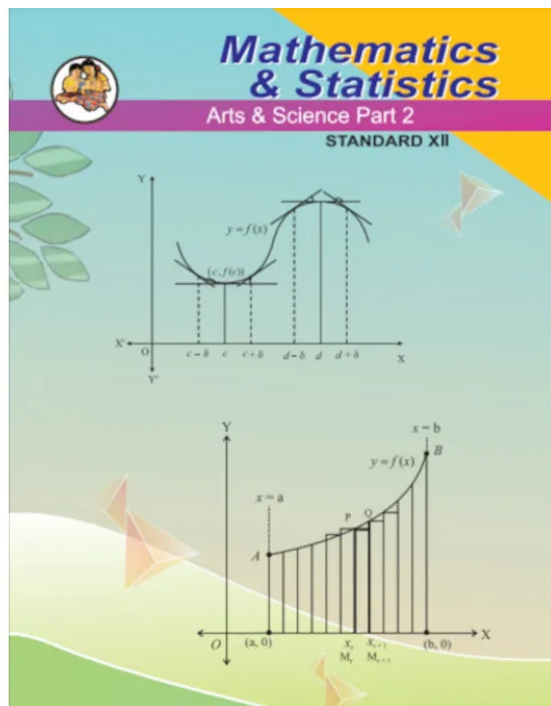


Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 2): Chapter 3- Indefinite Integration

Class 12 - Chapter 3 Indefinite Integration



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Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 2): Chapter 3- Indefinite Integration

Class 12: Maths Chapter 3 solutions. Complete Class 12 Maths Chapter 3 Notes.

Maharashtra Board Solutions Class 12-Arts & Science Maths (Part 2): Chapter 3- Indefinite Integration

Maharashtra Board 12th Maths Chapter 3, Class 12 Maths Chapter 3 solutions

Ex 3.1

I. Integrate the following functions w.r.t. x :

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(i) $x^3 + x^2 - x + 1$

Solution:

$$\begin{aligned}\int (x^3 + x^2 - x + 1) dx &= \int x^3 dx + \int x^2 dx - \int x dx + \int 1 dx \\ &= \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x + c.\end{aligned}$$

(ii) $x^2 \left(1 - \frac{2}{x}\right)^2$

Solution:

$$\begin{aligned}\int x^2 \left(1 - \frac{2}{x}\right)^2 dx &= \int x^2 \left(1 - \frac{4}{x} + \frac{4}{x^2}\right) dx \\ &= \int (x^2 - 4x + 4) dx \\ &= \int x^2 dx - 4 \int x dx + 4 \int 1 dx \\ &= \frac{x^3}{3} - 4 \left(\frac{x^2}{2}\right) + 4x + c \\ &= \frac{1}{3}x^3 - 2x^2 + 4x + c.\end{aligned}$$

(iii) $3 \sec^2 x - \frac{4}{x} + \frac{1}{x\sqrt{x}} - 7$

Solution:

$$\begin{aligned}& \int \left(3 \sec^2 x - \frac{4}{x} + \frac{1}{x\sqrt{x}} - 7 \right) dx \\&= 3 \int \sec^2 x \, dx - 4 \int \frac{1}{x} \, dx + \int x^{-\frac{3}{2}} \, dx - 7 \int 1 \, dx \\&= 3 \tan x - 4 \log |x| + \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} - 7x + c \\&= 3 \tan x - 4 \log |x| - \frac{2}{\sqrt{x}} - 7x + c\end{aligned}$$

(iv) $2x^3 - 5x + \frac{3}{x} + \frac{4}{x^5}$

Solution:

$$\begin{aligned}& \int \left(2x^3 - 5x + \frac{3}{x} + \frac{4}{x^5} \right) dx \\&= 2 \int x^3 \, dx - 5 \int x \, dx + 3 \int \frac{1}{x} \, dx + 4 \int x^{-5} \, dx \\&= 2 \left(\frac{x^4}{4} \right) - 5 \left(\frac{x^2}{2} \right) + 3 \log |x| + 4 \left(\frac{x^{-4}}{-4} \right) + c \\&= \frac{x^4}{2} - \frac{5}{2}x^2 + 3 \log |x| - \frac{1}{x^4} + c\end{aligned}$$

(v) $\frac{3x^3 - 2x + 5}{x\sqrt{x}}$

Solution:

$$\begin{aligned} & \int \frac{3x^3 - 2x + 5}{x\sqrt{x}} dx \\ &= \int x^{-\frac{3}{2}}(3x^3 - 2x + 5) dx \\ &= \int \left(3x^{\frac{3}{2}} - 2x^{-\frac{1}{2}} + 5x^{-\frac{3}{2}} \right) dx \\ &= 3 \int x^{\frac{3}{2}} dx - 2 \int x^{-\frac{1}{2}} dx + 5 \int x^{-\frac{3}{2}} dx \\ &= 3 \left(\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} \right) - 2 \left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right) + 5 \left(\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} \right) + c \\ &= \frac{6}{5} x^2 \sqrt{x} - 4 \sqrt{x} - \frac{10}{\sqrt{x}} + c. \end{aligned}$$

II. Evaluate:

(i) $\int \tan^2 x \cdot dx$

Solution:

$$\int \tan^2 x dx = \int (\sec^2 x - 1) dx$$

$$\begin{aligned} &= \int \sec^2 x \, dx - \int 1 \, dx \\ &= \tan x - x + c. \end{aligned}$$

(ii) $\int \frac{\sin 2x}{\cos x} \cdot dx$

Solution:

$$\begin{aligned} \int \frac{\sin 2x}{\cos x} dx &= \int \frac{2 \sin x \cos x}{\cos x} dx \\ &= 2 \int \sin x \, dx \\ &= -2 \cos x + c. \end{aligned}$$

(iii) $\int \frac{\sin x}{\cos^2 x} \cdot dx$

Solution:

$$\begin{aligned} \int \frac{\sin x}{\cos^2 x} dx &= \int \left(\frac{1}{\cos x} \right) \left(\frac{\sin x}{\cos x} \right) dx \\ &= \int \sec x \tan x \, dx \\ &= \sec x + c. \end{aligned}$$

(iv) $\int \frac{\cos 2x}{\sin^2 x} \cdot dx$

Solution:

$$\begin{aligned}\int \frac{\cos 2x}{\sin^2 x} dx &= \int \frac{(1 - 2 \sin^2 x)}{\sin^2 x} dx \\&= \int \left(\frac{1}{\sin^2 x} - \frac{2 \sin^2 x}{\sin^2 x} \right) dx \\&= \int \operatorname{cosec}^2 x \, dx - 2 \int dx \\&= -\cot x - 2x + c.\end{aligned}$$

(v) $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} \cdot dx$

Solution:

$$\begin{aligned}&\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx \\&= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cdot \cos^2 x} dx \\&= \int \left(\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} \right) dx \\&= \int \operatorname{cosec}^2 x \, dx - \int \sec^2 x \, dx \\&= -\cot x - \tan x + c\end{aligned}$$

(vi) $\int \frac{\sin x}{1 + \sin x} \cdot dx$

Solution:

$$\begin{aligned} & \int \frac{\sin x}{1 + \sin x} dx \\ &= \int \frac{\sin x}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} dx \\ &= \int \frac{\sin x - \sin^2 x}{1 - \sin^2 x} dx \\ &= \int \frac{\sin x - \sin^2 x}{\cos^2 x} dx \\ &= \int \left(\frac{\sin x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} \right) dx \\ &= \int \left(\frac{1}{\cos x} \right) \left(\frac{\sin x}{\cos x} \right) dx - \int \tan^2 x dx \\ &= \int \sec x \tan x dx - \int (\sec^2 x - 1) dx \\ &= \int \sec x \tan x dx - \int \sec^2 x dx + \int 1 dx \\ &= \sec x - \tan x + x + c. \end{aligned}$$

(vii) $\int \frac{\tan x}{\sec x + \tan x} \cdot dx$

Solution:

$$\begin{aligned} & \int \frac{\tan x}{\sec x + \tan x} dx \\ &= \int \frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x} dx \\ &= \int \frac{\sec x \tan x - \tan^2 x}{\sec^2 x - \tan^2 x} dx \\ &= \int \frac{\sec x \tan x - (\sec^2 x - 1)}{1} dx \\ &= \int \sec x \tan x dx - \int \sec^2 x dx + \int 1 dx \\ &= \sec x - \tan x + x + c. \end{aligned}$$

(viii) $\int \sqrt{1 + \sin 2x} \cdot dx$

Solution:

$$\begin{aligned} & \int \sqrt{1 + \sin 2x} dx \\ &= \int \sqrt{\cos^2 x + \sin^2 x + 2 \sin x \cos x} dx \\ &= \int \sqrt{(\cos x + \sin x)^2} dx \\ &= \int (\cos x + \sin x) dx \end{aligned}$$

$$\begin{aligned} &= \int \cos x \, dx + \int \sin x \, dx \\ &= \sin x - \cos x + c. \end{aligned}$$

(ix) $\int \sqrt{1 - \cos 2x} \cdot dx$

Solution:

$$\begin{aligned} &\int \sqrt{1 - \cos 2x} \, dx \\ &= \int \sqrt{2 \sin^2 x} \, dx = \sqrt{2} \int \sin x \, dx \\ &= -\sqrt{2} \cos x + c. \end{aligned}$$

(x) $\int \sin 4x \cos 3x \, dx$

Solution:

$$\begin{aligned} &\int \sin 4x \cos 3x \, dx \\ &= \frac{1}{2} \int \sin 4x \cos 3x \, dx \\ &= \frac{1}{2} \int [\sin (4x + 3x) + \sin (4x - 3x)] \, dx \\ &= \frac{1}{2} \int \sin 7x \, dx + \frac{1}{2} \int \sin x \, dx \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \left(\frac{-\cos 7x}{7} \right) - \frac{1}{2} \cos x + c \\ &= -\frac{1}{14} \cos 7x - \frac{1}{2} \cos x + c. \end{aligned}$$

III. Evaluate:

(i) $\int \frac{x}{x+2} \cdot dx$

Solution:

$$\begin{aligned} &\int \frac{x}{x+2} dx \\ &= \int \frac{(x+2)-2}{x+2} dx \\ &= \int \left(\frac{x+2}{x+2} - \frac{2}{x+2} \right) dx \\ &= \int 1 dx - 2 \int \frac{1}{x+2} dx \\ &= x - 2 \log |x+2| + c. \end{aligned}$$

(ii) $\int \frac{4x+3}{2x+1} \cdot dx$

Solution:

$$\begin{aligned}& \int \frac{4x+3}{2x+1} dx \\&= \int \frac{2(2x+1)+1}{2x+1} dx \\&= \int \left(\frac{2(2x+1)}{2x+1} + \frac{1}{2x+1} \right) dx \\&= 2 \int 1 dx + \int \frac{1}{2x+1} dx \\&= 2x + \frac{1}{2} \log |2x+1| + c.\end{aligned}$$

(iii) $\int \frac{5x+2}{3x-4} \cdot dx$

Solution:

$$\begin{aligned}& \int \frac{5x+2}{3x-4} dx \\&= \int \frac{\frac{5}{3}(3x-4) + \frac{20}{3} + 2}{3x-4} dx \\&= \int \frac{\frac{5}{3}(3x-4) + \frac{26}{3}}{3x-4} dx\end{aligned}$$

$$\begin{aligned} &= \int \left[\frac{5}{3} + \frac{\left(\frac{26}{3}\right)}{3x-4} \right] dx \\ &= \frac{5}{3} \int 1 \, dx + \frac{26}{3} \int \frac{1}{3x-4} \, dx \\ &= \frac{5x}{3} + \frac{26}{3} \cdot \frac{1}{3} \log |3x-4| + c \\ &= \frac{5x}{3} + \frac{26}{9} \log |3x-4| + c. \end{aligned}$$

(iv) $\int \frac{x-2}{\sqrt{x+5}} \cdot dx$

Solution:

$$\begin{aligned} \int \frac{x-2}{\sqrt{x+5}} \, dx &= \int \frac{(x+5)-7}{\sqrt{x+5}} \, dx \\ &= \int \left(\frac{x+5}{\sqrt{x+5}} - \frac{7}{\sqrt{x+5}} \right) \, dx \\ &= \int (x+5)^{\frac{1}{2}} \, dx - 7 \int (x+5)^{-\frac{1}{2}} \, dx \end{aligned}$$

$$\begin{aligned} &= \frac{(x+5)^{\frac{3}{2}}}{(3/2)} - \frac{7(x+5)^{\frac{1}{2}}}{(1/2)} + c \\ &= \frac{2}{3}(x+5)^{\frac{3}{2}} - 14\sqrt{x+5} + c. \end{aligned}$$

(v) $\int \frac{2x-7}{\sqrt{4x-1}} \cdot dx$

Solution:

$$\begin{aligned} &\int \frac{2x-7}{\sqrt{4x-1}} dx \\ &= \frac{1}{2} \int \frac{2(2x-7)}{\sqrt{4x-1}} dx \\ &= \frac{1}{2} \int \frac{(4x-1)-13}{\sqrt{4x-1}} dx \\ &= \frac{1}{2} \int \left(\frac{4x-1}{\sqrt{4x-1}} - \frac{13}{\sqrt{4x-1}} \right) dx \\ &= \frac{1}{2} \int (4x-1)^{\frac{1}{2}} dx - \frac{13}{2} \int (4x-1)^{-\frac{1}{2}} dx \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \cdot \frac{(4x-1)^{\frac{3}{2}}}{(4)\left(\frac{3}{2}\right)} - \frac{13}{2} \cdot \frac{(4x-1)^{\frac{1}{2}}}{(4)\left(\frac{1}{2}\right)} + c \\ &= \frac{1}{12} (4x-1)^{\frac{3}{2}} - \frac{13}{4} \sqrt{4x-1} + c. \end{aligned}$$

(vi) $\int \frac{\sin 4x}{\cos 2x} \cdot dx$

Solution:

$$\begin{aligned} &\int \frac{\sin 4x}{\cos 2x} dx \\ &= \int \frac{2 \sin 2x \cos 2x}{\cos 2x} dx \\ &= 2 \int \sin 2x dx \\ &= 2 \left(-\frac{\cos 2x}{2} \right) + c \\ &= -\cos 2x + c. \end{aligned}$$

(vii) $\int \sqrt{1 + \sin 5x} \cdot dx$

Solution:

$$\begin{aligned}& \int \sqrt{1 + \sin 5x} \cdot dx \\&= \int \sqrt{\sin^2 x + \cos^2 x + 5 \sin x \cos x} \cdot dx \\&= \int \sqrt{(\sin x + \cos x)^2} \cdot dx \\&= \int (\sin x + \cos x) \cdot dx \\&= \int \sin x \, dx + \int \cos x \cdot dx \\&= \left(\frac{2}{5} \sin \frac{5x}{2} - \cos \frac{5x}{2} \right) + c.\end{aligned}$$

(viii) $\int \cos^2 x \cdot dx$

Solution:

Recall the identity $\cos 2x = 2 \cos^2 x - 1$, which gives

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\begin{aligned}\text{Therefore, } & \int \cos^2 x \cdot dx \\ &= \frac{1}{2} \int (1 + \cos 2x) \cdot dx \\ &= \frac{1}{2} \int dx + \frac{1}{2} \int \cos 2x \cdot dx \\ &= \frac{x}{2} + \frac{1}{4} \sin 2x + C.\end{aligned}$$

$$\text{(ix) } \int \frac{2}{\sqrt{x}-\sqrt{x+3}} \cdot dx$$

Solution:

$$\begin{aligned}& \int \frac{2}{\sqrt{x}-\sqrt{x+3}} dx \\ &= \int \frac{2}{\sqrt{x}-\sqrt{x+3}} \times \frac{\sqrt{x}+\sqrt{x+3}}{\sqrt{x}+\sqrt{x+3}} dx \\ &= \int \frac{2(\sqrt{x}+\sqrt{x+3})}{x-(x+3)} dx \\ &= -\frac{2}{3} \int (\sqrt{x}+\sqrt{x+3}) dx\end{aligned}$$

$$\begin{aligned} &= -\frac{2}{3} \int x^{\frac{1}{2}} dx - \frac{2}{3} \int (x+3)^{\frac{1}{2}} dx \\ &= -\frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{2}{3} \cdot \frac{(x+3)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c \\ &= -\frac{4}{9} \left[x^{\frac{3}{2}} + (x+3)^{\frac{3}{2}} \right] + c. \end{aligned}$$

(x) $\int \frac{3}{\sqrt{7x-2}-\sqrt{7x-5}} \cdot dx$

Solution:

$$\begin{aligned} &\int \frac{3}{\sqrt{7x-2}-\sqrt{7x-5}} dx \\ &= \int \frac{3}{\sqrt{7x-2}-\sqrt{7x-5}} \times \frac{\sqrt{7x-2}+\sqrt{7x-5}}{\sqrt{7x-2}+\sqrt{7x-5}} dx \\ &= \int \frac{3(\sqrt{7x-2}+\sqrt{7x-5})}{(7x-2)-(7x-5)} dx \\ &= \int (\sqrt{7x-2}+\sqrt{7x-5}) dx \\ &= \int (7x-2)^{\frac{1}{2}} dx + \int (7x-5)^{\frac{1}{2}} dx \end{aligned}$$

$$\begin{aligned} &= \int (7x-2)^{\frac{1}{2}} dx + \int (7x-5)^{\frac{1}{2}} dx \\ &= \frac{(7x-2)^{\frac{3}{2}}}{3/2} \times \frac{1}{7} + \frac{(7x-5)^{\frac{3}{2}}}{3/2} \times \frac{1}{7} + c \\ &= \frac{2}{21} (7x-2)^{\frac{3}{2}} + \frac{2}{21} (7x-5)^{\frac{3}{2}} + c. \end{aligned}$$

IV.

Question 1.

If $f'(x) = x - \frac{3}{x^3}$, $f(1) = \frac{11}{2}$, find $f(x)$.

Solution:

By the definition of integral,

$$\begin{aligned} f(x) &= \int f'(x) dx \\ &= \int \left(x - \frac{3}{x^3} \right) dx \\ &= \int x dx - 3 \int x^{-3} dx \\ &= \frac{x^2}{2} - \frac{3x^{(-2)}}{(-2)} + c \end{aligned}$$

$$\begin{aligned} &= \int x \, dx - 3 \int x^{-3} \, dx \\ &= \frac{x^2}{2} - \frac{3x^{(-2)}}{(-2)} + c \\ &= \frac{x^2}{2} + \frac{3}{2x^2} + c \quad \dots\dots\dots(1) \end{aligned}$$

$$f(1) = \frac{11}{2} \quad \dots\dots\dots(\text{Given})$$

$$\therefore \frac{1}{2} + \frac{3}{2} + c = \frac{11}{2}$$

$$\therefore c = \frac{7}{2}$$

$$\therefore f(x) = \frac{x^2}{2} + \frac{3}{2x^2} + \frac{7}{2} \quad \dots\dots [\text{By (1)}]$$

Ex 3.2(A)

I. Integrate the following functions w.r.t. x:

Question 1.

$$\frac{(\log x)^n}{x}$$

Solution:

$$\text{Let } I = \int \frac{(\log x)^n}{x} dx$$

$$\text{Put } \log x = t. \quad \therefore \frac{1}{x} dx = dt$$

$$\therefore I = \int t^n dt = \frac{t^{n+1}}{n+1} + c$$

$$= \frac{1}{n+1} \cdot (\log x)^{n+1} + c.$$

Question 2.

$$\frac{(\sin^{-1} x)^{\frac{3}{2}}}{\sqrt{1-x^2}}$$

Solution:

$$\text{Let } I = \int \frac{(\sin^{-1} x)^{\frac{3}{2}}}{\sqrt{1-x^2}} dx$$

$$\text{Put } \sin^{-1} x = t. \quad \therefore \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\therefore I = \int t^{\frac{3}{2}} dt = \frac{t^{\frac{5}{2}}}{5/2} + c$$

Question 3.

$$\frac{1+x}{x \cdot \sin(x+\log x)}$$

Solution:

$$\text{Let } I = \int \frac{1+x}{x \cdot \sin(x+\log x)} dx$$

$$= \int \frac{1}{\sin(x+\log x)} \cdot \left(\frac{1+x}{x}\right) dx$$

$$= \int \frac{1}{\sin(x+\log x)} \cdot \left(\frac{1}{x} + 1\right) dx$$

$$\text{Put } x + \log x = t \quad \therefore \left(1 + \frac{1}{x}\right) dx = dt$$

$$\therefore I = \int \frac{1}{\sin t} dt = \int \operatorname{cosec} t dt$$

$$= \log |\operatorname{cosec} t - \cot t| + c$$

$$= \log |\operatorname{cosec}(x+\log x) - \cot(x+\log x)| + c.$$

Question 4.

$$\frac{x \cdot \sec^2(x^2)}{\sqrt{\tan^3(x^2)}}$$

Solution:

$$\text{Let } I = \int \frac{x \cdot \sec^2(x^2)}{\sqrt{\tan^3(x^2)}} dx$$

$$\text{Put } \tan(x^2) = t \quad \therefore \sec^2(x^2) \times 2x dx = dt$$

$$\therefore x \cdot \sec^2(x^2) dx = \frac{dt}{2}$$

$$\begin{aligned}\therefore I &= \int \frac{1}{\sqrt{t^3}} \cdot \frac{dt}{2} = \frac{1}{2} \int t^{-\frac{3}{2}} dt \\ &= \frac{1}{2} \cdot \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + c = \frac{-1}{\sqrt{t}} + c = \frac{-1}{\sqrt{\tan(x^2)}} + c.\end{aligned}$$

Question 5.

$$\frac{e^{3x}}{e^{3x}+1}$$

Solution:

$$\text{Let } I = \int \frac{e^{3x}}{e^{3x}+1} dx$$

$$\text{Put } e^{3x} + 1 = t.$$

$$\therefore 3e^{3x} dx = dt$$

$$\therefore e^{3x} dx = \frac{dt}{3}$$

$$\therefore I = \int \frac{1}{t} \cdot \frac{dt}{3} = \frac{1}{3} \int \frac{1}{t} dt$$

$$= \frac{1}{3} \log |t| + c = \frac{1}{3} \log |e^{3x} + 1| + c.$$

Question 6.

$$\frac{(x^2+2)}{(x^2+1)} \cdot a^{x+\tan^{-1} x}$$

Solution:

$$\text{Let } I = \int \frac{x^2 + 2}{(x^2 + 1)} \cdot a^{x + \tan^{-1} x} dx$$

$$= \int a^{x + \tan^{-1} x} \cdot \left(\frac{x^2 + 2}{x^2 + 1} \right) dx$$

Put $x + \tan^{-1} x = t$

$$\therefore \left(1 + \frac{1}{1 + x^2} \right) dx = dt$$

$$\therefore \left(\frac{1 + x^2 + 1}{1 + x^2} \right) dx = dt$$

$$\therefore \left(\frac{x^2 + 2}{x^2 + 1} \right) dx = dt$$

$$\therefore I = \int a^t dt = \frac{a^t}{\log a} + c$$

$$= \frac{a^{x + \tan^{-1} x}}{\log a} + c.$$

Question 7.

$$\frac{e^x \cdot \log(\sin e^x)}{\tan(e^x)}$$

Solution:

$$\text{Let } I = \int \frac{e^x \cdot \log(\sin e^x)}{\tan(e^x)} dx$$

$$= \int \log(\sin e^x) \cdot e^x \cot(e^x) dx$$

$$\text{Put } \log(\sin e^x) = t \quad \therefore \frac{1}{\sin(e^x)} \cdot \cos(e^x) \cdot e^x dx = dt$$

$$\therefore e^x \cdot \cot(e^x) dx = dt$$

$$\therefore I = \int t dt = \frac{t^2}{2} + c$$

$$= \frac{1}{2} [\log(\sin e^x)]^2 + c.$$

Question 8.

$$\frac{e^{2x}+1}{e^{2x}-1}$$

Solution:

$$\text{Let } I = \int \frac{e^{2x} + 1}{e^{2x} - 1} dx = \int \left(\frac{\frac{e^{2x} + 1}{e^x}}{\left(\frac{e^{2x} - 1}{e^x} \right)} \right) dx$$

$$= \int \left(\frac{e^x + e^{-x}}{e^x - e^{-x}} \right) dx$$

$$= \int \frac{\frac{d}{dx}(e^x - e^{-x})}{e^x - e^{-x}} dx$$

$$= \log|e^x - e^{-x}| + c \quad \dots \left[\because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]$$

Question 9.

$$\sin^4 x \cdot \cos^3 x$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \sin^4 x \cdot \cos^3 x \, dx \\ &= \int \sin^4 x \cdot \cos^2 x \cdot \cos x \, dx \\ &= \int \sin^4 x (1 - \sin^2 x) \cos x \, dx \\ \text{Put } \sin x &= t \quad \therefore \cos x \, dx = dt \\ \therefore I &= \int t^4 (1 - t^2) \, dt = \int (t^4 - t^6) \, dt \\ &= \int t^4 \, dt - \int t^6 \, dt \\ &= \frac{t^5}{5} - \frac{t^7}{7} + c \\ &= \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + c.\end{aligned}$$

Question 10.

$$\frac{1}{4x + 5x^{-11}}$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \frac{1}{4x + 5x^{-11}} \, dx \\ &= \int \frac{x^{11}}{x^{11}(4x + 5x^{-11})} \, dx \\ &= \int \frac{x^{11}}{4x^{12} + 5} \, dx\end{aligned}$$

$$\begin{aligned} &= \int \frac{x^{11}}{4x^{12} + 5} dx \\ &= \frac{1}{48} \int \frac{48x^{11}}{4x^{12} + 5} dx \\ &= \frac{1}{48} \int \frac{\frac{d}{dx}(4x^{12} + 5)}{4x^{12} + 5} dx \\ &= \frac{1}{48} \log |4x^{12} + 5| + c \quad \dots \left[\because \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right] \end{aligned}$$

Question 11.

$$x^9 \cdot \sec^2(x^{10})$$

Solution:

$$\text{Let } I = \int x^9 \cdot \sec^2(x^{10}) dx$$

$$\text{Put } x^{10} = t \quad \therefore 10x^9 dx = dt \quad \therefore x^9 dx = \frac{1}{10} dt$$

$$\begin{aligned} \therefore I &= \int \sec^2 t \cdot \frac{dt}{10} \\ &= \frac{1}{10} \int \sec^2 t dt \\ &= \frac{1}{10} \tan t + c \\ &= \frac{1}{10} \tan(x^{10}) + c. \end{aligned}$$

Question 12.

$$e^{3 \log x} \cdot (x^4 + 1)^{-1}$$

Solution:

$$\text{Let } I = e^{3 \log x} (x^4 + 1)^{-1} dx$$

$$= \int \frac{e^{\log x^3}}{x^4 + 1} dx$$

$$= \int \frac{x^3}{x^4 + 1} dx \quad \dots [\because e^{\log N} = N]$$

$$= \frac{1}{4} \int \frac{4x^3}{x^4 + 1} dx$$

$$= \frac{1}{4} \int \frac{\frac{d}{dx}(x^4 + 1)}{x^4 + 1} dx$$

$$= \frac{1}{4} \log |x^4 + 1| + c. \dots \left[\because \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right]$$

