(x)$

$(g - f)(x) = \sqrt{(9-x^2)} - \sqrt{(x+1)}$

Domain of g – f = Domain of g ∩ Domain of f

= [–3, 3] ∩ [–1, ∞)

= [–1, 3]

∴ g – f: [–1, 3] → R is given by (g – f) (x) = g(x) – f(x) = $\sqrt{(9-x^2)} - \sqrt{(x+1)}$

(iii) fg

We know, (fg) (x) = f(x)g(x)

(fg) (x) = $\sqrt{(x+1)}\ \sqrt{(9-x^2)}$

= $\sqrt{[(x+1)\ (9-x^2)]}$

= $\sqrt{[x(9-x^2) + (9-x^2)]}$

= $\sqrt{(9x-x^3+9-x^2)}$

= $\sqrt{(9+9x-x^2-x^3)}$

Domain of fg = Domain of f ∩ Domain of g

= [–1, ∞) ∩ [–3, 3]

= [–1, 3]

∴ fg: [–1, 3] → R is given by (fg) (x) = f(x) g(x) = $\sqrt{(x+1)}\ \sqrt{(9-x^2)} = \sqrt{(9+9x-x^2-x^3)}$

(iv) f/g

We know, (f/g) (x) = f(x)/g(x)

(f/g) (x) = $\sqrt{(x+1)} / \sqrt{(9-x^2)}$

= $\sqrt{[(x+1) / (9-x^2)]}$

Domain of f/g = Domain of f ∩ Domain of g

= [–1, ∞) ∩ [–3, 3]

= [–1, 3]

However, (f/g) (x) is defined for all real values of x ∈ [–1, 3], except for the case when $9 – x^2 = 0$ or x = ± 3

When x = ±3, (f/g) (x) will be undefined as the division result will be indeterminate.

Domain of f/g = [–1, 3] – {–3, 3}

Domain of f/g = [–1, 3)

∴ f/g: [–1, 3) → R is given by (f/g) (x) = f(x)/g(x) = $\sqrt{(x+1)} / \sqrt{(9-x^2)}$

(v) g/f

We know, (g/f) (x) = g(x)/f(x)

(g/f) (x) = $\sqrt{(9-x^2)} / \sqrt{(x+1)}$

= $\sqrt{[(9-x^2) / (x+1)]}$

Domain of g/f = Domain of f ∩ Domain of g

= [–1, ∞) ∩ [–3, 3]

= [–1, 3]

However, (g/f) (x) is defined for all real values of x ∈ [–1, 3], except for the case when x + 1 = 0 or x = –1

When x = –1, (g/f) (x) will be undefined as the division result will be indeterminate.

Domain of g/f = [–1, 3] – {–1}

Domain of g/f = (–1, 3]

∴ g/f: (–1, 3] → R is given by (g/f) (x) = g(x)/f(x) = $\sqrt{(9-x^2)} / \sqrt{(x+1)}$

(vi) 2f – √5g

We know, (2f – √5g) (x) = 2f(x) – √5g(x)

(2f – √5g) (x) = 2f (x) – √5g (x)

= $2\sqrt{(x+1)} - \sqrt{5}\sqrt{(9-x^2)}$

= $2\sqrt{(x+1)} - \sqrt{(45- 5x^2)}$

Domain of $2f - \sqrt{5}g$ = Domain of f ∩ Domain of g

= [–1, ∞) ∩ [–3, 3]

= [–1, 3]

∴ $2f - \sqrt{5}g$: [–1, 3] → R is given by $(2f - \sqrt{5}g)(x) = 2f(x) - \sqrt{5}g(x) = 2\sqrt{(x+1)} - \sqrt{(45 - 5x^2)}$

(vii) $f^2 + 7f$

We know, $(f^2 + 7f)(x) = f^2(x) + (7f)(x)$

$(f^2 + 7f)(x) = f(x) f(x) + 7f(x)$

$= \sqrt{(x+1)}\sqrt{(x+1)} + 7\sqrt{(x+1)}$

$= x + 1 + 7\sqrt{(x+1)}$

Domain of $f^2 + 7f$ is same as domain of f.

Domain of $f^2 + 7f$ = [–1, ∞)

∴ $f^2 + 7f$: [–1, ∞) → R is given by $(f^2 + 7f)(x) = f(x) f(x) + 7f(x) = x + 1 + 7\sqrt{(x+1)}$

(viii) 5/g

We know, $(5/g)(x) = 5/g(x)$

$(5/g)(x) = 5/\sqrt{(9-x^2)}$

Domain of 5/g = Domain of g = [–3, 3]

However, (5/g) (x) is defined for all real values of x ∈ [–3, 3], except for the case when $9 - x^2 = 0$ or x = ± 3

When x = ±3, (5/g) (x) will be undefined as the division result will be indeterminate.

