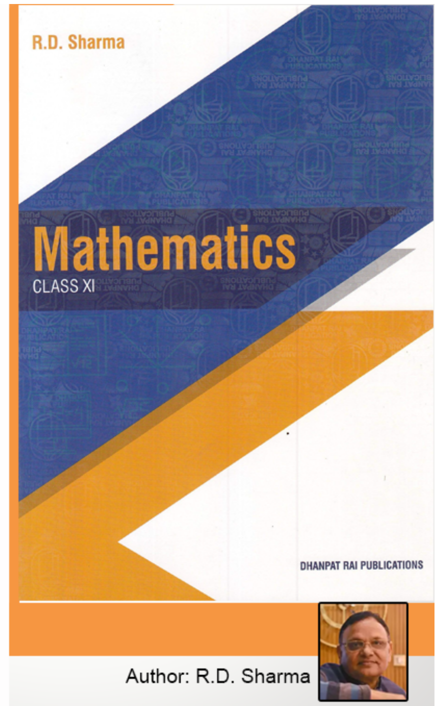
Class 11 - Chapter 9 Values of Trigonometric Functions at Multiples and Submultiples of an Angle



RD Sharma Solutions for Class 11 Maths Chapter 9–Values of Trigonometric Functions at Multiples and Submultiples of an Angle

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RD Sharma Solutions for Class 11 Maths Chapter 9–Values of Trigonometric Functions at Multiples and Submultiples of an Angle

RD Sharma 11th Maths Chapter 9, Class 11 Maths Chapter 9 solutions

EXERCISE 9.1 PAGE NO: 9.28

Prove the following identities:

1. $\sqrt{[(1 - \cos 2x) / (1 + \cos 2x)]} = \tan x$

Solution:

Let us consider LHS:

$\sqrt{[(1 - \cos 2x) / (1 + \cos 2x)]}$

We know that $\cos 2x = 1 - 2 \sin^2 x$

$= 2 \cos^2 x - 1$

So,

$\sqrt{[(1 - \cos 2x) / (1 + \cos 2x)]} = \sqrt{[(1 - (1 - 2\sin^2 x)) / (1 + (2\cos^2 x - 1))]}$

$= \sqrt{[(1 - 1 + 2\sin^2 x) / (1 + 2\cos^2 x - 1)]}$

$= \sqrt{[2 \sin^2 x / 2 \cos^2 x]}$

$= \sin x/\cos x$

$= \tan x$

= RHS

Hence proved.

2. $\sin 2x / (1 - \cos 2x) = \cot x$

Solution:

Let us consider LHS:

sin 2x / (1 – cos 2x)

We know that cos 2x = 1 – 2 $\sin^2$ x

Sin 2x = 2 sin x cos x

So,

sin 2x / (1 – cos 2x) = (2 sin x cos x) / (1 – (1 – 2$\sin^2$ x))

= (2 sin x cos x) / (1 – 1 + 2$\sin^2$ x)]

= [2 sin x cos x / 2 $\sin^2$ x]

= cos x/sin x

= cot x

= RHS

Hence proved.

3. sin 2x / (1 + cos 2x) = tan x

Solution:

Let us consider LHS:

sin 2x / (1 + cos 2x)

We know that cos 2x = 1 – 2 $\sin^2$ x

= 2 $\cos^2$ x – 1

Sin 2x = 2 sin x cos x

So,

sin 2x / (1 + cos 2x) = [2 sin x cos x / (1 + (2$\cos^2$x – 1))]

= [2 sin x cos x / (1 + 2$\cos^2$ x – 1)]

= [2 sin x cos x / 2 $\cos^2$ x]

= sin x/cos x

= tan x

= RHS

Hence proved.

4. $\sqrt{2+\sqrt{2+2\cos 4x}} = 2\cos x, 0 < x < \frac{\pi}{4}$

Solution:

Let us consider LHS:

$$\sqrt{2+\sqrt{2+2\cos 4x}} = \sqrt{2+\sqrt{2+2(2\cos^2 2x-1)}}$$

{since, cos 2x = 2 $\cos^2$ x – 1 ⇒ cos 4x = 2 $\cos^2$ 2x -1}

$$= \sqrt{2+\sqrt{2+4\cos^2 2x-2}}$$

$$= \sqrt{2+\sqrt{4\cos^2 2x}}$$

$$= \sqrt{2+2\cos 2x}$$

$$= \sqrt{2+2(2\cos^2 x-1)}$$ {since, cos 2x = 2 $\cos^2$ x – 1}

$$= \sqrt{2+4\cos^2 x-2}$$

$$= \sqrt{4\cos^2 x}$$

= 2 cos x

= RHS

Hence proved.

5. [1 – cos 2x + sin 2x] / [1 + cos 2x + sin 2x] = tan x

Solution:

Let us consider LHS:[1 – cos 2x + sin 2x] / [1 + cos 2x + sin 2x]

We know that, $\cos 2x = 1 – 2 \sin^2 x$

$= 2 \cos^2 x – 1$

$\sin 2x = 2 \sin x \cos x$

So,

$$= \frac{1 - (1 - 2\sin^2 x) + 2\sin x \cos x}{1 + (2\cos^2 x - 1) + 2\sin x \cos x}$$
$$= \frac{1 - 1 + 2\sin^2 x + 2\sin x \cos x}{1 + 2\cos^2 x - 1 + 2\sin x \cos x}$$
$$= \frac{2\sin^2 x + 2\sin x \cos x}{2\cos^2 x + 2\sin x \cos x}$$
$$= \frac{2\sin x(\sin x + \cos x)}{2\cos x(\cos x + \sin x)}$$
$$= \frac{\sin x}{\cos x}$$
$$= \tan x$$
$$= \text{RHS}$$

Hence proved.

6. [sin x + sin 2x] / [1 + cos x + cos 2x] = tan x

Solution:

Let us consider LHS:[sin x + sin 2x] / [1 + cos x + cos 2x]

We know that, $\cos 2x = \cos^2 x – \sin^2 x$

$\sin 2x = 2 \sin x \cos x$

So,

$$\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \frac{\sin x + 2\sin x \cos x}{1 + \cos x + (2\cos^2 x - 1)}$$

$$= \frac{\sin x + 2\sin x \cos x}{1 + \cos x + 2\cos^2 x - 1}$$

$$= \frac{\sin x + 2\sin x \cos x}{\cos x + 2\cos^2 x}$$

$$= \frac{\sin x\,(1 + 2\cos x)}{\cos x\,(1 + 2\cos x)}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

= RHS

Hence proved.