Question 13.

$$\frac{\sqrt{\tan x}}{\sin x \cdot \cos x}$$

Solution:

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} dx$$

Dividing numerator and denominator by $\cos^2 x$, we get

$$\begin{aligned} I &= \int \frac{\left(\frac{\sqrt{\tan x}}{\cos^2 x}\right)}{\left(\frac{\sin x}{\cos x}\right)} dx \\ &= \int \frac{\sqrt{\tan x} \cdot \sec^2 x}{\tan x} dx \\ &= \int \frac{\sec^2 x}{\sqrt{\tan x}} dx \end{aligned}$$

Put $\tan x = t \quad \therefore \sec^2 x \, dx = dt$

$$\begin{aligned} \therefore I &= \int \frac{1}{\sqrt{t}} dt = \int t^{-\frac{1}{2}} dt \\ &= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = 2\sqrt{t} + c \\ &= 2\sqrt{\tan x} + c. \end{aligned}$$

Question 14.

$$\frac{(x-1)^2}{(x^2+1)^2}$$

Solution:

$$\begin{aligned} \text{Let } I &= \int \frac{(x-1)^2}{(x^2+1)^2} dx \\ &= \int \frac{x^2 - 2x + 1}{(x^2+1)^2} dx \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{(x^2 + 1) - 2x}{(x^2 + 1)^2} dx \\
 &= \int \left[\frac{x^2 + 1}{(x^2 + 1)^2} - \frac{2x}{(x^2 + 1)^2} \right] dx \\
 &= \int \frac{1}{x^2 + 1} dx - \int \frac{2x}{(x^2 + 1)^2} dx \\
 &= I_1 - I_2 \quad \dots \text{ (Let)}
 \end{aligned}$$

In I_2 , Put $x^2 + 1 = t \quad \therefore 2x dx = dt$

$$\begin{aligned}
 \therefore I &= \int \frac{1}{x^2 + 1} dx - \int t^{-2} dt \\
 &= \tan^{-1} x - \frac{t^{-1}}{(-1)} + c \\
 &= \tan^{-1} x + \frac{1}{x^2 + 1} + c.
 \end{aligned}$$

Question 15.

$$\frac{2 \sin x \cos x}{3 \cos^2 x + 4 \sin^2 x}$$

Solution:

$$\text{Let } I = \int \frac{2 \sin x \cos x}{3 \cos^2 x + 4 \sin^2 x} dx$$

$$\text{Put } 3 \cos^2 x + 4 \sin^2 x = t$$

$$\therefore \left[3(2 \cos x) \frac{d}{dx}(\cos x) + 4(2 \sin x) \frac{d}{dx}(\sin x) \right] dx = dt$$

$$\therefore [-6 \cos x \sin x + 8 \sin x \cos x] dx = dt$$

$$\therefore 2 \sin x \cos x dx = dt$$

$$I = \int \frac{dt}{t} = \log |t| + c$$

$$= \log |3 \cos^2 x + 4 \sin^2 x| + c.$$

Question 16.

$$\frac{1}{\sqrt{x} + \sqrt{x^3}}$$

Solution:

$$\text{Let } I = \int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx$$

$$= \int \frac{1}{x^{\frac{1}{2}} + x^{\frac{3}{2}}} dx$$

$$\text{Put } x = t^2 \quad \therefore dx = 2t dt$$

$$\text{Also } x^{\frac{1}{2}} = (t^2)^{\frac{1}{2}} = t \text{ and } x^{\frac{3}{2}} = (t^2)^{\frac{3}{2}} = t^3$$

$$\therefore I = \int \frac{2t dt}{t + t^3}$$

$$= 2 \int \frac{t dt}{t(1 + t^2)}$$

$$= 2 \int \frac{1}{1 + t^2} dt$$

$$= 2 \tan^{-1} t + c$$

$$= 2 \tan^{-1} (\sqrt{x}) + c.$$

Question 17.

$$\frac{10x^9 + 10^x \cdot \log 10}{10^x + x^{10}}$$

Solution:

$$\text{Let } I = \int \frac{10x^9 + 10^x \cdot \log 10}{10^x + x^{10}} dx$$

$$\text{Put } 10^x + x^{10} = t$$

$$\therefore (10^x \cdot \log 10 + 10x^9) dx = dt$$

$$\therefore I = \int \frac{1}{t} dt = \log |t| + c$$

$$= \log |10^x + x^{10}| + c.$$

Question 18.

$$\frac{x^{n-1}}{\sqrt{1+4x^n}}$$

Solution:

$$\text{Let } I = \int \frac{x^{n-1}}{\sqrt{1+4x^n}} dx$$

$$\text{Put } x^n = t \quad \therefore nx^{n-1} dx = dt$$

$$\therefore x^{n-1} dx = \frac{dt}{n}$$



$$\begin{aligned}
 \therefore I &= \int \frac{1}{\sqrt{1+4t}} \cdot \frac{dt}{n} = \frac{1}{n} \int (1+4t)^{-\frac{1}{2}} dt \\
 &= \frac{1}{n} \cdot \frac{(1+4t)^{\frac{1}{2}}}{1/2} \times \frac{1}{4} + c \\
 &= \frac{1}{2n} \cdot \sqrt{1+4x^n} + c.
 \end{aligned}$$

Question 19.

$$(2x+1)\sqrt{x+2}$$

Solution:

$$\text{Let } I = \int (2x+1)\sqrt{x+2} \, dx$$

$$\text{Put } x+2=t \quad \therefore dx=dt$$

$$\text{Also, } x=t-2$$

$$\therefore 2x+1=2(t-2)+1=2t-3$$

$$\therefore I = \int (2t-3)\sqrt{t} \, dt$$

$$= \int (2t^{\frac{3}{2}} - 3t^{\frac{1}{2}}) \, dt$$

$$= 2 \int t^{\frac{3}{2}} \, dt - 3 \int t^{\frac{1}{2}} \, dt$$

$$= 2 \cdot \frac{t^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} - 3 \cdot \frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c$$

$$= \frac{4}{5}(x+2)^{\frac{5}{2}} - 2(x+2)^{\frac{3}{2}} + c.$$

Question 20.

$$x^5 \sqrt{a^2 + x^2}$$

Solution:

$$\text{Let } I = \int x^5 \sqrt{a^2 + x^2} dx$$

$$\text{Put, } a^2 + x^2 = t$$

$$\therefore 2x dx = dt \quad \therefore x dx = \frac{1}{2} dt$$

$$\text{Also, } x^2 = t - a^2$$

$$I = \int x^2 \cdot x^2 \sqrt{a^2 + x^2} x dx$$

$$= \frac{1}{2} \int (t - a^2)^2 \sqrt{t} dt$$

$$= \frac{1}{2} \int (t^2 - 2a^2 t + a^4) \sqrt{t} dt$$

$$= \frac{1}{2} \int (t^{\frac{5}{2}} - 2a^2 t^{\frac{3}{2}} + a^4 t^{\frac{1}{2}}) dt$$

$$= \frac{1}{2} \int t^{\frac{5}{2}} dt - a^2 \int t^{\frac{3}{2}} dt + \frac{a^4}{2} \int t^{\frac{1}{2}} dt$$

$$= \frac{1}{2} \cdot \frac{t^{\frac{7}{2}}}{\left(\frac{7}{2}\right)} - a^2 \cdot \frac{t^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + \frac{a^4}{2} \cdot \frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c$$

$$= \frac{1}{7} (a^2 + x^2)^{\frac{7}{2}} - \frac{2a^2}{5} (a^2 + x^2)^{\frac{5}{2}} + \frac{a^4}{3} (a^2 + x^2)^{\frac{3}{2}} + c.$$

Question 21.

$$(5 - 3x)(2 - 3x)^{-\frac{1}{2}}$$

Solution:

$$\text{Let } I = \int (5 - 3x)(2 - 3x)^{-\frac{1}{2}} dx$$

$$\text{Put } 2 - 3x = t$$

$$\therefore -3dx = dt$$

$$\therefore dx = \frac{-dt}{3}$$

$$\text{Also, } x = \frac{2-t}{3}$$

$$\therefore I = \int \left[5 - 3\left(\frac{2-t}{3}\right) \right] t^{-\frac{1}{2}} \cdot \left(\frac{-dt}{3}\right)$$

$$= -\frac{1}{3} \int (5 - 2 + t) t^{-\frac{1}{2}} dt$$

$$= -\frac{1}{3} \int (3 + t) t^{-\frac{1}{2}} dt$$

$$= -\frac{1}{3} \int (3t^{-\frac{1}{2}} + t^{\frac{1}{2}}) dt$$

$$= -\frac{3}{3} \int t^{-\frac{1}{2}} dt - \frac{1}{3} \int t^{\frac{1}{2}} dt$$

$$= -\frac{t^{\frac{1}{2}}}{(1/2)} - \frac{1}{3} \cdot \frac{t^{\frac{3}{2}}}{(3/2)} + c$$

$$= -2\sqrt{2-3x} - \frac{2}{9}(2-3x)^{\frac{3}{2}} + c.$$

Question 22.

$$\frac{7+4x+5x^2}{(2x+3)^{\frac{3}{2}}}$$

Solution:

$$\text{Let } I = \int \frac{7+4x+5x^2}{(2x+3)^{\frac{3}{2}}} dx = \int \frac{5x^2+4x+7}{(2x+3)^{\frac{3}{2}}} dx$$

$$\text{Put } 2x+3=t \quad \therefore 2dx=dt$$

$$\therefore dx = \frac{dt}{2}$$

$$\text{Also, } x = \frac{t-3}{2}$$

$$\begin{aligned} \therefore I &= \int \frac{5\left(\frac{t-3}{2}\right)^2 + 4\left(\frac{t-3}{2}\right) + 7}{t^{\frac{3}{2}}} \cdot \frac{dt}{2} \\ &= \frac{1}{2} \int \frac{5\left(\frac{t^2-6t+9}{4}\right) + 2(t-3) + 7}{t^{\frac{3}{2}}} dt \\ &= \frac{1}{2} \int \frac{5t^2 - 30t + 45 + 8t - 24 + 28}{4t^{\frac{3}{2}}} dt \\ &= \frac{1}{8} \int \frac{5t^2 - 22t + 49}{t^{\frac{3}{2}}} dt \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{8} \int (5t^{\frac{1}{2}} - 22t^{-\frac{1}{2}} + 49t^{-\frac{3}{2}}) dt \\
 &= \frac{5}{8} \int t^{\frac{1}{2}} dt - \frac{22}{8} \int t^{-\frac{1}{2}} dt + \frac{49}{8} \int t^{-\frac{3}{2}} dt \\
 &= \frac{5}{8} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{11}{4} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + \frac{49}{8} \cdot \frac{t^{-\frac{1}{2}}}{(-\frac{1}{2})} + c \\
 &= \frac{5}{12} (2x+3)^{\frac{3}{2}} - \frac{11}{2} \sqrt{2x+3} - \frac{49}{4} \cdot \frac{1}{\sqrt{2x+3}} + c.
 \end{aligned}$$

Question 23.

$$\frac{x^2}{\sqrt{9-x^6}}$$

Solution:

$$\text{Let } I = \int \frac{x^2}{\sqrt{9-x^6}} dx$$

$$\text{Put } x^3 = t \quad \therefore 3x^2 dx = dt \quad \therefore x^2 dx = \frac{1}{3} dt$$

$$\therefore I = \int \frac{1}{\sqrt{9-t^2}} \cdot \frac{dt}{3}$$

$$= \frac{1}{3} \int \frac{dt}{\sqrt{3^2-t^2}}$$

$$= \frac{1}{3} \sin^{-1} \left(\frac{t}{3} \right) + c$$

$$= \frac{1}{3} \sin^{-1} \left(\frac{x^3}{3} \right) + c.$$

Question 24.

$$\frac{1}{x(x^3-1)}$$

Solution:

$$\text{Let } I = \int \frac{1}{x(x^3-1)} dx$$

$$= \int \frac{x^{-4}}{x^{-4}x(x^3-1)} dx$$

$$= \int \frac{x^{-4}}{1-x^{-3}} dx$$

$$= \frac{1}{3} \int \frac{3x^{-4}}{1-x^{-3}} dx$$

$$= \frac{1}{3} \int \frac{\frac{d}{dx}(1-x^{-3})}{1-x^{-3}} dx$$

$$= \frac{1}{3} \log |1-x^{-3}| + c$$

$$\dots \left[\because \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right]$$

$$= \frac{1}{3} \log \left| 1 - \frac{1}{x^3} \right| + c$$

$$= \frac{1}{3} \log \left| \frac{x^3 - 1}{x^3} \right| + c.$$

Alternative Method :

$$\text{Let } I = \int \frac{1}{x(x^3 - 1)} dx$$

$$= \int \frac{x^2}{x^3(x^3 - 1)} dx$$

$$\text{Put } x^3 = t \quad \therefore 3x^2 dx = dt$$

$$\therefore x^2 dx = \frac{dt}{3}$$

$$\therefore I = \int \frac{1}{t(t-1)} \cdot \frac{dt}{3} = \frac{1}{3} \int \frac{1}{t(t-1)} dt$$

$$= \frac{1}{3} \int \frac{t - (t-1)}{t(t-1)} dt = \frac{1}{3} \int \left(\frac{1}{t-1} - \frac{1}{t} \right) dt$$

$$= \frac{1}{3} \left[\int \frac{1}{t-1} dt - \int \frac{1}{t} dt \right]$$

$$= \frac{1}{3} [\log |t-1| - \log |t|] + c$$

$$= \frac{1}{3} \log \left| \frac{t-1}{t} \right| + c$$

$$= \frac{1}{3} \log \left| \frac{x^3 - 1}{x^3} \right| + c.$$

Question 25.

$$\frac{1}{x \cdot \log x \cdot \log(\log x)}$$

Solution:

$$\text{Let } I = \int \frac{1}{x \cdot \log x \cdot \log(\log x)} dx$$

$$= \int \frac{1}{\log(\log x)} \cdot \frac{1}{x \cdot \log x} dx$$

$$\text{Put } \log(\log x) = t \quad \therefore \frac{1}{\log x} \cdot \frac{1}{x} dx = dt$$

$$\therefore \frac{1}{x \cdot \log x} dx = dt$$

$$\therefore I = \int \frac{1}{t} dt = \log |t| + c$$

$$= \log |\log(\log x)| + c.$$

II. Integrate the following functions w.r.t x:

Question 1.

$$\frac{\cos 3x - \cos 4x}{\sin 3x + \sin 4x}$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \frac{\cos 3x - \cos 4x}{\sin 3x + \sin 4x} dx \\&= \int \frac{-2 \sin\left(\frac{3x+4x}{2}\right) \sin\left(\frac{3x-4x}{2}\right)}{2 \sin\left(\frac{3x+4x}{2}\right) \cos\left(\frac{3x-4x}{2}\right)} dx \\&= \int -\frac{\sin\left(-\frac{x}{2}\right)}{\cos\left(-\frac{x}{2}\right)} dx = \int \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} dx \\&= \int \tan\left(\frac{x}{2}\right) dx \\&= \frac{\log \left| \sec\left(\frac{x}{2}\right) \right|}{\left(\frac{1}{2}\right)} + c \\&= 2 \log \left| \sec\left(\frac{x}{2}\right) \right| + c.\end{aligned}$$

Question 2.

$$\frac{\cos x}{\sin(x-a)}$$

Solution:

$$\text{Let } I = \int \frac{\cos x}{\sin(x-a)} dx$$

$$\begin{aligned} &= \int \frac{\cos[(x-a)+a]}{\sin(x-a)} dx \\ &= \int \frac{\cos(x-a)\cos a - \sin(x-a)\sin a}{\sin(x-a)} dx \\ &= \int \left[\frac{\cos(x-a)\cos a}{\sin(x-a)} - \frac{\sin(x-a)\sin a}{\sin(x-a)} \right] dx \\ &= \cos a \int \cot(x-a) dx - \sin a \int 1 dx \\ &= \cos a \log |\sin(x-a)| - x \sin a + c. \end{aligned}$$

Question 3.

$$\frac{\sin(x-a)}{\cos(x+b)}$$

Solution:

$$\begin{aligned} \text{Let } I &= \int \frac{\sin(x-a)}{\cos(x+b)} dx \\ &= \int \frac{\sin[(x+b)-(a+b)]}{\cos(x+b)} dx \\ &= \int \frac{\sin(x+b)\cos(a+b) - \cos(x+b)\sin(a+b)}{\cos(x+b)} dx \\ &= \int \left[\frac{\sin(x+b)\cos(a+b)}{\cos(x+b)} - \frac{\cos(x+b)\sin(a+b)}{\cos(x+b)} \right] dx \\ &= \cos(a+b) \int \tan(x+b) dx - \sin(a+b) \int 1 dx \end{aligned}$$

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-12-arts-science-maths-p-art-2-chapter-3-indefinite-integration/>

$$= \cos(a+b) \log |\sec(x+b)| - x \sin(a+b) + c.$$

Question 4.

$$\frac{1}{\sin x \cdot \cos x + 2 \cos^2 x}$$

Solution:

$$\text{Let } I = \int \frac{1}{\sin x \cdot \cos x + 2 \cos^2 x} dx$$

Dividing numerator and denominator of $\cos^2 x$, we get

$$I = \int \frac{\left(\frac{1}{\cos^2 x} \right)}{\frac{\sin x}{\cos x} + 2} dx$$

$$= \int \frac{\sec^2 x}{\tan x + 2} dx$$

$$\text{Put } \tan x = t \quad \therefore \sec^2 x dx = dt$$

$$\therefore I = \int \frac{1}{t+2} dt = \log |t+2| + c$$

$$= \log |\tan x + 2| + c.$$

Question 5.

$$\frac{\sin x + 2 \cos x}{3 \sin x + 4 \cos x}$$

Solution:

$$\text{Let } I = \int \frac{\sin x + 2 \cos x}{3 \sin x + 4 \cos x} dx$$

$$\text{Put, Numerator} = A (\text{Denominator}) + B \left[\frac{d}{dx} (\text{Denominator}) \right]$$

$$\therefore \sin x + 2 \cos x = A(3 \sin x + 4 \cos x) + B \left[\frac{d}{dx} (3 \sin x + 4 \cos x) \right]$$

$$\therefore \sin x + 2 \cos x = A(3 \sin x + 4 \cos x) + B \left[\frac{d}{dx} (3 \sin x + 4 \cos x) \right]$$

$$= A(3 \sin x + 4 \cos x) + B(3 \cos x - 4 \sin x)$$

$$\therefore \sin x + 2 \cos x = (3A - 4B) \sin x + (4A + 3B) \cos x$$

Equating the coefficients of $\sin x$ and $\cos x$ on both the sides, we get

$$3A - 4B = 1 \dots\dots (1)$$

$$\text{and } 4A + 3B = 2 \dots\dots (2)$$

Multiplying equation (1) by 3 and equation (2) by 4, we get

$$9A - 12B = 3$$

$$16A + 12B = 8$$

On adding, we get

$$25A = 11 \quad \therefore A = \frac{11}{25}$$

$$\therefore \text{from (2), } 4\left(\frac{11}{25}\right) + 3B = 2$$

$$\therefore 3B = 2 - \frac{44}{25} = \frac{6}{25} \quad \therefore B = \frac{2}{25}$$

$$\therefore \sin x + 2 \cos x = \frac{11}{25}(3 \sin x + 4 \cos x) + \frac{2}{25}(3 \cos x - 4 \sin x)$$

$$\therefore I = \int \left[\frac{\frac{11}{25}(3 \sin x + 4 \cos x) + \frac{2}{25}(3 \cos x - 4 \sin x)}{3 \sin x + 4 \cos x} \right] dx$$

$$= \int \left[\frac{11}{25} + \frac{\frac{2}{25}(3 \cos x - 4 \sin x)}{(3 \sin x + 4 \cos x)} \right] dx$$

$$= \frac{11}{25} \int 1 dx + \frac{2}{25} \int \frac{3 \cos x - 4 \sin x}{3 \sin x + 4 \cos x} dx$$

$$= \frac{11}{25}x + \frac{2}{25} \log |3 \sin x + 4 \cos x| + c$$

$$\dots \left[\because \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right]$$

Question 6.

$$\frac{1}{2+3 \tan x}$$

Solution:

$$\text{Let } I = \int \frac{1}{2+3 \tan x} dx$$

$$= \int \frac{1}{2+3 \left(\frac{\sin x}{\cos x} \right)} dx$$

$$= \int \frac{\cos x}{2 \cos x + 3 \sin x} dx$$

$$\text{Numerator} = A (\text{Denominator}) + B \left[\frac{d}{dx} (\text{Denominator}) \right]$$

$$\therefore \cos x = A(2 \cos x + 3 \sin x) + B \left[\frac{d}{dx} (2 \cos x + 3 \sin x) \right]$$

$$= A(2 \cos x + 3 \sin x) + B(-2 \sin x + 3 \cos x)$$

$$\therefore \cos x = (2A + 3B) \cos x + (3A - 2B) \sin x$$

Equating the coefficients of $\cos x$ and $\sin x$ on both the sides, we get

$$2A + 3B = 1 \dots\dots (1)$$

$$\text{and } 3A - 2B = 0 \dots\dots (2)$$

Multiplying equation (1) by 2 and equation (2) by 3, we get

$$4A + 6B = 2$$

$$9A - 6B = 0$$

On adding, we get

$$13A = 2 \quad \therefore A = \frac{2}{13}$$

$$\therefore \text{from (2), } 2B = 3A = 3\left(\frac{2}{13}\right) = \frac{6}{13}$$

$$\therefore B = \frac{3}{13}$$

$$\therefore \cos x = \frac{2}{13}(2 \cos x + 3 \sin x) + \frac{3}{13}(-2 \sin x + 3 \cos x)$$

$$\therefore I = \int \left[\frac{\frac{2}{13}(2 \cos x + 3 \sin x) + \frac{3}{13}(-2 \sin x + 3 \cos x)}{2 \cos x + 3 \sin x} \right] dx$$

$$= \int \left[\frac{2}{13} + \frac{\frac{3}{13}(-2 \sin x + 3 \cos x)}{2 \cos x + 3 \sin x} \right] dx$$

$$= \frac{2}{13} \int 1 dx + \frac{3}{13} \int \frac{-2 \sin x + 3 \cos x}{2 \cos x + 3 \sin x} dx$$

$$= \frac{2}{13}x + \frac{3}{13} \log |2 \cos x + 3 \sin x| + c$$

$$\dots \left[\because \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right]$$

Question 7.

$$\frac{4e^x - 25}{2e^x - 5}$$

Solution:

$$\text{Let } I = \int \frac{4e^x - 25}{2e^x - 5} dx$$

Put, Numerator = A (Denominator) + B $\left[\frac{d}{dx} (\text{Denominator})\right]$

$$\therefore 4e^x - 25 = A(2e^x - 5) + B\left[\frac{d}{dx} (2e^x - 5)\right]$$

$$= A(2e^x - 5) + B(2e^x - 0)$$

$$\therefore 4e^x - 25 = (2A + 2B)e^x - 5A$$

Equating the coefficient of e^x and constant on both sides, we get

$$2A + 2B = 4 \dots\dots\dots(1)$$

$$\text{and } 5A = 25$$

$$\therefore A = 5$$

$$\text{from (1), } 2(5) + 2B = 4$$

$$\therefore 2B = -6$$

$$\therefore B = -3$$

$$\therefore 4e^x - 25 = 5(2e^x - 5) - 3(2e^x)$$

$$\therefore I = \int \left[\frac{5(2e^x - 5) - 3(2e^x)}{2e^x - 5} \right] dx$$

$$= \int \left[5 - \frac{3(2e^x)}{2e^x - 5} \right] dx$$

$$= 5 \int 1 dx - 3 \int \frac{2e^x}{2e^x - 5} dx$$

$$= 5x - 3 \log |2e^x - 5| + c$$

$$\dots \left[\because \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right]$$

Question 8.

$$\frac{20+12e^x}{3e^x+4}$$

Solution:

$$\text{Let } I = \int \frac{20 + 12e^x}{3e^x + 4} \cdot dx$$

Put,

$$\text{Numerator} = A (\text{Denominator}) + B \left[\frac{d}{dx} (\text{Denominator}) \right]$$

$$\therefore 20 + 12e^x = A(3e^x + 4) + B \left[\frac{d}{dx} (3e^x + 4) \right]$$

$$= A(3e^x + 4) + B(3e^x + 0)$$

$$\therefore 20 + 12e^x = (2A + 2B)e^x - 5A$$

Equating the coefficient of e^x and constant on both sides, we get

$$2A + 2B = 4 \quad \dots(1)$$

and

$$5A = 20$$

$$\therefore A = 4$$

$$\therefore \text{from (1), } 2(4) + 2B = 4$$

$$\therefore 2B = -6$$

$$\therefore B = -3$$

$$\therefore 20 + 12e^x = 4(3e^x + 4) - 3(3e^x)$$

$$\begin{aligned}
 \therefore I &= \int \left[\frac{5(3e^x + 5) - 3(3e^x)}{3e^x + 4} \right] \cdot dx \\
 &= \int \left[5 - \frac{3(3e^x)}{3e^x + 4} \right] \cdot dx \\
 &= 5 \int 1 dx - 3 \int \frac{3e^x}{3e^x + 4} \cdot dx \\
 &= 5x - \log|3e^x + 4| + c.
 \end{aligned}$$

Question 9.

$$\frac{3e^{2x}+5}{4e^{2x}-5}$$

Solution:

$$\text{Let } I = \int \frac{3e^{2x}+5}{4e^{2x}-5} dx$$

Put, Numerator = A (Denominator) + B $\left[\frac{d}{dx} (\text{Denominator}) \right]$

$$\therefore 3e^{2x} + 5 = A(4e^{2x} - 5) + B \left[\frac{d}{dx} (4e^{2x} - 5) \right]$$

$$= A(4e^{2x} - 5) + B(4 \cdot e^{2x} \times 2 - 0)$$

$$\therefore 3e^{2x} + 5 = (4A + 8B) e^{2x} - 5A$$

Equating the coefficient of e^{2x} and constant on both sides, we get

$$4A + 8B = 3 \dots\dots\dots (1)$$

$$\text{and } -5A = 5$$

$$\therefore A = -1$$

$$\therefore \text{from (1), } 4(-1) + 8B = 3$$

$$\therefore 8B = 7$$

$$\therefore B = \frac{7}{8}$$

$$\therefore 3e^{2x} + 5 = -(4e^{2x} - 5) + \frac{7}{8}(8e^{2x})$$

$$\therefore I = \int \left[\frac{-(4e^{2x} - 5) + \frac{7}{8}(8e^{2x})}{4e^{2x} - 5} \right] dx$$

$$= \int \left[-1 + \frac{\frac{7}{8}(8e^{2x})}{4e^{2x} - 5} \right] dx$$

$$= -\int 1 dx + \frac{7}{8} \int \frac{8e^{2x}}{4e^{2x} - 5} dx$$

$$= -x + \frac{7}{8} \log |4e^{2x} - 5| + c$$

$$\dots \left[\because \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right]$$

Question 10.