Domain of 5/g = [–3, 3] – {–3, 3}

= (–3, 3)

∴ 5/g: (–3, 3) → R is given by $(5/g)(x) = 5/g(x) = 5/\sqrt{(9-x^2)}$

5. If $f(x) = \log_e(1 - x)$ and g(x) = [x], then determine each of the following functions:

(i) f + g

(ii) fg

(iii) f/g

(iv) g/f

Also, find (f + g) (–1), (fg) (0), (f/g) (1/2) and (g/f) (1/2).

Solution:

Given:

$f(x) = \log_e (1 - x)$ and $g(x) = [x]$

We know, f(x) takes real values only when $1 - x > 0$

$1 > x$

$x < 1$, ∴ $x \in (-\infty, 1)$

Domain of f = $(-\infty, 1)$

Similarly, g(x) is defined for all real numbers x.

Domain of g = $[x]$, $x \in R$

= R

(i) f + g

We know, $(f + g)(x) = f(x) + g(x)$

$(f + g)(x) = \log_e (1 - x) + [x]$

Domain of f + g = Domain of f ∩ Domain of g

Domain of f + g = $(-\infty, 1) \cap R$

= $(-\infty, 1)$

∴ $f + g: (-\infty, 1) \to R$ is given by $(f + g)(x) = \log_e (1 - x) + [x]$

(ii) fg

We know, (fg) (x) = f(x) g(x)

(fg) (x) = $\log_e (1 - x) \times [x]$

= $[x] \log_e (1 - x)$

Domain of fg = Domain of f ∩ Domain of g

= (–∞, 1) ∩ R

= (–∞, 1)

∴ fg: (–∞, 1) → R is given by (fg) (x) = $[x] \log_e (1 - x)$

(iii) f/g

We know, (f/g) (x) = f(x)/g(x)

(f/g) (x) = $\log_e (1 - x) / [x]$

Domain of f/g = Domain of f ∩ Domain of g

= (–∞, 1) ∩ R

= (–∞, 1)

However, (f/g) (x) is defined for all real values of x ∈ (–∞, 1), except for the case when [x] = 0.

We have, [x] = 0 when 0 ≤ x < 1 or x ∈ [0, 1)

When 0 ≤ x < 1, (f/g) (x) will be undefined as the division result will be indeterminate.

Domain of f/g = (–∞, 1) – [0, 1)

= (–∞, 0)

∴ f/g: (–∞, 0) → R is given by (f/g) (x) = $\log_e (1 - x) / [x]$

(iv) g/f

We know, (g/f) (x) = g(x)/f(x)

(g/f) (x) = $[x] / \log_e (1 - x)$

However, (g/f) (x) is defined for all real values of $x \in (-\infty, 1)$, except for the case when $\log_e(1 - x) = 0$.

$\log_e(1 - x) = 0 \Rightarrow 1 - x = 1$ or $x = 0$

When x = 0, (g/f) (x) will be undefined as the division result will be indeterminate.

Domain of g/f = $(-\infty, 1) - \{0\}$

$= (-\infty, 0) \cup (0, 1)$

$\therefore$ g/f: $(-\infty, 0) \cup (0, 1) \rightarrow R$ is given by $(g/f)(x) = [x] / \log_e(1 - x)$

(a) We need to find (f + g) (–1).

We have, $(f + g)(x) = \log_e(1 - x) + [x]$, $x \in (-\infty, 1)$

Substituting x = –1 in the above equation, we get

$(f + g)(-1) = \log_e(1 - (-1)) + [-1]$

$= \log_e(1 + 1) + (-1)$

$= \log_e 2 - 1$

$\therefore (f + g)(-1) = \log_e 2 - 1$

(b) We need to find (fg) (0).

We have, $(fg)(x) = [x] \log_e(1 - x)$, $x \in (-\infty, 1)$

Substituting x = 0 in the above equation, we get

$(fg)(0) = [0] \log_e(1 - 0)$

$= 0 \times \log_e 1$

$\therefore (fg)(0) = 0$

(c) We need to find (f/g) (1/2)

We have, $(f/g)(x) = \log_e(1 - x) / [x]$, $x \in (-\infty, 0)$

However, 1/2 is not in the domain of f/g.

∴ (f/g) (1/2) does not exist.

(d) We need to find (g/f) (1/2)

We have, (g/f) (x) = [x] / $\log_e$ (1 – x), x ∈ (–∞, 0) ∪ (0, ∞)

Substituting x=1/2 in the above equation, we get

(g/f) (1/2) = [x] / $\log_e$ (1 – x)

= (1/2)/ $\log_e$ (1 – 1/2)

= 0.5/ $\log_e$ (1/2)

= 0 / $\log_e$ (1/2)

= 0

∴ (g/f) (1/2) = 0

Chapterwise RD Sharma Solutions for Class 11 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.