7. cos 2x / (1 + sin 2x) = tan (π/4 – x)

Solution:

Let us consider LHS:

cos 2x / (1 + sin 2x)

We know that, cos 2x = $\cos^2 x - \sin^2 x$

Sin 2x = 2 sin x cos x

So,

$$\frac{\cos 2x}{1+\sin 2x} = \frac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x}$$

$$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\sin^2 x + \cos^2 x + 2\sin x \cos x}$$

(since, $a^2 - b^2 = (a - b)(a + b)$ & $\sin^2 x + \cos^2 x = 1$)

$$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2}$$

(since, $a^2 + b^2 + 2ab = (a + b)^2$)

$$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)(\sin x + \cos x)}$$

$$= \frac{(\cos x - \sin x)}{(\sin x + \cos x)}$$

Multiplying numerator and denominator by $1/\sqrt{2}$

We get,

$$= \frac{\frac{1}{\sqrt{2}}(\cos x - \sin x)}{\frac{1}{\sqrt{2}}(\sin x + \cos x)}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}{\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)}$$

$$= \frac{\left(\sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x\right)}{\left(\sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x\right)}$$ (since, $1/\sqrt{2} = \sin \pi/4$)

$$= \frac{\sin\left(\frac{\pi}{4} - x\right)}{\cos\left(\frac{\pi}{4} - x\right)}$$

By using the formulas,

sin (A – B) = sin A cos B – sin B cos A

cos (A – B) = cos A cos B + sin A sin B

= tan (π/4 - x)

= RHS

Hence proved.

8. cos x / (1 – sin x) = tan (π/4 + x/2)

Solution:

Let us consider LHS:

cos x / (1 – sin x)

We know that, cos 2x = $\cos^2$ x – $\sin^2$ x

Cos x = $\cos^2$ x/2 – $\sin^2$ x/2

Sin 2x = 2 sin x cos x

Sin x = 2 sin x/2 cos x/2

So,

$$\frac{\cos x}{1-\sin x}=\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{1-2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$=\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}$$

(By using the formula, $a^2 - b^2 = (a - b)(a + b)$ & $\sin^2 x + \cos^2 x = 1$)

$$=\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}$$

(By using the formula, $a^2 + b^2 + 2ab = (a + b)^2$)

$$=\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}$$

$$=\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}$$

$$= \frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)}$$

Let us multiply numerator and denominator by $1/\sqrt{2}$

We get,

$$= \frac{\frac{1}{\sqrt{2}}\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\frac{1}{\sqrt{2}}\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\cos\frac{x}{2} + \frac{1}{\sqrt{2}}\sin\frac{x}{2}\right)}{\left(\frac{1}{\sqrt{2}}\sin\frac{x}{2} - \frac{1}{\sqrt{2}}\cos\frac{x}{2}\right)}$$

$$= \frac{\left(\sin\frac{\pi}{4}\cos\frac{x}{2} + \cos\frac{\pi}{4}\sin\frac{x}{2}\right)}{\left(\sin\frac{\pi}{4}\sin\frac{x}{2} - \cos\frac{\pi}{4}\cos\frac{x}{2}\right)}$$ (since, $1/\sqrt{2} = \sin \pi/4$)

$$= \frac{\sin\left(\frac{\pi}{4} - x\right)}{\cos\left(\frac{\pi}{4} - x\right)}$$

$= \tan(\pi/4 - x)$

= RHS

Hence proved.

$$= \frac{1+\cos\frac{2\pi}{8}}{2} + \frac{1+\cos\frac{6\pi}{8}}{2} + \frac{1+\cos\frac{10\pi}{8}}{2} + \frac{1+\cos\frac{14\pi}{8}}{2}$$

$$= \frac{1+\cos\frac{2\pi}{8}}{2} + \frac{1+\cos\left(\pi-\frac{2\pi}{8}\right)}{2} + \frac{1+\cos\left(\pi+\frac{2\pi}{8}\right)}{2} + \frac{1+\cos\left(2\pi-\frac{2\pi}{8}\right)}{2}$$

$$\left\{\because \pi-\frac{2\pi}{8}=\frac{6\pi}{8}; \pi+\frac{2\pi}{8}=\frac{10\pi}{8}; 2\pi-\frac{2\pi}{8}=\frac{14\pi}{8}\right\}$$

$$= \frac{1+\cos\frac{2\pi}{8}}{2} + \frac{1-\cos\frac{2\pi}{8}}{2} + \frac{1-\cos\frac{2\pi}{8}}{2} + \frac{1+\cos\frac{2\pi}{8}}{2}$$

{we know, $\cos(\pi - A) = -\cos A$, $\cos(\pi + A) = -\cos A$ & $\cos(2\pi - A) = \cos A$}

$$= 2 \times \frac{1+\cos\frac{2\pi}{8}}{2} + 2 \times \frac{1-\cos\frac{2\pi}{8}}{2}$$

$$= 1 + \cos\frac{2\pi}{8} + 1 - \cos\frac{2\pi}{8}$$

= 2

= RHS

Hence proved.

$$= \frac{1+\cos\frac{2\pi}{8}}{2} + \frac{1+\cos\frac{6\pi}{8}}{2} + \frac{1+\cos\frac{10\pi}{8}}{2} + \frac{1+\cos\frac{14\pi}{8}}{2}$$

$$= \frac{1+\cos\frac{2\pi}{8}}{2} + \frac{1+\cos\left(\pi-\frac{2\pi}{8}\right)}{2} + \frac{1+\cos\left(\pi+\frac{2\pi}{8}\right)}{2} + \frac{1+\cos\left(2\pi-\frac{2\pi}{8}\right)}{2}$$

$$\left\{\because \pi-\frac{2\pi}{8}=\frac{6\pi}{8}; \pi+\frac{2\pi}{8}=\frac{10\pi}{8}; 2\pi-\frac{2\pi}{8}=\frac{14\pi}{8}\right\}$$

$$= \frac{1+\cos\frac{2\pi}{8}}{2} + \frac{1-\cos\frac{2\pi}{8}}{2} + \frac{1-\cos\frac{2\pi}{8}}{2} + \frac{1+\cos\frac{2\pi}{8}}{2}$$