$\cos^8 x \cdot \cot x$

Solution:

$$\text{Let } I = \int \cos^8 x \cot x dx$$

$$= \int \cos^8 x \cdot \frac{\cos x}{\sin x} dx$$

$$\text{Put } \sin x = t \quad \therefore \cos x dx = dt$$

$$\cos^8 x = (\cos^2 x)^4 = (1 - \sin^2 x)^4$$

$$= (1 - t^2)^4 = 1 - 4t^2 + 6t^4 - 4t^6 + t^8$$

$$I = \int \frac{1 - 4t^2 + 6t^4 - 4t^6 + t^8}{t} dt$$

$$\begin{aligned}
 &= \int \left[\frac{1}{t} - 4t + 6t^3 - 4t^5 + t^7 \right] dt \\
 &= \int \frac{1}{t} dx - 4 \int t dt + 6 \int t^3 dt - 4 \int t^5 dt + \int t^7 dt \\
 &= \log |t| - 4 \left(\frac{t^2}{2} \right) + 6 \left(\frac{t^4}{4} \right) - 4 \left(\frac{t^6}{6} \right) + \frac{t^8}{8} + c \\
 &= \log |\sin x| - 2 \sin^2 x + \frac{3}{2} \sin^4 x - \frac{2}{3} \sin^6 x + \frac{\sin^8 x}{8} + c.
 \end{aligned}$$

Question 11.

$\tan^5 x$

Solution:

Let $I = \int \tan^5 x dx$

$$\begin{aligned}
 &= \int \tan^3 x \tan^2 x dx \\
 &= \int \tan^3 x (\sec^2 x - 1) dx \\
 &= \int (\tan^3 x \sec^2 x - \tan^3 x) dx \\
 &= \int (\tan^3 x \sec^2 x - \tan x \cdot \tan^2 x) dx \\
 &= \int [\tan^3 x \sec^2 x - \tan x (\sec^2 x - 1)] dx \\
 &= \int (\tan^3 x \sec^2 x - \tan x \sec^2 x + \tan x) dx \\
 &= \int [(\tan^3 x - \tan x) \sec^2 x + \tan x] dx \\
 &= \int (\tan^3 x - \tan x) \sec^2 x dx + \int \tan x dx \\
 &= I_1 + I_2 \quad \dots \text{ (Let)}
 \end{aligned}$$

In I_1 , put $\tan x = t \quad \therefore \sec^2 x dx = dt$

$$\begin{aligned}\therefore I &= \int (t^3 - t) dt + \int \tan x dx \\ &= \frac{t^4}{4} - \frac{t^2}{2} + \log |\sec x| + c \\ &= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log |\sec x| + c.\end{aligned}$$

Question 12.

$$\cos^7 x$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \cos^7 x dx \\ &= \int \cos^6 x \cdot \cos x dx \\ &= \int (1 - \sin^2 x)^3 \cos x dx \\ \text{Put, } \sin x &= t \quad \therefore \cos x dx = dt \\ I &= \int (1 - t^2)^3 dt \\ &= \int (1 - 3t^2 + 3t^4 - t^6) dt \\ &= \int 1 dt - 3 \int t^2 dt + 3 \int t^4 dt - \int t^6 dt \\ &= t - 3 \left(\frac{t^3}{3} \right) + 3 \left(\frac{t^5}{5} \right) - \frac{t^7}{7} + c \\ &= \sin x - \sin^3 x + \frac{3}{5} \sin^5 x - \frac{1}{7} \sin^7 x + c.\end{aligned}$$

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-12-arts-science-maths-part-2-chapter-3-indefinite-integration/>

Question 13.

$$\tan 3x \tan 2x \tan x$$

Solution:

$$\text{Let } I = \int \tan 3x \tan 2x \tan x \, dx$$

$$\text{Consider } \tan 3x = \tan (2x + x) = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$$\tan 3x (1 - \tan 2x \tan x) = \tan 2x + \tan x$$

$$\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$$

$$\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$$

$$I = \int (\tan 3x - \tan 2x - \tan x) \, dx$$

$$= \int \tan 3x \, dx - \int \tan 2x \, dx - \int \tan x \, dx$$

$$= \frac{1}{3} \log |\sec 3x| - \frac{1}{2} \log |\sec 2x| - \log |\sec x| + c.$$

Question 14.

$$\sin^5 x \cos^8 x$$

Solution:

$$\text{Let } I = \int \sin^5 x \cos^8 x \, dx$$

$$= \int \sin^4 x \cos^8 x \sin x \, dx$$

$$= \int (1 - \cos^2 x)^2 \cos^8 x \sin x \, dx$$

$$\text{Put } \cos x = t \quad \therefore -\sin x \, dx = dt$$

$$\therefore \sin x \, dx = -dt$$

$$I = - \int (1 - t^2)^2 t^8 \, dt$$

$$= - \int (1 - 2t^2 + t^4) t^8 \, dt$$

$$= - \int (t^8 - 2t^{10} + t^{12}) \, dt$$

$$= - \int t^8 \, dt + 2 \int t^{10} \, dt - \int t^{12} \, dt$$

$$= -\frac{t^9}{9} + 2\left(\frac{t^{11}}{11}\right) - \frac{t^{13}}{13} + c$$

$$= -\frac{1}{9}\cos^9 x + \frac{2}{11}\cos^{11} x - \frac{1}{13}\cos^{13} x + c.$$

Question 15.

$$3^{\cos^2 x} \cdot \sin 2x$$

Solution:

$$\text{Let } I = \int 3^{\cos^2 x} \sin 2x \, dx$$

$$\text{Put } \cos^2 x = t$$

$$\therefore [2 \cos x \frac{d}{dx}(\cos x)] \, dx = dt$$

$$\therefore -2 \sin x \cos x \, dx = dt$$

$$\therefore \sin 2x \, dx = -dt$$

$$I = -\int 3^t \, dt = -\frac{1}{\log 3} \cdot 3^t + c$$

$$= -\frac{1}{\log 3} \cdot 3^{\cos^2 x} + c.$$

Question 16.

$$\frac{\sin 6x}{\sin 10x \sin 4x}$$

Solution:

$$\text{Let } I = \int \frac{\sin 6x}{\sin 10x \sin 4x} \, dx = \int \frac{\sin (10x - 4x)}{\sin 10x \sin 4x} \, dx$$

$$= \int \frac{\sin 10x \cos 4x - \cos 10x \sin 4x}{\sin 10x \sin 4x} \, dx$$

$$\begin{aligned} &= \int \left[\frac{\sin 10x \cos 4x}{\sin 10x \sin 4x} - \frac{\cos 10x \sin 4x}{\sin 10x \sin 4x} \right] dx \\ &= \int \cot 4x \, dx - \int \cot 10x \, dx \\ &= \frac{1}{4} \log |\sin 4x| - \frac{1}{10} \log |\sin 10x| + c. \end{aligned}$$

Question 17.

$$\frac{\sin x \cos^3 x}{1 + \cos^2 x}$$

Solution:

$$\text{Let } I = \int \frac{\sin x \cos^3 x}{1 + \cos^2 x} \, dx$$

Put $\cos x = t$

$$\therefore -\sin x \, dx = dt \quad \therefore \sin x \, dx = -dt$$

$$\begin{aligned} I &= - \int \frac{t^3}{t^2 + 1} \, dt \\ &= - \int \frac{t(t^2 + 1) - t}{t^2 + 1} \, dt \\ &= - \int \left[\frac{t(t^2 + 1)}{t^2 + 1} - \frac{t}{t^2 + 1} \right] \, dt \\ &= - \int t \, dt + \int \frac{t}{t^2 + 1} \, dt \end{aligned}$$

$$\begin{aligned} &= -\int t \, dt + \frac{1}{2} \int \frac{2t}{t^2+1} \, dt \\ &= -\frac{t^2}{2} + \frac{1}{2} \log|t^2+1| + c \\ \dots &\left[\because \frac{d}{dt}(t^2+1) = 2t \text{ and } \int \frac{f'(x)}{f(x)} \, dx = \log[f(x)] + c \right] \\ &= -\frac{1}{2} \cos^2 x + \frac{1}{2} \log|\cos^2 x + 1| + c \\ &= \frac{1}{2} [\log|\cos^2 x + 1| - \cos^2 x] + c. \end{aligned}$$

Ex 3.2(B)

I. Evaluate the following:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-12-arts-science-maths-p-art-2-chapter-3-indefinite-integration/>

Question 1.

$$\int \frac{1}{4x^2-3} \cdot dx$$

Solution:

$$I = \int \frac{1}{4x^2-3} dx = \frac{1}{4} \int \frac{1}{x^2-\frac{3}{4}} dx$$

$$= \frac{1}{4} \int \frac{1}{x^2 - \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

$$= \frac{1}{4} \frac{1}{2\left(\frac{\sqrt{3}}{2}\right)} \log \left| \frac{x - \frac{\sqrt{3}}{2}}{x + \frac{\sqrt{3}}{2}} \right| + c$$

$$= \frac{1}{4\sqrt{3}} \log \left| \frac{2x - \sqrt{3}}{2x + \sqrt{3}} \right| + c.$$

Question 2.

$$\int \frac{1}{25-9x^2} \cdot dx$$

Solution:

$$I = \int \frac{1}{25-9x^2} dx = \int \frac{1}{5^2-(3x)^2} dx$$

$$= \frac{1}{2(5)} \log \left| \frac{5+3x}{5-3x} \right| \cdot \frac{1}{3} + c$$

$$= \frac{1}{30} \log \left| \frac{5+3x}{5-3x} \right| + c.$$

Alternative Method :

$$\int \frac{1}{25-9x^2} dx = \frac{1}{9} \int \frac{1}{\frac{25}{9}x^2} dx$$

$$= \frac{1}{9} \int \frac{1}{\left(\frac{5}{3}\right)^2 - x^2} dx = \frac{1}{9} \times \frac{1}{2 \times \frac{5}{3}} \log \left| \frac{\frac{5}{3} + x}{\frac{5}{3} - x} \right| + c$$
$$= \frac{1}{30} \log \left| \frac{5+3x}{5-3x} \right| + c.$$

Question 3.

$$\int \frac{1}{7+2x^2} \cdot dx$$

Solution:

$$I = \int \frac{1}{7+2x^2} dx = \frac{1}{2} \int \frac{1}{\frac{7}{2} + x^2} dx$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{1}{\left(\sqrt{\frac{7}{2}}\right)^2 + x^2} dx \\ &= \frac{1}{2} \cdot \frac{1}{\left(\sqrt{\frac{7}{2}}\right)} \tan^{-1} \left| \frac{x}{\sqrt{\frac{7}{2}}} \right| + c \\ &= \frac{1}{\sqrt{14}} \tan^{-1} \left| \frac{\sqrt{2}x}{\sqrt{7}} \right| + c. \end{aligned}$$

Question 4.

$$\int \frac{1}{\sqrt{3x^2+8}} \cdot dx$$

Solution:

$$\begin{aligned} \int \frac{1}{\sqrt{3x^2+8}} dx &= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + \frac{8}{3}}} dx \\ &= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + \left(\sqrt{\frac{8}{3}}\right)^2}} dx \\ &= \frac{1}{\sqrt{3}} \log \left| x + \sqrt{x^2 + \left(\sqrt{\frac{8}{3}}\right)^2} \right| + c_1 \end{aligned}$$

$$\begin{aligned} &= \frac{1}{\sqrt{3}} \log \left| x + \sqrt{x^2 + \frac{8}{3}} \right| + c_1 \\ &= \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{3}x + \sqrt{3x^2 + 8}}{\sqrt{3}} \right| + c_1 \\ &= \frac{1}{\sqrt{3}} \log |\sqrt{3}x + \sqrt{3x^2 + 8}| - \log \sqrt{3} + c_1 \\ &= \frac{1}{\sqrt{3}} \log |\sqrt{3}x + \sqrt{3x^2 + 8}| + c, \text{ where } c = c_1 - \log \sqrt{3} \end{aligned}$$

Alternative Method :

$$\begin{aligned} \int \frac{1}{\sqrt{3x^2 + 8}} dx &= \int \frac{1}{\sqrt{(\sqrt{3}x)^2 + (\sqrt{8})^2}} dx \\ &= \frac{\log |\sqrt{3}x + \sqrt{(\sqrt{3}x)^2 + (\sqrt{8})^2}| + c}{\sqrt{3}} \\ &= \frac{1}{\sqrt{3}} \log |\sqrt{3}x + \sqrt{3x^2 + 8}| + c. \end{aligned}$$

Question 5.

$$\int \frac{1}{\sqrt{11-4x^2}} \cdot dx$$

Solution:

$$\int \frac{1}{\sqrt{11-4x^2}} dx$$

$$\begin{aligned} &= \int \frac{1}{\sqrt{(\sqrt{11})^2 - (2x)^2}} dx \\ &= \frac{1}{2} \sin^{-1} \left(\frac{2x}{\sqrt{11}} \right) + c. \end{aligned}$$

Question 6.

$$\int \frac{1}{\sqrt{2x^2 - 5}} \cdot dx$$

Solution:

$$\begin{aligned} \int \frac{1}{\sqrt{2x^2 - 5}} dx &= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 - \frac{5}{2}}} dx \\ &= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 - \left(\sqrt{\frac{5}{2}}\right)^2}} dx \\ &= \frac{1}{\sqrt{2}} \log \left| x + \sqrt{x^2 - \frac{5}{2}} \right| + c. \end{aligned}$$

Question 7.

$$\int \sqrt{\frac{9+x}{9-x}} \cdot dx$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \sqrt{\frac{9+x}{9-x}} dx \\&= \int \sqrt{\frac{9+x}{9-x} \times \frac{9+x}{9+x}} dx \\&= \int \frac{9+x}{\sqrt{81-x^2}} dx \\&= \int \frac{9}{\sqrt{81-x^2}} dx + \int \frac{x}{\sqrt{81-x^2}} dx \\&= 9 \int \frac{1}{\sqrt{9^2-x^2}} dx + \frac{1}{2} \int \frac{2x}{\sqrt{81-x^2}} dx \\&= I_1 + I_2 \quad \dots (\text{Let})\end{aligned}$$

$$I_1 = 9 \int \frac{1}{\sqrt{9^2-x^2}} dx = 9 \sin^{-1}\left(\frac{x}{9}\right) + c_1$$

In I_2 , put $81-x^2 = t$

$$\therefore -2x dx = dt \quad \therefore 2x dx = -dt$$

$$I_2 = -\frac{1}{2} \int t^{-\frac{1}{2}} dt = -\frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_2$$

$$= -\sqrt{81-x^2} + c_2$$

$$I = 9 \sin^{-1}\left(\frac{x}{9}\right) - \sqrt{81-x^2} + c, \text{ where } c = c_1 + c_2$$

Question 8.

$$\int \sqrt{\frac{2+x}{2-x}} \cdot dx$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \sqrt{\frac{2+x}{2-x}} \cdot dx \\&= \int \sqrt{\frac{2+x}{2-x} \times \frac{2+x}{2+x}} \cdot dx \\&= \int \frac{2+x}{\sqrt{4-x^2}} \cdot dx \\&= \int \frac{2}{\sqrt{4-x^2}} \cdot dx + \int \frac{x}{\sqrt{4-x^2}} \cdot dx \\&= 2 \int \frac{1}{\sqrt{2^2-x^2}} \cdot dx + \frac{1}{2} \int \frac{2x}{\sqrt{4-x^2}} \cdot dx \\&= I_1 + I_2 \quad \dots(\text{Let})\end{aligned}$$

$$\begin{aligned}I_1 &= 2 \int \frac{1}{\sqrt{2^2-x^2}} \cdot dx \\&= 2 \sin^{-1}\left(\frac{x}{2}\right) + c_1\end{aligned}$$

In I_2 , put $4-x^2 = t$

$$\therefore -2x \, dx = dt$$

$$\therefore 2x \, dx = -dt$$

$$I_2 = -\frac{1}{2} \int t^{-\frac{1}{2}} dt$$

$$\begin{aligned} &= -\frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_2 \\ &= -\sqrt{4-x^2} + c_2 \\ I &= 2 \sin^{-1}\left(\frac{x}{2}\right) - \sqrt{4-x^2} + c. \end{aligned}$$

Question 9.

$$\int \sqrt{\frac{10+x}{10-x}} \cdot dx$$

Solution:

$$\begin{aligned} \text{Let } I &= \int \sqrt{\frac{10+x}{10-x}} \cdot dx \\ &= \int \sqrt{\frac{10+x}{10-x}} \times \frac{10+x}{10+x} \cdot dx \\ &= \int \frac{10+x}{\sqrt{100-x^2}} \cdot dx \\ &= \int \frac{10}{\sqrt{100-x^2}} \cdot dx + \int \frac{x}{\sqrt{100-x^2}} \cdot dx \\ &= 10 \int \frac{1}{\sqrt{10^2-x^2}} \cdot dx + \frac{1}{2} \int \frac{2x}{\sqrt{100-x^2}} \cdot dx \\ &= I_1 + I_2 \quad \dots(\text{Let}) \end{aligned}$$

$$I_1 = 10 \int \frac{1}{\sqrt{10^2 - x^2}} \cdot dx$$

$$= 10 \sin^{-1}\left(\frac{x}{10}\right) + c_1$$

In I_2 , put $100 - x^2 = t$

$$\therefore -2x \, dx = dt$$

$$\therefore 2x \, dx = -dt$$

$$I_2 = -\frac{1}{2} \int t^{-\frac{1}{2}} dt$$

$$= -\frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_2$$

$$= -\sqrt{100 - x^2} + c_2$$

$$I = 10 \sin^{-1}\left(\frac{x}{10}\right) - \sqrt{100 - x^2} + c.$$

Question 10.

$$\int \frac{1}{x^2 + 8x + 12} \cdot dx$$

Solution:

$$\int \frac{1}{x^2 + 8x + 12} dx$$

$$\begin{aligned} &= \int \frac{1}{(x^2 + 8x + 16) - 16 + 12} dx \\ &= \int \frac{1}{(x + 4)^2 - 2^2} dx \\ &= \frac{1}{2(2)} \log \left| \frac{(x + 4) - 2}{(x + 4) + 2} \right| + c \\ &= \frac{1}{4} \log \left| \frac{x + 2}{x + 6} \right| + c. \end{aligned}$$

Question 11.

$$\int \frac{1}{1+x-x^2} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{1+x-x^2} dx$$

$$1+x-x^2 = 1 - (x^2 - x)$$

$$= 1 - \left(x^2 - x + \frac{1}{4} \right) + \frac{1}{4}$$

$$= \frac{5}{4} - \left(x^2 - x + \frac{1}{4} \right)$$

$$= \left(\frac{\sqrt{5}}{2} \right)^2 - \left(x - \frac{1}{2} \right)^2$$

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$$\begin{aligned}\therefore I &= \int \frac{1}{\left(\frac{\sqrt{5}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2} dx \\ &= \frac{1}{2\left(\frac{\sqrt{5}}{2}\right)} \log \left| \frac{\frac{\sqrt{5}}{2} + \left(x - \frac{1}{2}\right)}{\frac{\sqrt{5}}{2} - \left(x - \frac{1}{2}\right)} \right| + c \\ &= \frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{5} - 1 + 2x}{\sqrt{5} + 1 - 2x} \right| + c.\end{aligned}$$

Question 12.

$$\int \frac{1}{4x^2 - 20x + 17} \cdot dx$$

Solution:

$$\begin{aligned}& \int \frac{1}{4x^2 - 20x + 17} dx \\ &= \frac{1}{4} \int \frac{1}{x^2 - 5x + \frac{17}{4}} dx \\ &= \frac{1}{4} \int \frac{1}{\left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} + \frac{17}{4}} dx\end{aligned}$$

$$\begin{aligned}&= \frac{1}{4} \int \frac{1}{\left(x - \frac{5}{2}\right)^2 - (\sqrt{2})^2} dx \\&= \frac{1}{4} \times \frac{1}{2\sqrt{2}} \log \left| \frac{x - \frac{5}{2} - \sqrt{2}}{x - \frac{5}{2} + \sqrt{2}} \right| + c \\&= \frac{1}{8\sqrt{2}} \log \left| \frac{2x - 5 - 2\sqrt{2}}{2x - 5 + 2\sqrt{2}} \right| + c.\end{aligned}$$

Question 13.

$$\int \frac{1}{5-4x-3x^2} \cdot dx$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \frac{1}{5-4x-3x^2} dx \\5-4x-3x^2 &= 3 \left[\frac{5}{3} - \left(x^2 + \frac{4}{3}x \right) \right] \\&= 3 \left[\frac{5}{3} - \left(x^2 + \frac{4}{3}x + \frac{4}{9} \right) + \frac{4}{9} \right] \\&= 3 \left[\frac{19}{9} - \left(x^2 + \frac{4x}{3} + \frac{4}{9} \right) \right] \\&= 3 \left[\left(\frac{\sqrt{19}}{3} \right)^2 - \left(x + \frac{2}{3} \right)^2 \right]\end{aligned}$$

$$\begin{aligned}
 I &= \int \frac{1}{3 \left[\left(\frac{\sqrt{19}}{3} \right)^2 - \left(x + \frac{2}{3} \right)^2 \right]} dx \\
 &= \frac{1}{3} \frac{1}{2 \left(\frac{\sqrt{19}}{3} \right)} \log \left| \frac{\frac{\sqrt{19}}{3} + \left(x + \frac{2}{3} \right)}{\frac{\sqrt{19}}{3} - \left(x + \frac{2}{3} \right)} \right| + c \\
 &= \frac{1}{2\sqrt{19}} \log \left| \frac{\sqrt{19} + 2 + 3x}{\sqrt{19} - 2 - 3x} \right| + c \\
 &= \frac{1}{2\sqrt{19}} \log \left| \frac{3x + 2 + \sqrt{19}}{-(3x + 2 - \sqrt{19})} \right| + c \\
 &= \frac{1}{2\sqrt{19}} \log \left| \frac{3x + 2 + \sqrt{19}}{3x + 2 - \sqrt{19}} \right| + c \quad \dots [\because |-x| = x]
 \end{aligned}$$

Question 14.

$$\int \frac{1}{\sqrt{3x^2 + 5x + 7}} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{\sqrt{3x^2 + 5x + 7}} dx$$

$$3x^2 + 5x + 7 = 3 \left[x^2 + \frac{5}{3}x + \frac{7}{3} \right]$$

$$= 3 \left[\left(x^2 + \frac{5x}{3} + \frac{25}{36} \right) + \left(\frac{7}{3} - \frac{25}{36} \right) \right]$$

$$= 3 \left[\left(x + \frac{5}{6} \right)^2 + \left(\frac{\sqrt{59}}{6} \right)^2 \right]$$

$$\therefore \sqrt{3x^2 + 5x + 7} = \sqrt{3} \sqrt{\left(x + \frac{5}{6} \right)^2 + \left(\frac{\sqrt{59}}{6} \right)^2}$$

$$\therefore I = \frac{1}{\sqrt{3}} \int \frac{1}{\left(x + \frac{5}{6} \right)^2 + \left(\frac{\sqrt{59}}{6} \right)^2} dx$$

$$= \frac{1}{\sqrt{3}} \log \left| x + \frac{5}{6} + \sqrt{\left(x + \frac{5}{6} \right)^2 + \left(\frac{\sqrt{59}}{6} \right)^2} \right| + c$$

$$= \frac{1}{\sqrt{3}} \log \left| x + \frac{5}{6} + \sqrt{x^2 + \frac{5x}{3} + \frac{7}{3}} \right| + c.$$

Question 15.

$$\int \frac{1}{\sqrt{x^2 + 8x - 20}} \cdot dx$$

Solution:

$$\int \frac{1}{\sqrt{x^2 + 8x - 20}} dx$$

$$= \int \frac{1}{\sqrt{(x^2 + 8x + 16) - 16 - 20}} dx$$

$$\begin{aligned} &= \int \frac{1}{\sqrt{(x+4)^2 - (6)^2}} dx \\ &= \log |(x+4) + \sqrt{(x+4)^2 - (6)^2}| + c \\ &= \log |(x+4) + \sqrt{x^2 - 8x - 20}| + c. \end{aligned}$$

Question 16.

$$\int \frac{1}{\sqrt{8-3x+2x^2}} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{\sqrt{8-3x+2x^2}} \cdot dx$$

$$8-3x+2x^2 = 8 \left[x^2 + \frac{3}{2}x + \frac{2}{2} \right]$$

$$= 8 \left[\left(x^2 + \frac{3x}{2} + \frac{6}{4} \right) + \left(\frac{2}{2} - \frac{6}{4} \right) \right]$$

$$= 8 \left[\left(x + \frac{3}{4} \right)^2 + \left(\frac{\sqrt{1}}{4} \right)^2 \right]$$

$$\therefore \sqrt{3x^2 + 5x + 7} = \sqrt{3} \sqrt{\left(x + \frac{3}{4} \right)^2 + \left(\frac{\sqrt{1}}{4} \right)^2}$$

$$\begin{aligned}\therefore I &= \frac{1}{\sqrt{3}} \int \frac{1}{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{1}}{4}\right)^2} \cdot dx \\&= \frac{1}{\sqrt{2}} \log \left| x - \frac{3}{4} + \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{1}}{4}\right)^2} \right| + c \\&= \frac{1}{\sqrt{2}} \log \left| x - \frac{3}{4} + \sqrt{x^2 - \frac{3x}{2} + 4} \right| + c.\end{aligned}$$

Question 17.

$$\int \frac{1}{\sqrt{(x-3)(x+2)}} \cdot dx$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \frac{1}{\sqrt{(x-3)(x+2)}} dx \\&= \int \frac{1}{\sqrt{x^2 - x - 6}} dx \\&= \int \frac{1}{\sqrt{\left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} - 6}} dx\end{aligned}$$

$$\begin{aligned} &= \int \frac{1}{\sqrt{\left(x - \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2}} dx \\ &= \log \left| \left(x - \frac{1}{2}\right) + \sqrt{\left(x - \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2} \right| + c \\ &= \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 6} \right| + c. \end{aligned}$$

Question 18.

$$\int \frac{1}{4 + 3 \cos^2 x} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{4 + 3 \cos^2 x} dx$$

Dividing both numerator and denominator by $\cos^2 x$,
we get

$$\begin{aligned} I &= \int \frac{\sec^2 x}{4 \sec^2 x + 3} dx \\ &= \int \frac{\sec^2 x}{4(1 + \tan^2 x) + 3} dx \\ &= \int \frac{\sec^2 x}{4 \tan^2 x + 7} dx \end{aligned}$$

Put $\tan x = t \quad \therefore \sec^2 x \, dx = dt$

$$\begin{aligned} I &= \int \frac{dt}{4t^2 + 7} \\ &= \int \frac{dt}{(2t)^2 + (\sqrt{7})^2} \\ &= \frac{1}{\sqrt{7}} \tan^{-1} \left(\frac{2t}{\sqrt{7}} \right) \cdot \frac{1}{2} + c \\ &= \frac{1}{2\sqrt{7}} \tan^{-1} \left(\frac{2 \tan x}{\sqrt{7}} \right) + c. \end{aligned}$$

Question 19.

$$\int \frac{1}{\cos 2x + 3 \sin^2 x} \cdot dx$$

Solution:

$$\begin{aligned} \text{Let } I &= \int \frac{1}{\cos 2x + 3 \sin^2 x} dx \\ &= \int \frac{1}{1 - 2 \sin^2 x + 3 \sin^2 x} dx \\ &= \int \frac{1}{1 + \sin^2 x} dx \end{aligned}$$

Dividing both numerator and denominator by $\cos^2 x$,
we get

$$I = \int \frac{\sec^2 x \, dx}{\sec^2 x + \tan^2 x} = \int \frac{\sec^2 x \, dx}{1 + \tan^2 x + \tan^2 x}$$

$$= \int \frac{\sec^2 x \, dx}{2 \tan^2 x + 1}$$

Put $\tan x = t \quad \therefore \sec^2 x \, dx = dt$

$$\therefore I = \int \frac{1}{2t^2 + 1} dt = \frac{1}{2} \int \frac{1}{t^2 + \left(\frac{1}{\sqrt{2}}\right)^2} dt$$

$$= \frac{1}{2} \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} \tan^{-1} \left(\frac{t}{\left(\frac{1}{\sqrt{2}}\right)} \right) + c$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} (\sqrt{2} \tan x) + c.$$

Question 20.

$$\int \frac{\sin x}{\sin 3x} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{\sin x}{\sin 3x} dx$$

$$= \int \frac{\sin x}{3 \sin x - 4 \sin^3 x} dx$$

$$= \int \frac{1}{3 - 4 \sin^2 x} dx$$

Dividing both numerator and denominator by $\cos^2 x$,
we get

$$\begin{aligned} I &= \int \frac{\sec^2 x}{3 \sec^2 x - 4 \tan^2 x} dx \\ &= \int \frac{\sec^2 x}{3(1 + \tan^2 x) - 4 \tan^2 x} dx \\ &= \int \frac{\sec^2 x}{3 - \tan^2 x} dx \end{aligned}$$

Put $\tan x = t \quad \therefore \sec^2 x dx = dt$

$$\begin{aligned} I &= \int \frac{dt}{(\sqrt{3})^2 - t^2} \\ &= \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + t}{\sqrt{3} - t} \right| + c \\ &= \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right| + c. \end{aligned}$$

II. Integrate the following functions w. r. t. x:

Question 1.

$$\int \frac{1}{3+2 \sin x} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{3 + 2 \sin x} dx$$

$$\text{Put } \tan(x/2) = t \quad \therefore x = 2 \tan^{-1} t$$

$$\therefore dx = \frac{2dt}{1+t^2} \text{ and } \sin x = \frac{2t}{1+t^2}$$

$$\therefore I = \int \frac{1}{3 + 2\left(\frac{2t}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2}$$

$$= \int \frac{1+t^2}{3+3t^2+4t} \cdot \frac{2dt}{1+t^2}$$

$$= 2 \int \frac{1}{3t^2+4t+3} dt$$

$$= \frac{2}{3} \int \frac{1}{t^2 + \frac{4}{3}t + 1} dt$$

$$= \frac{2}{3} \int \frac{1}{\left(t^2 + \frac{4}{3}t + \frac{4}{9}\right) - \frac{4}{9} + 1} dt$$

$$= \frac{2}{3} \int \frac{1}{\left(t + \frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} dt$$

$$\begin{aligned} &= \frac{2}{3} \times \frac{1}{\left(\frac{\sqrt{5}}{3}\right)} \tan^{-1} \left[\frac{t + \frac{2}{3}}{\frac{\sqrt{5}}{3}} \right] + c \\ &= \frac{2}{\sqrt{5}} \tan^{-1} \left(\frac{3t + 2}{\sqrt{5}} \right) + c \\ &= \frac{2}{\sqrt{5}} \tan^{-1} \left[\frac{3 \tan \left(\frac{x}{2} \right) + 2}{\sqrt{5}} \right] + c. \end{aligned}$$

Question 2.

$$\int \frac{1}{4-5 \cos x} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{4-5 \cos x} dx$$

$$\text{Put } \tan \left(\frac{x}{2} \right) = t \quad \therefore x = 2 \tan^{-1} t$$

$$\therefore dx = \frac{2dt}{1+t^2} \quad \text{and} \quad \cos x = \frac{1-t^2}{1+t^2}$$

$$\therefore I = \int \frac{1}{4-5 \left(\frac{1-t^2}{1+t^2} \right)} \cdot \frac{2dt}{1+t^2}$$

$$= \int \frac{1+t^2}{4(1+t^2)-5(1-t^2)} \cdot \frac{2dt}{1+t^2}$$

$$= \int \frac{2dt}{9t^2-1} = \frac{2}{9} \int \frac{1}{t^2 - \frac{1}{9}} dt$$

$$= \frac{2}{9} \int \frac{1}{t^2 - \left(\frac{1}{3}\right)^2} dt$$

$$= \frac{2}{9} \times \frac{1}{2 \times \frac{1}{3}} \log \left| \frac{t - \frac{1}{3}}{t + \frac{1}{3}} \right| + c$$

$$= \frac{1}{3} \log \left| \frac{3 \tan\left(\frac{x}{2}\right) - 1}{3 \tan\left(\frac{x}{2}\right) + 1} \right| + c.$$

Question 3.

$$\int \frac{1}{2 + \cos x - \sin x} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{2 + \cos x - \sin x} dx$$

$$\text{Put } \tan(x/2) = t \quad \therefore x = 2 \tan^{-1} t$$

$$\therefore dx = \frac{2dt}{1+t^2} \text{ and } \sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}$$

$$\begin{aligned}\therefore I &= \int \frac{1}{2 + \left(\frac{1-t^2}{1+t^2}\right) - \left(\frac{2t}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2} \\&= \int \frac{1+t^2}{2+2t^2+1-t^2-2t} \cdot \frac{2dt}{1+t^2} \\&= 2 \int \frac{1}{t^2-2t+3} dt = 2 \int \frac{1}{(t^2-2t+1)+2} dt \\&= 2 \int \frac{1}{(t-1)^2 + (\sqrt{2})^2} dt \\&= 2 \times \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{t-1}{\sqrt{2}} \right) + c \\&= \sqrt{2} \tan^{-1} \left[\frac{\tan\left(\frac{x}{2}\right) - 1}{\sqrt{2}} \right] + c.\end{aligned}$$

Question 4.

$$\int \frac{1}{3+2 \sin x - \cos x} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{3+2 \sin x - \cos x} dx$$

$$\text{Put } \tan\left(\frac{x}{2}\right) = t \quad \therefore x = 2 \tan^{-1} t$$

$$\therefore dx = \frac{2}{1+t^2} dt \text{ and}$$

$$\sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}$$

$$\begin{aligned} \therefore I &= \int \frac{1}{3+2\left(\frac{2t}{1+t^2}\right)-\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2} \\ &= \int \frac{1+t^2}{3(1+t^2)+4t-(1-t^2)} \cdot \frac{2dt}{1+t^2} \\ &= 2 \int \frac{dt}{4t^2+4t+2} \\ &= 2 \int \frac{dt}{4t^2+4t+1+1} \\ &= 2 \int \frac{dt}{(2t+1)^2+1^2} \\ &= \frac{2}{2} \tan^{-1}\left(\frac{2t+1}{1}\right) + c \\ &= \tan^{-1}\left[2 \tan^{-1}\left(\frac{x}{2}\right)+1\right] + c. \end{aligned}$$

Question 5.

$$\int \frac{1}{3-2 \cos 2x} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{3 - 2 \cos 2x} dx$$

$$\text{Put } \tan x = t \quad \therefore x = \tan^{-1} t$$

$$\therefore dx = \frac{dt}{1+t^2} \text{ and } \cos 2x = \frac{1-t^2}{1+t^2}$$

$$\therefore I = \int \frac{1}{3 - 2 \left(\frac{1-t^2}{1+t^2} \right)} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{1+t^2}{3+3t^2-2+2t^2} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{1}{1+5t^2} dt$$

$$= \frac{1}{5} \int \frac{1}{\left(\frac{1}{\sqrt{5}} \right)^2 + t^2} dt$$

$$= \frac{1}{5} \times \frac{1}{(1/\sqrt{5})} \tan^{-1} \left(\frac{t}{1/\sqrt{5}} \right) + c$$

$$= \frac{1}{\sqrt{5}} \tan^{-1} (\sqrt{5} \tan x) + c.$$

Question 6.

$$\int \frac{1}{2 \sin 2x - 3} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{2 \sin 2x - 3} dx$$

$$\text{Put } \tan x = t \quad \therefore x = \tan^{-1} t$$

$$\therefore dx = \frac{dt}{1+t^2} \quad \text{and} \quad \sin 2x = \frac{2t}{1+t^2}$$

$$\begin{aligned} \therefore I &= \int \frac{1}{2 \left(\frac{2t}{1+t^2} \right) - 3} \cdot \frac{dt}{1+t^2} \\ &= \int \frac{1+t^2}{4t-3-3t^2} \cdot \frac{dt}{1+t^2} \\ &= \int \frac{1}{-3t^2+4t-3} dt \\ &= -\frac{1}{3} \int \frac{1}{t^2 - \frac{4}{3}t + 1} dt \\ &= -\frac{1}{3} \int \frac{1}{\left(t^2 - \frac{4}{3}t + \frac{4}{9}\right) - \frac{4}{9} + 1} dt \\ &= -\frac{1}{3} \int \frac{1}{\left(t - \frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} dt \end{aligned}$$

$$\begin{aligned}
 &= -\frac{1}{3} \times \frac{1}{(\sqrt{5}/3)} \tan^{-1} \left(\frac{t - \frac{2}{3}}{\sqrt{5}/3} \right) + c \\
 &= -\frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{3t - 2}{\sqrt{5}} \right) + c \\
 &= -\frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{3 \tan x - 2}{\sqrt{5}} \right) + c.
 \end{aligned}$$

Question 7.

$$\int \frac{1}{3 + 2 \sin 2x + 4 \cos 2x} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{3 + 2 \sin 2x + 4 \cos 2x} dx$$

$$\text{Put } \tan x = t \quad \therefore x = \tan^{-1} t$$

$$\therefore dx = \frac{dt}{1+t^2} \text{ and } \sin 2x = \frac{2t}{1+t^2}, \cos 2x = \frac{1-t^2}{1+t^2}$$

$$\begin{aligned}
 \therefore I &= \int \frac{1}{3 + 2 \left(\frac{2t}{1+t^2} \right) + 4 \left(\frac{1-t^2}{1+t^2} \right)} \cdot \frac{dt}{1+t^2} \\
 &= \int \frac{1+t^2}{3(1+t^2) + 4t + 4(1-t^2)} \cdot \frac{dt}{1+t^2} \\
 &= \int \frac{1}{7 + 4t - t^2} dt = \int \frac{1}{7 - (t^2 - 4t + 4) + 4} dt
 \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{1}{(\sqrt{11})^2 - (t-2)^2} dt \\
 &= \frac{1}{2\sqrt{11}} \log \left| \frac{\sqrt{11} + t - 2}{\sqrt{11} - t + 2} \right| + c \\
 &= \frac{1}{2\sqrt{11}} \log \left| \frac{\sqrt{11} + \tan x - 2}{\sqrt{11} - \tan x + 2} \right| + c.
 \end{aligned}$$

Question 8.

$$\int \frac{1}{\cos x - \sin x} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{\cos x - \sin x} dx$$

Dividing each term by $\sqrt{1^2 + (-1)^2} = \sqrt{2}$, we get

$$\begin{aligned}
 I &= \frac{1}{\sqrt{2}} \int \frac{1}{\cos x \cdot \frac{1}{\sqrt{2}} - \sin x \cdot \frac{1}{\sqrt{2}}} dx \\
 &= \frac{1}{\sqrt{2}} \int \frac{1}{\cos x \cdot \cos \frac{\pi}{4} - \sin x \cdot \sin \frac{\pi}{4}} dx \\
 &= \frac{1}{\sqrt{2}} \int \frac{1}{\cos \left(x + \frac{\pi}{4} \right)} dx
 \end{aligned}$$

$$= \frac{1}{\sqrt{2}} \int \sec\left(x + \frac{\pi}{4}\right) dx$$

$$= \frac{1}{\sqrt{2}} \log \left| \sec\left(x + \frac{\pi}{4}\right) + \tan\left(x + \frac{\pi}{4}\right) \right| + c.$$

Question 9.

$$\int \frac{1}{\cos x - \sqrt{3} \sin x} \cdot dx$$

Solution:

$$\text{Let } I = \int \frac{1}{\cos x - \sqrt{3} \sin x} \cdot dx$$

Dividing each term by $\sqrt{1^2 + (-1)^2} = \sqrt{2}$, we get

$$I = \frac{1}{2} \int \frac{1}{\cos x \cdot \frac{1}{\sqrt{2}} - \sin x \cdot \frac{1}{\sqrt{2}}} \cdot dx$$

$$= \frac{1}{2} \int \frac{1}{\cos x \cdot \cos \frac{\pi}{4} - \sin x \cdot \sin \frac{\pi}{4}} \cdot dx$$

$$= \frac{1}{2} \int \frac{1}{\cos\left(x + \frac{\pi}{4}\right)} \cdot dx$$

$$= \frac{1}{2} \int \sec\left(x + \frac{\pi}{4}\right) \cdot dx$$

$$= \frac{1}{2} \log \left| \sec\left(x + \frac{\pi}{4}\right) + \tan\left(x + \frac{\pi}{4}\right) \right| + c.$$

Ex 3.2(C)

I. Evaluate:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-12-arts-science-maths-p-art-2-chapter-3-indefinite-integration/>

Question 1.

$$\int \frac{3x+4}{x^2+6x+5} dx$$

Solution:

$$\text{Let } I = \int \frac{3x+4}{x^2+6x+5} dx$$

$$\text{Let } 3x + 4 = A \left[\frac{d}{dx} (x^2 + 6x + 5) \right] + B$$

$$= A(2x + B) + B$$

$$\therefore 3x + 4 = 2Ax + (6A + B)$$

Comparing the coefficient of x and constant on both sides, we get

$$2A = 3 \text{ and } 6A + B = 4$$

$$\therefore A = \frac{3}{2} \text{ and } 6\left(\frac{3}{2}\right) + B = 4$$

$$\therefore B = -5$$

$$3x + 4 = \frac{3}{2} (2x + 6) - 5$$

$$\therefore I = \int \frac{\frac{3}{2}(2x+6) - 5}{x^2 + 6x + 5} dx$$

$$= \frac{3}{2} \int \frac{2x+6}{x^2+6x+5} dx - 5 \int \frac{1}{x^2+6x+5} dx$$

$$= \frac{3}{2} I_1 - 5 I_2$$

$$I_1 \text{ is of the type } \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c$$

$$\therefore I_1 = \log |x^2 + 6x + 5| + c_1$$

$$I_2 = \int \frac{1}{x^2 + 6x + 5} dx = \int \frac{1}{(x^2 + 6x + 9) - 4} dx$$

$$\begin{aligned}
 &= \int \frac{1}{(x+3)^2 - 2^2} dx \\
 &= \frac{1}{2 \times 2} \log \left| \frac{x+3-2}{x+3+2} \right| + c_2 = \frac{1}{4} \log \left| \frac{x+1}{x+5} \right| + c_2 \\
 \therefore I &= \frac{3}{2} \log |x^2 + 6x + 5| - \frac{5}{4} \log \left| \frac{x+1}{x+5} \right| + c, \\
 &\text{where } c = c_1 + c_2.
 \end{aligned}$$

Question 2.

$$\int \frac{2x+1}{x^2+4x-5} dx$$

Solution:

$$\text{Let } I = \int \frac{2x+1}{x^2+4x-5} dx$$

$$\text{Let } 2x + 1 = A \left[\frac{d}{dx} (x^2 + 4x - 5) \right] + B$$

$$2x + 1 = A(2x + 1) + B$$

$$\therefore 2x + 1 = 2Ax + (4A + B)$$

Comparing the coefficient of x and constant on both sides, we get

$$4A = 2 \text{ and } 4A + B = 4$$

$$\therefore A = \frac{3}{2} \text{ and } 6\left(\frac{3}{2}\right) + B = 4$$

$$\therefore B = -5$$

$$\therefore 2x + 1 = \frac{3}{2}(2x + 1) - 5$$

$$\therefore I = \int \frac{\frac{3}{2}(2x + 1) - 5}{x^2 + 6x + 5} \cdot dx$$

$$= \frac{3}{2} \int \frac{2x + 1}{x^2 + 4x - 5} \cdot dx - 5 \int \frac{1}{x^2 + 4x + 5} \cdot dx$$

$$= \frac{3}{2}I_1 - 5I_2$$

$$I_1 \text{ is of the type } \int \frac{f'(x)}{f(x)} \cdot dx = \log|f(x)| + c$$

$$\therefore I_1 = \log|x^2 + 4x - 5| + c_1$$

$$I_2 = \int \frac{1}{x^2 + 4x - 5} \cdot dx$$

$$= \int \frac{1}{(x^2 + 4x - 9) - 4} \cdot dx$$

$$= \int \frac{1}{(x + 3)^2 - 2^2} \cdot dx$$

$$= \log \left| \frac{x + 3 - 2}{x + 3 + 2} \right| + c_2$$

$$= \log \left| \frac{x - 1}{x + 5} \right| + c_2$$

$$\therefore I = \log|x^2 + 4x - 5| - \frac{1}{2} \log \left| \frac{x - 1}{x + 5} \right| + c,$$

Question 3.

$$\int \frac{2x+3}{2x^2+3x-1} dx$$

Solution:

$$\text{Let } I = \int \frac{2x+3}{2x^2+3x-1} dx$$

$$\text{Let } 2x+3 = A\left[\frac{d}{dx}(2x^2+3x-1)\right] + B$$

$$2x+3 = A(4x+3) + B$$

$$\therefore 2x+3 = 4Ax + (3A+B)$$

Comparing the coefficient of x and constant on both sides, we get

$$4A = 2 \text{ and } 3A + B = 3$$

$$\therefore A = \frac{1}{2} \text{ and } 3\left(\frac{1}{2}\right) + B = 3$$

$$\therefore B = \frac{3}{2}$$

$$\therefore 2x+3 = \frac{1}{2}(4x+3) + \frac{3}{2}$$

$$\therefore I = \int \frac{\frac{1}{2}(4x+3) + \frac{3}{2}}{2x^2+3x-1} dx$$

$$= \frac{1}{2} \int \frac{4x+3}{2x^2+3x-1} dx + \frac{3}{2} \int \frac{1}{2x^2+3x-1} dx$$

$$= \frac{1}{2} I_1 + \frac{3}{2} I_2$$

$$I_1 \text{ is of the type } \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$$

$$\therefore I_1 = \log|2x^2+3x-1| + c_1$$

$$I_2 = \int \frac{1}{2x^2+3x-1} dx = \frac{1}{2} \int \frac{1}{x^2 + \frac{3}{2}x - \frac{1}{2}} dx$$

$$\begin{aligned}
 &= \frac{1}{2} \int \frac{1}{\left(x^2 + \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16} - \frac{1}{2}} dx \\
 &= \frac{1}{2} \int \frac{1}{\left(x + \frac{3}{4}\right)^2 - \left(\frac{\sqrt{17}}{4}\right)^2} dx \\
 &= \frac{1}{2} \times \frac{1}{2 \times \frac{\sqrt{17}}{4}} \log \left| \frac{x + \frac{3}{4} - \frac{\sqrt{17}}{4}}{x + \frac{3}{4} + \frac{\sqrt{17}}{4}} \right| + c_2 \\
 &= \frac{1}{\sqrt{17}} \log \left| \frac{4x + 3 - \sqrt{17}}{4x + 3 + \sqrt{17}} \right| + c_2 \\
 \therefore I &= \frac{1}{2} \log |2x^2 + 3x - 1| + \frac{3}{2\sqrt{17}} \log \left| \frac{4x + 3 - \sqrt{17}}{4x + 3 + \sqrt{17}} \right| + c, \\
 &\text{where } c = c_1 + c_2.
 \end{aligned}$$

Question 4.

$$\int \frac{3x+4}{\sqrt{2x^2+2x+1}} dx$$

Solution:

$$\text{Let } I = \int \frac{3x+4}{\sqrt{2x^2+2x+1}} dx$$

$$\text{Let } 3x + 4 = A \left[\frac{d}{dx} (2x^2 + 2x + 1) \right] + B$$

$$\therefore 3x + 4 = A(4x + 2) + B$$

$$\therefore 3x + 4 = 4Ax + (2A + B)$$

Comparing the coefficient of x and the constant on both the sides, we get

$$4A = 3 \text{ and } 2A + B = 4$$

$$\therefore A = \frac{3}{4} \text{ and } 2\left(\frac{3}{4}\right) + B = 4$$

$$\therefore B = \frac{5}{2}$$

$$\therefore 3x + 4 = \frac{3}{4}(4x + 2) + \frac{5}{2}$$

$$\begin{aligned}\therefore I &= \int \frac{\frac{3}{4}(4x + 2) + \frac{5}{2}}{\sqrt{2x^2 + 2x + 1}} dx \\ &= \frac{3}{4} \int \frac{4x + 2}{\sqrt{2x^2 + 2x + 1}} dx + \frac{5}{2} \int \frac{1}{\sqrt{2x^2 + 2x + 1}} dx \\ &= \frac{3}{4} I_1 + \frac{5}{2} I_2\end{aligned}$$

$$\text{In } I_1, \text{ put } 2x^2 + 2x + 1 = t \quad \therefore (4x + 2)dx = dt$$

$$\begin{aligned}\therefore I_1 &= \int \frac{1}{\sqrt{t}} dt = \int t^{-\frac{1}{2}} dt \\ &= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_1 = 2\sqrt{2x^2 + 2x + 1} + c\end{aligned}$$

$$\begin{aligned}I_2 &= \frac{5}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 + x + \frac{1}{2}}} dx \\ &= \frac{5}{\sqrt{2}} \int \frac{1}{\sqrt{\left(x^2 + x + \frac{1}{4}\right) + \frac{1}{4}}} dx\end{aligned}$$

$$\begin{aligned}
 &= \frac{5}{\sqrt{2}} \int \frac{1}{\sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}} dx \\
 &= \frac{5}{\sqrt{2}} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \right| + c_2 \\
 &= \frac{5}{\sqrt{2}} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + \frac{1}{2}} \right| + c_2 \\
 \therefore I &= \frac{3}{2} \sqrt{2x^2 + 2x + 1} + \frac{5}{2\sqrt{2}} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + \frac{1}{2}} \right| + c, \\
 &\text{where } c = c_1 + c_2.
 \end{aligned}$$

Question 5.

$$\int \frac{7x+3}{\sqrt{3+2x-x^2}} dx$$

Solution:

$$\text{Let } I = \int \frac{7x+3}{\sqrt{3+2x-x^2}} dx$$

$$\text{Let } 7x + 3 = A \left[\frac{d}{dx} (3 + 2x - x^2) \right] + B$$

$$7x + 3 = A(2 - 2x) + B$$

$$\therefore 7x + 3 = -2Ax + (2A + B)$$

Comparing the coefficient of x and constant on both the sides, we get

$$-2A = 7 \text{ and } 2A + B = 3$$

$$\therefore A = -\frac{7}{2} \text{ and } 2\left(-\frac{7}{2}\right) + B = 3 \quad \therefore B = 10$$

$$\therefore 7x + 3 = \frac{-7}{2}(2 - 2x) + 10$$

$$\begin{aligned}\therefore I &= \int \frac{\frac{-7}{2}(2 - 2x) + 10}{\sqrt{3 + 2x - x^2}} dx \\ &= \frac{-7}{2} \int \frac{(2 - 2x)}{\sqrt{3 + 2x - x^2}} dx + 10 \int \frac{1}{\sqrt{3 + 2x - x^2}} dx \\ &= \frac{-7}{2} I_1 + 10 I_2\end{aligned}$$

In I_1 , put $3 + 2x - x^2 = t \quad \therefore (2 - 2x)dx = dt$

$$\begin{aligned}\therefore I_1 &= \int \frac{1}{\sqrt{t}} dt = \int t^{-1/2} dt \\ &= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_1 = 2\sqrt{3 + 2x - x^2} + c_1\end{aligned}$$

$$\begin{aligned}I_2 &= \int \frac{1}{\sqrt{3 - (x^2 - 2x + 1) + 1}} dx \\ &= \int \frac{1}{\sqrt{(2)^2 - (x - 1)^2}} dx \\ &= \sin^{-1}\left(\frac{x - 1}{2}\right) + c_2\end{aligned}$$

$$\therefore I = -7\sqrt{3+2x-x^2} + 10\sin^{-1}\left(\frac{x-1}{2}\right) + c,$$

where $c = c_1 + c_2$.