{we know, $\cos(\pi - A) = -\cos A$, $\cos(\pi + A) = -\cos A$ & $\cos(2\pi - A) = \cos A$}

$$= 2 \times \frac{1+\cos\frac{2\pi}{8}}{2} + 2 \times \frac{1-\cos\frac{2\pi}{8}}{2}$$

$$= 1 + \cos\frac{2\pi}{8} + 1 - \cos\frac{2\pi}{8}$$

= 2

= RHS

Hence proved.

$$\left\{\because \pi - \frac{2\pi}{8} = \frac{6\pi}{8}; \pi + \frac{2\pi}{8} = \frac{10\pi}{8}; 2\pi - \frac{2\pi}{8} = \frac{14\pi}{8}\right\}$$

$$= \frac{1 - \cos\frac{2\pi}{8}}{2} + \frac{1 - \left(-\cos\frac{2\pi}{8}\right)}{2} + \frac{1 - \left(-\cos\frac{2\pi}{8}\right)}{2} + \frac{1 - \cos\frac{2\pi}{8}}{2}$$

{we know, cos (π – A) = - cos A, cos (π + A) = - cos A & cos (2π – A) = cos A}

$$= \frac{1 - \cos\frac{2\pi}{8}}{2} + \frac{1 + \cos\frac{2\pi}{8}}{2} + \frac{1 + \cos\frac{2\pi}{8}}{2} + \frac{1 - \cos\frac{2\pi}{8}}{2}$$

$$= 2 \times \frac{1 - \cos\frac{2\pi}{8}}{2} + 2 \times \frac{1 + \cos\frac{2\pi}{8}}{2}$$

$$= 1 - \cos\frac{2\pi}{8} + 1 + \cos\frac{2\pi}{8}$$

= 2

= RHS

Hence proved.

11. (cos α + cos β)^2 + (sin α + sin β)^2 = 4 cos^2 (α – β)/2

Solution:

Let us consider LHS:

(cos α + cos β)^2 + (sin α + sin β)^2

Upon expansion, we get,

(cos α + cos β)^2 + (sin α + sin β)^2 =

= cos^2 α + cos^2 β + 2 cos α cos β + sin^2 α + sin^2 β + 2 sin α sin β

= 2 + 2 cos α cos β + 2 sin α sin β

= 2 (1 + cos α cos β + sin α sin β)

= 2 (1 + cos (α – β)) [since, cos (A – B) = cos A cos B + sin A sin B]

$= 2 (1 + 2 \cos^2 (\alpha – \beta)/2 – 1)$ [since, $\cos 2x = 2\cos^2 x – 1$]

$= 2 (2 \cos^2 (\alpha – \beta)/2)$

$= 4 \cos^2 (\alpha – \beta)/2$

= RHS

Hence Proved.

12. $\sin^2 (\pi/8 + x/2) – \sin^2 (\pi/8 – x/2) = 1/\sqrt{2} \sin x$

Solution:

Let us consider LHS:

$\sin^2 (\pi/8 + x/2) – \sin^2 (\pi/8 – x/2)$

we know, $\sin^2 A – \sin^2 B = \sin (A+B) \sin (A-B)$

so,

$\sin^2 (\pi/8 + x/2) – \sin^2 (\pi/8 – x/2) = \sin (\pi/8 + x/2 + \pi/8 – x/2) \sin (\pi/8 + x/2 – (\pi/8 – x/2))$

$= \sin (\pi/8 + \pi/8) \sin (\pi/8 + x/2 – \pi/8 + x/2)$

$= \sin \pi/4 \sin x$

$= 1/\sqrt{2} \sin x$ [since, since $\pi/4 = 1/\sqrt{2}$]

= RHS

Hence proved.

13. $1 + \cos^2 2x = 2 (\cos^4 x + \sin^4 x)$

Solution:

Let us consider LHS:

$1 + \cos^2 2x$

We know, $\cos 2x = \cos^2 x – \sin^2 x$

$\cos^2 x + \sin^2 x = 1$

so,

$1 + \cos^2 2x = (\cos^2 x + \sin^2 x)^2 + (\cos^2 x - \sin^2 x)^2$

$= (\cos^4 x + \sin^4 x + 2\cos^2 x \sin^2 x) + (\cos^4 x + \sin^4 x - 2\cos^2 x \sin^2 x)$

$= \cos^4 x + \sin^4 x + \cos^4 x + \sin^4 x$

$= 2\cos^4 x + 2\sin^4 x$

$= 2(\cos^4 x + \sin^4 x)$

= RHS

Hence proved.

14. $\cos^3 2x + 3\cos 2x = 4(\cos^6 x - \sin^6 x)$

Solution:

Let us consider RHS:

$4(\cos^6 x - \sin^6 x)$

Upon expansion we get,

$4(\cos^6 x - \sin^6 x) = 4[(\cos^2 x)^3 - (\sin^2 x)^3]$

$= 4(\cos^2 x - \sin^2 x)(\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x)$

By using the formula,

$a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

$= 4\cos 2x (\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x + \cos^2 x \sin^2 x - \cos^2 x \sin$

We know, $\cos 2x = \cos^2 x - \sin^2 x$

So,

$= 4\cos 2x (\cos^4 x + \sin^4 x + 2\cos^2 x \sin^2 x - \cos^2 x \sin^2 x)$

$= 4 \cos 2x [(\cos^2 x)^2 + (\sin^2 x)^2 + 2 \cos^2 x \sin^2 x – \cos^2 x \sin^2 x]$

We know, $a^2 + b^2 + 2ab = (a + b)^2$

$= 4 \cos 2x [(1)^2 – 1/4 (4 \cos^2 x \sin^2 x)]$

$= 4 \cos 2x [(1)^2 – 1/4 (2 \cos x \sin x)^2]$

We know, $\sin 2x = 2\sin x \cos x$

$= 4 \cos 2x [(1^2) – 1/4 (\sin 2x)^2]$

$= 4 \cos 2x (1 – 1/4 \sin^2 2x)$

We know, $\sin^2 x = 1 – \cos^2 x$

$= 4 \cos 2x [1 – 1/4 (1 – \cos^2 2x)]$

$= 4 \cos 2x [1 – 1/4 + 1/4 \cos^2 2x]$

$= 4 \cos 2x [3/4 + 1/4 \cos^2 2x]$

$= 4 (3/4 \cos 2x + 1/4 \cos^3 2x)$

$= 3 \cos 2x + \cos^3 2x$

$= \cos^3 2x + 3 \cos 2x$

= LHS

Hence proved.