Question 6.

$$\int \sqrt{\frac{x-7}{x-9}} dx$$

Solution:

$$\text{Let } I = \int \sqrt{\frac{x-7}{x-9}} dx = \int \sqrt{\frac{x-7}{x-9}} \times \frac{x-7}{x-7} dx$$

$$\int \sqrt{\frac{(x-7)^2}{x^2-16x+63}} dx$$

$$\int \frac{x-7}{\sqrt{x^2-16x+63}} dx$$

$$\begin{aligned}\text{Let } x-7 &= A \left[\frac{d}{dx}(x^2-16x+63) \right] + B \\ &= A(2x-16) + B \\ &= 2Ax + (B-16A)\end{aligned}$$

Comparing the coefficients of x and constant term on both sides, we get

$$2A = 1 \quad \therefore A = \frac{1}{2} \text{ and}$$

$$B - 16A = -7 \quad \therefore B - 16\left(\frac{1}{2}\right) = -7$$

$$\therefore B = 1$$

$$\therefore x - 7 = \frac{1}{2}(2x - 16) + 1$$

$$\begin{aligned}\therefore I &= \int \frac{\frac{1}{2}(2x - 16) + 1}{\sqrt{x^2 - 16x + 63}} dx \\ &= \frac{1}{2} \int \frac{2x - 16}{\sqrt{x^2 - 16x + 63}} dx + \int \frac{1}{\sqrt{x^2 - 16x + 63}} dx \\ &= \frac{1}{2} I_1 + I_2\end{aligned}$$

In I_1 , put $x^2 - 16x + 63 = t$

$$\therefore (2x - 16) dx = dt$$

$$\begin{aligned}\therefore I_1 &= \frac{1}{2} \int \frac{1}{\sqrt{t}} dt = \frac{1}{2} \int t^{-\frac{1}{2}} dt \\ &= \frac{1}{2} \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_1 = \sqrt{x^2 - 16x + 63} + c_1\end{aligned}$$

$$\begin{aligned}I_2 &= \int \frac{1}{\sqrt{x^2 - 16x + 63}} dx \\ &= \int \frac{1}{\sqrt{(x^2 - 16x + 64) - 1}} dx\end{aligned}$$

$$\begin{aligned}
 &= \int \frac{1}{\sqrt{(x-8)^2 - 1^2}} dx \\
 &= \log |x-8 + \sqrt{(x-8)^2 - 1^2}| + c_2 \\
 &= \log |x-8 + \sqrt{x^2 - 16x + 63}| + c_2 \\
 \therefore I &= \sqrt{x^2 - 16x + 63} + \log |x-8 + \sqrt{x^2 - 16x + 63}| + c, \\
 &\text{where } c = c_1 + c_2.
 \end{aligned}$$

Question 7.

$$\int \sqrt{\frac{9-x}{x}} dx$$

Solution:

$$\begin{aligned}
 \text{Let } I &= \int \sqrt{\frac{9-x}{x}} dx = \int \sqrt{\frac{9-x}{x} \cdot \frac{9-x}{9-x}} dx \\
 &= \int \frac{9-x}{\sqrt{9x-x^2}} dx
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } 9-x &= A \left[\frac{d}{dx} (9x-x^2) \right] + B \\
 &= A(9-2x) + B
 \end{aligned}$$

$$\therefore 9-x = (9A+B) - 2Ax$$

Comparing the coefficient of x and constant on both the sides, we get

$$-2A = -1 \text{ and } 9A + B = 9$$

$$\therefore A = \frac{1}{2} \text{ and } 9\left(\frac{1}{2}\right) + B = 9 \quad \therefore B = \frac{9}{2}$$

$$\therefore 9 - x = \frac{1}{2}(9 - 2x) + \frac{9}{2}$$

$$\begin{aligned}\therefore I &= \int \frac{\frac{1}{2}(9 - 2x) + \frac{9}{2}}{\sqrt{9x - x^2}} dx \\ &= \frac{1}{2} \int \frac{9 - 2x}{\sqrt{9x - x^2}} dx + \frac{9}{2} \int \frac{1}{\sqrt{9x - x^2}} dx \\ &= \frac{1}{2} I_1 + \frac{9}{2} I_2\end{aligned}$$

$$\text{In } I_1, \text{ put } 9x - x^2 = t \quad \therefore (9 - 2x)dx = dt$$

$$\begin{aligned}\therefore I_1 &= \int \frac{1}{\sqrt{t}} dt = \int t^{-1/2} dt \\ &= \frac{t^{1/2}}{1/2} + c_1 = 2\sqrt{9x - x^2} + c_1\end{aligned}$$

$$I_2 = \int \frac{1}{\sqrt{\frac{81}{4} - \left(x^2 - 9x + \frac{81}{4}\right)}} dx$$

$$\begin{aligned}
 &= \int \frac{1}{\sqrt{\left(\frac{9}{2}\right)^2 - \left(x - \frac{9}{2}\right)^2}} dx \\
 &= \sin^{-1} \left(\frac{x - \frac{9}{2}}{9/2} \right) + c_2 = \sin^{-1} \left(\frac{2x - 9}{9} \right) + c_2 \\
 \therefore I &= \sqrt{9x - x^2} + \frac{9}{2} \sin^{-1} \left(\frac{2x - 9}{9} \right) + c,
 \end{aligned}$$

where $c = c_1 + c_2$.

Question 8.

$$\int \frac{3 \cos x}{4 \sin^2 x + 4 \sin x - 1} dx$$

Solution:

$$\text{Let } I = \int \frac{3 \cos x}{4 \sin^2 x + 4 \sin x - 1} dx$$

$$\text{Put } \sin x = t \quad \therefore \cos x dx = dt$$

$$\therefore I = 3 \int \frac{dt}{4t^2 + 4t - 1}$$

$$= 3 \int \frac{dt}{(4t^2 + 4t + 1) - 2}$$

$$= 3 \int \frac{dt}{(2t + 1)^2 - (\sqrt{2})^2}$$

$$= \frac{3}{2(2\sqrt{2})} \log \left| \frac{2t + 1 - \sqrt{2}}{2t + 1 + \sqrt{2}} \right| + c$$

$$= \frac{3}{4\sqrt{2}} \log \left| \frac{2\sin x + 1 - \sqrt{2}}{2\sin x + 1 + \sqrt{2}} \right| + c.$$

Question 9.

$$\int \sqrt{\frac{e^{3x} - e^{2x}}{e^x + 1}} dx$$

Solution:

$$\begin{aligned} \text{Let } I &= \int \sqrt{\frac{e^{3x} - e^{2x}}{e^x + 1}} dx \\ &= \int \sqrt{\frac{e^{2x}(e^x - 1)}{e^x + 1}} dx \\ &= \int e^x \sqrt{\frac{e^x - 1}{e^x + 1}} dx \end{aligned}$$

$$\text{Put } e^x = t \quad \therefore e^x dx = dt$$

$$\begin{aligned} \therefore I &= \int \sqrt{\frac{t-1}{t+1}} dt \\ &= \int \sqrt{\frac{t-1}{t+1} \times \frac{t-1}{t+1}} dt \\ &= \int \sqrt{\frac{(t-1)^2}{t^2-1}} dt \end{aligned}$$

$$\begin{aligned} &= \int \frac{t-1}{\sqrt{t^2-1}} dt \\ &= \frac{1}{2} \int \frac{2t}{\sqrt{t^2-1}} dt - \int \frac{1}{\sqrt{t^2-1}} dt \\ &= I_1 - I_2 \end{aligned}$$

In I_1 , put $t^2 - 1 = \theta$

$$\therefore 2t dt = d\theta$$

$$\therefore I_1 = \frac{1}{2} \int \frac{d\theta}{\sqrt{\theta}} = \frac{1}{2} \int \theta^{-\frac{1}{2}} d\theta$$

$$= \frac{1}{2} \frac{\theta^{\frac{1}{2}}}{(1/2)} + c_1 = \sqrt{\theta} + c_1$$

$$= \sqrt{t^2-1} + c_1 = \sqrt{e^{2x}-1} + c_1$$

$$I_2 = \int \frac{1}{\sqrt{t^2-1}} dt$$

$$= \log|t + \sqrt{t^2-1}| + c_2$$

$$= \log|e^x + \sqrt{e^{2x}-1}| + c_2$$

$$\therefore I = \sqrt{e^{2x}-1} - \log|e^x + \sqrt{e^{2x}-1}| + c, \text{ where } c = c_1 + c_2.$$

Ex 3.3

I. Evaluate the following:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-12-arts-science-maths-p-art-2-chapter-3-indefinite-integration/>

Question 1.

$$\int x^2 \log x \, dx$$

Solution:

$$\begin{aligned}\text{Let } I &= \int x^2 \log x \, dx = \int \log x \cdot x^2 \, dx \\&= (\log x) \int x^2 \, dx - \int \left[\left\{ \frac{d}{dx} (\log x) \right\} \int x^2 \, dx \right] dx \\&= (\log x) \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} \, dx \\&= \frac{x^3 \log x}{3} - \frac{1}{3} \int x^2 \, dx \\&= \frac{x^3 \log x}{3} - \frac{1}{3} \left(\frac{x^3}{3} \right) + c \\&= \frac{x^3}{9} (3 \log x - 1) + c.\end{aligned}$$

Question 2.

$$\int x^2 \sin 3x \, dx$$

Solution:

$$\begin{aligned}\text{Let } I &= \int x^2 \sin 3x \, dx \\&= x^2 \int \sin 3x \, dx - \int \left[\left\{ \frac{d}{dx} (x^2) \right\} \int \sin 3x \, dx \right] dx \\&= x^2 \left(-\frac{\cos 3x}{3} \right) - \int 2x \left(-\frac{\cos 3x}{3} \right) dx\end{aligned}$$

$$\begin{aligned}
 &= \frac{x^2 \cos 3x}{3} + \frac{2}{3} \int x \cos 3x \, dx \\
 &= -\frac{x^2 \cos 3x}{3} + \\
 &\quad \frac{2}{3} \left[x \int \cos 3x \, dx - \int \left\{ \frac{d}{dx}(x) \int \cos 3x \, dx \right\} dx \right] \\
 &= -\frac{x^2 \cos 3x}{3} + \frac{2}{3} \left[\frac{x \sin 3x}{3} - \int 1 \cdot \frac{\sin 3x}{3} \, dx \right] \\
 &= -\frac{x^2 \cos 3x}{3} + \frac{2}{9} x \sin 3x - \frac{2}{9} \int \sin 3x \, dx \\
 &= -\frac{x^2 \cos 3x}{3} + \frac{2}{9} x \sin 3x - \frac{2}{9} \int \left(\frac{-\cos 3x}{3} \right) + c \\
 &= -\frac{x^2 \cos 3x}{3} + \frac{2}{9} x \sin 3x + \frac{2}{27} \cos 3x + c.
 \end{aligned}$$

Question 3.

$\int x \tan^{-1} x \, dx$

Solution:

$$\begin{aligned}
 \text{Let } I &= \int x \tan^{-1} x \, dx = \int (\tan^{-1} x) \cdot x \, dx \\
 &= (\tan^{-1} x) \int x \, dx - \int \left[\left\{ \frac{d}{dx} (\tan^{-1} x) \right\} \int x \, dx \right] dx
 \end{aligned}$$

$$\begin{aligned}&= (\tan^{-1} x) \left(\frac{x^2}{2} \right) - \int \left(\frac{1}{1+x^2} \right) \left(\frac{x^2}{2} \right) dx \\&= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{x^2+1} dx \\&= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{(x^2+1)-1}{x^2+1} dx \\&= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \left[\int \left(1 - \frac{1}{x^2+1} \right) dx \right] \\&= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \left[\int 1 dx - \int \frac{1}{x^2+1} dx \right] \\&= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x) + c.\end{aligned}$$

Question 4.

$\int x^2 \tan^{-1} x \, dx$

Solution:

$$\begin{aligned}\text{Let } I &= \int x^2 \tan^{-1} x \, dx = \int (\tan^{-1} x) \cdot x^2 \, dx \\&= (\tan^{-1} x) \int x^2 \, dx - \int \left[\frac{d}{dx} (\tan^{-1} x) \int x^2 \, dx \right] dx \\&= (\tan^{-1} x) \left(\frac{x^3}{3} \right) - \int \left(\frac{1}{1+x^2} \right) \left(\frac{x^3}{3} \right) dx \\&= \frac{x^3 \tan^{-1} x}{3} - \frac{1}{3} \int \frac{x(x^2+1)-x}{x^2+1} dx\end{aligned}$$

$$\begin{aligned}
 &= \frac{x^3 \tan^{-1} x}{3} - \frac{1}{3} \left[\int \left\{ x - \frac{x}{x^2 + 1} \right\} dx \right] \\
 &= \frac{x^3 \tan^{-1} x}{3} - \frac{1}{3} \left[\int x dx - \frac{1}{2} \int \frac{2x}{x^2 + 1} dx \right] \\
 &= \frac{x^3 \tan^{-1} x}{3} - \frac{1}{3} \left[\frac{x^2}{2} - \frac{1}{2} \log |x^2 + 1| \right] + c \\
 &\quad \dots \left[\because \frac{d}{dx}(x^2 + 1) = 2x \text{ and } \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right] \\
 &= \frac{x^3 \tan^{-1} x}{3} - \frac{x^2}{6} + \frac{1}{6} \log |x^2 + 1| + c.
 \end{aligned}$$

Question 5.

$$\int x^3 \tan^{-1} x \, dx$$

Solution:

$$\text{Let } I = \int x^3 \tan^{-1} x \, dx$$

$$= \int (\tan^{-1} x) \cdot x^3 \, dx$$

$$= (\tan^{-1} x) \int x^3 \, dx - \int \left[\frac{d}{dx} (\tan^{-1} x) \int x^3 \, dx \right] dx$$

$$= (\tan^{-1} x) \left(\frac{x^4}{4} \right) - \int \left(\frac{1}{1+x^2} \right) \frac{x^4}{4} dx$$

$$= \frac{x^4 \tan^{-1} x}{4} - \frac{1}{4} \int \frac{(x^4 - 1) + 1}{x^2 + 1} dx$$

$$= \frac{x^4 \tan^{-1} x}{4} - \frac{1}{4} \int \frac{(x^2 - 1)(x^2 + 1) + 1}{x^2 + 1} dx$$

$$\begin{aligned}
 &= \frac{x^4 \tan^{-1} x}{4} - \frac{1}{4} \int \left[x^2 - 1 + \frac{1}{x^2 + 1} \right] dx \\
 &= \frac{x^4 \tan^{-1} x}{4} - \frac{1}{4} \left[\int x^2 dx - \int 1 dx + \int \frac{1}{x^2 + 1} dx \right] \\
 &= \frac{x^4 \tan^{-1} x}{4} - \frac{1}{4} \left[\frac{x^3}{3} - x + \tan^{-1} x \right] + c \\
 &= \frac{x^4 \tan^{-1} x}{4} - \frac{\tan^{-1} x}{4} - \frac{x^3}{12} + \frac{x}{4} + c \\
 &= \frac{1}{4} (\tan^{-1} x) (x^4 - 1) - \frac{x}{12} (x^2 - 3) + c.
 \end{aligned}$$

Question 6.

$$\int (\log x)^2 dx$$

Solution:

$$\text{Let } I = \int (\log x)^2 dx$$

$$\text{Put } \log x = t$$

$$\therefore x = e^t \quad \therefore dx = e^t dt$$

$$\therefore I = \int t^2 e^t dt$$

$$= t^2 \int e^t dt - \int \left[\frac{d}{dt}(t^2) \int e^t dt - dt \right] dt$$

$$= t^2 e^t - \int 2t e^t dt$$

$$= t^2 e^t - 2 \left[t \int e^t dt - \int \left\{ \frac{d}{dt}(t) \int e^t dt \right\} dt \right]$$

$$= t^2 e^t - 2 [t e^t - \int 1 \cdot e^t dt]$$

$$= t^2 e^t - 2 t e^t + 2 e^t + c$$

$$= e^t [t^2 - 2t + 2] + c$$

$$= x [(\log x)^2 - 2(\log x) + 2] + c.$$

Alternative Method :

$$\text{Let } I = \int (\log x)^2 dx = \int (\log x)^2 \cdot 1 dx$$

$$= (\log x)^2 \int 1 dx - \int \left[\frac{d}{dx} (\log x)^2 \cdot \int 1 dx \right] dx$$

$$= (\log x)^2 \cdot x - \int 2 \log x \cdot \frac{d}{dx} (\log x) \cdot x dx$$

$$= x (\log x)^2 - \int 2 \log x \times \frac{1}{x} \times x dx$$

$$= x (\log x)^2 - 2 \int (\log x) \cdot 1 dx$$

$$= x (\log x)^2 - 2 [(\log x) \int 1 dx - \int \left\{ \frac{d}{dx} (\log x) \int 1 dx \right\} dx]$$

$$= x (\log x)^2 - 2 [(\log x)x - \int \frac{1}{x} \times x dx]$$

$$= x (\log x)^2 - 2x \log x + 2 \int 1 dx$$

$$= x (\log x)^2 - 2x \log x + 2x + c$$

$$= x [(\log x)^2 - 2 \log x + 2] + c.$$

Question 7.

$$\int \sec^3 x dx$$

$$\int \sec^3 x \, dx$$

Solution:

$$\text{Let } I = \int \sec^3 x \, dx$$

$$= \int \sec x \sec^2 x \, dx$$

$$= \sec x \int \sec^2 x \, dx - \int \left[\frac{d}{dx}(\sec x) \int \sec^2 x \, dx \right] dx$$

$$= \sec x \tan x - \int (\sec x \tan x)(\tan x) \, dx$$

$$= \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$$

$$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$\therefore I = \sec x \tan x - I + \log|\sec x + \tan x|$$

$$\therefore 2I = \sec x \tan x + \log|\sec x + \tan x|$$

$$\therefore I = \frac{1}{2} [\sec x \tan x + \log|\sec x + \tan x|] + c.$$

Question 8.

$$\int x \cdot \sin^2 x \, dx$$

Solution:

$$\int x \cdot \sin^2 x \, dx = \int x \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$= \frac{1}{2} \int (x - x \cos 2x) dx$$

$$= \frac{1}{2} \int x \, dx - \frac{1}{2} \int x \cos 2x \, dx$$

$$= \frac{1}{2} \cdot \frac{x^2}{2} - \frac{1}{2} \left[x \int \cos 2x \, dx - \int \left\{ \frac{d}{dx}(x) \int \cos 2x \, dx \right\} dx \right]$$

$$\begin{aligned}&= \frac{x^2}{4} - \frac{1}{2} \left[x \cdot \frac{\sin 2x}{2} - \int 1 \cdot \frac{\sin 2x}{2} dx \right] \\&= \frac{x^2}{4} - \frac{1}{4} x \cdot \sin 2x + \frac{1}{4} \int \sin 2x dx \\&= \frac{x^2}{4} - \frac{1}{4} x \cdot \sin 2x + \frac{1}{4} \cdot \frac{(-\cos 2x)}{2} + c \\&= \frac{x^2}{4} - \frac{1}{4} x \cdot \sin 2x - \frac{1}{8} \cos 2x + c \\&= \frac{1}{4} [x^2 - x \sin 2x - \frac{1}{2} \cos 2x] + c.\end{aligned}$$

Question 9.

$$\int x^3 \log x \, dx$$

Solution:

$$\begin{aligned}\text{Let } I &= \int x^3 \cdot \log x \cdot dx \\&= \int \log x \cdot x^3 \cdot dx \\&= (\log x) \int x^3 \cdot dx - \int \left[\left\{ \frac{d}{dx} (\log x) \int x^3 \cdot dx \right\} \right] \cdot dx \\&= (\log x) \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} \cdot dx \\&= \frac{x^4}{4} \log x - \frac{1}{4} \int x^3 \cdot dx\end{aligned}$$

$$\begin{aligned} &= \frac{x^4}{4} \log x - \frac{1}{4} \left(\frac{x^4}{4} \right) + c \\ &= \frac{x^4}{4} \log x - \frac{x^4}{16} + c. \end{aligned}$$

Question 10.

$$\int e^{2x} \cos 3x \, dx$$

Solution:

$$\begin{aligned} \text{Let } I &= \int e^{2x} \cos 3x \, dx \\ &= e^{2x} \int \cos 3x \, dx - \int \left[\frac{d}{dx} (e^{2x}) \int \cos 3x \, dx \right] dx \\ &= e^{2x} \cdot \frac{\sin 3x}{3} - \int e^{2x} \times 2 \times \frac{\sin 3x}{3} \, dx \\ &= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x \, dx \\ &= \frac{1}{3} e^{2x} \sin 3x - \\ &\quad \frac{2}{3} \left[e^{2x} \int \sin 3x \, dx - \int \left\{ \frac{d}{dx} (e^{2x}) \int \sin 3x \, dx \right\} dx \right] \\ &= \frac{1}{3} e^{2x} \sin 3x - \\ &\quad \frac{2}{3} \left[e^{2x} \cdot \left(\frac{-\cos 3x}{3} \right) - \int e^{2x} \times 2 \times \left(\frac{-\cos 3x}{3} \right) dx \right] \\ &= \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x - \frac{4}{9} \int e^{2x} \cos 3x \, dx \end{aligned}$$

$$\therefore I = \frac{1}{3}e^{2x} \sin 3x + \frac{2}{9}e^{2x} \cos 3x - \frac{4}{9}I$$

$$\therefore \left(1 + \frac{4}{9}\right)I = \frac{1}{3}e^{2x} \sin 3x + \frac{2}{9}e^{2x} \cos 3x$$

$$\therefore \frac{13}{9}I = \frac{e^{2x}}{9}(3 \sin 3x + 2 \cos 3x)$$

$$\therefore I = \frac{e^{2x}}{13}(2 \cos 3x + 3 \sin 3x) + c.$$

Question 11.

$\int x \sin^{-1} x \, dx$

Solution:

$$\begin{aligned}\text{Let } I &= \int x \sin^{-1} x \, dx = \int (\sin^{-1} x) \cdot x \, dx \\&= (\sin^{-1} x) \int x \, dx - \int \left[\frac{d}{dx} (\sin^{-1} x) \int x \, dx \right] dx \\&= (\sin^{-1} x) \left(\frac{x^2}{2} \right) - \int \left(\frac{1}{\sqrt{1-x^2}} \right) \left(\frac{x^2}{2} \right) dx \\&= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx \\&= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{(1-x^2)-1}{\sqrt{1-x^2}} dx \\&= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left[\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right] dx\end{aligned}$$

$$\begin{aligned}
 &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \sqrt{1-x^2} dx - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx \\
 &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left[\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right] - \frac{1}{2} \sin^{-1} x + c \\
 &= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{4} x \sqrt{1-x^2} - \frac{1}{4} \sin^{-1} x + c.
 \end{aligned}$$

Question 12.

$$\int x^2 \cos^{-1} x \, dx$$

Solution:

$$\begin{aligned}
 \text{Let } I &= \int x^2 \cos^{-1} x \, dx = \int (\cos^{-1} x) \cdot x^2 \, dx \\
 &= (\cos^{-1} x) \int x^2 \, dx - \int \left[\frac{d}{dx} (\cos^{-1} x) \int x^2 \, dx \right] dx \\
 &= (\cos^{-1} x) \left(\frac{x^3}{3} \right) - \int \left(\frac{-1}{\sqrt{1-x^2}} \right) \left(\frac{x^3}{3} \right) dx \\
 &= \frac{x^3 \cos^{-1} x}{3} + \frac{1}{3} \int \frac{x^2 \cdot x}{\sqrt{1-x^2}} dx
 \end{aligned}$$

$$\text{In } \int \frac{x^3}{\sqrt{1-x^2}} dx, \text{ put } 1-x^2 = t$$

$$\therefore -2x \, dx = dt \quad \therefore x \, dx = -\frac{1}{2} dt$$

$$\text{Also, } x^2 = 1-t$$

$$\begin{aligned}
 \therefore I &= \frac{x^3 \cos^{-1} x}{3} + \frac{1}{3} \int \frac{(1-t)}{\sqrt{t}} \left(-\frac{1}{2}\right) dt \\
 &= \frac{x^3 \cos^{-1} x}{3} - \frac{1}{6} \int \left(\frac{1}{\sqrt{t}} - \sqrt{t}\right) dt \\
 &= \frac{x^3 \cos^{-1} x}{3} - \frac{1}{6} \int t^{-\frac{1}{2}} dt + \frac{1}{6} \int t^{\frac{1}{2}} dt \\
 &= \frac{x^3 \cos^{-1} x}{3} - \frac{1}{6} \left(\frac{t^{\frac{1}{2}}}{1/2}\right) + \frac{1}{6} \frac{t^{\frac{3}{2}}}{(3/2)} + c \\
 &= \frac{x^3 \cos^{-1} x}{3} - \frac{1}{3} \sqrt{1-x^2} + \frac{1}{9} (1-x^2)^{\frac{3}{2}} + c.
 \end{aligned}$$

Question 13.

$$\int \frac{\log(\log x)}{x} dx$$

Solution:

$$\begin{aligned}
 \text{Let } I &= \int \frac{\log(\log x)}{x} dx \\
 &= \int \log(\log x) \cdot \frac{1}{x} dx \\
 \text{Put } \log x &= t \quad \therefore \frac{1}{x} dx = dt \\
 \therefore I &= \int \log t dt = \int (\log t) \cdot 1 dt \\
 &= (\log t) \int 1 dt - \int \left[\frac{d}{dt} (\log t) \int 1 dt \right] dt
 \end{aligned}$$

$$= (\log t) t - \int \frac{1}{t} \times t dt$$

$$= t \log t - \int 1 dt$$

$$= t \log t - t + c$$

$$= t(\log t - 1) + c$$

$$= (\log x) \cdot [\log(\log x) - 1] + c.$$

Question 14.

$$\int \frac{t \cdot \sin^{-1} t}{\sqrt{1-t^2}} dt$$

Solution:

$$\text{Let } I = \int \frac{t \cdot \sin^{-1} t}{\sqrt{1-t^2}} dt = \int t \cdot \sin^{-1} t \cdot \frac{1}{\sqrt{1-t^2}} dt$$

$$\text{Put } \sin^{-1} t = \theta \quad \therefore \frac{1}{\sqrt{1-t^2}} dt = d\theta$$

$$\text{and } t = \sin \theta$$

$$\therefore I = \int (\sin \theta) \cdot \theta d\theta = \int \theta \sin \theta d\theta$$

$$= \theta \int \sin \theta d\theta - \int \left[\frac{d}{d\theta}(\theta) \int \sin \theta d\theta \right] d\theta$$

$$= \theta (-\cos \theta) - \int 1 \cdot (-\cos \theta) d\theta$$

$$= -\theta \cos \theta + \int \cos \theta d\theta$$

$$= -\theta \cos \theta + \sin \theta + c$$

$$= -\theta \cdot \sqrt{1-\sin^2 \theta} + \sin \theta + c$$

$$= -\sin^{-1} t \cdot \sqrt{1-t^2} + t + c$$

$$= -\sqrt{1-t^2} \cdot \sin^{-1} t + t + c.$$

Question 15.

$$\int \cos \sqrt{x} \, dx$$

Solution:

$$\text{Let } I = \int \cos \sqrt{x} \, dx$$

$$\text{Put } \sqrt{x} = t$$

$$\therefore x = t^2$$

$$\therefore dx = 2t \, dt$$

$$\therefore I = \int (\cos t) 2t \, dt$$

$$= \int 2t \cos t \, dt$$

$$= 2t \int \cos t \, dt - \int \left[\frac{d}{dt} (2t) \int \cos t \, dt \right] dt$$

$$= 2t \sin t - \int 2 \sin t \, dt$$

$$= 2t \sin t + 2 \cos t + c$$

$$= 2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c.$$

Question 16.

$$\int \sin \theta \cdot \log(\cos \theta) \, d\theta$$

Solution:

$$\text{Let } I = \int \sin \theta \cdot \log(\cos \theta) \, d\theta$$

$$= \int \log(\cos \theta) \cdot \sin \theta \, d\theta$$

$$\text{Put } \cos \theta = t$$

$$\therefore -\sin \theta \, d\theta = dt$$

$$\therefore \sin \theta \, d\theta = -dt$$

$$\therefore I = \int \log t \cdot (-dt)$$

$$= - \int (\log t) \cdot 1 \, dt$$

$$= - \left[(\log t) \int 1 \, dt - \int \left\{ \frac{d}{dt} (\log t) \int 1 \, dt \right\} dt \right]$$

$$= - \left[(\log t) t - \int \frac{1}{t} \cdot t \, dt \right]$$

$$= -t \log t + \int 1 \, dt$$

$$= -t \log t + t + c$$

$$= -\cos \theta \cdot \log(\cos \theta) + \cos \theta + c$$

$$= -\cos \theta [\log(\cos \theta) - 1] + c.$$

Question 17.

$$\int x \cos^3 x \, dx$$

Solution:

$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

$$\therefore \cos^3 x + 3 \cos x = 4 \cos 3x$$

$$\therefore \cos^3 x = \frac{1}{4} \cos 3x + \frac{3}{4} \cos x$$

$$\therefore \int \cos^3 x \, dx = \frac{1}{4} \int \cos 3x \, dx + \frac{3}{4} \int \cos x \, dx$$

$$= \frac{1}{4} \left(\frac{\sin 3x}{3} \right) + \frac{3}{4} \sin x$$

$$= \frac{\sin 3x}{12} + \frac{3 \sin x}{4} \quad \dots (1)$$

$$\text{Let } I = \int x \cos^3 x \, dx$$

$$= x \int \cos^3 x \, dx - \int \left[\left\{ \frac{d}{dx}(x) \right\} \int \cos^3 x \, dx \right] dx$$

$$= x \left[\frac{\sin 3x}{12} + \frac{3 \sin x}{4} \right] - \int 1 \cdot \left(\frac{\sin 3x}{12} + \frac{3 \sin x}{4} \right) dx$$

... [By (1)]

$$= \frac{x \sin 3x}{12} + \frac{3x \sin x}{4} - \frac{1}{12} \int \sin 3x \, dx - \frac{3}{4} \int \sin x \, dx$$

$$= \frac{x \sin 3x}{12} + \frac{3x \sin x}{4} - \frac{1}{12} \left(\frac{-\cos 3x}{3} \right) - \frac{3}{4} (-\cos x) + c$$

$$= \frac{1}{4} \left[\frac{x}{3} \sin 3x + \frac{1}{9} \cos 3x + 3x \sin x + 3 \cos x \right] + c.$$

Question 18.

$$\int \frac{\sin(\log x)^2}{x} \cdot \log x \, dx$$

Solution:

$$\text{Let } I = \int \frac{\sin(\log x)^2}{x} \cdot \log x \, dx$$

$$\text{Put } (\log x)^2 = t$$

$$\therefore 2 \log x \cdot \frac{1}{x} dx = dt$$

$$\therefore \frac{1}{x} \log x \, dx = \frac{1}{2} dt$$

$$\begin{aligned}\therefore I &= \frac{1}{2} \int \sin t \, dt \\ &= -\frac{1}{2} \cos t + c \\ &= -\frac{1}{2} \cos [(\log x)^2] + c.\end{aligned}$$

Question 19.

$$\int \frac{\log x}{x} dx$$

Solution:

$$\text{Let } I = \int \frac{\log x}{x} dx$$

Put $\log x = t$

$$\frac{1}{x} dx = dt$$

$$\therefore I = \int t \, dt$$

$$= \frac{1}{2} t^2 + c$$

$$= \frac{1}{2} (\log x)^2 + c$$

Question 20.

$$\int x \sin 2x \cos 5x \, dx.$$

Solution:

$$\text{Let } I = \int x \sin 2x \cos 5x \, dx$$

$$\sin 2x \cos 5x = \frac{1}{2} [2 \sin 2x \cos 5x]$$

$$\begin{aligned}
 &= \frac{1}{2} [\sin(2x + 5x) + \sin(2x - 5x)] \\
 &= \frac{1}{2} [\sin 7x - \sin 3x] \\
 \therefore \int \sin 2x \cos 5x \, dx &= \frac{1}{2} [\int \sin 7x \, dx - \int \sin 3x \, dx] \\
 &= \frac{1}{2} \left(\frac{-\cos 7x}{7} \right) - \frac{1}{2} \left(\frac{-\cos 3x}{3} \right) \\
 &= -\frac{1}{14} \cos 7x + \frac{1}{6} \cos 3x \quad \dots (1)
 \end{aligned}$$

$$\begin{aligned}
 I &= \int x \sin 2x \cos 5x \, dx \\
 &= x \int \sin 2x \cos 5x \, dx - \int \left[\frac{d}{dx}(x) \int \sin 2x \cos 5x \, dx \right] dx \\
 &= x \left[-\frac{1}{14} \cos 7x + \frac{1}{6} \cos 3x \right] - \\
 &\quad \int 1 \cdot \left(-\frac{1}{14} \cos 7x + \frac{1}{6} \cos 3x \right) dx \quad \dots [\text{By (1)}] \\
 &= -\frac{x}{14} \cos 7x + \frac{x}{6} \cos 3x + \frac{1}{14} \int \cos 7x \, dx - \frac{1}{6} \int \cos 3x \, dx \\
 &= -\frac{x}{14} \cos 7x + \frac{x}{6} \cos 3x + \frac{1}{14} \left(\frac{\sin 7x}{7} \right) - \frac{1}{6} \left(\frac{\sin 3x}{3} \right) + c \\
 &= -\frac{x}{14} \cos 7x + \frac{x}{6} \cos 3x + \frac{\sin 7x}{98} - \frac{\sin 3x}{18} + c.
 \end{aligned}$$

Question 21.

$$\int \cos(\sqrt[3]{x}) \, dx$$

Solution:

$$\text{Let } I = \int \cos(\sqrt[3]{x}) dx$$

$$\text{Put } \sqrt[3]{x} = t \quad \therefore x = t^3 \quad \therefore dx = 3t^2 dt$$

$$\therefore I = \int 3t^2 \cos t dt$$

$$= 3t^2 \int \cos t dt - \int \left[\frac{d}{dt}(3t^2) \int \cos t dt \right] dt$$

$$= 3t^2 \sin t - \int 6t \sin t dt$$

$$= 3t^2 \sin t - \left[6t \int \sin t dt - \int \left\{ \frac{d}{dt}(6t) \int \sin t dt \right\} dt \right]$$

$$= 3t^2 \sin t - [6t(-\cos t) - \int 6(-\cos t) dt]$$

$$= 3t^2 \sin t + 6t \cos t - 6 \sin t + c$$

$$= 3(t^2 - 2) \sin t + 6t \cos t + c$$

$$= 3(x^{\frac{2}{3}} - 2) \sin(\sqrt[3]{x}) + 6\sqrt[3]{x} \cos(\sqrt[3]{x}) + c.$$

II. Integrate the following functions w.r.t. x:

Question 1.

$$e^{2x} \sin 3x$$

Solution:

$$\text{Let } I = \int e^{2x} \cdot \sin 3x$$

$$= e^{2x} \cdot \sin 3x - \int \left[\frac{d}{dx}(e^{2x}) \int \sin 3x \cdot dx \right] \cdot dx$$

$$\begin{aligned} &= e^{2x} \cdot \frac{\sin 3x}{3} - \int e^{2x} \times 2 \times \frac{\sin 3x}{3} \cdot dx \\ &= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x \cdot dx \\ &= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left[e^{2x} \int \sin 3x \cdot dx \right] \\ &= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left[e^{2x} \cdot \left(\frac{-\cos 3x}{3} \right) - \int e^{2x} \times 2 \times \left(\frac{-\sin 3x}{3} \right) \cdot dx \right] \\ &= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{13} e^{2x} \cos 3x - \frac{4}{13} \int e^{2x} \cos 3x \cdot dx \\ \therefore I &= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{13} e^{2x} \cos 3x - \frac{4}{13} I \\ \therefore \left(1 + \frac{4}{13} \right) I &= \frac{1}{3} e^{2x} \sin 3x - \frac{2}{13} e^{2x} \cos 3x \\ \therefore \frac{13}{13} I &= \frac{e^{2x}}{13} (2 \sin 3x - 3 \cos 3x) \\ \therefore I &= \frac{e^{2x}}{13} (2 \sin 3x - 3 \cos 3x) + c. \end{aligned}$$

Question 2.

$e^{-x} \cos 2x$

Solution:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-12-arts-science-maths-p-art-2-chapter-3-indefinite-integration/>

$$\begin{aligned}\text{Let } I &= \int e^{-x} \cos 2x \cdot dx \\&= e^{-x} \int \cos 2x \cdot dx - \int \left[\frac{d}{dx} (e^{-x}) \int \sin 2x \cdot dx \right] \cdot dx \\&= e^{-x} \cdot \frac{\cos 2x}{3} - \int e^{-x} \times 2 \times \frac{\sin 2x}{3} \cdot dx \\&= \frac{1}{3} e^{-x} \cos 2x - \frac{2}{3} \int e^{-x} \sin 2x \cdot dx \\&= \frac{1}{3} e^{-x} \cos 2x - \frac{2}{3} \left[e^{-x} \int \sin 2x \cdot dx \right] \\&= \frac{1}{3} e^{-x} \cos 2x - \frac{2}{3} \left[e^{-x} \cdot \left(\frac{-\cos 2x}{3} \right) - \int e^{-x} \times 2 \times \left(\frac{-\cos 2x}{3} \right) \cdot dx \right] \\&= \frac{1}{3} e^{-x} \cos 2x + \frac{2}{5} e^{-x} \cos 2x - \frac{2}{5} \int e^{-x} \sin 2x \cdot dx \\ \therefore I &= \frac{1}{3} e^{-x} \cos 2x + \frac{2}{5} e^{-x} \cos 2x - \frac{3}{5} I \\ \therefore \left(1 + \frac{4}{5} \right) I &= \frac{1}{3} e^{-x} \cos 2x + \frac{2}{5} e^{-x} \sin 2x \\ \therefore \frac{e^{-x}}{5} I &= \frac{e^{-x}}{5} (2 \cos 2x + 2 \sin 2x) \\ \therefore I &= \frac{e^{-x}}{5} (2 \cos 2x + 2 \sin 2x) + c.\end{aligned}$$

Question 3.

$\sin(\log x)$

Solution:

$$\text{Let } I = \int \sin(\log x) dx$$

$$\text{Put } \log x = t \quad \therefore x = e^t \quad \therefore dx = e^t dt$$

$$\therefore I = \int \sin t \times e^t dt = \int e^t \sin t dt$$

$$= e^t \int \sin t dt - \int \left[\frac{d}{dt}(e^t) \int \sin t dt \right] dt$$

$$= e^t (-\cos t) - \int e^t (-\cos t) dt$$

$$= -e^t \cos t + \int e^t \cos t dt$$

$$= -e^t \cos t + e^t \int \cos t dt - \int \left[\frac{d}{dt}(e^t) \int \cos t dt \right] dt$$

$$= -e^t \cos t + e^t \sin t - \int e^t \sin t dt$$

$$\therefore I = -e^t \cos t + e^t \sin t - I$$

$$\therefore 2I = e^t (\sin t - \cos t)$$

$$\therefore I = \frac{e^t}{2} (\sin t - \cos t) + c$$

$$= \frac{x}{2} [\sin(\log x) - \cos(\log x)] + c.$$

Question 4.

$$\sqrt{5x^2 + 3}$$

Solution:

$$\text{Let } I = \int \sqrt{5x^2 + 3} dx$$

$$\begin{aligned} &= \sqrt{5} \int \sqrt{x^2 + \frac{3}{5}} dx \\ &= \sqrt{5} \left[\frac{x}{2} \sqrt{x^2 + \frac{3}{5}} + \frac{\left(\frac{3}{5}\right)}{2} \log \left| x + \sqrt{x^2 + \frac{3}{5}} \right| \right] + c \\ &= \frac{\sqrt{5}}{2} \left[x \sqrt{x^2 + \frac{3}{5}} + \frac{3}{5} \log \left| x + \sqrt{x^2 + \frac{3}{5}} \right| \right] + c. \end{aligned}$$

Question 5.

$$x^2 \sqrt{a^2 - x^6}$$

Solution:

$$\text{Let } I = \int x^2 \sqrt{a^2 - x^6} dx$$

$$\text{Put } x^3 = t \quad \therefore 3x^2 dx = dt \quad \therefore x^2 dx = \frac{1}{3} dt$$

$$\begin{aligned} \therefore I &= \int \sqrt{a^2 - t^2} \cdot \frac{dt}{3} = \frac{1}{3} \int \sqrt{a^2 - t^2} dt \\ &= \frac{1}{3} \left[\frac{t}{2} \sqrt{a^2 - t^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{t}{a} \right) \right] + c \\ &= \frac{1}{6} \left[x^3 \sqrt{a^2 - x^6} + a^2 \sin^{-1} \left(\frac{x^3}{a} \right) \right] + c. \end{aligned}$$

Question 6.

$$\sqrt{(x-3)(7-x)}$$

Solution:

$$\begin{aligned}
 \text{Let } I &= \int \sqrt{(x-3)(7-x)} \, dx \\
 &= \int \sqrt{-x^2 + 10x - 21} \, dx \\
 &= \int \sqrt{-(x^2 - 10x + 21)} \, dx \\
 &= \int \sqrt{4 - (x^2 - 10x + 25)} \, dx \\
 &= \int \sqrt{2^2 - (x-5)^2} \, dx \\
 &= \left(\frac{x-5}{2} \right) \sqrt{2^2 - (x-5)^2} + \frac{2^2}{2} \sin^{-1} \left(\frac{x-5}{2} \right) + c \\
 &= \left(\frac{x-5}{2} \right) \sqrt{(x-3)(7-x)} + 2 \sin^{-1} \left(\frac{x-5}{2} \right) + c.
 \end{aligned}$$

Question 7.

$$\sqrt{4^x (4^x + 4)}$$

Solution:

$$\begin{aligned}
 \text{Let } I &= \int \sqrt{4^x (4^x + 4)} \, dx \\
 &= \int 2^x \sqrt{(2^x)^2 + 2^2} \, dx \\
 \text{Put } 2^x &= t \quad \therefore 2^x \log 2 \, dx = dt \quad \therefore 2^x \, dx = \frac{1}{\log 2} \, dt \\
 \therefore I &= \int \sqrt{t^2 + 2^2} \cdot \frac{dt}{\log 2} = \frac{1}{\log 2} \int \sqrt{t^2 + 2^2} \, dt \\
 &= \frac{1}{\log 2} \left[\frac{t}{2} \sqrt{t^2 + 2^2} + \frac{2^2}{2} \log |t + \sqrt{t^2 + 2^2}| \right] + c
 \end{aligned}$$

$$= \frac{1}{\log 2} \left[\frac{2^x}{2} \sqrt{4^x + 4} + 2 \log |2^x + \sqrt{4^x + 4}| \right] + c$$

Question 8.

$$(x+1) \sqrt{2x^2+3}$$

Solution:

$$\text{Let } I = \int (x+1) \sqrt{2x^2+3} \, dx$$

$$\text{Let } x+1 = A \left[\frac{d}{dx} (2x^2+3) \right] + B$$

$$= A(4x) + B$$

$$= 4Ax + B$$

Comparing the coefficients of x and constant term on both the sides, we get

$$4A = 1, B = 1$$

$$\therefore A = \frac{1}{4}, B = 1$$

$$\therefore x+1 = \frac{1}{4}(4x) + 1$$

$$\therefore I = \int \left[\frac{1}{4}(4x) + 1 \right] \sqrt{2x^2+3} \, dx$$

$$= \frac{1}{4} \int 4x \sqrt{2x^2+3} \, dx + \int \sqrt{2x^2+3} \, dx.$$

$$= I_1 + I_2$$

$$\text{In } I_1, \text{ put } 2x^2+3 = t \quad \therefore 4x \, dx = dt$$

$$\therefore I_1 = \frac{1}{4} \int t^{\frac{1}{2}} \, dt = \frac{1}{4} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + c_1 = \frac{1}{6} (2x^2+3)^{\frac{3}{2}} + c_1$$

$$\begin{aligned}
 I_2 &= \int \sqrt{2x^2 + 3} dx \\
 &= \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}} dx \\
 &= \sqrt{2} \left[\frac{x}{2} \sqrt{x^2 + \frac{3}{2}} + \frac{\left(\frac{3}{2}\right)}{2} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| \right] + c_2 \\
 &= \sqrt{2} \left[\frac{x}{2} \sqrt{x^2 + \frac{3}{2}} + \frac{3}{4} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| \right] + c_2 \\
 \therefore I &= \frac{1}{6} (2x^2 + 3)^{\frac{3}{2}} + \\
 &\quad \sqrt{2} \left[\frac{x}{2} \sqrt{x^2 + \frac{3}{2}} + \frac{3}{4} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| \right] + c, \\
 &\quad \text{where } c = c_1 + c_2.
 \end{aligned}$$

Question 9.

$$x\sqrt{5-4x-x^2}$$

Solution:

$$\text{Let } I = \int x\sqrt{5-4x-x^2} dx$$

$$\text{Let } x = A\left[\frac{d}{dx}(5-4x-x^2)\right] + B$$

$$= A[-4-2x] + B$$

$$= -2Ax + (B-4A)$$

Comparing the coefficients of x and the constant term on both sides, we get

$$-2A = 1, B - 4A = 0$$

$$\therefore A = -\frac{1}{2}, B = 4A = 4\left(-\frac{1}{2}\right) = -2$$

$$\therefore x = -\frac{1}{2}(-4-2x)-2$$

$$\therefore I = \int \left[-\frac{1}{2}(-4-2x)-2 \right] \sqrt{5-4x-x^2} dx$$

$$= -\frac{1}{2} \int (-4-2x) \sqrt{5-4x-x^2} dx - 2 \int \sqrt{5-4x-x^2} dx$$

$$= I_1 - I_2$$

In I_1 , put $5-4x-x^2 = t$

$$\therefore (-4-2x) dx = dt$$

$$\therefore I_1 = -\frac{1}{2} \int t^{\frac{1}{2}} dt = -\frac{1}{2} \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + c_1$$

$$= -\frac{1}{3} (5-4x-x^2)^{\frac{3}{2}} + c_1$$

$$I_2 = 2 \int \sqrt{5-4x-x^2} dx$$

$$= 2 \int \sqrt{5-(x^2+4x)} dx$$

$$= 2 \int \sqrt{9-(x^2+4x+4)} dx$$

$$= 2 \int \sqrt{3^2-(x+2)^2} dx$$

$$= 2 \left[\left(\frac{x+2}{2} \right) \sqrt{3^2-(x+2)^2} + \frac{3^2}{2} \sin^{-1} \left(\frac{x+2}{3} \right) \right] + c_2$$

$$= (x+2) \sqrt{5-4x-x^2} + 9 \sin^{-1} \left(\frac{x+2}{3} \right) + c_2$$

$$\therefore I = -\frac{1}{3} (5-4x-x^2)^{\frac{3}{2}} - (x+2) \sqrt{5-4x-x^2} -$$

$$9 \sin^{-1}\left(\frac{x+2}{3}\right) + c, \text{ where } c = c_1 + c_2$$

Question 10.

$$\sec^2 x \sqrt{\tan^2 x + \tan x - 7}$$

Solution:

$$\text{Let } I = \int \sec^2 x \sqrt{\tan^2 x + \tan x - 7} \, dx$$

$$\text{Put } \tan x = t \quad \therefore \sec^2 x \, dx = dt$$

$$\therefore I = \int \sqrt{t^2 + t - 7} \, dt$$

$$= \int \sqrt{t^2 + t + \frac{1}{4} - \frac{29}{4}} \, dt$$

$$= \int \sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{\sqrt{29}}{2}\right)^2} \, dt$$

$$= \left(\frac{t + \frac{1}{2}}{2}\right) \sqrt{\left(t + \frac{1}{2}\right)^2 - \frac{29}{4}}$$

$$- \frac{\left(\frac{29}{4}\right)}{2} \log \left| \left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 - \frac{29}{4}} \right| + c$$

$$= \frac{(2t+1)}{4} \sqrt{t^2 + t - 7} -$$

$$\begin{aligned}
 &= \frac{(2t+1)}{4} \sqrt{t^2+t-7} - \\
 &\quad \frac{29}{8} \log \left| \left(t + \frac{1}{2} \right) + \sqrt{t^2+t-7} \right| + c \\
 &= \left(\frac{2 \tan x + 1}{4} \right) \sqrt{\tan^2 x + \tan x - 7} - \\
 &\quad \frac{29}{8} \log \left| \left(\tan x + \frac{1}{2} \right) + \sqrt{\tan^2 x + \tan x - 7} \right| + c.
 \end{aligned}$$

Question 11.

$$\sqrt{x^2 + 2x + 5}$$

Solution:

$$\begin{aligned}
 \text{Let } I &= \int \sqrt{x^2 + 2x + 5} \, dx \\
 &= \int \sqrt{x^2 + 2x + 1 + 4} \, dx \\
 &= \int \sqrt{(x+1)^2 + 2^2} \, dx \\
 &= \left(\frac{x+1}{2} \right) \int \sqrt{(x+1)^2 + 2^2} + \\
 &\quad \frac{2^2}{2} \log |(x+1) + \sqrt{(x+1)^2 + 2^2}| + c \\
 &= \left(\frac{x+1}{2} \right) \sqrt{x^2 + 2x + 5} + \\
 &\quad 2 \log |(x+1) + \sqrt{x^2 + 2x + 5}| + c.
 \end{aligned}$$

Question 12.

$$\sqrt{2x^2 + 3x + 4}$$

Question 12.

$$\sqrt{2x^2 + 3x + 4}$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \sqrt{2x^2 + 3x + 4} \, dx \\&= \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}x + 2} \, dx \\&= \sqrt{2} \int \sqrt{\left(x^2 + \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{16} + 2} \, dx \\&= \sqrt{2} \int \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2} \, dx \\&= \sqrt{2} \left[\frac{\left(x + \frac{3}{4}\right)}{2} \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2} \right. \\&\quad \left. + \frac{\left(\frac{23}{16}\right)}{2} \log \left| \left(x + \frac{3}{4}\right) + \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2} \right| \right] + c \\&= \sqrt{2} \left[\left(\frac{4x+3}{8}\right) \sqrt{x^2 + \frac{3}{2}x + 2} \right. \\&\quad \left. + \frac{23}{32} \log \left| \left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2} \right| \right] + c.\end{aligned}$$

III. Integrate the following functions w.r.t. x:

Question 1.

$$[2 + \cot x - \operatorname{cosec}^2 x] e^x$$

Solution:

$$\text{Let } I = \int e^x [2 + \cot x - \operatorname{cosec}^2 x] dx$$

$$\text{Put } f(x) = 2 + \cot x$$

$$\therefore f'(x) = \frac{d}{dx}(2 + \cot x)$$

$$= \frac{d}{dx}(2) + \frac{d}{dx}(\cot x)$$

$$= 0 - \operatorname{cosec}^2 x$$

$$= -\operatorname{cosec}^2 x$$

$$\therefore I = \int e^x [f(x) + f'(x)] dx$$

$$= e^x f(x) + c$$

$$= e^x (2 + \cot x) + c.$$

Question 2.

$$\left(\frac{1 + \sin x}{1 + \cos x} \right) e^x$$

Solution:

$$\text{Let } I = \int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx$$

$$= \int e^x \left[\frac{1 + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right] dx$$

$$= \int e^x \left[\frac{1}{2 \cos^2 \frac{x}{2}} + \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right] dx$$

$$= \int e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \left(\frac{x}{2} \right) \right] dx$$

$$\text{Put } f(x) = \tan \left(\frac{x}{2} \right)$$

$$\therefore f'(x) = \frac{d}{dx} \left[\tan \frac{x}{2} \right] = \sec^2 \frac{x}{2} \cdot \frac{1}{2} = \frac{1}{2} \sec^2 \frac{x}{2}$$

$$\therefore I = \int e^x [f(x) + f'(x)] dx$$

$$= e^x f(x) + c$$

$$= e^x \cdot \tan \left(\frac{x}{2} \right) + c.$$

Question 3.

$$e^x \left(\frac{1}{x} - \frac{1}{x^2} \right)$$

Solution:

$$\text{Let } I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right)$$

$$\text{Let } f(x) = \frac{1}{x}, f'(x) = -\frac{1}{x^2}$$

$$\therefore I = \int e^x [f(x) + f'(x)] dx$$

$$= e^x f(x) + c$$

$$= e^x \cdot \frac{1}{x} + c$$

Question 4.

$$\left[\frac{x}{(x+1)^2} \right] e^x$$

Solution:

$$\begin{aligned} \text{Let } I &= \int e^x \left[\frac{x}{(x+1)^2} \right] dx \\ &= \int e^x \left[\frac{(x+1)-1}{(x+1)^2} \right] dx \\ &= \int e^x \left[\frac{1}{x+1} - \frac{1}{(x+1)^2} \right] dx \end{aligned}$$

$$\text{Let } f(x) = \frac{1}{x+1} = (x+1)^{-1}$$

$$\therefore f'(x) = \frac{d}{dx} (x+1)^{-1} = -(x+1)^{-2} \frac{d}{dx} (x+1)$$

$$= \frac{-1}{(x+1)^2} \times 1 = \frac{-1}{(x+1)^2}$$

$$\therefore I = \int e^x [f(x) + f'(x)] dx$$

$$= e^x \cdot f(x) + c$$

$$= \frac{e^x}{x+1} + c.$$

Question 5.

$$\frac{e^x}{x} \cdot [x(\log x)^2 + 2 \log x]$$

Solution:

$$\text{Let } I = \int \frac{e^x}{x} [x(\log x)^2 + 2 \log x] dx$$

$$= \int e^x \left[(\log x)^2 + \frac{2 \log x}{x} \right] dx$$

$$\text{Put } f(x) = (\log x)^2$$

$$\begin{aligned} \therefore f'(x) &= \frac{d}{dx} (\log x)^2 = 2 (\log x) \cdot \frac{d}{dx} (\log x) \\ &= \frac{2 \log x}{x} \end{aligned}$$

$$\begin{aligned} \therefore I &= \int e^x [f(x) + f'(x)] dx \\ &= e^x \cdot f(x) + c = e^x \cdot (\log x)^2 + c. \end{aligned}$$

Question 6.

$$e^{5x} \left[\frac{5x \log x + 1}{x} \right]$$

Solution:

$$\text{Let } I = \int e^{5x} \left[\frac{5x \log x + 1}{x} \right]$$

$$= \int e^{5x} \left[5 \log x + \frac{1}{x} \right] dx$$

$$\text{Put } 5x = t \quad \therefore 5 dx = dt \quad \therefore dx = \frac{1}{5} dt$$

$$\text{Also, } x = \frac{t}{5}$$

$$\therefore I = \frac{1}{5} \int e^t \left[5 \log \left(\frac{t}{5} \right) + \frac{5}{t} \right] dt$$

$$\text{Let } f(t) = 5 \log\left(\frac{t}{5}\right) = 5 \log t - 5 \log 5$$

$$\therefore f'(t) = \frac{d}{dt}[5 \log t - 5 \log 5] = \frac{5}{t} - 0 = \frac{5}{t}$$

$$\therefore I = \frac{1}{5} \int e^t [f(t) + f'(t)] dt$$

$$= \frac{1}{5} e^t f(t) + c$$

$$= \frac{1}{5} e^t \cdot 5 \log\left(\frac{t}{5}\right) + c$$

$$= e^{5x} \log x + c.$$

Question 7.

$$e^{\sin^{-1} x} \left[\frac{x + \sqrt{1-x^2}}{\sqrt{1-x^2}} \right]$$

Solution:

$$\text{Let } I = \int e^{\sin^{-1} x} \left[\frac{x + \sqrt{1-x^2}}{\sqrt{1-x^2}} \right] dx$$

$$= \int e^{\sin^{-1} x} [x + \sqrt{1-x^2}] \cdot \frac{1}{\sqrt{1-x^2}} dx$$

$$\text{Put } \sin^{-1} x = t \quad \therefore \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\text{and } x = \sin t$$

$$\begin{aligned}\therefore I &= \int e^t [\sin t + \sqrt{1 - \sin^2 t}] dt \\ &= \int e^t [\sin t + \sqrt{\cos^2 t}] dt \\ &= \int e^t (\sin t + \cos t) dt\end{aligned}$$

$$\text{Let } f(t) = \sin t \quad \therefore f'(t) = \cos t$$

$$\begin{aligned}\therefore I &= \int e^t [f(t) + f'(t)] dt \\ &= e^t \cdot f(t) + c \\ &= e^t \cdot \sin t + c \\ &= e^{\sin^{-1} x} \cdot x + c = x \cdot e^{\sin^{-1} x} + c.\end{aligned}$$

Question 8.

$$\log(1+x)^{(1+x)}$$

Solution :

$$\begin{aligned}\text{Let } I &= \int \log(1+x)^{(1+x)} dx \\ &= \int (1+x) \log(1+x) dx \\ &= \int [\log(1+x)] (1+x) dx \\ &= [\log(1+x)] \int (1+x) dx - \\ &\quad \int \left[\frac{d}{dt} \{ \log(1+x) \} \int (1+x) dx \right] dx \\ &= [\log(1+x)] \left[\frac{(1+x)^2}{2} \right] - \int \frac{1}{x+1} \cdot \frac{(x+1)^2}{2} dx\end{aligned}$$

$$\begin{aligned} &= \frac{(x+1)^2}{2} \cdot \log(1+x) - \frac{1}{2} \int (x+1) dx \\ &= \frac{(x+1)^2}{2} \cdot \log(1+x) - \frac{1}{2} \cdot \frac{(x+1)^2}{2} + c \\ &= \frac{(x+1)^2}{2} \left[\log(1+x) - \frac{1}{2} \right] + c. \end{aligned}$$

Question 9.