15. (sin 3x + sin x) sin x + (cos 3x – cos x) cos x = 0

Solution:

Let us consider LHS:

$(\sin 3x + \sin x) \sin x + (\cos 3x – \cos x) \cos x$

$= (\sin 3x) (\sin x) + \sin^2 x + (\cos 3x) (\cos x) – \cos^2 x$

$= [(\sin 3x) (\sin x) + (\cos 3x) (\cos x)] + (\sin^2 x – \cos^2 x)$

= [(sin 3x) (sin x) + (cos 3x) (cos x)] – ($\cos^2 x - \sin^2 x$)

= cos (3x – x) – cos 2x

We know, cos 2x = $\cos^2 x - \sin^2 x$

cos A cos B + sin A sin B = cos(A – B)

So,

= cos 2x – cos 2x

= 0

= RHS

Hence Proved.

16. $\cos^2 (\pi/4 - x) - \sin^2 (\pi/4 - x) = \sin 2x$

Solution:

Let us consider LHS:

$\cos^2 (\pi/4 - x) - \sin^2 (\pi/4 - x)$

We know, $\cos^2 A - \sin^2 A = \cos 2A$

So,

$\cos^2 (\pi/4 - x) - \sin^2 (\pi/4 - x) = \cos 2 (\pi/4 - x)$

= cos (π/2 – 2x)

= sin 2x [since, cos (π/2 – A) = sin A]

= RHS

Hence proved.

17. $\cos 4x = 1 - 8 \cos^2 x + 8 \cos^4 x$

Solution:

Let us consider LHS:

cos 4x

We know, cos 2x = 2 $\cos^2$ x – 1

So,

cos 4x = 2 $\cos^2$ 2x – 1

= 2(2 $\cos^2$ 2x – 1)^2 – 1

= 2[(2 $\cos^2$ 2x)^2 + 1^2 – 2×2 $\cos^2$ x] – 1

= 2(4 $\cos^4$ 2x + 1 – 4 $\cos^2$ x) – 1

= 8 $\cos^4$ 2x + 2 – 8 $\cos^2$ x – 1

= 8 $\cos^4$ 2x + 1 – 8 $\cos^2$ x

= RHS

Hence Proved.

18. sin 4x = 4 sin x $\cos^3$ x – 4 cos x $\sin^3$ x

Solution:

Let us consider LHS:

sin 4x

We know, sin 2x = 2 sin x cos x

cos 2x = $\cos^2$ x – $\sin^2$ x

So,

sin 4x = 2 sin 2x cos 2x

= 2 (2 sin x cos x) ($\cos^2$ x – $\sin^2$ x)

= 4 sin x cos x ($\cos^2$ x – $\sin^2$ x)

= 4 sin x $\cos^3$ x – 4 $\sin^3$ x cos x

= RHS

Hence proved.

19. 3(sin x – cos x)^4 + 6 (sin x + cos x)^2 + 4 ($\sin^6$ x + $\cos^6$ x) = 13

Solution:

Let us consider LHS:

3(sin x – cos x)^4 + 6 (sin x + cos x)^2 + 4 ($\sin^6$ x + $\cos^6$ x)

We know, $(a + b)^2 = a^2 + b^2 + 2ab$

$(a – b)^2 = a^2 + b^2 – 2ab$

$a^3 + b^3 = (a + b) (a^2 + b^2 – ab)$

So,

3(sin x – cos x)^4 + 6 (sin x + cos x)^2 + 4 ($\sin^6$ x + $\cos^6$ x) = 3{(sin x – cos x)^2}^{2} + 6 {(sin x)^2 + (cos x)^2 + 2 sin x cos x)} + 4 {($\sin^2$ x)^3 + ($\cos^2$ x)^3}

= 3{(sin x)^2 + (cos x)^2 – 2 sin x cos x)}^{2} + 6 ($\sin^2$ x + $\cos^2$ x + 2 sin x cos x) + 4{($\sin^2$ x + $\cos^2$ x) ($\sin^4$ x + $\cos^4$ x – $\sin^2$ x $\cos^2$ x)}

= 3(1 – 2 sin x cos x)^2 + 6 (1 + 2 sin x cos x) + 4{(1) ($\sin^4$ x + $\cos^4$ x – $\sin^2$ x $\cos^2$ x)}

We know, $\sin^2$ x + $\cos^2$ x = 1

So,

= 3{1^2 + (2 sin x cos x)^2 – 4 sin x cos x} + 6 (1 + 2 sin x cos x) + 4{($\sin^2$ x)^2 + ($\cos^2$ x)^2 + 2 $\sin^2$ x $\cos^2$ x – 3 $\sin^2$ x $\cos^2$ x)}

= 3{1 + 4 $\sin^2$ x $\cos^2$ x – 4 sin x cos x} + 6 (1 + 2 sin x cos x) + 4{($\sin^2$ x + $\cos^2$ x)^2 – 3 $\sin^2$ x $\cos^2$ x)}

= 3 + 12 $\sin^2$ x $\cos^2$ x – 12 sin x cos x + 6 + 12 sin x cos x + 4{(1)^2 – 3 $\sin^2$ x $\cos^2$ x)}

= 9 + 12 $\sin^2$ x $\cos^2$ x + 4(1 – 3 $\sin^2$ x $\cos^2$ x)

$= 9 + 12 \sin^2 x \cos^2 x + 4 – 12 \sin^2 x \cos^2 x$

$= 13$

= RHS

Hence proved.

20. $2(\sin^6 x + \cos^6 x) – 3(\sin^4 x + \cos^4 x) + 1 = 0$

Solution:

Let us consider LHS:

$2(\sin^6 x + \cos^6 x) – 3(\sin^4 x + \cos^4 x) + 1$

We know, $(a + b)^2 = a^2 + b^2 + 2ab$

$a^3 + b^3 = (a + b) (a^2 + b^2 – ab)$

So,

$2(\sin^6 x + \cos^6 x) – 3(\sin^4 x + \cos^4 x) + 1 = 2\{(\sin^2 x)^3 + (\cos^2 x)^3\} – 3\{(\sin^2 x)^2 + (\cos^2 x)^2\} + 1$

$= 2\{(\sin^2 x + \cos^2 x) (\sin^4 x + \cos^4 x – \sin^2 x \cos^2 x\} – 3\{(\sin^2 x)^2 + (\cos^2 x)^2 + 2\sin^2 x \cos^2 x – 2\sin^2 x \cos^2 x\} + 1$

$= 2\{(1) (\sin^4 x + \cos^4 x + 2 \sin^2 x \cos^2 x – 3 \sin^2 x \cos^2 x\} – 3\{(\sin^2 x + \cos^2 x)^2 – 2\sin^2 x \cos^2 x\} + 1$

We know, $\sin^2 x + \cos^2 x = 1$

$= 2\{(\sin^2 x + \cos^2 x)^2 – 3 \sin^2 x \cos^2 x\} – 3\{(1)^2 – 2\sin^2 x \cos^2 x\} + 1$

$= 2\{(1)^2 – 3 \sin^2 x \cos^2 x\} – 3(1 – 2\sin^2 x \cos^2 x) + 1$

$= 2(1 – 3 \sin^2 x \cos^2 x) – 3 + 6 \sin^2 x \cos^2 x + 1$

$= 2 – 6 \sin^2 x \cos^2 x – 2 + 6 \sin^2 x \cos^2 x$

$= 0$

= RHS

Hence proved.

21. $\cos^6 x - \sin^6 x = \cos 2x (1 - 1/4 \sin^2 2x)$

Solution:

Let us consider LHS:

$\cos^6 x - \sin^6 x$

We know, $(a + b)^2 = a^2 + b^2 + 2ab$

$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

So,

$\cos^6 x - \sin^6 x = (\cos^2 x)^3 - (\sin^2 x)^3$

$= (\cos^2 x - \sin^2 x)(\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x)$

We know, $\cos 2x = \cos^2 x - \sin^2 x$

So,

$= \cos 2x [(\cos^2 x)^2 + (\sin^2 x)^2 + 2 \cos^2 x \sin^2 x - \cos^2 x \sin^2 x]$

$= \cos 2x [(\cos^2 x)^2 + (\sin^2 x)^2 - 1/4 \times 4 \cos^2 x \sin^2 x]$

We know, $\sin^2 x + \cos^2 x = 1$

So,

$= \cos 2x [(1)^2 - 1/4 \times (2 \cos x \sin x)^2]$

We know, $\sin 2x = 2 \sin x \cos x$

So,

$= \cos 2x [1 - 1/4 \times (\sin 2x)^2]$

$= \cos 2x [1 - 1/4 \times \sin^2 2x]$

= RHS

Hence proved.

22. tan (π/4 + x) + tan (π/4 – x) = 2 sec 2x

Solution:

Let us consider LHS:

tan (π/4 + x) + tan (π/4 – x)

We know,

tan (A+B) = (tan A + tan B)/(1- tan A tan B)

tan (A-B) = (tan A – tan B)/(1+ tan A tan B)

So,

$$tan(\frac{\pi}{4}+x)+tan(\frac{\pi}{4}-x)=\frac{tan\frac{\pi}{4}+tanx}{1-tan\frac{\pi}{4}tanx}+\frac{tan\frac{\pi}{4}-tanx}{1+tan\frac{\pi}{4}tanx}$$

We know, tan π/4 = 1

So,

$$=\frac{1+\tan x}{1-\tan x}+\frac{1-\tan x}{1+\tan x}$$

$$=\frac{(1+\tan x)^2+(1-\tan x)^2}{(1-\tan x)(1+\tan x)}$$

We know, $(a-b)(a+b)=a^2-b^2$;

$(a+b)^2=a^2+b^2+2ab$ &

$(a-b)^2=a^2+b^2-2ab$

So,

$$=\frac{1^2+\tan^2 x+2\tan x+1^2+\tan^2 x-2\tan x}{1^2-\tan^2 x}$$

$$=\frac{1+\tan^2 x+1+\tan^2 x}{1-\tan^2 x}$$

$$=\frac{2(1+\tan^2 x)}{1-\tan^2 x}$$

We know, tan x = sin x/cos x

So,

$$=\frac{2\left(1+\left(\frac{\sin x}{\cos x}\right)^2\right)}{1-\left(\frac{\sin x}{\cos x}\right)^2}$$

$$=\frac{2\left(1+\frac{\sin^2 x}{\cos^2 x}\right)}{1-\frac{\sin^2 x}{\cos^2 x}}$$

$$=\frac{2\left(\frac{\cos^2 x+\sin^2 x}{\cos^2 x}\right)}{\frac{\cos^2 x-\sin^2 x}{\cos^2 x}}$$

We know, $\cos^2 x + \sin^2 x = 1$ & $\cos 2x = \cos^2 x - \sin^2 x$

So,

$$= \frac{2\left(\frac{1}{\cos^2 x}\right)}{\frac{\cos 2x}{\cos^2 x}}$$

$$= \frac{2}{\cos 2x}$$

$= 2 \sec 2x$ (since, $1/\cos 2x = \sec 2x$)

= RHS

Hence proved.