$\operatorname{cosec}(\log x)[1 - \cot(\log x)]$

Solution:

Let $I = \int \operatorname{cosec}(\log x)[1 - \cot(\log x)] dx$

Put $\log x = t$

$x = e^t$

$dx = e^t dt$

$I = \int \operatorname{cosec} t (1 - \cot t) \cdot e^t dt$

$= \int e^t [\operatorname{cosec} t - \operatorname{cosec} t \cot t] dt$

$= \int e^t \left[\operatorname{cosec} t + \frac{d}{dt} (\operatorname{cosec} t) \right] dt$

$= e^t \operatorname{cosec} t + c \dots [\because e^t [f(t) + f'(t)] dt = e^t f(t) + c]$

$= x \cdot \operatorname{cosec}(\log x) + c.$

Ex 3.4

I. Integrate the following w. r. t. x:

<https://www.indcareer.com/schools/maharashtra-board-solutions-class-12-arts-science-maths-part-2-chapter-3-indefinite-integration/>

Question 1.

$$\frac{x^2+2}{(x-1)(x+2)(x+3)}$$

Solution:

$$\text{Let } I = \int \frac{x^2+2}{(x-1)(x+2)(x+3)} dx$$

$$\text{Let } \frac{x^2+2}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3}$$

$$\therefore x^2+2 = A(x+2)(x+3) + B(x-1)(x+3) + C(x-1)(x+2)$$

Put $x-1=0$, i.e. $x=1$, we get

$$1+2 = A(3)(4) + B(0)(4) + C(0)(3)$$

$$\therefore 3 = 12A \quad \therefore A = \frac{1}{4}$$

Put $x+2=0$, i.e. $x=-2$, we get

$$4+2 = A(0)(1) + B(-3)(1) + C(-3)(0)$$

$$\therefore 6 = -3B \quad \therefore B = -2$$

Put $x+3=0$, i.e. $x=-3$, we get

$$9+2 = A(-1)(0) + B(-4)(0) + C(-4)(-1)$$

$$\therefore 11 = 4C$$

$$\therefore C = \frac{11}{4}$$

$$\therefore \frac{x^2+2}{(x-1)(x+2)(x+3)} = \frac{(1/4)}{x-1} + \frac{(-2)}{x+2} + \frac{(11/4)}{x+3}$$

$$\begin{aligned}\therefore I &= \int \left[\frac{(1/4)}{x-1} + \frac{(-2)}{x+2} + \frac{(11/4)}{x+3} \right] dx \\ &= \frac{1}{4} \int \frac{1}{x-1} dx - 2 \int \frac{1}{x+2} dx + \frac{11}{4} \int \frac{1}{x+3} dx \\ &= \frac{1}{4} \log|x-1| - 2 \log|x+2| + \frac{11}{4} \log|x+3| + c.\end{aligned}$$

Question 2.

$$\frac{x^2}{(x^2+1)(x^2-2)(x^2+3)}$$

Solution:

$$\text{Let } I = \int \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} dx$$

$$\text{Consider, } \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)}$$

For finding partial fractions only, put $x^2 = t$.

$$\begin{aligned}\therefore \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} &= \frac{t}{(t+1)(t-2)(t+3)} \\ &= \frac{A}{t+1} + \frac{B}{t-2} + \frac{C}{t+3} \quad \dots \text{ (Say)}\end{aligned}$$

$$\therefore t = A(t-2)(t+3) + B(t+1)(t+3) + C(t+1)(t-2)$$

Put $t+1=0$, i.e. $t=-1$, we get

$$-1 = A(-3)(2) + B(0)(2) + C(0)(-3)$$

$$\therefore -1 = -6A \quad \therefore A = \frac{1}{6}$$

Put $t - 2 = 0$, i.e. $t = 2$, we get

$$2 = A(0)(5) + B(3)(5) + C(3)(0)$$

$$\therefore 2 = 15B \quad \therefore B = \frac{2}{15}$$

Put $t + 3 = 0$, i.e. $t = -3$, we get

$$-3 = A(-5)(0) + B(-2)(0) + C(-2)(-5)$$

$$-3 = 10C \quad \therefore C = -\frac{3}{10}$$

$$\therefore \frac{t}{(t+1)(t-2)(t+3)} = \frac{(1/6)}{t+1} + \frac{(2/15)}{t-2} + \frac{(-3/10)}{t+3}$$

$$\therefore \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} = \frac{(1/6)}{x^2+1} + \frac{(2/15)}{x^2-2} + \frac{(-3/10)}{x^2+3}$$

$$\begin{aligned} \therefore I &= \int \left[\frac{(1/6)}{x^2+1} + \frac{(2/15)}{x^2-2} + \frac{(-3/10)}{x^2+3} \right] dx \\ &= \frac{1}{6} \int \frac{1}{1+x^2} dx + \frac{2}{15} \int \frac{1}{x^2 - (\sqrt{2})^2} dx - \frac{3}{10} \int \frac{1}{x^2 + (\sqrt{3})^2} dx \\ &= \frac{1}{6} \tan^{-1} x + \frac{2}{15} \times \frac{1}{2\sqrt{2}} \log \left| \frac{x-\sqrt{2}}{x+\sqrt{2}} \right| - \\ &\quad \frac{3}{10} \times \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) + c \\ &= \frac{1}{6} \tan^{-1} x + \frac{1}{15\sqrt{2}} \log \left| \frac{x-\sqrt{2}}{x+\sqrt{2}} \right| - \frac{\sqrt{3}}{10} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) + c. \end{aligned}$$

Question 3.

$$\frac{12x+3}{6x^2+13x-63}$$

Solution:

$$\text{Let } I = \int \frac{12x+3}{6x^2+13x-63} dx$$

$$\text{Let } \frac{12x+3}{6x^2+13x-63} = \frac{12x+3}{(2x+9)(3x-7)} = \frac{A}{2x+9} + \frac{B}{3x-7}$$

$$\therefore 12x+3 = A(3x-7) + B(2x+9)$$

$$\text{Put } 2x+9=0, \text{ i.e. } x = -\frac{9}{2}, \text{ we get}$$

$$12\left(-\frac{9}{2}\right) + 3 = A\left(-\frac{27}{2} - 7\right) + B(0)$$

$$\therefore -51 = \frac{-41}{2}A \quad \therefore A = \frac{102}{41}$$

$$\text{Put } 3x-7=0, \text{ i.e. } x = \frac{7}{3}, \text{ we get}$$

$$12\left(\frac{7}{3}\right) + 3 = A(0) + B\left(\frac{14}{3} + 9\right)$$

$$\therefore 31 = \frac{41}{3}B \quad \therefore B = \frac{93}{41}$$

$$\therefore \frac{12x+3}{6x^2+13x-63} = \frac{(102/41)}{2x+9} + \frac{(93/41)}{3x-7}$$

$$\therefore I = \int \left[\frac{(102/41)}{2x+9} + \frac{(93/41)}{3x-7} \right] dx$$

$$\begin{aligned} &= \frac{102}{41} \int \frac{1}{2x+9} dx + \frac{93}{41} \int \frac{1}{3x-7} dx \\ &= \frac{102}{41} \cdot \frac{\log|2x+9|}{2} + \frac{93}{41} \cdot \frac{\log|3x-7|}{3} + c \\ &= \frac{51}{41} \log|2x+9| + \frac{31}{41} \log|3x-7| + c. \end{aligned}$$

Question 4.

$$\frac{2x}{4-3x-x^2}$$

Solution:

$$\text{Let } I = \int \frac{2x}{4-3x-x^2} dx$$

$$\text{Let } \frac{2x}{4-3x-x^2} = \frac{2x}{(4+x)(1-x)} = \frac{A}{4+x} + \frac{B}{1-x}$$

$$\therefore 2x = A(1-x) + B(4+x)$$

Put $4+x=0$, i.e. $x=-4$, we get

$$-8 = A(5) + B(0) \quad \therefore A = -\frac{8}{5}$$

Put $1-x=0$, i.e. $x=1$, we get

$$2 = A(0) + B(5) \quad \therefore B = \frac{2}{5}$$

$$\therefore \frac{2x}{4-3x-x^2} = \frac{(-8/5)}{4+x} + \frac{(2/5)}{1-x}$$

$$\begin{aligned}\therefore I &= \int \left[\frac{(-8/5)}{4+x} + \frac{(2/5)}{1-x} \right] dx \\ &= -\frac{8}{5} \int \frac{1}{4+x} dx + \frac{2}{5} \int \frac{1}{1-x} dx \\ &= -\frac{8}{5} \log|4+x| + \frac{2}{5} \cdot \frac{\log|1-x|}{-1} + c \\ &= -\frac{8}{5} \log|4+x| - \frac{2}{5} \log|1-x| + c.\end{aligned}$$

Question 5.

$$\frac{x^2+x-1}{x^2+x-6}$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \frac{x^2+x-1}{x^2+x-6} dx \\ &= \int \frac{(x^2+x-6)+5}{x^2+x-6} dx \\ &= \int \left[1 + \frac{5}{x^2+x-6} \right] dx \\ &= \int 1 dx + 5 \int \frac{1}{x^2+x-6} dx \\ \text{Let } \frac{1}{x^2+x-6} &= \frac{1}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2} \\ \therefore 1 &= A(x-2) + B(x+3) \\ \text{Put } x+3 &= 0, \text{ i.e. } x = -3, \text{ we get}\end{aligned}$$

$$1 = A(-5) + B(0) \quad \therefore A = \frac{-1}{5}$$

Put $x - 2 = 0$, i.e. $x = 2$, we get

$$1 = A(0) + B(5) \quad \therefore B = \frac{1}{5}$$

$$\therefore \frac{1}{x^2 + x - 6} = \frac{(-1/5)}{x + 3} + \frac{(1/5)}{x - 2}$$

$$\therefore I = \int 1 \, dx + 5 \int \left[\frac{(-1/5)}{x + 3} + \frac{(1/5)}{x - 2} \right] dx$$

$$= \int 1 \, dx - \int \frac{1}{x + 3} \, dx + \int \frac{1}{x - 2} \, dx$$

$$= x - \log|x + 3| + \log|x - 2| + c$$

$$= x + \log \left| \frac{x - 2}{x + 3} \right| + c.$$

Question 6.

$$\frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1}$$

Solution:

$$\text{Let } I = \int \frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1} \, dx$$

$$3x^2 - 2x - 1 \overline{) 6x^3 + 5x^2 - 7} \quad (2x + 3$$

$$\begin{array}{r} 6x^3 - 4x^2 - 2x \\ - \quad + \quad + \\ \hline 9x^2 + 2x - 7 \end{array}$$

$$\frac{9x^2 - 6x - 3}{8x - 4}$$

$$\begin{aligned}\therefore I &= \int \left[(2x + 3) + \frac{8x - 4}{3x^2 - 2x - 1} \right] dx \\ &= \int 2x + 3 + \int \frac{8x - 4}{(x - 1)(3x + 1)} dx\end{aligned}$$

$$\text{Let } \frac{8x - 4}{(x - 1)(3x + 1)} = \frac{A}{x - 1} + \frac{B}{3x + 1}$$

$$\therefore 8x - 4 = A(3x + 1) + B(x - 1)$$

Put $x - 1 = 0$, i.e. $x = 1$, we get

$$8 - 4 = A(4) + B(0) \quad \therefore A = 1$$

Put $3x + 1 = 0$, i.e. $x = -\frac{1}{3}$, we get

$$8\left(-\frac{1}{3}\right) - 4 = A(0) + B\left(-\frac{4}{3}\right)$$

$$\therefore \frac{-8 - 12}{3} = -\frac{4B}{3} \quad \therefore B = 5$$

$$\therefore \frac{8x - 4}{(x - 1)(3x + 1)} = \frac{1}{x - 1} + \frac{5}{3x + 1}$$

$$\begin{aligned}\therefore I &= 2 \int x dx + 3 \int 1 dx + \int \left[\frac{1}{x - 1} + \frac{5}{3x + 1} \right] dx \\ &= 2\left(\frac{x^2}{2}\right) + 3x + \int \frac{1}{x - 1} dx + 5 \int \frac{1}{3x + 1} dx \\ &= x^2 + 3x + \log|x - 1| + \frac{5}{3} \log|3x + 1| + c.\end{aligned}$$

Question 7.

$$\frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)}$$

Solution:

$$\text{Let } I = \int \frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)} \cdot dx$$

$$\text{Let } \frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)} = \frac{A}{4x^2 - 1} + \frac{B}{x + 3}$$

$$\therefore 12x^2 - 2x - 9 = A(x + 3) + B(4x^2 - 1)$$

$$\text{Put } 4x^2 - 1 = 0, \text{ i.e. } x^2 = \frac{1}{4}, \text{ i.e. } x = \frac{1}{2} \text{ we get}$$

$$12 \times \left(\frac{1}{2}\right)^2 - 2 \times \left(\frac{1}{2}\right) - 9 = A\left(\frac{7}{2}\right) + B(0)$$

$$\therefore -7 = \frac{7A}{2}$$

$$\therefore A = -2$$

$$\text{Put } x + 3 = 0, \text{ i.e. } x = -3, \text{ we get}$$

$$12(-3)^2 - 2(-3) - 9 = A(0) + B(4(3^2) - 1)$$

$$\therefore 105 = 35B$$

$$\therefore B = 3$$

$$\frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)} = \frac{-2}{4x^2 - 1} + \frac{3}{x + 3}$$

$$\begin{aligned}\therefore I &= \int \left[\frac{-2}{4x^2 - 1} + \frac{3}{x + 3} \right] \cdot dx \\ &= (-2) \int \frac{1}{(2x)^2 - 1} \cdot dx + 3 \int \frac{1}{x + 3} \cdot dx \\ &= \frac{1}{2} \log \left| \frac{2x + 1}{2x - 1} \right| + 3 \log |x + 3| + c.\end{aligned}$$

Question 8.

$$\frac{1}{x(x^5+1)}$$

Solution:

$$\text{Let } I = \int \frac{1}{x(x^5+1)} dx = \int \frac{x^4}{x^5(x^5+1)} dx$$

Put $x^5 = t$. Then $5x^4 dx = dt$

$$\therefore x^4 dx = \frac{dt}{5}$$

$$\begin{aligned}\therefore I &= \int \frac{1}{t(t+1)} \cdot \frac{dt}{5} \\ &= \frac{1}{5} \int \frac{(t+1) - t}{t(t+1)} dt \\ &= \frac{1}{5} \int \left(\frac{1}{t} - \frac{1}{t+1} \right) dt \\ &= \frac{1}{5} \left[\int \frac{1}{t} dt - \int \frac{1}{t+1} dt \right]\end{aligned}$$

$$= \frac{1}{5} [\log |t| - \log |t+1|] + c$$

$$= \frac{1}{5} \log \left| \frac{t}{t+1} \right| + c.$$

$$= \frac{1}{5} \log \left| \frac{x^5}{x^5+1} \right| + c.$$

Question 9.

$$\frac{2x^2-1}{x^4+9x^2+20}$$

Solution:

$$\text{Let } I = \int \frac{2x^2-1}{x^4+9x^2+20} \cdot dx$$

$$\text{Consider, } \frac{2x^2-1}{x^4+9x^2+20}$$

For finding partial fractions only, put $x^2 = t$.

$$\therefore \frac{2x^2-1}{x^4+9x^2+20} = \frac{t}{(t-1)(t-2)(t+3)}$$

$$= \frac{A}{t+1} + \frac{B}{t-2} + \frac{C}{t+3} \quad \dots(\text{Say})$$

$$\therefore t = A(t-2)(t+3) + B(t+1)(t+3) + C(t+1)(t-2)$$

Put $t+1 = 0$, i.e. $t = -1$, we get

$$-1 = A(-3)(2) + B(0)(2) + C(0)(-3)$$

$$\therefore -1 = -6A$$

$$\therefore A = \frac{1}{6}$$

Put $t - 2 = 0$, i.e. $t = 2$, we get

$$2 = A(0)(5) + B(3)(5) + C(3)(0)$$

$$\therefore 2 = 15B$$

$$\therefore B = \frac{2}{15}$$

Put $t + 3 = 0$, i.e. $t = -3$, we get

$$-3 = A(-5)(0) + B(-2)(0) + C(-2)(-5)$$

$$-3 = 10C$$

$$\therefore C = -\frac{3}{10}$$

$$\therefore \frac{t}{(t+1)(t-2)(t+3)} = \frac{\left(\frac{1}{6}\right)}{t+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}$$

$$\therefore \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} = \frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}$$

$$\therefore I = \int \left[\frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3} \right] \cdot dx$$

$$\begin{aligned}
 &= \frac{1}{6} \int \frac{1}{1+x^2} \cdot dx + \frac{2}{15} \int \frac{1}{x^2 - (\sqrt{2})^2} \cdot dx - \frac{3}{10} \int \frac{1}{x^2 + (\sqrt{3})^2} \cdot dx \\
 &= \frac{1}{6} \tan^{-1} x + \frac{2}{15} \times \frac{1}{2\sqrt{2}} \log \left| \frac{x - \sqrt{2}}{x + \sqrt{2}} \right| - \frac{3}{10} \times \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) + c \\
 &= \frac{11}{\sqrt{5}} \tan^{-1} \left(\frac{x}{\sqrt{5}} \right) - \frac{9}{2} \tan^{-1} \left(\frac{x}{2} \right) + c.
 \end{aligned}$$

Question 10.

$$\frac{x^2+3}{(x^2-1)(x^2-2)}$$

Solution:

$$\text{Let } I = \int \frac{x^2+3}{(x^2-1)(x^2-2)} \cdot dx$$

$$\text{Consider, } \frac{x^2+3}{(x^2-1)(x^2-2)}$$

For finding partial fractions only, put $x^2 = t$.

$$\begin{aligned}
 \therefore \frac{x^2+3}{(x^2-1)(x^2-2)} &= \frac{t}{(t+1)(t-2)} \\
 &= \frac{A}{t+1} + \frac{B}{t-2} \quad \dots(\text{Say})
 \end{aligned}$$

$$\therefore t = A(t-2)(t+3) + B(t+1)(t+3) + C(t+1)(t-2)$$

Put $t+1=0$, i.e. $t=-1$, we get

$$-1 = A(-3)(2) + B(0)(2) + C(0)(-3)$$

$$\therefore -1 = -6A$$

$$\therefore A = \frac{1}{6}$$

Put $t-2=0$, i.e. $t=2$, we get

$$2 = A(0)(5) + B(3)(5) + C(3)(0)$$

$$\therefore 2 = 15B$$

$$\therefore B = \frac{2}{15}$$

Put $t+3=0$, i.e. $t=-3$, we get

$$-3 = A(-5)(0) + B(-2)(0) + C(-2)(-5)$$

$$-3 = 10C$$

$$\therefore C = -\frac{3}{10}$$

$$\therefore \frac{t}{(t+1)(t-2)(t+3)} = \frac{\left(\frac{1}{6}\right)}{t+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}$$

$$\therefore \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} = \frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}$$

$$\begin{aligned}
 \therefore I &= \int \left[\frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3} \right] \cdot dx \\
 &= \frac{1}{6} \int \frac{1}{1+x^2} \cdot dx + \frac{2}{15} \int \frac{1}{x^2 - (\sqrt{2})^2} \cdot dx - \frac{3}{10} \int \frac{1}{x^2 + (\sqrt{3})^2} \cdot dx \\
 &= \frac{1}{6} \tan^{-1} x + \frac{2}{15} \times \frac{1}{2\sqrt{2}} \log \left| \frac{x - \sqrt{2}}{x + \sqrt{2}} \right| - \frac{3}{10} \times \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) + c \\
 &= 2 \log \left| \frac{x+1}{x-1} \right| + \frac{5}{2\sqrt{2}} \log \left| \frac{x - \sqrt{2}}{x + \sqrt{2}} \right| + c.
 \end{aligned}$$

Question 11.

$$\frac{2x}{(2+x^2)(3+x^2)}$$

Solution:

$$\text{Let } I = \int \frac{2x}{(2+x^2)(3+x^2)} dx$$

$$\text{Put } x^2 = t \quad \therefore 2x dx = dt$$

$$\begin{aligned}
 \therefore I &= \int \frac{1}{(2+t)(3+t)} dt \\
 &= \int \frac{(3+t) - (2+t)}{(2+t)(3+t)} dt \\
 &= \int \left[\frac{1}{2+t} - \frac{1}{3+t} \right] dt
 \end{aligned}$$

$$\begin{aligned} &= \int \frac{1}{2+t} dt - \int \frac{1}{3+t} dt \\ &= \log|2+t| - \log|3+t| + c \\ &= \log \left| \frac{2+t}{3+t} \right| + c \\ &= \log \left| \frac{2+x^2}{3+x^2} \right| + c. \end{aligned}$$

Question 12.

$$\frac{2^x}{4^x - 3 \cdot 2^x - 4}$$

Solution:

$$\begin{aligned} \text{Let } I &= \int \frac{2^x}{4^x - 3 \cdot 2^x - 4} dx \\ &= \int \frac{2^x}{(2^x)^2 - 3 \cdot 2^x - 4} \end{aligned}$$

Put $2^x = t$

$$\therefore 2^x \log 2 dx = dt \quad \therefore 2^x dx = \frac{1}{\log 2} dt$$

$$\begin{aligned} \therefore I &= \frac{1}{\log 2} \int \frac{dt}{t^2 - 3t - 4} \\ &= \frac{1}{\log 2} \int \frac{1}{(t+1)(t-4)} dt \end{aligned}$$

$$\begin{aligned} &= \frac{1}{5 \log 2} \int \frac{(t+1) - (t-4)}{(t+1)(t-4)} dt \\ &= \frac{1}{5 \log 2} \int \left[\frac{1}{t-4} - \frac{1}{t+1} \right] dt \\ &= \frac{1}{5 \log 2} \left[\int \frac{1}{t-4} dt - \int \frac{1}{t+1} dt \right] \\ &= \frac{1}{5 \log 2} [\log |t-4| - \log |t+1|] + c \\ &= \frac{1}{5 \log 2} \log \left| \frac{2^x - 4}{2^x + 1} \right| + c. \end{aligned}$$

Question 13.

$$\frac{3x-2}{(x+1)^2(x+3)}$$

Solution:

$$\text{Let } I = \int \frac{3x-2}{(x+1)^2(x+3)} dx$$

$$\text{Let } \frac{3x-2}{(x+1)^2(x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}$$

$$\therefore 3x-2 = A(x+1)(x+3) + B(x+3) + C(x+1)^2$$

Put $x+1=0$, i.e. $x=-1$, we get

$$-3-2 = A(0)(2) + B(2) + C(0)$$

$$\therefore -5 = 2B \quad \therefore B = -\frac{5}{2}$$

Put $x+3=0$, i.e. $x=-3$, we get

$$-9 - 2 = A(-2)(0) + B(0) + C(-2)^2$$

$$\therefore -11 = 4C \quad \therefore C = -\frac{11}{4}$$

Put $x = 0$, we get

$$-2 = A(1)(3) + B(3) + C(1)$$

$$\therefore -2 = 3A + 3B + C$$

$$\therefore -2 = 3A - \frac{15}{2} - \frac{11}{4}$$

$$\therefore 3A = -2 + \frac{15}{2} + \frac{11}{4} = \frac{-8 + 30 + 11}{4} = \frac{33}{4} \quad \therefore A = \frac{11}{4}$$

$$\therefore \frac{3x-2}{(x+1)^2(x+3)} = \left(\frac{11}{4}\right) \frac{1}{x+1} + \left(-\frac{5}{2}\right) \frac{1}{(x+1)^2} + \left(-\frac{11}{4}\right) \frac{1}{x+3}$$

$$\begin{aligned} \therefore I &= \int \left[\frac{(11/4)}{x+1} + \frac{(-5/2)}{(x+1)^2} + \frac{(-11/4)}{x+3} \right] dx \\ &= \frac{11}{4} \int \frac{1}{x+1} dx - \frac{5}{2} \int (x+1)^{-2} dx - \frac{11}{4} \int \frac{1}{x+3} dx \\ &= \frac{11}{4} \log|x+1| - \frac{5}{2} \cdot \frac{(x+1)^{-1}}{-1} \cdot \frac{1}{1} - \frac{11}{4} \log|x+3| + c \\ &= \frac{11}{4} \log \left| \frac{x+1}{x+3} \right| + \frac{5}{2(x+1)} + c. \end{aligned}$$

Question 14.

$$\frac{5x^2+20x+6}{x^3+2x^2+x}$$

Solution:

$$\text{Let } I = \int \frac{5x^2+20x+6}{x^3+2x^2+x} dx$$

$$= \int \frac{5x^2+20x+6}{x(x^2+2x+1)} dx$$

$$= \int \frac{5x^2+20x+6}{x(x+1)^2} dx$$

$$\text{Let } \frac{5x^2+20x+6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$\therefore 5x^2+20x+6 = A(x+1)^2 + Bx(x+1) + Cx$$

Put $x=0$, we get

$$0+0+6 = A(1) + B(0)(1) + C(0) \quad \therefore A=6$$

Put $x+1=0$, i.e. $x=-1$, we get

$$5(1)+20(-1)+6 = A(0) + B(-1)(0) + C(-1)$$

$$\therefore -9 = -C \quad \therefore C=9$$

Put $x=1$, we get

$$5(1)+20(1)+6 = A(4) + B(1)(2) + C(1)$$

But $A=6$ and $C=9$

$$\therefore 31 = 24 + 2B + 9 \quad \therefore B = -1$$

$$\therefore \frac{5x^2+20x+6}{x(x+1)^2} = \frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2}$$

$$\therefore I = \int \left[\frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2} \right] dx$$

$$\begin{aligned} &= 6 \int \frac{1}{x} dx - \int \frac{1}{x+1} dx + 9 \int (x+1)^{-2} dx \\ &= 6 \log|x| - \log|x+1| + 9 \cdot \frac{(x+1)^{-1}}{-1} + c \\ &= \log|x^6| - \log|x+1| - \frac{9}{(x+1)} + c \\ &= \log \left| \frac{x^6}{x+1} \right| - \frac{9}{(x+1)} + c. \end{aligned}$$