EXERCISE 9.2 PAGE NO: 9.36

Prove that:

1. $\sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x$

Solution:

Let us consider LHS:

sin 5x

Now,

sin 5x = sin (3x + 2x)

But we know,

Sin (x + y) = sin x cos y + cos x sin y…..(i)

So,

sin 5x = sin 3x cos 2x + cos 3x sin 2x

= sin (2x + x) cos 2x + cos (2x + x) sin 2x……..(ii)

And

$\cos (x + y) = \cos x \cos y - \sin x \sin y$……(iii)

Now substituting equation (i) and (iii) in equation (ii), we get

$\sin 5x = (\sin 2x \cos x + \cos 2x \sin x) \cos 2x + (\cos 2x \cos x - \sin 2x \sin x) \sin 2x$

$= \sin 2x \cos 2x \cos x + \cos^2 2x \sin x + (\sin 2x \cos 2x \cos x - \sin^2 2x \sin x)$

$= 2\sin 2x \cos 2x \cos x + \cos^2 2x \sin x - \sin^2 2x \sin x$ …….(iv)

Now $\sin 2x = 2\sin x \cos x$………(v)

And $\cos 2x = \cos^2 x - \sin^2 x$………(vi)

Substituting equation (v) and (vi) in equation (iv), we get

$\sin 5x = 2(2\sin x \cos x) (\cos^2 x - \sin^2 x) \cos x + (\cos^2 x - \sin^2 x)^2 \sin x - (2\sin x \cos x)^2 \sin x$

$= 4(\sin x \cos^2 x) ([1 - \sin^2 x] - \sin^2 x) + ([1 - \sin^2 x] - \sin^2 x)^2 \sin x - (4\sin^2 x \cos^2 x)\sin x$

(as $\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$)

$\sin 5x = 4(\sin x [1 - \sin^2 x]) (1 - 2\sin^2 x) + (1 - 2\sin^2 x)^2 \sin x - 4\sin^3 x [1 - \sin^2 x]$

$= 4\sin x (1 - \sin^2 x) (1 - 2\sin^2 x) + (1 - 4\sin^2 x + 4\sin^4 x) \sin x - 4\sin^3 x + 4\sin^5 x$

$= (4\sin x - 4\sin^3 x) (1 - 2\sin^2 x) + \sin x - 4\sin^3 x + 4\sin^5 x - 4\sin^3 x + 4\sin^5 x$

$= 4\sin x - 8\sin^3 x - 4\sin^3 x + 8\sin^5 x + \sin x - 8\sin^3 x + 8\sin^5 x$

$= 5\sin x - 20\sin^3 x + 16\sin^5 x$

= RHS

Hence proved.

2. 4 ($\cos^3 10^\circ + \sin^3 20^\circ$) = 3 ($\cos 10^\circ + \sin 20^\circ$)

Solution:

Let us consider LHS:

$4 (\cos^3 10^\circ + \sin^3 20^\circ)$

We know that, sin 60° = √3/2 = cos 30°

Sin 30° = cos 60° = 1/2

So,

Sin (3×20°) = cos (3×10°)

$3\sin 20° – 4\sin^3 20° = 4\cos^3 10° – 3\cos 10°$

(we know, $\sin 3\theta = 3\sin \theta – 4\sin^3 \theta$ and $\cos 3\theta = 4\cos^3\theta – 3\cos\theta$)

So,

$4(\cos^3 10°+\sin^3 20°) = 3(\sin 20°+\cos 10°)$

= RHS

Hence proved.

3. $\cos^3 x \sin 3x + \sin^3 x \cos 3x = 3/4 \sin 4x$

Solution:

We know that,

$\cos 3\theta = 4\cos^3\theta – 3\cos\theta$

So, $4 \cos^3\theta = \cos 3\theta + 3\cos\theta$

$\cos^3 \theta = [\cos 3\theta + 3\cos\theta]/4$ …… (i)

Similarly,

$\sin 3\theta = 3\sin \theta – 4\sin^3 \theta$

$4 \sin^3\theta = 3\sin\theta – \sin 3\theta$

$\sin^3\theta = [3\sin\theta – \sin 3\theta]/4$ …….. (ii)

Now,

Let us consider LHS:

$\cos^3 x \sin 3x + \sin^3 x \cos 3x$

Substituting the values from equation (i) and (ii), we get

$\cos^3 x \sin 3x + \sin^3 x \cos 3x = (\cos 3x + 3 \cos x)/4 \sin 3x + (3\sin x – \sin 3x)/4 \cos 3x$

= 1/4 (sin 3x cos 3x + 3 sin 3x cox x + 3sin x cos 3x – sin 3x cos 3x)

= 1/4 (3(sin 3x cos x + sin x cos 3x) + 0)

= 1/4 (3 sin (3x + x))

(We know, sin(x + y) = sin x cos y + cos x sin y)

= 3/4 sin 4x

= RHS

Hence proved.

4. sin 5x = 5 $\cos^4 x$ sin x – 10 $\cos^2 x \sin^3 x + \sin^5 x$

Solution:

Let us consider LHS:

sin 5x

Now,

sin 5x = sin (3x + 2x)

But we know,

Sin (x + y) = sin x cos y + cos x sin y…..(i)

So,

sin 5x = sin 3x cos 2x + cos 3x sin 2x

= sin (2x + x) cos 2x + cos (2x + x) sin 2x……..(ii)

And

$\cos(x + y) = \cos x \cos y - \sin x \sin y$......(iii)

Now substituting equation (i) and (iii) in equation (ii), we get

$\sin 5x = (\sin 2x \cos x + \cos 2x \sin x) \cos 2x + (\cos 2x \cos x - \sin 2x \sin x) \sin 2x$ … (iv)

Now $\sin 2x = 2\sin x \cos x$.........(v)

And $\cos 2x = \cos^2 x - \sin^2 x$.........(vi)

Substituting equation (v) and (vi) in equation (iv), we get

$\sin 5x = [(2 \sin x \cos x) \cos x + (\cos^2 x - \sin^2 x) \sin x] (\cos^2 x - \sin^2 x) + [(\cos^2 x - \sin^2 x) \cos x - (2 \sin x \cos x) \sin x)] (2 \sin x \cos x)$

$= [2 \sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x] (\cos^2 x - \sin^2 x) + [\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x] (2 \sin x \cos x)$

$= \cos^2 x [3 \sin x \cos^2 x - \sin^3 x] - \sin^2 x [3 \sin x \cos^2 x - \sin^3 x] + 2 \sin x \cos^4 x - 2 \sin^3 x \cos^2 x - 4 \sin^3 x \cos^2 x$

$= 3 \sin x \cos^4 x - \sin^3 x \cos^2 x - 3 \sin^3 x \cos^2 x - \sin^5 x + 2 \sin x \cos^4 x - 2 \sin^3 x \cos^2 x - 4 \sin^3 x \cos^2 x$

$= 5 \sin x \cos^4 x - 10\sin^3 x\cos^2 x + \sin^5 x$

= RHS

Hence proved.

5. $\sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x$

Solution:

Let us consider LHS:

$\sin 5x$

Now,

$\sin 5x = \sin(3x + 2x)$

But we know,

$\sin(x + y) = \sin x \cos y + \cos x \sin y$.....(i)

So,

$\sin 5x = \sin 3x \cos 2x + \cos 3x \sin 2x$

$= \sin(2x + x) \cos 2x + \cos(2x + x) \sin 2x$........(ii)

And

$\cos(x + y) = \cos x \cos y – \sin x \sin y$......(iii)

Now substituting equation (i) and (iii) in equation (ii), we get

$\sin 5x = (\sin 2x \cos x + \cos 2x \sin x) \cos 2x + (\cos 2x \cos x – \sin 2x \sin x) \sin 2x$

$= \sin 2x \cos 2x \cos x + \cos^2 2x \sin x + (\sin 2x \cos 2x \cos x – \sin^2 2x \sin x)$

$= 2\sin 2x \cos 2x \cos x + \cos^2 2x \sin x – \sin^2 2x \sin x$ ....(iv)

Now $\sin 2x = 2\sin x \cos x$.........(v)

And $\cos 2x = \cos^2 x – \sin^2 x$.........(vi)

Substituting equation (v) and (vi) in equation (iv), we get

$\sin 5x = 2(2\sin x \cos x) (\cos^2 x – \sin^2 x) \cos x + (\cos^2 x – \sin^2 x)^2 \sin x – (2\sin x \cos x)^2 \sin x$

$= 4(\sin x \cos^2 x) ([1– \sin^2 x] – \sin^2 x) + ([1–\sin^2 x] – \sin^2 x)^2 \sin x – (4\sin^2 x \cos^2 x)\sin x$

(as $\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1– \sin^2 x$)

$\sin 5x = 4(\sin x [1 – \sin^2 x]) (1 – 2\sin^2 x) + (1 – 2\sin^2 x)^2 \sin x – 4\sin^3 x [1 – \sin^2 x]$

$= 4\sin x (1 – \sin^2 x) (1 – 2\sin^2 x) + (1 – 4\sin^2 x + 4\sin^4 x) \sin x – 4\sin^3 x + 4\sin^5 x$

$= (4\sin x – 4\sin^3 x) (1 – 2\sin^2 x) + \sin x – 4\sin^3 x + 4\sin^5 x – 4\sin^3 x + 4\sin^5 x$

$= 4\sin x – 8\sin^3 x – 4\sin^3 x + 8\sin^5 x + \sin x – 8\sin^3 x + 8\sin^5 x$

$= 5\sin x – 20\sin^3 x + 16\sin^5 x$

= RHS

Hence proved.

7. $\tan x+\tan\left(\frac{\pi}{3}+x\right)-\tan\left(\frac{\pi}{3}-x\right)=3\tan 3x$

Solution:

Let us consider LHS:

$\tan x+\tan\left(\frac{\pi}{3}+x\right)-\tan\left(\frac{\pi}{3}-x\right)$

$$=\tan x+\left(\frac{\tan\frac{\pi}{3}+\tan x}{1-\tan x\tan\frac{\pi}{3}}\right)-\left(\frac{\tan\frac{\pi}{3}-\tan x}{1+\tan x\tan\frac{\pi}{3}}\right)$$

We know that,

$$\tan(A+B)=\left(\frac{\tan A+\tan B}{1-\tan A\tan B}\right)\text{ and }\tan(A-B)=\left(\frac{\tan A-\tan B}{1+\tan A\tan B}\right)$$

So,

$$=\tan x+\left(\frac{\sqrt{3}+\tan x}{1-\sqrt{3}\tan x}\right)-\left(\frac{\sqrt{3}-\tan x}{1+\sqrt{3}\tan x}\right)$$

$$=\tan x+\left(\frac{(1+\sqrt{3}\tan x)(\sqrt{3}+\tan x)-(1-\sqrt{3}\tan x)(\sqrt{3}-\tan x)}{\left(1-\tan x\left(\sqrt{3}\right)\right)\left(1+\tan x\left(\sqrt{3}\right)\right)}\right)$$

Simplify and cancel the similar terms of different sign in the above expression we get,

$$= \tan x + \left(\frac{(0 + 6\tan x + 2\tan x + 0)}{(1 - 3\tan^2 x)}\right)$$

$$= \tan x + \left(\frac{8\tan x}{(1 - 3\tan^2 x)}\right)$$

$$= \left(\frac{\tan x\,(1 - 3\tan^2 x) + 8\tan x}{(1 - 3\tan^2 x)}\right)$$

$$= \left(\frac{(\tan x - 3\tan^3 x) + 8\tan x}{(1 - 3\tan^2 x)}\right)$$

$$= \left(\frac{9\tan x - 3\tan^3 x}{(1 - 3\tan^2 x)}\right)$$

$$= 3\left(\frac{3\tan x - \tan^3 x}{(1 - 3\tan^2 x)}\right)$$

= 3 tan 3x (since, $\tan 3x = (3\tan x - \tan^3 x)/(1-3\tan^2 x)$)

= RHS

Hence proved.

EXERCISE 9.3 PAGE NO: 9.42

Prove that:

1. $\sin^2 2\pi/5 - \sin^2 \pi/3 = (\sqrt{5} - 1)/8$

Solution:

Let us consider LHS:

$\sin^2 2\pi/5 - \sin^2 \pi/3 = \sin^2 (\pi/2 - \pi/10) - \sin^2 \pi/3$

we know, sin (90°– A) = cos A

So, $\sin^2 (\pi/2 – \pi/10) = \cos^2 \pi/10$

Sin $\pi/3 = \sqrt{3}/2$

Then the above equation becomes,

$= \cos^2 \pi/10 – (\sqrt{3}/2)^2$

We know, $\cos \pi/10 = \sqrt{(10+2\sqrt{5})}/4$

the above equation becomes,

$= [\sqrt{(10+2\sqrt{5})}/4]^2 – 3/4$

$= [10 + 2\sqrt{5}]/16 – 3/4$

$= [10 + 2\sqrt{5} – 12]/16$

$= [2\sqrt{5} – 2]/16$

$= [\sqrt{5} – 1]/8$

= RHS

Hence proved.