Question 15.

$$\frac{1}{x(1+4x^3+3x^6)}$$

Solution:

$$\begin{aligned} \text{Let } I &= \int \frac{1}{x(1+4x^3+3x^6)} dx \\ &= \int \frac{x^2}{x^3(1+4x^3+3x^6)} dx \end{aligned}$$

$$\text{Put } x^3 = t \quad \therefore 3x^2 dx = dt \quad \therefore x^2 dx = \frac{1}{3} dt$$

$$\begin{aligned} \therefore I &= \frac{1}{3} \int \frac{1}{t(1+4t+3t^2)} dt \\ &= \frac{1}{3} \int \frac{1}{t(t+1)(3t+1)} dt \end{aligned}$$

$$\text{Let } \frac{1}{t(t+1)(3t+1)} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{3t+1}$$

$$\therefore 1 = A(t+1)(3t+1) + Bt(3t+1) + Ct(t+1)$$

Put $t = 0$, we get

$$1 = A(1) + B(0) + C(0) \quad \therefore A = 1$$

Put $t + 1 = 0$, i.e. $t = -1$, we get

$$1 = A(0) + B(-1)(-2) + C(0) \quad \therefore B = \frac{1}{2}$$

Put $3t + 1 = 0$, i.e. $t = -\frac{1}{3}$, we get

$$1 = A(0) + B(0) + C\left(-\frac{1}{3}\right)\left(\frac{2}{3}\right) \quad \therefore C = -\frac{9}{2}$$

$$\therefore \frac{1}{t(t+1)(3t+1)} = \frac{1}{t} + \frac{\left(\frac{1}{2}\right)}{t+1} + \frac{\left(-\frac{9}{2}\right)}{3t+1}$$

$$\therefore I = \frac{1}{3} \int \left[\frac{1}{t} + \frac{\left(\frac{1}{2}\right)}{t+1} + \frac{\left(-\frac{9}{2}\right)}{3t+1} \right] dt$$

$$= \frac{1}{3} \left[\int \frac{1}{t} dt + \frac{1}{2} \int \frac{1}{t+1} dt - \frac{9}{2} \int \frac{1}{3t+1} dt \right]$$

$$= \frac{1}{3} \left[\log|t| + \frac{1}{2} \log|t+1| - \frac{9}{2} \cdot \frac{1}{3} \log|3t+1| \right] + c$$

$$= \frac{1}{3} \log |x^3| + \frac{1}{2} \log |x^3 + 1| - \frac{3}{2} \log |3x^3 + 1| + c$$

$$= \log |x| + \frac{1}{2} \log |x^3 + 1| - \frac{3}{2} \log |3x^3 + 1| + c.$$

Question 16.

$$\frac{1}{x^3-1}$$

Solution:

$$\begin{aligned} \text{Let } I &= \int \frac{1}{x^3-1} dx \\ &= \int \frac{1}{(x-1)(x^2+x+1)} dx \end{aligned}$$

$$\text{Let } \frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

$$\therefore 1 = A(x^2+x+1) + (Bx+C)(x-1)$$

Put $x-1=0$ i.e. $x=1$, we get

$$1 = A(3) + (B+C)(0) \quad \therefore A = \frac{1}{3}$$

Put $x=0$, we get

$$1 = A(1) + C(-1) \quad \therefore C = A - 1 = -\frac{2}{3}$$

Comparing the coefficients of x^2 on both the sides, we get

$$0 = A + B \quad \therefore B = -A = -\frac{1}{3}$$

$$\begin{aligned}\therefore \frac{1}{(x-1)(x^2+x+1)} &= \frac{\left(\frac{1}{3}\right)}{x-1} + \frac{\left(-\frac{1}{3}x - \frac{2}{3}\right)}{x^2+x+1} \\ &= \frac{1}{3} \left[\frac{1}{x-1} - \frac{x+2}{x^2+x+1} \right]\end{aligned}$$

$$\begin{aligned}\text{Let } x+2 &= p \left[\frac{d}{dx}(x^2+x+1) \right] + q \\ &= p(2x+1) + q = 2px + (p+q)\end{aligned}$$

Comparing coefficients of x and the constant term on both the sides, we get

$$2p = 1 \text{ i.e. } p = \frac{1}{2} \text{ and } p + q = 2$$

$$\therefore q = 2 - p = 2 - \frac{1}{2} = \frac{3}{2}$$

$$\therefore x+2 = \frac{1}{2}(2x+1) + \frac{3}{2}$$

$$\begin{aligned}\therefore \frac{1}{(x+1)(x^2+x+1)} &= \frac{1}{3} \left[\frac{1}{x-1} - \frac{\frac{1}{2}(2x+1) + \frac{3}{2}}{(x^2+x+1)} \right] \\ &= \frac{1}{3} \left[\frac{1}{x-1} - \frac{1}{2} \left(\frac{2x+1}{x^2+x+1} \right) - \frac{\left(\frac{3}{2}\right)}{(x^2+x+1)} \right]\end{aligned}$$

$$\begin{aligned}\therefore I &= \frac{1}{3} \int \left[\frac{1}{x-1} - \frac{1}{2} \left(\frac{2x+1}{x^2+x+1} \right) - \frac{\left(\frac{3}{2}\right)}{(x^2+x+1)} \right] dx \\&= \frac{1}{3} \int \frac{1}{x-1} dx - \frac{1}{6} \int \frac{2x+1}{x^2+x+1} dx - \frac{1}{2} \int \frac{1}{x^2+x+\frac{1}{4}+\frac{3}{4}} dx \\&= \frac{1}{3} \log|x-1| - \\&\quad \frac{1}{6} \int \frac{\frac{d}{dx}(x^2+x+1)}{x^2+x+1} dx - \frac{1}{2} \int \frac{1}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx \\&= \frac{1}{3} \log|x-1| - \frac{1}{6} \log|x^2+x+1| - \\&\quad \frac{1}{2} \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1} \left[\frac{\left(x+\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} \right] + c \\&= \frac{1}{3} \log|x-1| - \frac{1}{6} \log|x^2+x+1| - \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right) + c.\end{aligned}$$

Question 17.

$$\frac{(3 \sin x - 2) \cdot \cos x}{5 - 4 \sin x - \cos^2 x}$$

Solution:

$$\text{Let } I = \int \frac{(3 \sin x - 2) \cos x}{5 - 4 \sin x - \cos^2 x} dx$$

$$= \int \frac{(3 \sin x - 2) \cos x}{5 - (1 - \sin^2 x) - 4 \sin x} dx$$

$$= \int \frac{(3 \sin x - 2) \cos x}{5 - 1 + \sin^2 x - 4 \sin x} dx$$

$$= \int \frac{(3 \sin x - 2) \cos x}{\sin^2 x - 4 \sin x + 4} dx$$

$$\text{Put } \sin x = t \quad \therefore \cos x dx = dt$$

$$\therefore I = \int \frac{3t - 2}{t^2 - 4t + 4} dt$$

$$= \int \frac{3t - 2}{(t - 2)^2} dt$$

$$\text{Let } \frac{3t - 2}{(t - 2)^2} = \frac{A}{t - 2} + \frac{B}{(t - 2)^2}$$

$$\therefore 3t - 2 = A(t - 2) + B$$

Put $t - 2 = 0$, i.e. $t = 2$, we get

$$4 = A(0) + B \quad \therefore B = 4$$

Put $t = 0$, we get

$$-2 = A(-2) + B$$

$$\therefore -2 = -2A + 4$$

$$\therefore 2A = 6 \quad \therefore A = 3$$

$$\therefore \frac{3t-2}{(t-2)^2} = \frac{3}{t-2} + \frac{4}{(t-2)^2}$$

$$\therefore I = \int \left[\frac{3}{t-2} + \frac{4}{(t-2)^2} \right] dt$$

$$= 3 \int \frac{1}{t-2} dt + 4 \int (t-2)^{-2} dt$$

$$= 3 \log|t-2| + 4 \cdot \frac{(t-2)^{-1}}{-1} \cdot \frac{1}{1} + c$$

$$= 3 \log|t-2| - \frac{4}{(t-2)} + c$$

$$= 3 \log|\sin x - 2| - \frac{4}{(\sin x - 2)} + c.$$

Question 18.

$$\frac{1}{\sin x + \sin 2x}$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \frac{1}{\sin x + \sin 2x} dx \\&= \int \frac{1}{\sin x + 2 \sin x \cos x} dx \\&= \int \frac{dx}{\sin x (1 + 2 \cos x)} = \int \frac{\sin x dx}{\sin^2 x (1 + 2 \cos x)} \\&= \int \frac{\sin x dx}{(1 - \cos^2 x)(1 + 2 \cos x)} \\&= \int \frac{\sin x dx}{(1 - \cos x)(1 + \cos x)(1 + 2 \cos x)}\end{aligned}$$

Put $\cos x = t \quad \therefore -\sin x dx = dt$

$$\therefore \sin x dx = -dt$$

$$\begin{aligned}\therefore I &= \int \frac{-dt}{(1-t)(1+t)(1+2t)} \\&= - \int \frac{dt}{(1-t)(1+t)(1+2t)}\end{aligned}$$

$$\text{Let } \frac{1}{(1-t)(1+t)(1+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1+2t}$$

$$\therefore 1 = A(1+t)(1+2t) + B(1-t)(1+2t) + C(1-t)(1+t)$$

Putting $1 - t = 0$, i.e. $t = 1$, we get

$$1 = A(2)(3) + B(0)(3) + C(0)(2) \quad \therefore A = 1/6$$

Putting $1 + t = 0$, i.e. $t = -1$, we get

$$1 = A(0)(-1) + B(2)(-1) + C(2)(0)$$

$$\therefore B = -1/2$$

Putting $1 + 2t = 0$, i.e. $t = -1/2$, we get

$$1 = A(0) + B(0) + C\left(\frac{3}{2}\right)\left(\frac{1}{2}\right) \quad \therefore C = 4/3$$

$$\therefore \frac{1}{(1-t)(1+t)(1+2t)} = \frac{(1/6)}{1-t} + \frac{(-1/2)}{1+t} + \frac{(4/3)}{1+2t}$$

$$\therefore I = - \int \left[\frac{(1/6)}{1-t} + \frac{(-1/2)}{1+t} + \frac{(4/3)}{1+2t} \right] dt$$

$$= -\frac{1}{6} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{1+t} dt - \frac{4}{3} \int \frac{1}{1+2t} dt$$

$$= -\frac{1}{6} \cdot \frac{\log|1-t|}{-1} + \frac{1}{2} \log|1+t| - \frac{4}{3} \cdot \frac{\log|1+2t|}{2} + c$$

$$= \frac{1}{6} \log|1 - \cos x| + \frac{1}{2} \log|1 + \cos x| - \frac{2}{3} \log|1 + 2 \cos x| + c$$

$$= \frac{1}{2} \log|\cos x + 1| + \frac{1}{6} \log|\cos x - 1| - \frac{2}{3} \log|2 \cos x + 1| + c.$$

Question 19.

$$\frac{1}{2 \sin x + \sin 2x}$$

Solution:

$$\begin{aligned}\text{Let } I &= \int \frac{1}{2 \sin x + \sin 2x} dx \\&= \int \frac{1}{2 \sin x + 2 \sin x \cos x} dx \\&= \int \frac{1}{2 \sin x (1 + \cos x)} dx \\&= \int \frac{\sin x}{2 \sin^2 x (1 + \cos x)} dx \\&= \int \frac{\sin x dx}{2 (1 - \cos^2 x)(1 + \cos x)} \\&= \int \frac{\sin x dx}{2 (1 - \cos x)(1 + \cos x)(1 + \cos x)} \\&= \int \frac{\sin x dx}{2 (1 - \cos x)(1 + \cos x)^2}\end{aligned}$$

$$\text{Put } \cos x = t \quad \therefore -\sin x dx = dt$$

$$\therefore \sin x dx = -dt$$

$$\therefore I = -\frac{1}{2} \int \frac{1}{(1-t)(1+t)^2} dt$$

$$= \frac{1}{2} \int \frac{1}{(t-1)(t+1)^2} dt$$

$$\text{Let } \frac{1}{(t-1)(t+1)^2} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{(t+1)^2}$$

$$\therefore 1 = A(t+1)^2 + B(t-1)(t+1) + C(t-1)$$

Put $t+1=0$, i.e., $t=-1$, we get

$$\therefore 1 = A(0) + B(0) + C(-2)$$

$$\therefore C = -\frac{1}{2}$$

Put $t-1=0$, i.e., $t=1$, we get

$$\therefore 1 = A(4) + B(0) + C(0) \quad \therefore A = \frac{1}{4}$$

Comparing coefficients of t^2 on both the sides, we get

$$0 = A + B$$

$$\therefore B = -A = -\frac{1}{4}$$

$$\therefore \frac{1}{(t-1)(t+1)^2} = \frac{\left(\frac{1}{4}\right)}{t-1} + \frac{\left(-\frac{1}{4}\right)}{t+1} + \frac{\left(-\frac{1}{2}\right)}{(t+1)^2}$$

$$\therefore I = \frac{1}{2} \int \left[\frac{\left(\frac{1}{4}\right)}{t-1} + \frac{\left(-\frac{1}{4}\right)}{t+1} + \frac{\left(-\frac{1}{2}\right)}{(t+1)^2} \right] dt$$

$$= \frac{1}{8} \int \frac{1}{t-1} dt - \frac{1}{8} \int \frac{1}{t+1} dt - \frac{1}{4} \int \frac{1}{(t+1)^2} dt$$

$$= \frac{1}{8} \log |t-1| - \frac{1}{8} \log |t+1| - \frac{1}{4} \frac{(t+1)^{-1}}{(-1)} + c$$

$$= \frac{1}{8} \log \left| \frac{t-1}{t+1} \right| + \frac{1}{4} \cdot \frac{1}{t+1} + c$$

$$= \frac{1}{8} \log \left| \frac{\cos x - 1}{\cos x + 1} \right| + \frac{1}{4(\cos x + 1)} + c.$$

Question 20.

$$\frac{1}{\sin 2x + \cos x}$$

Solution:

$$\begin{aligned} \text{Let } I &= \int \frac{1}{\sin 2x + \cos x} \cdot dx \\ &= \int \frac{1}{\sin x + \sin 2x \cos x} \cdot dx \\ &= \int \frac{dx}{\sin x(1 + 2 \cos x)} \\ &= \int \frac{\sin x \cdot dx}{\sin^2 x(1 + 2 \cos x)} \\ &= \int \frac{\sin \cdot dx}{(1 - \cos^2 x)(1 + 2 \cos x)} \\ &= \int \frac{\sin \cdot dx}{(1 - \cos x)(1 + \sin 2x)(1 + \cos x)} \end{aligned}$$

Put $\cos x = t$

$$\therefore -\sin x \cdot dx = dt$$

$$\therefore \sin x \cdot dx = -dt$$

$$\therefore I = \int \frac{-dt}{(1-t)(1+t)(1+2t)}$$

$$= - \int \frac{dt}{(1-t)(1+t)(1+2t)}$$

$$\text{Let } \frac{1}{(1-t)(1+t)(1+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1+2t}$$

$$\therefore 1 = A(1+t)(1+2t) + B(1-t)(1+2t) + C(1-t)(1+t)$$

Putting $1-t=0$, i.e. $t=1$, we get

$$1 = A(2)(3) + B(0)(3) + C(0)(2)$$

$$\therefore A = \frac{1}{6}$$

Putting $1-t=0$, i.e. $t=-1$, we get

$$1 = A(0)(-1) + B(2)(-1) + C(2)(0)$$

$$\therefore B = -\frac{1}{2}$$

Putting $1+2t=0$, i.e. $t=-\frac{1}{2}$, we get

$$1 = A(0) + B(0) + C\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$$

$$\therefore C = \frac{4}{3}$$

$$\therefore \frac{1}{(1-t)(1+t)(1+2t)} = \frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(\frac{-1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t}$$

$$\therefore I = \int \left[\frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(\frac{-1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t} \right] \cdot dt$$

$$= -\frac{1}{6} \int \frac{1}{1-t} \cdot dt + \frac{1}{2} \int \frac{1}{1+t} \cdot dt - \frac{4}{3} \int \frac{1}{1+2t} \cdot dt$$

$$= -\frac{1}{6} \cdot \frac{\log|1-t|}{-1} + \frac{1}{2} \log|1+t| - \frac{4}{3} \cdot \frac{\log|1+2t|}{2} + c$$

$$= -\frac{1}{6} \log|\sin x + 1| + \frac{1}{2} \log|\sin x - 1| - \frac{2}{3} \log|\sin x + 2| + c$$

$$= -\frac{1}{6} \log|1 - \sin x| - \frac{1}{2} \log|1 + \sin x| + \frac{2}{3} \log|1 + 2 \sin x| + c.$$

Question 21.

$$\frac{1}{\sin x (3+2 \cos x)}$$

Solution:

$$\text{Let } I = \int \frac{1}{\sin x (3+2 \cos x)} dx$$

$$= \int \frac{\sin x}{\sin^2 x (3+2 \cos x)} dx$$

$$= \int \frac{\sin x}{(1 - \cos^2 x)(3 + 2 \cos x)} dx$$

$$= \int \frac{\sin x}{(1 - \cos x)(1 + \cos x)(3 + 2 \cos x)} dx$$

Put $\cos x = t \quad \therefore -\sin x \, dx = dt$

$$\therefore \sin x \, dx = -dt$$

$$\therefore I = \int \frac{1}{(1-t)(1+t)(3+2t)} (-dt)$$

$$= \int \frac{-1}{(1-t)(1+t)(3+2t)} dt$$

Let $\frac{-1}{(1-t)(1+t)(3+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{3+2t}$

$$\therefore -1 = A(1+t)(3+2t) + B(1-t)(3+2t) + C(1-t)(1+t)$$

Put $1-t=0$, i.e. $t=1$, we get

$$-1 = A(2)(5) + B(0)(5) + C(0)(2)$$

$$\therefore -1 = 10A \quad \therefore A = \frac{-1}{10}$$

Put $1+t=0$, i.e. $t=-1$, we get

$$-1 = A(0)(1) + B(2)(1) + C(2)(0)$$

$$\therefore -1 = 2B \quad \therefore B = -\frac{1}{2}$$

Put $3+2t=0$, i.e. $t=-\frac{3}{2}$, we get

$$-1 = A\left(-\frac{1}{2}\right)(0) + B\left(\frac{5}{2}\right)(0) + C\left(\frac{5}{2}\right)\left(-\frac{1}{2}\right)$$

$$\therefore -1 = -\frac{5}{4}C \quad \therefore C = \frac{4}{5}$$

$$\therefore \frac{-1}{(1-t)(1+t)(3+2t)} = \frac{\left(\frac{-1}{10}\right)}{1-t} + \frac{\left(-\frac{1}{2}\right)}{1+t} + \frac{\left(\frac{4}{5}\right)}{3+2t}$$

$$\therefore I = \int \left[\frac{\left(\frac{-1}{10}\right)}{1-t} + \frac{\left(-\frac{1}{2}\right)}{1+t} + \frac{\left(\frac{4}{5}\right)}{3+2t} \right] dt$$

$$= -\frac{1}{10} \int \frac{1}{1-t} dt - \frac{1}{2} \int \frac{1}{1+t} dt + \frac{4}{5} \int \frac{1}{3+2t} dt$$

$$= -\frac{1}{10} \frac{\log|1-t|}{-1} - \frac{1}{2} \log|1+t| + \frac{4}{5} \frac{\log|3+2t|}{2} + c$$

$$= \frac{1}{10} \log|1 - \cos x| - \frac{1}{2} \log|1 + \cos x| +$$

$$\frac{2}{5} \log|3 + 2 \cos x| + c.$$

Question 22.

$$\frac{5 \cdot e^x}{(e^x+1)(e^{2x}+9)}$$

Solution:

$$\text{Let } I = \int \frac{5e^x}{(e^x + 1)(e^{2x} + 9)} dx$$

$$\text{Put } e^x = t \quad \therefore e^x dx = dt$$

$$\therefore I = 5 \int \frac{1}{(t+1)(t^2+9)} dt$$

$$\text{Let } \frac{1}{(t+1)(t^2+9)} = \frac{A}{t+1} + \frac{Bt+C}{t^2+9}$$

$$\therefore 1 = A(t^2+9) + (Bt+C)(t+1)$$

$$\text{Put } t+1=0, \text{ i.e. } t=-1, \text{ we get}$$

$$1 = A(1+9) + C(0) \quad \therefore A = \frac{1}{10}$$

$$\text{Put } t=0, \text{ we get}$$

$$1 = A(9) + C(1) \quad \therefore C = 1 - 9A = 1 - \frac{9}{10} = \frac{1}{10}$$

Comparing coefficients of t^2 on both the sides, we get

$$0 = A + B \quad \therefore B = -A = -\frac{1}{10}$$

$$\therefore \frac{1}{(t+1)(t^2+9)} = \frac{\left(\frac{1}{10}\right)}{t+1} + \frac{\left(-\frac{1}{10}t + \frac{1}{10}\right)}{t^2+9}$$

$$\therefore I = 5 \int \left[\frac{\left(\frac{1}{10}\right)}{t+1} + \frac{\left(-\frac{1}{10}t + \frac{1}{10}\right)}{t^2+9} \right] dt$$



$$\begin{aligned}
 &= \frac{1}{2} \int \frac{1}{t+1} dt - \frac{1}{2} \int \frac{t}{t^2+9} dt + \frac{1}{2} \int \frac{t}{t^2+9} dt \\
 &= \frac{1}{2} \log|t+1| - \frac{1}{4} \int \frac{2t}{t^2+9} dt + \frac{1}{2} \cdot \frac{1}{3} \tan^{-1}\left(\frac{t}{3}\right) \\
 &= \frac{1}{2} \log|t+1| - \frac{1}{4} \int \frac{\frac{d}{dt}(t^2+9)}{t^2+9} dt + \frac{1}{6} \tan^{-1}\left(\frac{t}{3}\right) \\
 &= \frac{1}{2} \log|t+1| - \frac{1}{4} \log|t^2+9| + \frac{1}{6} \tan^{-1}\left(\frac{t}{3}\right) + c \\
 &= \frac{1}{2} \log|e^x+1| - \frac{1}{4} \log|e^{2x}+9| + \frac{1}{6} \tan^{-1}\left(\frac{e^x}{3}\right) + c.
 \end{aligned}$$

Question 23.

$$\frac{2 \log x + 3}{x(3 \log x + 2)[(\log x)^2 + 1]}$$

Solution:

$$\text{Let } I = \int \frac{2 \log x + 3}{x(3 \log x + 2)[(\log x)^2 + 1]} dx$$

$$\text{Put } \log x = t \quad \therefore \frac{1}{x} dx = dt$$

$$\therefore I = \int \frac{2t + 3}{(3t + 2)(t^2 + 1)} dt$$

$$\text{Let } \frac{2t+3}{(3t+2)(t^2+1)} = \frac{A}{3t+2} + \frac{Bt+C}{t^2+1}$$

$$\therefore 2t+3 = A(t^2+1) + (Bt+C)(3t+2)$$

$$\text{Put } 3t+2=0 \text{ i.e. } t = -\frac{2}{3}, \text{ we get}$$

$$2\left(-\frac{2}{3}\right) + 3 = A\left(\frac{4}{9} + 1\right) + \left(-\frac{2}{3}B + C\right) \quad (0)$$

$$\therefore \frac{5}{3} = A\left(\frac{13}{9}\right) \quad \therefore A = \frac{15}{13}$$

$$\text{Put } t=0, \text{ we get}$$

$$3 = A(1) + C(2) = \frac{15}{13} + 2C$$

$$\therefore 2C = 3 - \frac{15}{13} = \frac{24}{13} \quad \therefore C = \frac{12}{13}$$

Comparing coefficients of t^2 on both the sides, we get

$$0 = A + 3B \quad \therefore B = -\frac{A}{3} = -\frac{5}{13}$$

$$\therefore \frac{2t+3}{(3t+2)(t^2+1)} = \frac{\left(\frac{15}{13}\right)}{3t+2} + \frac{\left(-\frac{5}{13}t + \frac{12}{13}\right)}{t^2+1}$$

$$\begin{aligned} \therefore I &= \int \left[\frac{\left(\frac{15}{13}\right)}{3t+2} + \frac{\left(-\frac{5}{13}t + \frac{12}{13}\right)}{t^2+1} \right] dt \\ &= \frac{15}{13} \int \frac{1}{3t+2} dt - \frac{5}{26} \int \frac{2t}{t^2+1} dt + \frac{12}{13} \int \frac{1}{t^2+1} dt \end{aligned}$$

$$= \frac{15}{13} \cdot \frac{1}{3} \log |3t+2| - \frac{5}{26} \log |t^2+1| + \frac{12}{13} \tan^{-1}(t) + c$$

$$\dots \left[\because \frac{d}{dt}(t^2+1) = 2t \text{ and } \int \frac{f'(t)}{f(t)} dt = \log |f(t)| + c \right]$$

$$\begin{aligned} &= \frac{5}{13} \log |3 \log x + 2| - \frac{5}{26} \log |(\log x)^2 + 1| + \\ &\quad \frac{12}{13} \tan^{-1}(\log x) + c. \end{aligned}$$



Maharashtra Board Solutions

Class 12 Arts & Science Maths

(Part 2)

- Chapter 1- Differentiation
- Chapter 2- Applications of Derivatives
- Chapter 3- Indefinite Integration
- Chapter 4- Definite Integration
- Chapter 5- Application of Definite Integration
- Chapter 6- Differential Equations
- Chapter 7- Probability Distributions
- Chapter 8- Binomial Distribution

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The Maharashtra State Board of Secondary and Higher Secondary Education or MSBSHSE (Marathi: महाराष्ट्र राज्य माध्यमिक आणि उच्च माध्यमिक शिक्षण मंडळ), is an **autonomous and statutory body established in 1965**. The board was amended in the year 1977 under the provisions of the Maharashtra Act No. 41 of 1965.

The Maharashtra State Board of Secondary & Higher Secondary Education (MSBSHSE), Pune is an independent body of the Maharashtra Government. There are more than 1.4 million students that appear in the examination every year. The Maha State Board conducts the board examination twice a year. This board conducts the examination for SSC and HSC.

The Maharashtra government established the Maharashtra State Bureau of Textbook Production and Curriculum Research, also commonly referred to as Ebalbharati, in 1967 to take up the responsibility of providing quality textbooks to students from all classes studying under the Maharashtra State Board. MSBHSE prepares and updates the curriculum to provide holistic development for students. It is designed to tackle the difficulty in understanding the concepts with simple language with simple illustrations. Every year around 10 lakh students are enrolled in schools that are affiliated with the Maharashtra State Board.

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