2. $\sin^2 24^\circ – \sin^2 6^\circ = (\sqrt{5} – 1)/8$

Solution:

Let us consider LHS:

$\sin^2 24^\circ – \sin^2 6^\circ$

we know, $\sin (A + B) \sin (A – B) = \sin^2 A – \sin^2 B$

Then the above equation becomes,

$\sin^2 24^\circ – \sin^2 6^\circ = \sin (24^\circ + 6^\circ) – \sin (24^\circ – 6^\circ)$

$= \sin 30^\circ – \sin 18^\circ$

$= \sin 30^\circ – (\sqrt{5} – 1)/4$ [since, $\sin 18^\circ = (\sqrt{5} – 1)/4$]

$= 1/2 \times (\sqrt{5} - 1)/4$

$= (\sqrt{5} - 1)/8$

= RHS

Hence proved.

3. $\sin^2 42^\circ - \cos^2 78^\circ = (\sqrt{5} + 1)/8$

Solution:

Let us consider LHS:

$\sin^2 42^\circ - \cos^2 78^\circ = \sin^2 (90^\circ - 48^\circ) - \cos^2 (90^\circ - 12^\circ)$

$= \cos^2 48^\circ - \sin^2 12^\circ$ [since, sin (90 – A) = cos A and cos (90 – A) = sin A]

We know, $\cos (A + B) \cos (A - B) = \cos^2 A - \sin^2 B$

Then the above equation becomes,

$= \cos^2 (48^\circ + 12^\circ) \cos (48^\circ - 12^\circ)$

$= \cos 60^\circ \cos 36^\circ$ [since, $\cos 36^\circ = (\sqrt{5} + 1)/4$]

$= 1/2 \times (\sqrt{5} + 1)/4$

$= (\sqrt{5} + 1)/8$

= RHS

Hence proved.

4. $\cos 78^\circ \cos 42^\circ \cos 36^\circ = 1/8$

Solution:

Let us consider LHS:

$\cos 78^\circ \cos 42^\circ \cos 36^\circ$

Let us multiply and divide by 2 we get,

$\cos 78^\circ \cos 42^\circ \cos 36^\circ = 1/2 (2 \cos 78^\circ \cos 42^\circ \cos 36^\circ)$

We know, $2 \cos A \cos B = \cos (A + B) + \cos (A – B)$

Then the above equation becomes,

$= 1/2 (\cos (78^\circ + 42^\circ) + \cos (78^\circ – 42^\circ)) \times \cos 36^\circ$

$= 1/2 (\cos 120^\circ + \cos 36^\circ) \times \cos 36^\circ$

$= 1/2 (\cos (180^\circ – 60^\circ) + \cos 36^\circ) \times \cos 36^\circ$

$= 1/2 (-\cos (60^\circ) + \cos 36^\circ) \times \cos 36^\circ$ [since, $\cos(180° – A) = – A$]

$= 1/2 (-1/2 + (\sqrt{5} + 1)/4) ((\sqrt{5} + 1)/4)$ [since, $\cos 36^\circ = (\sqrt{5} + 1)/4$]

$= 1/2 (\sqrt{5} + 1 – 2)/4 ((\sqrt{5} + 1)/4)$

$= 1/2 (\sqrt{5} – 1)/4) ((\sqrt{5} + 1)/4)$

$= 1/2 ((\sqrt{5})^2 – 1^2)/16$

$= 1/2 (5-1)/16$

$= 1/2 (4/16)$

$= 1/8$

= RHS

Hence proved.

5. cos π/15 cos 2π/15 cos 4π/15 cos 7π/15 = 1/16

Solution:

Let us consider LHS:

$\cos \pi/15 \cos 2\pi/15 \cos 4\pi/15 \cos 7\pi/15$

Let us multiply and divide by $2 \sin \pi/15$, we get,

$= [2 \sin \pi/15 \cos \pi/15] \cos 2\pi/15 \cos 4\pi/15 \cos 7\pi/15] / 2 \sin \pi/15$

We know, 2sin A cos A = sin 2A

Then the above equation becomes,

= [(sin 2π/15) cos 2π/15 cos 4π/15 cos 7π/15] / 2 sin π/15

Now, multiply and divide by 2 we get,

= [(2 sin 2π/15 cos 2π/15) cos 4π/15 cos 7π/15] / 2 × 2 sin π/15

We know, 2sin A cos A = sin 2A

Then the above equation becomes,

= [(sin 4π/15) cos 4π/15 cos 7π/15] / 4 sin π/15

Now, multiply and divide by 2 we get,

= [(2 sin 4π/15 cos 4π/15) cos 7π/15] / 2 × 4 sin π/15

We know, 2sin A cos A = sin 2A

Then the above equation becomes,

= [(sin 8π/15) cos 7π/15] / 8 sin π/15

Now, multiply and divide by 2 we get,

= [2 sin 8π/15 cos 7π/15] / 2 × 8 sin π/15

We know, 2sin A cos B = sin (A+B) + sin (A–B)

Then the above equation becomes,

= [sin (8π/15 + 7π/15) + sin (8π/15 – 7π/15)] / 16 sin π/15

= [sin (π) + sin (π/15)] / 16 sin π/15

= [0 + sin (π/15)] / 16 sin π/15

= sin (π/15) / 16 sin π/15

= 1/16

= RHS

Hence proved.

Chapterwise RD Sharma Solutions for Class 11 Maths :

About RD Sharma

RD Sharma i*sn't the kind of author you'd bump into at lit fests. But his bestselling books have helped many* CBSE *students lose their dread of* maths. *Sunday Times profiles the tutor turned internet star*

He dreams of algorithms that would give most people nightmares. And, spends every waking hour thinking of ways to explain concepts like 'series solution of linear differential equations'. Meet Dr Ravi Dutt Sharma — mathematics teacher and author of 25 reference books — whose name evokes as much awe as the subject he teaches. And though students have used his thick tomes for the last 31 years to ace the dreaded maths exam, it's only recently that a spoof video turned the tutor into a YouTube star.

R D Sharma had a good laugh but said he shared little with his on-screen persona except for the love for maths. "I like to spend all my time thinking and writing about maths problems. I find it relaxing," he says. When he is not writing books explaining mathematical concepts for classes 6 to 12 and engineering students, Sharma is busy dispensing his duty as vice-principal and head of department of science and humanities at Delhi government's Guru Nanak Dev Institute of Technology.